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Integrity of Inverse-Based Multivariable Controllers: Conventional Feedback Structure

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Increased availability of inexpensive and powerful process control computers does not lead to widespread acceptance of multivariable controllers. One of the reasons is that the failure conditions of multivariable controllers are not well understood. This paper analyzes the failure tolerance and integral controllability of inverse-based controllers under the conventional feedback structure. Failure tolerance is addressed on the basis of the stability of the subsystems, e.g., to avoid instability as the result of positive feedback. Complete integral controllability (CIC) is defined according to the operability of the subsystems, i.e., without causing instability as the controller gains in the tuning matrix are reduced arbitrarily. Necessary and/or sufficient conditions for CIC are derived. The results show that, similar to multiloop single input-single output systems, some sort of pairing, e.g., taking the corresponding sensor off under actuator failure, is required for the inverse-based multivariable controllers. Furthermore, the well-known interaction measures, e.g., Niederlinski index, relative gain array, and block relative gain are useful in determining the appropriate pairings. Rules for the pairing are summarized that ensure complete integral controllability for the inverse-based controller of the size 2 × 2, 3 × 3, and 4 × 4.

1. Introduction

Advances in computer hardware and the availability of inexpensive process control computers make the application of multivariable controllers attractive to process industries. However, widespread acceptance of multivariable controllers is limited by the fact that the integrity (failure tolerance) of multivariable controllers is not well understood (Tzouanas et al., 1988). In practice, the multimillion chemical processes are operated, if not optimally, at least safely under abnormal operating conditions. From an operating point of view, typical abnormal operating conditions are characterized by sensor failure and/or actuator (valve) failure.

Conventionally, in the design of multiloop single input-single output (SISO) (fully decentralized) controllers, the controlled and manipulated variables are paired such that the control system remains stable against sensor and/or actuator failures (with combinations of loops on manual). Relative gain array (RGA) (Bristol, 1966; McAvoy, 1983), Niederlinski index (NI) (Niederlinski, 1971), and block relative gain (BRG) (Manousiouthakis et al., 1986) are widely used for eliminating pairings that produce unstable closed-loop systems under failure conditions (Grosdider et al., 1985; Yu and Luyben, 1986; Skogestad and Morari, 1988; Chiu and Arkun, 1990; Yu and Fan, 1990). Notice that for failure tolerance, generally, only the static problem is addressed. This means that the controller gains either take the nominal value or have zero value under failure conditions. Morari and co-workers (Skogestad and Morari, 1988; Morari and Zafiriou, 1989) go a step further to discuss the operational problem by defining the decentralized integral controllability (DIC). Physically, a decentralized integral controllable system allows the operator to reduce the controller gains independently to zero without introducing instability (as a result of positive feedback). Certainly, this is a desirable property for any control system. Some necessary or sufficient conditions for the DIC are introduced (Skogestad and Morari, 1988; Yamanaka and Kawasaki, 1989; Yu and Fan, 1990). Yu and Fan (1990) point out that the DIC of a square system is equivalent to a well-known mathematical problem, D-stability (Johnson, 1974). The necessary and sufficient conditions for DIC for 2 × 2 and 3 × 3 systems are given by Skogestad and Morari (1988) and Yu and Fan (1990), respectively. Furthermore, they can be verified by examining the diagonal elements of the RGA.

For multivariable controllers, the problems of failure tolerance and integral controllability still exist. However, it is not clear what a failure condition may lead to. For example, if an actuator fails, in order to maintain an offset-free subsystem, which controlled variable should be left uncontrolled? Will the subsystem remain stable and/or retain integral controllability? In an operating plant, these questions have brought little concern for decentralized control systems since, in the design phase, the manipulated and controlled variables are paired to achieve failure tolerance or DIC.

The purpose of this work is to study the failure tolerance and integral controllability of multivariable control systems with inverse-based controllers. Generally, the inverse-based multivariable controllers can be classified into the conventional feedback structure and the internal model control (IMC) structure according to the ways of implementation. In the conventional feedback structure, only the sensor signals are fed back into the controller, and in the IMC structure (Morari and Zafiriou, 1989), both the sensor and actuator signals are fed back into the con-

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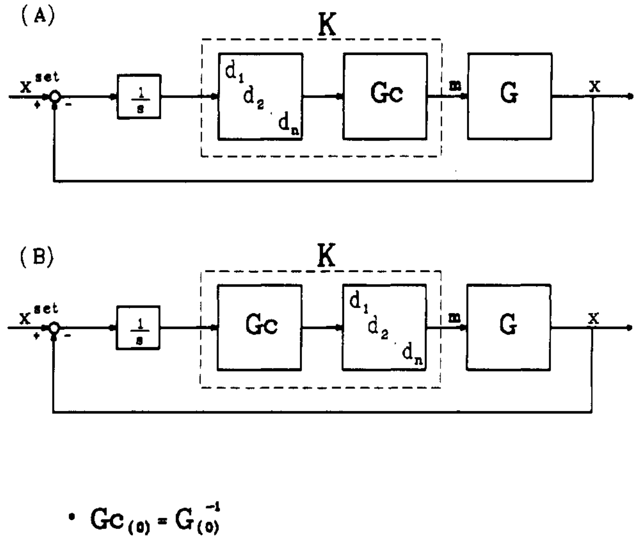


Figure 1. Inverse-based controllers with diagonal tuning matrices: (A) output-decoupling scheme and (B) input-decoupling scheme.

trollers. Despite the fact that these two structures give exactly the same behavior at normal operation, under failure conditions, the behavior of these two structures are quite different. This can be understood from the ways the controllers function. For example, quadratic dynamic matrix control (QDMC) (Prett and Garcia, 1988) treats a known actuator failure as a constrained optimization problem. Modular multivariable control (MMC) (Brosilow and Laiseca, 1989), which is implemented in an intuitively appealing modular form, provides another alternative. MMC can be viewed as a "smart" IMC in which the priorities in the controlled and manipulated variables are assigned and ensured via some logic functions. Unlike QDMC, integral action may remain for the primary controlled variables under actuator failures. However, in the implementation phase, these two types of controllers require the signals of the actuators, which can be imprecise or unavailable in many occasions. In this work, the inverse-based multivariable controllers considered are of the conventional feedback structure.

Typical inverse-based multivariable controllers in chemical process control, $\mathbf{K}_{(s)} = \mathbf{Gc}_{(s)}\mathbf{D}_{(s)}$ and $\mathbf{K}_{(s)} = \mathbf{D}_{(s)}\mathbf{Gc}_{(s)}$ (Figure 1), are considered. Here, **Gc** is the approximated inverse, i.e., $\mathbf{Gc}_{(0)} = \mathbf{G}_{(0)}^{-1}$, **D** is a diagonal tuning matrix, and the integrator $(1/s)$ is factored out as shown in Figure 1. These two inverse-based controllers are, generally, referred to as output decoupling (Luyben, 1970; Arkun et al., 1984) and input decoupling, respectively. The process considered in this work is a stable, linear, and time-invariant square system. The $n \times n$ process transfer matrix, $\mathbf{G}_{(s)}$, is restricted to a strictly proper system. The inverse-based controller, $\mathbf{K}_{(s)}/s$, has integral action with a diagonal tuning matrix **D**. Note that the instability considered here is a low (zero) frequency type of instability, i.e., instability as the result of positive feedback.

This paper is organized as follows. Section 2 gives some notations and definitions of useful operators RGA, NI, and BRG. The failure tolerance and DIC for decentralized control systems are also derived. The inverse-based controllers are defined in section 3. Section 4 discusses the sensor and actuator failure conditions for the inverse-based controllers. A necessary condition for failure tolerant system is also derived. The notion of complete integral controllability (CIC) is introduced for multivariable control systems and the necessary and sufficient conditions for CIC for systems smaller than 5 × 5 are derived in section 5. Some type of pairing (taking corresponding sensors off under actuator failures) is suggested for the inverse-based controllers and rules for such pairing are summarized in section 6 followed by the conclusion.

2. Preliminaries and Notations

Since the development of the failure tolerance and operability for the inverse-based controllers is closely related to that of the decentralized controllers, the failure tolerance and DIC for multiloop SISO controllers are summarized here. Some necessary notations are given below. Let N be the set of integers $N = \{1,2,...,n\}$ and I be the subset of N with strictly increasing indices $I = \{i_1,i_2,...,i_m\}$ where $1 \le i_1 \le i_2 \le ... \le i_m \le n$. Let I' denote the complement of the subset I. For a square $n \times n$ matrix, **A**, $\mathbf{A}_{I,J}$ denotes the corresponding submatrix of **A** with rows and columns defined by the indices in I and J, respectively.

2.1. Failure Tolerance. 1. Single-Loop Stability and Overall Stability. NI (Niederlinski, 1971) is an important measure concerning the single-loop and overall stability of closed-loop systems under integral control. The operator is defined as

$$\mathrm{NI}\{\mathbf{G}_{(0)}\} = \frac{\det(\mathbf{G}_{(0)})}{\prod_i g_{ii}} \tag{1}$$

where $\mathbf{G}_{(0)}$ is the steady-state transfer function matrix. Since only the steady-state aspect is considered, in this paper, the subscript (0) is dropped for clarity. A negative NI, NI < 0, is an indication of either some single loops or the overall system is not integral stabilizable. This is clear from the determinant condition for the integral stability (Niederlinski, 1971; Grosdidier et al., 1985). Note that NI is simply the det (**G**) with signs of diagonal elements properly adjusted.

2. Single-Loop Failure. RGA (Bristol, 1966) probably is the most studied interaction measure for variable pairing. Considering a $n \times n$ real matrix, **G**, the RGA is defined as

$$\mathrm{RGA}\{\mathbf{G}\} = \mathbf{G} \otimes (\mathbf{G}^{-1})^{\mathrm{T}} = [\lambda_{ij}] \tag{2}$$

where ⊗ is the element by element multiplication and the ijth entry of the RGA can be expressed as

$$\lambda_{ij} = g_{ij} \cdot \hat{g}_{ji} \tag{3}$$

where $\hat{g}_{ji}$ is the jith entry of $\mathbf{G}^{-1}$. The operator RGA{·} denotes the relative gain array of {·} and λ_{ij} is the ijth entry. The diagonal elements of the RGA provide some information on single-loop failure. For example, a decentralized control system paired with a negative diagonal element of RGA, i.e., $\lambda_{ii} < 0$, will have at least one of the following properties: (1) the overall closed-loop system is unstable, (2) loop i is unstable by itself, and (3) the closed-loop system with loop i removed is unstable (Grosdidier et al., 1985; Yu and Fan, 1990). Again, this is the determinant condition for integral stabilizability for the subsystem (e.g., when loop i is removed) which becomes obvious from the alternative expression for λ_{ii}:

$$\lambda_{ii} = \frac{g_{ii} \cdot \det(\mathbf{G}^{ii})}{\det(\mathbf{G})} \tag{4}$$

where $\mathbf{G}^{ii}$ is **G** with the ith row and the ith column removed.

3. Multiple-Loop Failure. Block relative gain (BRG) (Manousiouthakis et al., 1986) is useful when dealing with multiple-loop failure (Yu and Fan, 1990). The BRG for **G** is defined via the operator BRG{·} according to a block structure I.

$$\mathrm{BRG}^{\ell}\{\mathbf{G}\} = \Lambda^{\ell}_{II} = \mathbf{G}_{II'}(\mathbf{G}^{-1})_{II} \qquad (5)$$

$$\mathrm{BRG}^{r}\{\mathbf{G}\} = \Lambda^{r}_{II} = (\mathbf{G}^{-1})_{II'}\mathbf{G}_{II} \qquad (6)$$

where $\mathbf{G}_{II}$ is a principal submatrix of $\mathbf{G}$ defined by I and $(\mathbf{G}^{-1})_{II}$ is the principal submatrix of $\mathbf{G}^{-1}$ with the block structure defined by I. The superscripts ℓ and r denote the left and right BRG, respectively (Manousiouthakis et al., 1986). For a decentralized control system, any pairing with a negative determinant of BRG, i.e., det $(\Lambda_{II}) < 0$, will have at least one of following properties: (1) the closed-loop system is unstable; (2) the closed-loop system with loops in the subset I removed is unstable, (3) the closed-loop system with loops in the subset I' removed is unstable (Grosdidier and Morari, 1987; Yu and Fan, 1990). An alternative expression for the BRG is (Grosdidier and Morari, 1987)

$$\det(\Lambda_{II}) = \frac{\det(\mathbf{G}_{II})\cdot\det(\mathbf{G}_{I'I'})}{\det(\mathbf{G})} \qquad (7)$$

Furthermore

$$\det(\Lambda_{II}) = \det(\Lambda_{I'I'}) \qquad (8)$$

Again, the result is obvious, since we are checking the determinant condition for integral stabilizability for the principal submatrices.

In summary, the failure tolerance test is simply testing the nonnegativeness of the principal minors of $\mathbf{G}$ (with the sign adjusted to have positive diagonal elements), despite different expressions in the literatures (e.g., Lunze (1988), p 199; Chiu and Arkun (1990); Yu and Fan (1990)).

2.2. Decentralized Integral Controllability (DIC). Consider the decentralized control structure in Figure 2 with the diagonal controller $\mathbf{K}_{(s)}/s$. The following property is very desirable from operation point of view.

Definition 1: A process $\mathbf{G}_{(s)}$ is decentralized integral controllable (DIC) if there exists a decentralized controller $\mathbf{K}_{(s)}/s$ such that the closed-loop system in Figure 2 with the nominal tuning, $d_i = 1$, $\forall\, i$, is stable and has the property that each loop gain can be reduced independently $(0 \le d_i \le 1)$ without introducing instability (Skogestad and Morari, 1988; Morari and Zafiriou, 1989; Yu and Fan, 1990).

It means that the controller gain of individual loop can be reduced independently without introducing instability. DIC requires all the eigenvalues of $\mathbf{GK}_{(0)}$ stay in the open RHP (right half-plane) for all positive scalings. Yu and Fan (1990) point out that the mathematical formulation for DIC is the same as D stability in most cases. It is desirable to pair input and output variables such that the system is DIC. Unfortunately, no necessary and sufficient condition exists for DIC except for 2 × 2 and 3 × 3 systems. The necessary and sufficient conditions for DIC for 2 × 2 and 3 × 3 systems are as follows:

1. 2 × 2 system (Skogestad and Morari, 1988)

$$\lambda_{ii} > 0, \quad \forall\, i$$

2. 3 × 3 system (Yu and Fan, 1990)

(i) $\quad \mathrm{NI}\{\mathbf{G}\} > 0$

(ii) $\quad \lambda_{ii} > 0, \quad \forall\, i$

(iii) $\quad \mathrm{SRG}\{\mathbf{G}\} > 1$

where SRG denotes $\sum_{i=1}^{3}\lambda_{ii}^{1/2}$ and λ_{ii} is the diagonal element of the RGA{G}.

3. Inverse-Based Controllers

Despite the importance of issues such as failure tolerance and integral controllability, little research has been done on failure tolerance (or integral controllability) for commonly used multivariable controllers. Shimenura and Fujita (1985) proposed a procedure to design a failure-tolerant state feedback system based on a solution of a LQ problem. Since multivariable controllers may take many possible forms, the scope of this work is to study the failure tolerance of the so-called inverse-based controllers. The general restrictions on the controllers are (1) they have integral action and (2) $\mathbf{K}_{(s)}/s$ (Figure 1) is proper. Depending on the location of the diagonal adjustable parameters (diagonal tuning matrix), the inverse-based controllers are classified further into an output-side tuning scheme (Figure 1A) where

$$\mathbf{K}_{(s)} = \mathbf{Gc}_{(s)}\cdot\mathbf{D}_{(s)} \qquad (9)$$

Here $\mathbf{Gc}_{(s)}$ is the inverse-based part and $\mathbf{D}_{(s)}$ is a diagonal tuning matrix. This is often referred to as "output decoupling" (Luyben, 1970; Arkun et al., 1984). The other type is with an input-side tuning scheme (Figure 1B) where

$$\mathbf{K}_{(s)} = \mathbf{D}_{(s)}\cdot\mathbf{Gc}_{(s)} \qquad (10)$$

This of often called the "input decoupling". The only restriction on the inverse-based part $\mathbf{Gc}_{(s)}$ is

$$\lim_{s\to 0}\mathbf{Gc}_{(s)} = \mathbf{G}_{(0)}^{-1} \qquad (11)$$

It means that the steady-state controller transfer function matrix, $\mathbf{Gc}$, is the inverse of the process transfer function matrix. In other words, the inverse-based controllers are, at least, equipped with steady-state decoupler.

For the sake of simplicity, the controllers employed in the simulation studies are designed according to

$$\mathbf{Gc}_{(s)} = \mathbf{G}_{(0)}^{-1} \qquad (12)$$

and the diagonal tuning matrix $\mathbf{D}_{(s)}/s$ has the form of PI controllers in the diagonal.

4. Failure Tolerance of Inverse-Based Controllers

In order to characterize failure conditions of a feedback system, two diagonal switching matrices (Fujita and Shimemura, 1988), Δ_A and Δ_S, are introduced (Figure 3). Δ_A and Δ_S are used to describe the actuator and sensor failures, respectively. The Δ matrix is defined as follows

$$\Delta = \mathrm{diag}\,(\delta_1,\delta_2,\ldots,\delta_n) \qquad (13)$$

with

$$\delta_i = \begin{cases} 1 & \text{(normal operation)} \\ 0 & \text{(under failure)} \end{cases}$$

Therefore, under normal operation Δ_A and Δ_S are simply identity matrices.

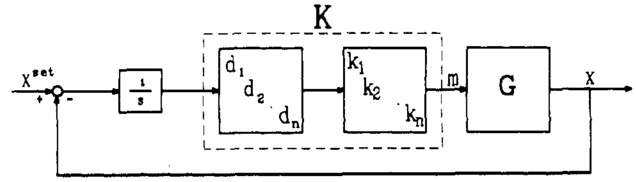


Figure 2. Decentralized control system.

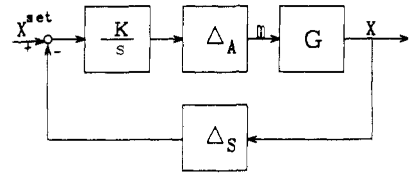


Figure 3. Feedback system under possible failure conditions.

Table I. Process and Load Transfer Functions for WB and CL Columns

A. Wood and Berry (WB)

process		load
$\dfrac{12.8e^{-s}}{16.7s + 1}$	$\dfrac{-18.9e^{-3s}}{21s + 1}$	$\dfrac{3.8e^{-8.1s}}{14.9s + 1}$
$\dfrac{6.6e^{-7s}}{10.9s + 1}$	$\dfrac{-19.4e^{-3s}}{14.4s + 1}$	$\dfrac{4.9e^{-3.4s}}{13.2s + 1}$

B. Chiang and Luyben (CL)

process				load
$\dfrac{4.45}{(14s + 1)(4s + 1)}$	$\dfrac{-7.4}{(16s + 1)(4s + 1)}$	0	$\dfrac{0.35}{(25.7s + 1)(2s + 1)}$	$\dfrac{1.02e^{-4.5s}}{(25s + 1)(2s + 1)^2}$
$\dfrac{17.3e^{-0.9s}}{(17s + 1)(0.5s + 1)}$	$\dfrac{-41}{(21s + 1)(s + 1)}$	0	$\dfrac{9.2e^{-03s}}{20s + 1}$	$\dfrac{19.7e^{-0.3s}}{(25s + 1)(s + 1)}$
$\dfrac{0.22e^{-1.2s}}{(17.5s + 1)(4s + 1)}$	$\dfrac{-4.66}{(13s + 1)(4s + 1)}$	$\dfrac{3.6}{(13s + 1)(4s + 1)}$	$\dfrac{0.042(78.7s + 1)}{(21s + 1)(11.6s + 1)(3s + 1)}$	$\dfrac{0.75e^{-5s}}{(15.6s + 1)(2s + 1)^2}$
$\dfrac{1.82e^{-s}}{(21s + 1)(s + 1)}$	$\dfrac{-34.5}{(20s + 1)(s + 1)}$	$\dfrac{12.2e^{-0.9s}}{(18.5s + 1)(s + 1)}$	$\dfrac{-6.92e^{-0.6s}}{20s + 1}$	$\dfrac{16.61e^{-0.6s}}{(25s + 1)(2s + 1)}$

4.1. Sensor Failure. Consider the case when sensors from subset I' fail (or the sensors in the subset I remain in service). Equivalently, we set indices δ_i's in the switching matrix according to I', i.e., $\delta_i = 0$, $\forall\ i \in I'$. The closed-loop characteristic equation becomes

$$\det\,(\mathbf{I} + \Delta_S\mathbf{GK}/s) = \det\,(\mathbf{I} + (\mathbf{GK})_{II}/s) = 0 \quad (14)$$

Definition 2. A stable multivariable feedback system $\mathbf{Q} = \mathbf{GK}$ (Figure 3) with integral action is SFT (sensor failure tolerant) if the reduced systems are stable for all possible combinations of sensor failures, i.e., all possible 0 and 1 combinations in Δ_S.

This defines the system integrity against sensor failure which is an extended version of the j-SFT of Grosdidier et al. (1985). Since the closed-loop characteristic equation becomes $\det\,(\mathbf{I} + (\mathbf{GK})_{II}/s) = 0$, the stability under failure condition requires checking all principal submatrices of $\mathbf{GK}$, i.e., $(\mathbf{GK})_{II}$, $\forall\ I \subset N$. To avoid instability as a result of positive feedback under integral control, the stability can be checked by looking at the location of the eigenvalues $(\mathbf{GK})_{II}$ (Grosdidier et al., 1985; Yu and Fan, 1990).

Theorem 1. A multivariable feedback system $\mathbf{Q} = \mathbf{GK}$ (Figure 3) with integral action is stable and all principal minors of $\mathbf{Q}$ are nonzero. The feedback system is SFT if, and only if, all eigenvalues of $(\mathbf{GK})_{II}$ lie on the RHP for all I's.

Proof: Simply apply the eigenvalue test for integral controllability (theorem 7, Grossdidier et al. (1985)) on all principal submatrices (Yamanaka and Kawasaki, 1989).

For a feedback system with a known controller, SFT can be checked by examining the eigenvalues of all principal submatrices. For the inverse-based controllers (Figure 1), much general results can be obtained. The next corollary shows the output-decoupling scheme is always SFT.

Corollary 1. A nominally stable feedback system $\mathbf{GK}$ in Figure 1A with $\mathbf{K}_{(0)} = \mathbf{G}_{(0)}^{-1}\mathbf{D}_{(0)}$ is SFT.

Proof: This is a direct consequence of theorem 1. Since $\mathbf{D}$ is a positive diagonal matrix, we have

$$\lambda_i[(\mathbf{GK})_{II}] = \lambda_i[(\mathbf{GG}^{-1}\mathbf{D})_{II}] = \lambda_i[(\mathbf{D})_{II}] > 0, \quad \forall\ i$$

However, the input-decoupling scheme is not quite as tolerant to sensor failures. Here, the matrix under consideration is

$$\mathbf{GK} = \mathbf{GDG}^{-1} \quad (15)$$

Despite the fact that, under normal operation, the eigenvalues of input decoupling are the same as the eigenvalues of output decoupling (similar matrices), the eigenvalues of principal submatrices, $(\mathbf{GDG}^{-1})_{II}$, can have a negative real part. Let us take a 2×2 system as an example with $\mathbf{D} = \mathrm{diag}\,(d_1,d_2)$ (Figure 1B).

$$\mathbf{GDG}^{-1} = \begin{bmatrix} \lambda_{11}d_1 + \lambda_{12}d_2 & -\lambda_{12}(g_{12}/g_{22})(d_1 - d_2) \\ \lambda_{11}(g_{21}/g_{11})(d_1 - d_2) & \lambda_{21}d_1 + \lambda_{22}d_2 \end{bmatrix} \quad (16)$$

Certainly, this, generally, is not a diagonal matrix. When sensor failure occurs in the first output, the subsystem is the (2,2) entry in $\mathbf{GDG}^{-1}$

$$\lambda_{21}d_1 + \lambda_{22}d_2 \quad (17)$$

Even with a positive diagonal element of RGA, i.e., $\lambda_{22} > 0$, SFT is not always guaranteed. A distillation example (WB column, Wood and Berry (1973); Table IA) is used to illustrate this. The RGA is

$$\mathrm{RGA}(\mathbf{G}) = \begin{bmatrix} 2.01 & -1.01 \\ -1.01 & 2.01 \end{bmatrix}$$

Figure 4 shows the load responses under normal operation initially. However, if a sensor in the first output fails, the closed-loop system becomes unstable. The reason is simple since $\lambda_{21}d_1 + \lambda_{22}d_2 = -0.01$ ($d_1 = 0.05$ and $d_2 = 0.02$), the failure results in a positive feedback system which, consequently, leads to instability.

Despite the fact that the closed-loop system may become unstable under sensor failure (even for a simple 2×2 system), the generally used inverse-based controller, the output decoupling (Figure 1A), is always SFT. This is one way to design an inverse-based controller, e.g., output-decoupling scheme, such that the closed-loop system possesses integrity against sensor failures. However, this is not the case for actuator failures.

4.2. Actuator Failure. When the actuators from the subset I' fail (or the actuators from the subset I remain in service), we, generally, take the same number of sensors off in order to maintain integral action in the subsystem. In this paper, unless otherwise mentioned, when the actuators from a subset I' fail, we take the corresponding sensors (sensors in the same subset I') off. This is corresponding to the ith row and ith column of the controller

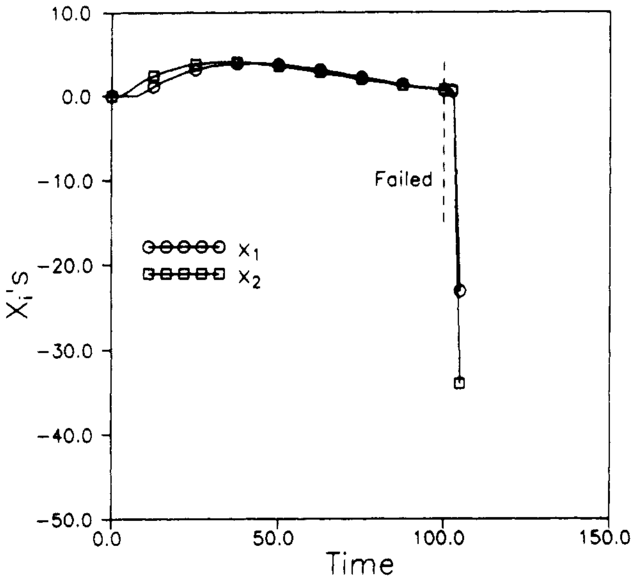


Figure 4. Load responses of WB column under input-decoupling control with sensor failure in the first output with $d_1 = 0.05$ and $d_2 = 0.02$.

matrix set to zero for $i \in I'$. If the actuators in the subset I' fail, the closed-loop characteristic equation becomes (Figure 3)

$$\det\,(\mathbf{I} + \Delta_S \mathbf{G} \Delta_A \mathbf{K}/s) = 0 \tag{18}$$

Here, Δ_A ($=\Delta_S$) is the switching matrix with δ_i set to zero, i.e., $\delta_i = 0$ for all $i \in I'$. As far as the instability (as the result of positive feedback) is concerned, we are interested in the matrix $\mathbf{G}_{II}\boldsymbol{K}_{II}$.

Definition 3. A stable multivariable feedback system $\mathbf{Q} = \mathbf{GK}$ (Figure 3) with integral action is AFT (actuator failure tolerant) if the reduced systems are stable for all possible 0 and 1 combinations in Δ_A.

Similar to the case of sensor failure, the eigenvalues of $\mathbf{G}_{II}\boldsymbol{K}_{II}$ play an important role in AFT.

Theorem 2. A multivariable feedback system $\mathbf{Q} = \mathbf{GK}$ (Figure 3) with integral action is stable and all principal minors of $\mathbf{Q}$ are nonzero. The feedback system is AFT if, and only if, all eigenvalues of $\mathbf{G}_{II}\boldsymbol{K}_{II}$ lie on the RHP for all I's.

Proof: The proof is similar to theorem 1 except that a different matrix is employed.

For the inverse-based controller considered (Figure 1), the conditions for AFT become:

Corollary 2. A nominally stable feedback system with $\mathbf{K}_{(0)} = \mathbf{G}_{(0)}^{-1}\mathbf{D}_{(0)}$ (Figure 1A) and nonzero Λ^{ℓ}_{II} is AFT if, and only if

$$\lambda_i[\Lambda^{\ell}_{II}\mathbf{D}_{II}] > 0, \quad \forall\, i \quad \text{and} \quad \forall\, I \subset N$$

Proof: Since

$$\begin{aligned}\lambda_i[\mathbf{G}_{II}\boldsymbol{K}_{II}] &= \lambda_i[\mathbf{G}_{II}(\mathbf{G}^{-1}\mathbf{D})_{II}] \\ &= \lambda_i[\mathbf{G}_{II}(\mathbf{G}^{-1})_{II}\mathbf{D}_{II}] \\ &= \lambda_i[\Lambda^{\ell}_{II}\mathbf{D}_{II}]\end{aligned}$$

The result is a direct consequence of theorem 2.

Corollary 3. A nominally stable feedback system with $\mathbf{K}_{(0)} = \mathbf{D}_{(0)}\mathbf{G}_{(0)}^{-1}$ (Figure 1B) and nonzero Λ^{r}_{II} is AFT if, and only if

$$\lambda_i[\Lambda^{r}_{II}\mathbf{D}_{II}] > 0, \quad \forall\, i \quad \text{and} \quad \forall\, I \subset N$$

Proof: Since

$$\begin{aligned}\lambda_i[\mathbf{G}_{II}\boldsymbol{K}_{II}] &= \lambda_i[\mathbf{G}_{II}(\mathbf{D}\mathbf{G}^{-1})_{II}] \\ &= \lambda_i[\mathbf{G}_{II}\boldsymbol{D}_{II}(\mathbf{G}^{-1})_{II}] \\ &= \lambda_i[(\mathbf{G}^{-1})_{II}\mathbf{G}_{II}\boldsymbol{D}_{II}] \\ &= \lambda_i[\Lambda^{r}_{II}\mathbf{D}_{II}]\end{aligned}$$

The result is directly available from theorem 2.

It is interesting to see that the AFT for the inverse-based controller is closely related to the BRG. Furthermore, the output- and input-decoupling schemes related to left and right BRG (Λ^{ℓ}_{II} and Λ^{r}_{II}), respectively. Despite the fact that the controller parameters, e.g., d_i's, are needed in the eigenvalue test (corollaries 2 and 3), the simple determinant test provides a quick assessment of lack of integrity against actuator failure.

Theorem 3. For a nominally stable feedback system with an inverse-based controller, ($\mathbf{Gc}_{(0)} = \mathbf{G}_{(0)}^{-1}$ in Figure 1), if $\det\,(\Lambda_{II}) < 0$ (left and right BRG), then the following are true: (1) the subsystem with the actuators from the subset I removed is unstable and (2) the subsystem with the actuators from the subset I' removed is unstable.

Proof: A negative determinant in Λ_{II} indicates at least a negative eigenvalue in $\mathbf{G}_{II}\boldsymbol{K}_{II}$ which, in turn, provides a positive feedback system. Since, $\det\,(\Lambda_{II}) = \det\,(\Lambda_{I'I'})$, a similar argument can be applied to the subsystem $\mathbf{G}_{I'I'}\mathbf{K}_{I'I'}$.

Since $\det\,(\Lambda_{II}) = \det\,(\Lambda_{I'I'})$, the RGA provides useful information for single actuator failure (probably the failure condition with the highest probability). For example, if $N = \{1,2,3,4\}$ and $I = \{2,3,4\}$, we have

$$\det\,(\Lambda_{II}) = \lambda_{11} \tag{19}$$

Let us take a 4 × 4 distillation column studied by Chiang and Luyben (1988) as an example (CL column). The process and load transfer functions are listed in Table IB. The RGA is

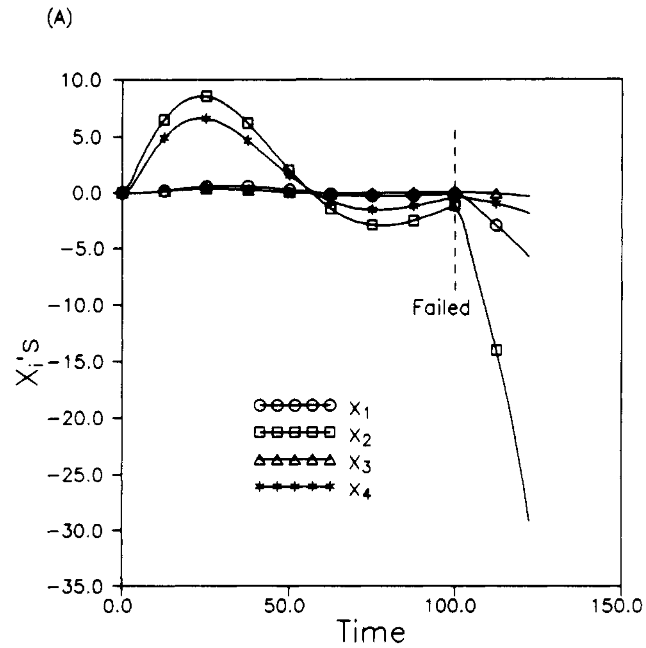
$$\mathrm{RGA}\{\mathbf{G}\} = \begin{array}{c} \begin{array}{cccc} m_1 & m_2 & m_3 & m_4 \end{array} \\ \begin{bmatrix} 2.098 & -0.998 & 0 & -0.100 \\ -1.039 & 1.332 & 0 & 0.707 \\ 0.041 & -0.563 & 1.514 & 0.008 \\ -0.100 & 1.229 & -0.514 & 0.385 \end{bmatrix} \end{array} \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \tag{20}$$

An inverse-based controller with output decoupling is employed. When the first actuator (m_1) fails, if we take the second sensor (x_2) off, the closed-loop system becomes unstable (Figure 5A). The reason is obvious $\lambda_{21} = \det\,(\Lambda_{IJ}) = -1.039 < 0$ ($I = \{1,3,4\}$ and $J = \{2,3,4\}$). However, if we take the first sensor (x_1) off, the closed-loop system remains stable (Figure 5B; $\lambda_{11} = 2.098$). Therefore, in case of single actuator failure, the RGA provides a simple solution to what sensor not to take off. It is obvious that the RGA provides useful information beyond the scope of fully decentralized controller.

In addition to the stability of the system, we would like the feedback system to be also operable under failure condition.

5. Complete Integral Controllability

In chemical process control, the operators often adjust tuning constants on-line in the face of disturbances. Therefore, it is desirable that the nominal feedback system and all of its principal subsystem remain stable as we adjust the tuning constants, e.g., d_i's in Figure 1, inde-



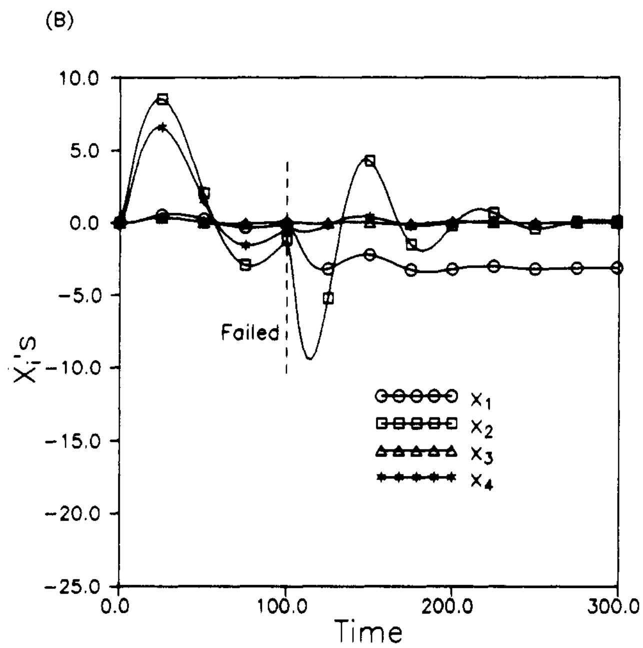


Figure 5. Load responses of CL column under output-decoupling control with (A) m_1 failure by taking x_2 off and (B) m_1 failure by taking x_1 off.

pendently. Since the inverse-based controller is employed, instead of a decentralized controller, a new definition, CIC (complete integral controllability), is introduced to distinguish from DIC.

Definition 4. A process $\mathbf{G}_{(s)}$ with an inverse-based controller $\mathbf{K}_{(s)}/s = \mathbf{Gc}_{(s)}\cdot\mathbf{D}_{(s)}/s$ (Figure 1A; or $\mathbf{K}_{(s)}/s = \mathbf{D}_{(s)}\mathbf{Gc}_{(s)}/s$ in Figure 1B) is stable. The feedback system is CIC if it has the property that each tuning constant d_i can be reduced independently (i.e., $0 < d_i \le \bar{d}_i$) without introducing instability for normal operation as well as all possible combinations of actuator failures.

The CIC addresses both the failure tolerance and the operability of a feedback system controlled by an inverse-based controller. For example, for an output-decoupling scheme (Figure 1A), the CIC requires that all subsystems remain stable under actuator failures (AFT) and allow operators to reduce the tuning constants (d_i's) arbitrarily without introducing instability under normal operation as well as failure conditions. Let $\mathcal{D}$ denote the set of positive diagonal matrices. The next theorem formulates CIC for inverse-based controller mathematically.

Theorem 4. A process $\mathbf{G}_{(s)}$ with an inverse-based controller (Figure 1) is CIC if, and only if, all the eigenvalues of $\mathbf{G}_{II}\boldsymbol{K}_{II}$ lie on the open RHP for all $\mathbf{D} \in \mathcal{D}$ and $I \subset N$.

Proof: Simply apply the eigenvalue test (for positive feedback) for all possible subsystems under all positive scalings.

Theorem 4 gives the necessary and sufficient condition for CIC for the inverse-based controller with general structure. For a given structure, the conditions in theorem 4 can be simplified further as shown in the next two corollaries.

Corollary 4. A process $\mathbf{G}_{(s)}$ with an inverse-based controller $\mathbf{K} = \mathbf{G}^{-1}\mathbf{D}$ (output-decoupling scheme, Figure 1A) is CIC if, and only if, all of the eigenvalues of $(\boldsymbol{\Lambda}^{\ell}_{II}\mathbf{D}_{II})$ lie on the RHP for all $\mathbf{D} \in \mathcal{D}$ and $I \subset N$.

Proof: Since

$$\lambda_i[\mathbf{G}_{II}\boldsymbol{K}_{II}] = \lambda_i[\mathbf{G}_{II}(\mathbf{G}^{-1})_{II}\mathbf{D}_{II}] = \lambda_i[\boldsymbol{\Lambda}^{\ell}_{II}\mathbf{D}_{II}]$$

the result can readily be derived from theorem 4.

Corollary 5. A process $\mathbf{G}$ with an inverse-based controller $\mathbf{K} = \mathbf{D}\mathbf{G}^{-1}$ (input-decoupling scheme, Figure 1B) is CIC if, and only if, all of the eigenvalues of $(\boldsymbol{\Lambda}^{r}_{II}\mathbf{D}_{II})$ lie on the RHP for all $\mathbf{D} \in \mathcal{D}$ and $I \subset N$.

Proof: Since

$$\begin{aligned}\lambda_i[\mathbf{G}_{II}\boldsymbol{K}_{II}] &= \lambda_i[\mathbf{G}_{II}\boldsymbol{D}_{II}(\mathbf{G}^{-1})_{II}] \\ &= \lambda_i[(\mathbf{G}^{-1})_{II}\mathbf{G}_{II}\boldsymbol{D}_{II}(\mathbf{G}^{-1})_{II}((\mathbf{G}^{-1})_{II})^{-1}] \\ &= \lambda_i[(\mathbf{G}^{-1})_{II}\mathbf{G}_{II}\boldsymbol{D}_{II}] \\ &= \lambda_i[\boldsymbol{\Lambda}^{r}_{II}\mathbf{D}_{II}]\end{aligned}$$

the result is a direct consequence of theorem 4.

Corollaries 4 and 5 are the important results of this paper. CIC of the inverse-based controllers related to the left and right BRG for the output- and input-decoupling schemes, respectively. As pointed out by Yu and Fan (1990), the positiveness of the eigenvalues under positive diagonal scaling is generally referred to as the problem of D stability. For decentralized control (DIC), we check whether $\mathbf{G}$ is D stable. For inverse-based controllers (CIC), the D stability of BRG's is our concern.

Before deriving simpler necessary and sufficient conditions for CIC for 2 × 2, 3 × 3, and 4 × 4 systems, some necessary conditions are given here. Generally, we can use the simple test of necessity for CIC to eliminate some undesirable pairings, e.g., the actuator-and-sensor pair under a failure condition followed by more rigorous tests for necessity and sufficiency. The results of theorem 3 indicate that a negative det (Λ_{II}) (right or left) implies that the system is not CIC. Another necessary condition is based on the diagonal element of Λ_{II}.

Theorem 5. The process $\mathbf{G}$ with inverse-based controller in Figure 1A (or Figure 1B) is not CIC if any diagonal element $\boldsymbol{\Lambda}^{\ell}_{II}$ (or $\boldsymbol{\Lambda}^{r}_{II}$) is negative.

Proof: See lemma 1 of Yamanaka and Kawasaki (1989).

The importance of this necessary condition for CIC is that it can be checked by looking at the RGA. The reason is: the diagonal element of left (or right) BRG, $\boldsymbol{\Lambda}^{\ell}_{II}$ (or $\boldsymbol{\Lambda}^{r}_{II}$) is equal to the row (or column) sum of the RGA defined by the block structure I (Manousiouthakis et al., 1986). The CL column is used to illustrate the lack of CIC. When

Table II. CIC Tests for the Output-Decoupling Scheme for CL Column

subsystem	Λ^f_{II}	det (Λ^f_{II})	RGA{Λ^f_{II}}[a]	SRG{Λ^f_{II}}[b]
(1,2)	$\begin{matrix} 1.10 & -0.03 \\ 2.63 & 0.29 \end{matrix}$	0.39	0.82	
(1,3)	$\begin{matrix} 2.10 & 0.83 \\ 0.64 & 1.55 \end{matrix}$	2.73	1.19	
(1,4)	$\begin{matrix} 2.00 & -0.26 \\ 2.83 & 0.28 \end{matrix}$	1.32	0.43	
(2,3)	$\begin{matrix} 1.33 & -4.95 \\ 0.01 & 0.95 \end{matrix}$	1.32	0.96	
(2,4)	$\begin{matrix} 2.04 & 0.95 \\ 0.59 & 1.61 \end{matrix}$	2.73	1.20	
(3,4)	$\begin{matrix} 1.52 & -0.15 \\ 3.83 & -0.13^{c} \end{matrix}$	0.39	−0.50	N/A[d]
(1,2,3)	$\begin{matrix} 1.10 & -0.03 & -0.07 \\ 2.63 & 0.29 & -1.73 \\ 0.01 & 0.00 & 0.99 \end{matrix}$	0.38	0.81 0.83 1.01	2.82
(1,2,4)	$\begin{matrix} 1.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 \\ -1.82 & 0.48 & 1.51 \end{matrix}$	1.51	1.00 1.00 1.00	3.00
(1,3,4)	$\begin{matrix} 2.00 & 0.89 & -0.26 \\ 0.63 & 1.56 & -0.17 \\ 4.65 & 4.16 & -0.23^{c} \end{matrix}$	1.33	0.50 0.90 −0.44	N/A[d]
(2,3,4)	$\begin{matrix} 2.04 & 3.22 & 0.95 \\ -0.01 & 0.96 & -0.01 \\ 0.11 & 0.34 & 1.10 \end{matrix}$	2.10	1.03 0.98 1.05	3.03

[a] Diagonal element of the RGA of Λ^f_{II}. [b] Summation of the square root of the diagonal elements of the RGA of Λ^f_{II}. [c] Negative diagonal element in the BRG. [d] Not applicable.

the second actuator (m_2) and sensor (x_2) fail ($I = \{1,3,4\}$), the diagonal element of Λ^f_{II} can be calculated from the RGA (eq 20) and $\Lambda^f_{II}(3,3)$ becomes

$$\Lambda^f_{II}(3,3) = (-0.100) + (-0.514) + (0.385) = -0.229$$

Table II shows the left BRG's for this system. Therefore, the feedback system is not CIC despite the fact that the determinant of the BRG is positive.

$$\det(\Lambda^f_{II}) = \lambda_{22} = 1.322$$

An output-decoupling control is designed with $\mathbf{D}_{(0)} = \mathrm{diag}\,(0.09,0.09,0.09,0.09)$. The closed-loop system is stable for a load change under normal operation (Figure 6). At $t = 100$ min, the second actuator fails (taking x_2–m_2 off) and the closed-loop system remains stable with oscillatory responses (Figure 6). To manage the situation, if we reduce d_1 to 0.005 and d_3 to 0.009, the closed-loop system becomes unstable (Figure 6). The reason is obvious: the subsystem with x_2–m_2 removed is not CIC as shown from the columnwise summation of the RGA. However, this particular failure will not produce an unstable system if the input-decoupling control is used, since Λ^r_{II} is checked in this situation (instead of Λ^f_{II}) as shown in Table III.

For the inverse-based controller, the necessary and sufficient conditions for CIC for 2 × 2, 3 × 3, and 4 × 4 systems can be checked with simple tests on the BRG's. Notice that the inverse-based controllers (Figure 1) under normal operation are always integral controllable and remain stable as we adjust diagonal tuning constants d_i's independently. The reason is: $\Lambda_{II} = \mathbf{I}$ for $I = N = \{1,2,...,n\}$ and the eigenvalues of **GK** are the diagonal elements of **D** (corollaries 4 and 5).

Theorem 6. For a 2 × 2 system, the feedback system in Figure 1A or 1B is CIC if, and only if, the process **G** has the following property:

P1 (1 × 1 blocks). All diagonal elements of RGA{G} are positive ($\lambda_{ii} > 0$, $\forall\, i$).

Proof: The BRG of block size 1 is simply the iith entry of RGA{G}. Therefore, $\lambda_{ii} > 0$ is necessary and sufficient condition for positive $\lambda_{ii}d_i$ (or $d_i\lambda_{ii}$) for all positive d_i's.

Theorem 7. For a 3 × 3 system, the feedback system in Figure 1A or 1B is CIC if, and only if, the process **G** has properties P1 and P2.

P2 (2 × 2 blocks). (1) All diagonal elements of Λ_{II} are positive; (2) det (Λ_{II}) > 0; (3) the diagonal elements of RGA{Λ_{II}} are positive.

Proof: The necessary and sufficient condition for the D stability (Johnson, 1974; Skogestad and Morari, 1988) for the BRG (corollaries 4 and 5) with the block size 2 is P2 (i.e., conditions 1, 2, and 3).

Notice that Λ^f_{II} is employed for the output-decoupling scheme (Figure 1A) and Λ^r_{II} is used for the input-decoupling scheme (Figure 1B).

Theorem 8. For a 4 × 4 system, the feedback system in Figure 1A or 1B is CIC if, and only if, the process **G** has the properties P1, P2, and P3.

P3 (3 × 3 blocks). (1) All diagonal elements of Λ_{II} are positive; (2) det (Λ_{II}) > 0; (3) the diagonal elements of RGA{Λ_{II}} are positive; (4) SRG{RGA{Λ_{II}}} > 1.

Proof: The necessary and sufficient condition for the D stability (Yu and Fan, 1990) for the BRG (corollaries 4 and 5) with the block size 3 is P3 (i.e., conditions 1–4).

The tests (properties P1, P2, and P3) for CIC with the inverse-based controller (Figure 1) is presented for the system with dimensions up to 4 × 4. This probably covers most multivariable processes in chemical process control. More importantly, the tests for CIC are based on the steady-state process transfer function matrix $\mathbf{G}_{(0)}$ only (independent of controller parameter except for the inverse-based structure). Furthermore, the tests can be carried out using familiar operators such as RGA, BRG, and determinant. Table II shows that the CL column with output decoupling (Figure 1A) is not CIC. For the case of the second actuator (m_2) failure, the subsystem fails to meet conditions 1 and 3 of property P3 (as shown in Figure 6). When actuators m_1 and m_2 fail, adjusting the tuning constants d_3 and d_4 may lead to instability as shown in Table II. Therefore, in addition to validating (or invali-

Table III. CIC Tests for the Input-Decoupling Scheme for CL Column

subsystem	Λ^r_{II}	det (Λ^r_{II})	RGA{Λ^r_{II}}[a]	SRG{Λ^r_{II}}[b]
(1,2)	$\begin{matrix}1.06 & -1.03\\ 0.04 & 0.33\end{matrix}$	0.39	0.90	
(1,3)	$\begin{matrix}2.14 & 0.67\\ 0.76 & 1.51\end{matrix}$	2.73	1.19	
(1,4)	$\begin{matrix}2.00 & 0.54\\ -1.37 & 0.28\end{matrix}$	1.32	0.43	
(2,3)	$\begin{matrix}0.77 & 0.43\\ -0.35 & 1.51\end{matrix}$	1.32	0.88	
(2,4)	$\begin{matrix}2.56 & -0.05\\ -1.23 & 1.09\end{matrix}$	2.73	1.02	
(3,4)	$\begin{matrix}1.00 & 0.31\\ 0.00 & 0.39\end{matrix}$	0.39	1.00	
(1,2,3)	$\begin{matrix}1.10 & -1.89 & 0.67\\ 0.06 & -0.23^{c} & 0.43\\ 0.08 & -1.45 & 1.51\end{matrix}$	0.38	0.81 −0.96 −0.51	N/A[d]
(1,2,4)	$\begin{matrix}0.96 & 0.87 & 0.00\\ -0.03 & 1.56 & 0.00\\ -0.04 & 0.88 & 0.99\end{matrix}$	1.51	0.99 0.98 1.00	2.98
(1,3,4)	$\begin{matrix}2.04 & 0.00 & 0.55\\ 0.68 & 1.00 & 0.36\\ -1.33 & 0.00 & 0.29\end{matrix}$	1.33	0.45 1.00 0.45	2.34
(2,3,4)	$\begin{matrix}2.00 & 0.00 & -0.05\\ 1.10 & 1.00 & -0.05\\ -2.11 & 0.00 & 1.10\end{matrix}$	2.10	1.05 1.00 1.05	3.05

[a] Diagonal element of the RGA of Λ^r_{II}. [b] Summation of the square root of the diagonal elements of the RGA of Λ^r_{II}. [c] Negative diagonal element in the BRG. [d] Not applicable.

dating) CIC, the results of the tests (Table II or III) also provide information on when and how the instability may occur. It is interesting to note that the RGA of the BRG is always symmetric. For example, when $I = \{1,3,4\}$, the RGA of Λ^{ℓ}_{II} becomes

$$\mathrm{RGA}\{\Lambda^{\ell}_{II}\} = \begin{bmatrix} 0.50 & -0.42 & 0.92 \\ -0.42 & 0.90 & 0.52 \\ 0.92 & 0.52 & -0.44 \end{bmatrix}$$

6. Summary

In order to ensure failure tolerance and operability of the inverse-based multivariable controllers (Figure 1), some form of "pairing" is proposed in the design of a multivariable controller. Unlike the pairing in the decentralized control, the pairing discussed here is more or less pairing off (e.g., to select an appropriate sensor taking off service). The objective of pairing here is to ensure that each individual element or block is stable and operable (just the same as the pairing requirement in decentralized control). Based on the results of theorems 6, 7, and 8 (e.g., properties P1, P2, and P3), the rules for the pairing are summarized for 2 × 2, 3 × 3, and 4 × 4 systems.

1. 2 × 2 systems:

R1. Eliminate pairings with negative diagonal element in RGA{**G**}.

2. 3 × 3 systems:

R1. Eliminate pairings with negative diagonal element in RGA{**G**}.

R2. Eliminate pairings with negative diagonal element in Λ_{II} (for block size 2).

R3. Eliminate pairings with det (Λ_{II}) < 0.

R4. Eliminate pairings with negative diagonal element in RGA{Λ_{II}} (for block size 2).

Remarks: (1) R3 is listed here for the sake of consistency with theorem 7. Here, R3 provides the same information as R1. (2) R2 can be checked by adding up elements of RGA{**G**} rowwise or columnwise according to the block structure.

3. 4 × 4 systems:

R1. Eliminate pairings with negative diagonal element in RGA{**G**}.

R2. Eliminate pairings with negative diagonal element in Λ_{II} (block sizes 2 and 3).

R3. Eliminate pairings with det (Λ_{II}) < 0 (for block size 2).

R4. Eliminate pairings with negative diagonal element in RGA{Λ_{II}} (for block sizes 2 and 3).

R5. Eliminate pairings with SRG{RGA{Λ_{II}}} < 1 (for block size 3).

Remarks: (1) For the output-decoupling scheme (Figure 1A), Λ^{ℓ}_{II} is used, and for the input decoupling control (Figure 1B) Λ^r_{II} is used. (2) The process that passes the test is CIC.

The pairing rules presented here show a very interesting aspect of multivariable control. The measures such as RGA and BRG are the inherent properties of a given process. Pairings (e.g., pairing on or off) showing a negative measure in the diagonal element of RGA{**G**} or det (Λ_{II}) are not desirable for decentralized controllers or for the inverse-based controllers.

7. Conclusion

The failure tolerance and operability of inverse-based controllers with integral action are discussed. Sensor failure tolerance (SFT) and actuator failure tolerance (AFT) are defined. Some necessary as well as necessary and sufficient conditions are derived for SFT and AFT. A more general problem, operability, is addressed with the introduction of CIC. Necessary and sufficient conditions for CIC for 2 × 2, 3 × 3, and 4 × 4 systems are derived. More importantly, the tests for CIC are based on well-known interaction measures such as RGA and BRG, which are very simple to apply. In order to have a feedback system that is both stable and operable under failure conditions, some form of "pairing" is proposed for the inverse-based multivariable controllers. The rules for the pairing are summarized to ensure CIC, if possible. The results (SFT, AFT, and CIC) of this paper provide a better understanding for the inverse-based controllers under

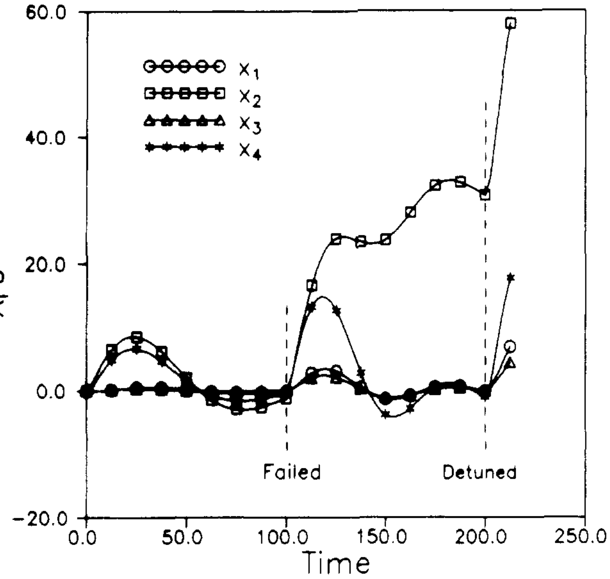


Figure 6. Load responses of CL column under output-decoupling control with (1) normal operation ($0 \leq t < 100$); (2) actuator failure by taking $x_2 - m_2$ off at t = 100 min; (3) detuning $\mathbf{D}_{II}$ from diag (0.09,0.09,0.09) to diag (0.005,0.009,0.09) with $I = \{1,3,4\}$ at t = 200 min.

failure conditions and, hopefully, will lead to better acceptance of multivariable controllers.

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Nomenclature

AFT = actuator failure tolerance
BRG = block relative gain
CIC = complete integral controllability
$\mathbf{D}$ = diagonal tuning matrix for the inverse-based controllers
$\mathcal{D}$ = set of positive diagonal matrices
$\mathbf{D}_{II}$ = principal submatrix of $\mathbf{D}$ with rows and columns defined by I
d_i = tuning constant in the iith entry of diagonal matrix $\mathbf{D}$
$\bar{d}_i$ = upper bound of d_i
det = determinant
DIC = decentralized integral controllability
$\mathbf{G}$ = Process transfer function matrix
$\mathbf{Gc}$ = approximated inverse of $\mathbf{G}$ ($\mathbf{Gc}_{(0)} = \mathbf{G}_{(0)}^{-1}$)
$\mathbf{G}_{II}$ = principal submatrix of $\mathbf{G}$ with rows and columns defined by I
g_{ij} = ijth entry of $\mathbf{G}$
$\hat{g}_{ij}$ = ijth entry of $\mathbf{G}^{-1}$
$\mathbf{K}_{(s)}$ = inverse-based controller matrix with integrator factored out
$\mathbf{K}_{II}$ = principal submatrix of $\mathbf{K}$ with rows and columns defined by I
MMC = modular multivariable control
m = manipulated variable
NI = Niederlinski index as defined by (1)
$\mathbf{Q} = \mathbf{G}\cdot\mathbf{K}$
QDMC = quadratic dynamic matrix control
RGA = relative gain array
RHP = right half-plane
SFT = sensor failure tolerance
SRG = summation of square root of the diagonal elements of the RGA for 3 × 3 systems
s = Laplace transform variable
x = controlled variable

Greek Letters

Δ = switching matrix
δ_i = ith entry of Δ taking the value of 0 or 1
Λ_{II} = block relative gain array defined by I
λ_{ij} = ijth relative gain
λ_i = ith eigenvalue

Subscripts

A = actuator
I = subset of N
I' = complement of the subset of I with respect to N
N = the integer set $N = \{1,2,...,n\}$
S = sensor

Superscripts

ℓ = left
r = right
T = transpose
−1 = inverse

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