Nonlinear Nonclassical Waves for Dense Gase Flows: Preliminary Numerical Results *

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This brief note reports preliminary results for the numerical study of nonlinear nonclassical waves for dense gas flows. We use the Euler equations of gas dynamics as a model system (cf. [5]), and take the popular van der Waals equation of state of the form,

$$p(V, e) = \left(\frac{\gamma - 1}{V - \beta}\right)\left(e + \frac{\alpha}{V}\right) - \frac{\alpha}{V^2},$$

as a model for the thermodynamic behavior of real gases [7], Here p denotes the pressure, V denotes the specific volume, e denotes the specific internal energy, and α, β, and γ denotes the molecular cohesive forces, finite size of the molecules, and the ratio of specific heats, respectively. For the type of problems considered here, it is very useful to define the so-called fundamental derivative of gas dynamics G as follows:

$$G = -\frac{V}{2}\frac{(\partial^2 p/\partial V^2)_S}{(\partial p/\partial V)_S},$$

and use it as a way to determine whether a fluid state is in a dense gas regime or not. That is, if $G > 0$;

$$\left(\frac{\partial^2 p}{\partial V^2}\right)_S > 0 \quad \text{and} \quad \left(\frac{\partial p}{\partial V}\right)_S < 0,$$

we say that a fluid state is in the perfect gas regime, and if $G \le 0$;

$$\left(\frac{\partial^2 p}{\partial V^2}\right)_S \le 0 \quad \text{and} \quad \left(\frac{\partial p}{\partial V}\right)_S < 0,$$

we say that a fluid state is in the dense gas regime, where S is the specific entropy. It is well-known that in the former case, there exhibits positive nonlinearity, and exists classical entropy-satisfying nonlinear waves, while in the latter case, there exhibits negative nonlinearity, and exists nonclassical entropy-violating nonlinear waves such as expansion shock and compression rarefaction. Figure 1 shows the graph of the isothermal lines, saturation curve, and $G = 0$, for the van der Waals equation of state in the $p - V$ plane.

To simulate problems arising from dense gas flows, as a first step, we devised a variant of the Roe Riemann solver [9], and incorporated that into a Godunov-type method based on the wave-propagation formulation [8]. We then performed a series of one- and two-dimensional test problems considered by Argrow *et al.* [1, 2, 3, 4, 6] so as to validate the proposed method. Numerical results

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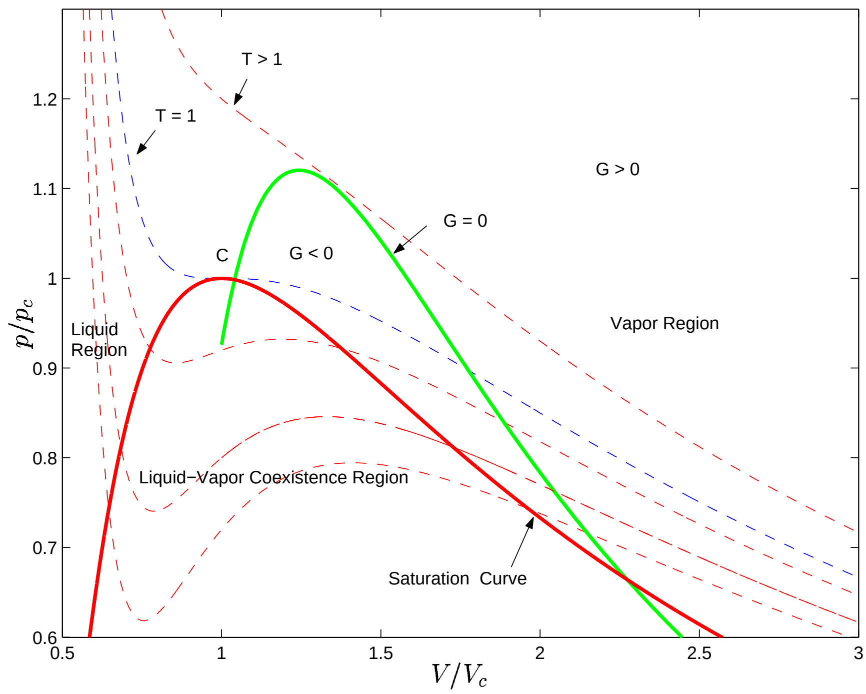

Figure 1: Graphs of the isothermal lines, saturation curves, and $G = 0$, for a van der Waals gas in $p - V$ plane. Here T is the temperature, and p_c and V_c are the pressure and specific volume at the critial states, respectively.

of some of the runs are present in Figs. 2–12. As far as the global structure is concerned, we observe good agreement of the solution as compared to those appeared in the literature. Noe that to limit the size of the report, we do not write down all the initial conditions (cf. [1, 2]) and the other related parameters for the runs.

To end this note, we want to say that the work present here is far from complete. For the shock-ramp interaction problem, for example, the resolution near the Mach stem is too low to be able to draw any concrete conclusions. For that, the technique of local adaptive mesh refinemnet should be included in the algorithm. Moreover, multicomponent version of the algorithm (cf. [10]) should be fully tested for multicomponent dense gas flows.

References

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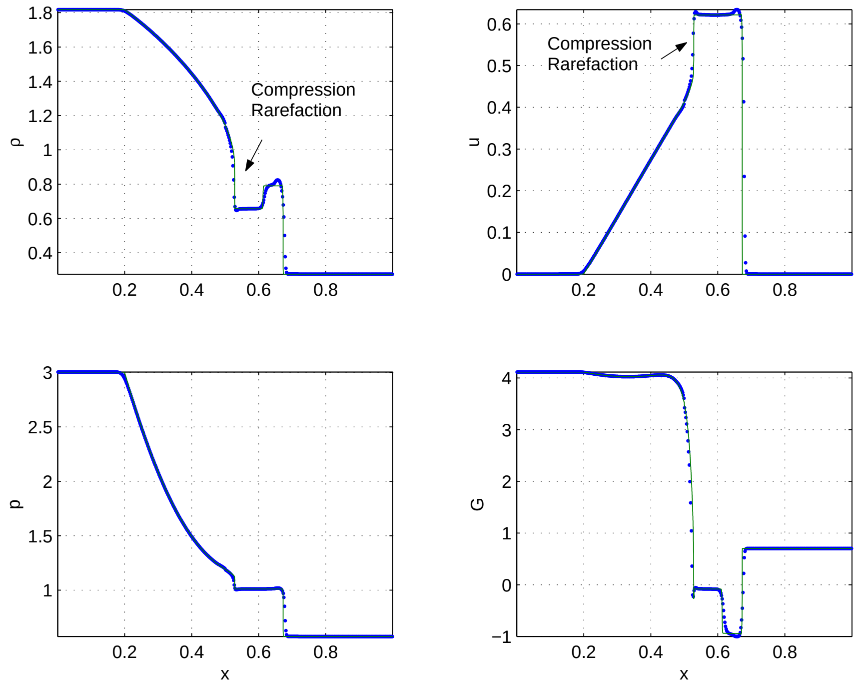

Figure 2: One-dimensional densed gas computation: Example 1. Plot of the density ρ, velocity u, pressure p, and the fundamental derivative of gas dynamics G.

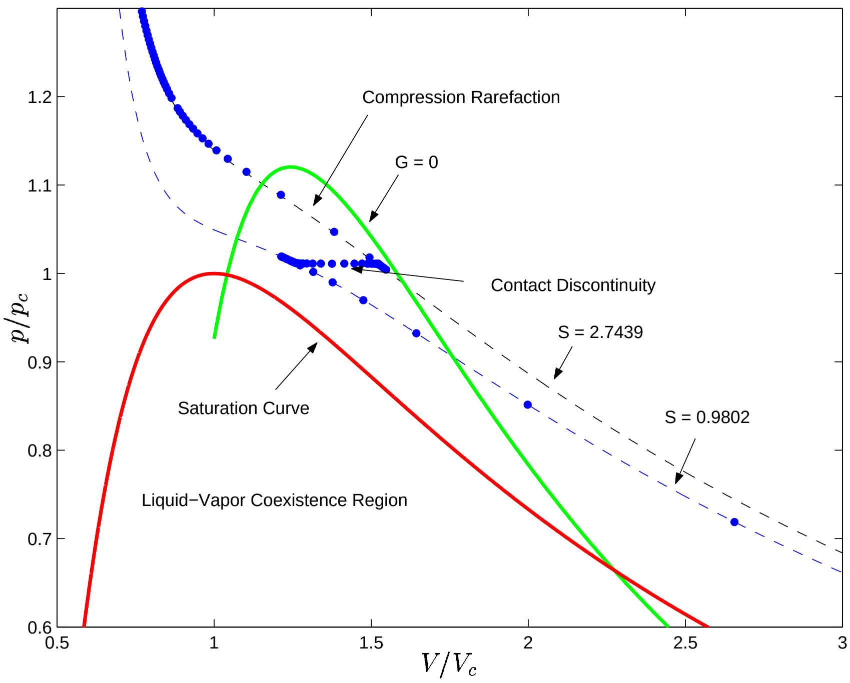

Figure 3: One-dimensional densed gas computation: Example 1. Plot of the solution in the $p-V$ plane. Here S denotes the specific entropy.

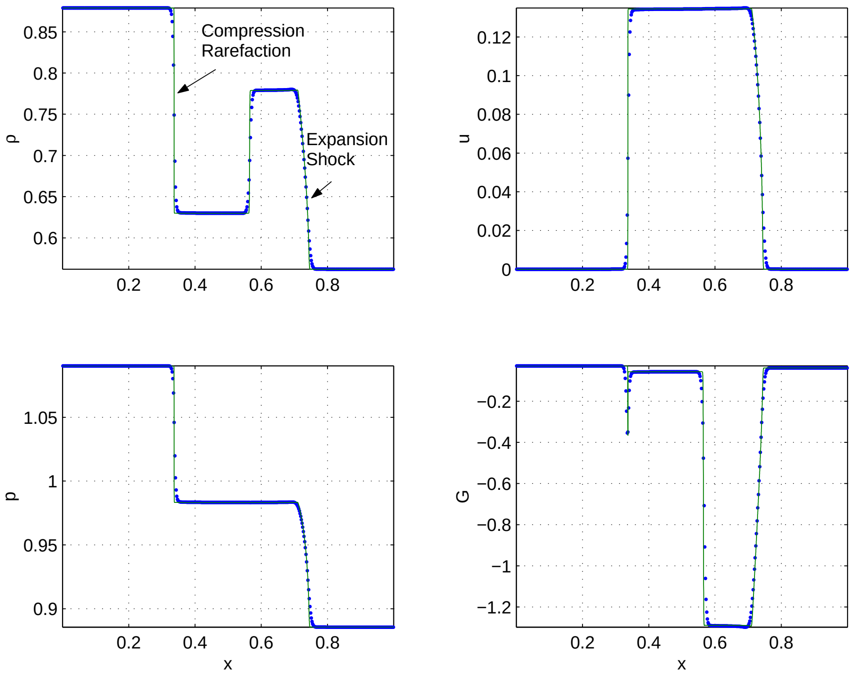

Figure 4: One-dimensional densed gas computation: Example 2. Plot of the density ρ, velocity u, pressure p, and the fundamental derivative of gas dynamics G.

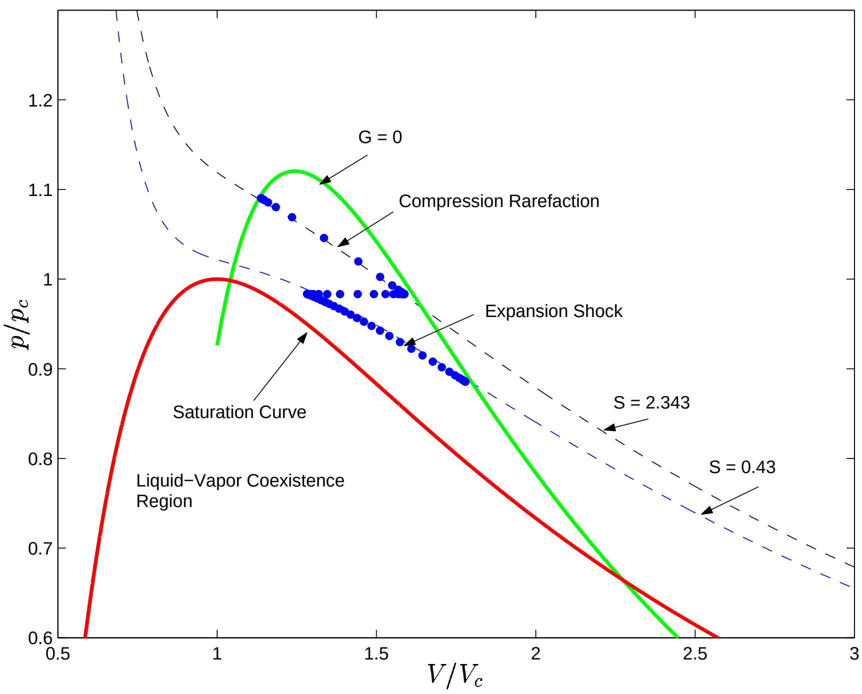

Figure 5: One-dimensional densed gas computation: Example 2. Plot of the solution in the $p-V$ plane. Here S denotes the specific entropy.

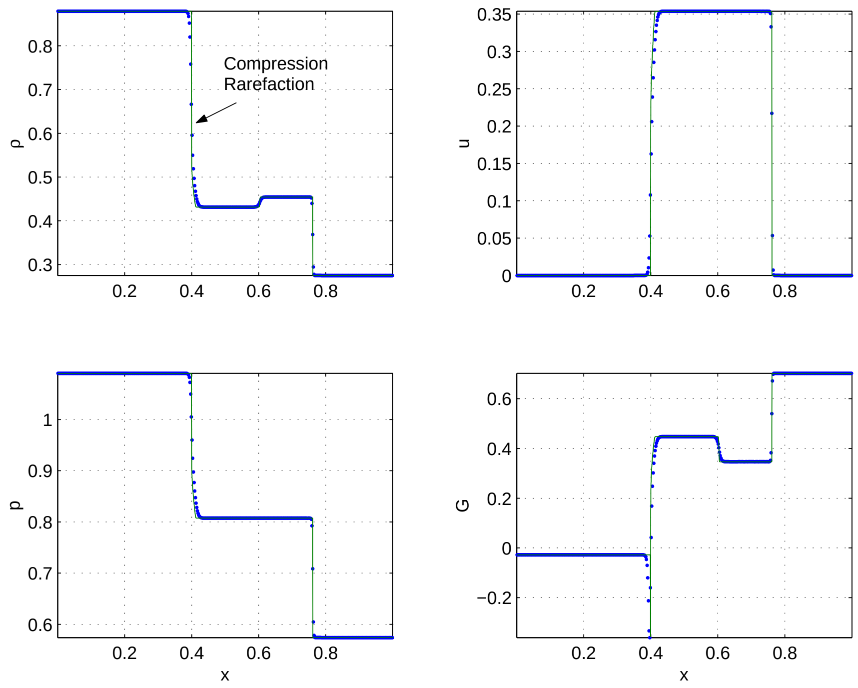

Figure 6: One-dimensional dense gas computation: Example 3. Plot of the density ρ, velocity u, pressure p, and the fundamental derivative of gas dynamics G.

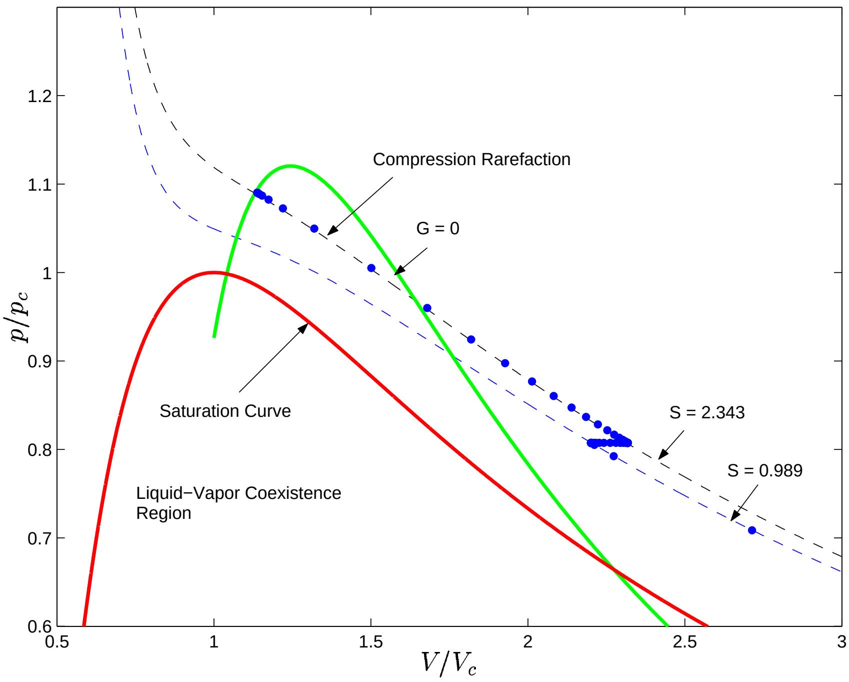

Figure 7: One-dimensional densed gas computation: Example 3. Plot of the solution in the $p - V$ plane. Here S denotes the specific entropy.

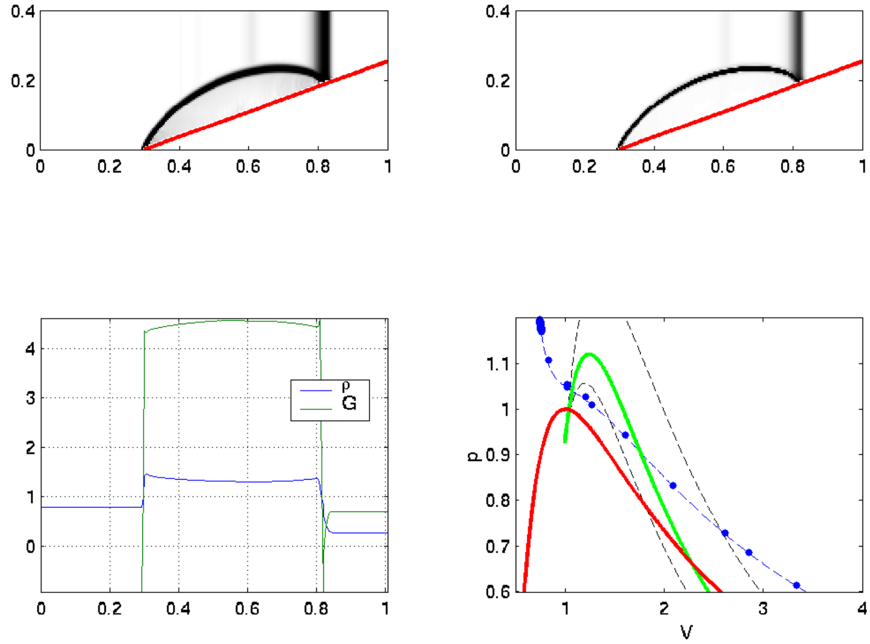

Figure 8: Two-dimensional dense gas computation: Shock-ramp reflection example 1. Contour plots of the density and pressure, cross-section plot of the solution along the ramp, and plot of the solution in the $p-V$ plane are shown.

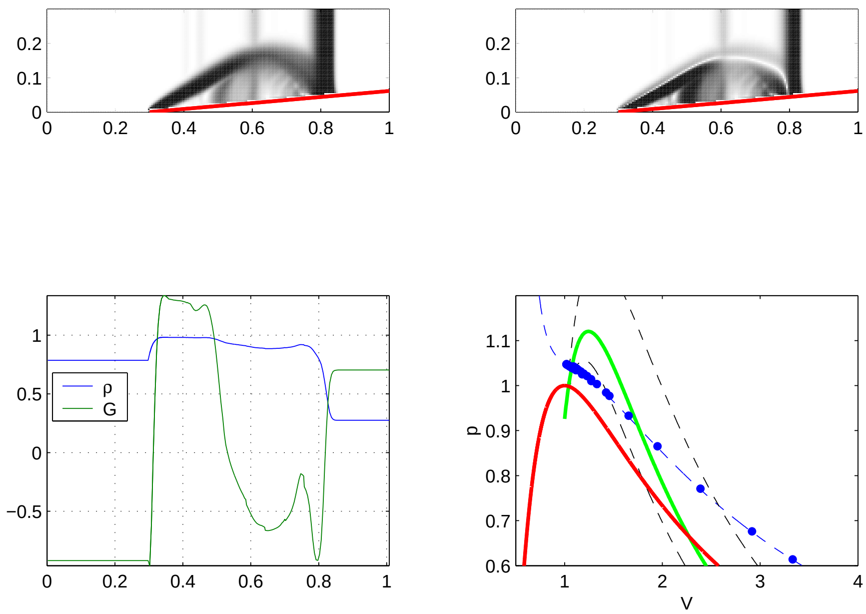

Figure 9: Two-dimensional dense gas computation: Shock-ramp reflection example 2. Contour plots of the density and pressure, cross-section plot of the solution along the ramp, and plot of the solution in the $p-V$ plane are shown.

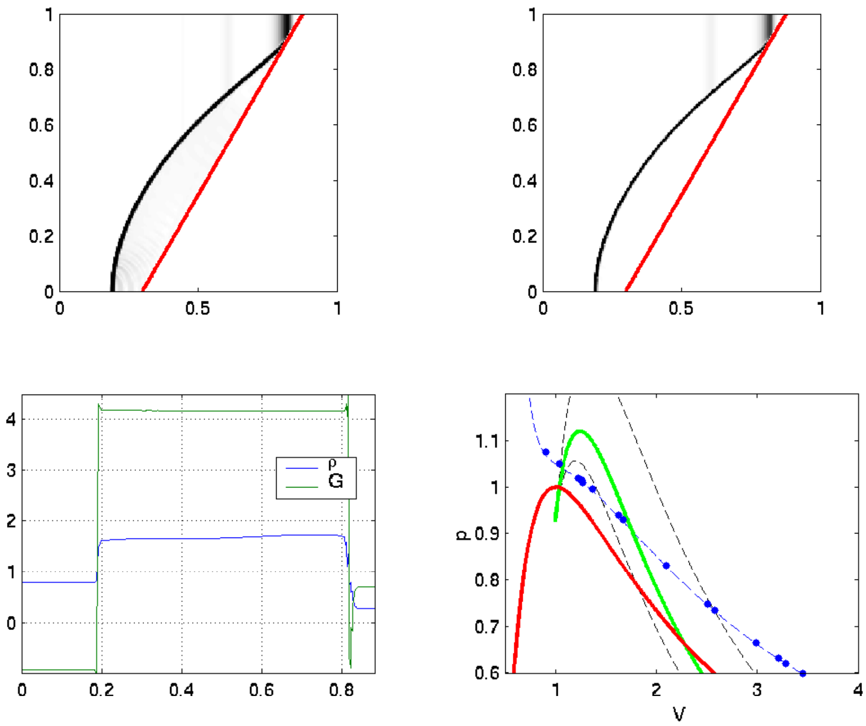

Figure 10: Two-dimensional dense gas computation: Shock-ramp reflection example 3. Contour plots of the density and pressure, cross-section plot of the solution along the ramp, and plot of the solution in the $p-V$ plane are shown.

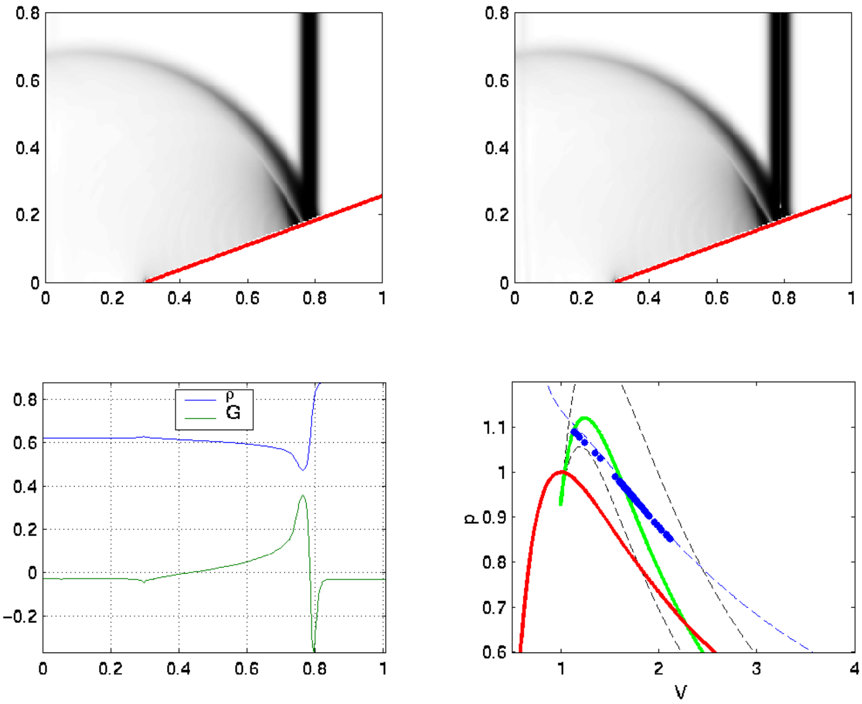

Figure 11: Two-dimensional dense gas computation: Shock-ramp reflection example 4. Contour plots of the density and pressure, cross-section plot of the solution along the ramp, and plot of the solution in the $p-V$ plane are shown.

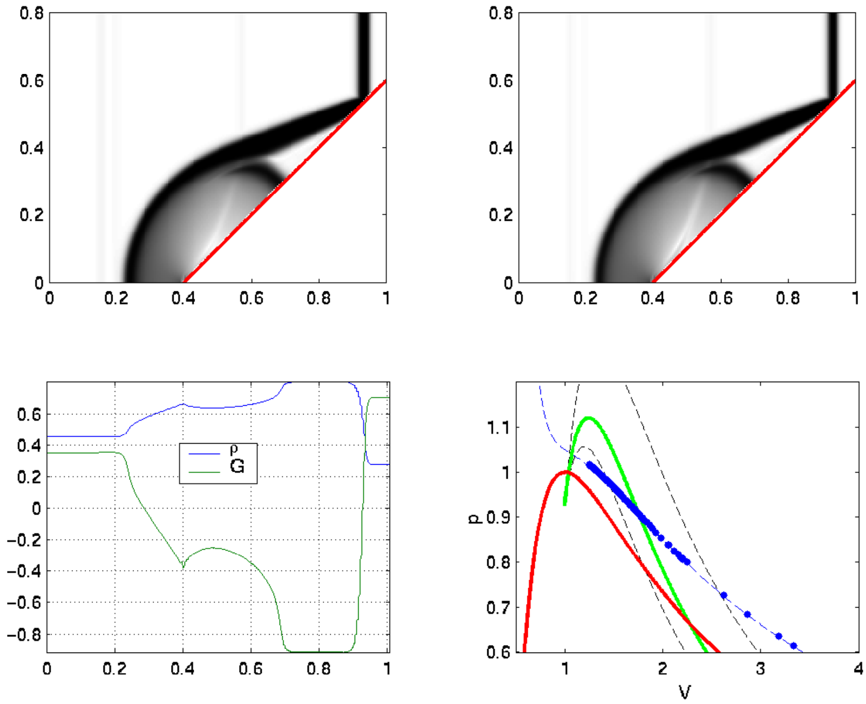

Figure 12: Two-dimensional dense gas computation: Shock-ramp reflection example 5. Contour plots of the density and pressure, cross-section plot of the solution along the ramp, and plot of the solution in the $p-V$ plane are shown.

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