Direction-Finding Methods for Cyclostationary Signals in the Presence of Coherent Sources

Yung-Ting Lee and Ju-Hong Lee

***Abstract*—By exploiting the signal cyclostationarity, conjugate Cyclic MUSIC presented in [8], has been shown to be effective in performing signal-selective direction finding. However, a direct application of conjugate Cyclic MUSIC in the presence of coherent signals of interest (SOIs) will result in ambiguity. To tackle the drawback of conjugate Cyclic MUSIC, we present a scheme called Hankel approximation method (HAM) in conjunction with conjugate Cyclic MUSIC to cope with the performance deterioration due to coherent SOIs for a uniform linear array. Theoretical analysis shows that at least $2K$ sensor elements are required to resolve K coherent SOIs. Making use of the forward/backward (F/B) technique presented in [6], F/B HAM is further developed, which requires $[3K/2]$ sensor elements to decorrelate K coherent SOIs, where $[x]$ represents the integer part of x. Several simulation examples are also presented for illustrating the effectiveness of the proposed methods.**

***Index Terms*—Cyclostationary signals, direction finding, uniform linear array.**

I. Introduction

SINCE the works of Pisarenko [1], Schmidt [2], Roy and Kailath [3], etc., eigenspace-based (ESB) methods have been a topic of great interest in estimating the directions of signals. These ESB methods, in general, utilize certain eigenstructure properties resulting from the special structure of array output covariance matrix for planar wavefronts to generate spectral peaks or nulls in the directions of arrival (DOAs). Moreover, they yield high resolution even when some of signals are partially correlated. In coherent situations such as multipath propagation, a direct application of these ESB methods, however, results in ambiguity. Several techniques have been developed to remedy this problem, of which the spatial smoothing (SS) scheme presented in [4], [5], for the case of a uniform linear array (ULA) is specially noteworthy. The solution is based on a preprocessing scheme that groups the total array of sensor elements into several overlapping subarrays and then averages the subarray output covariance matrices to form the spatially smoothed covariance matrix. It has been shown in [5] that at least $2K$ sensor elements are required to decorrelate and resolve K coherent signals. In order to reduce the number of sensor elements required by the SS scheme, the forward/backward (F/B) SS scheme is then investigated in [6], which requires $[3K/2]$ sensor elements to estimate K coherent signals, where $[x]$ stands for the integer part of x. Moreover, the SS and F/B SS schemes can obtain asymptotical unbiased estimation of DOA irrespective of signal-to-noise ratio (SNR) and angular separations of signals. On the other hand, a novel algorithm based on the Teoplitz approximation method (TAM) was developed in [14] for decorrelating the coherent signals even when a small number of data snapshots is used. However, the theoretical analysis on TAM is still an open issue.

Recently, cyclostationary properties have been widely considered for signal processing. Cyclostationarity [8] which is a statistical property possessed by most man-made communication signals corresponds to the underlying periodicity arising from cycle frequencies or baud rates. Cyclic MUSIC and conjugate Cyclic MUSIC presented in [8], [9] accommodate multiple signals having the same cycle frequency by using the same type of subspace fitting as MUSIC [2]. Moreover, in certain cases as shown in [10], the mean-square error (MSE) of the direction estimates obtained by Cyclic MUSIC is less than the Cramér-Rao lower bound for stationary signals. However, a direct application of Cyclic MUSIC and conjugate Cyclic MUSIC in the presence of coherent *signals-of-interest* (SOIs) also suffers the performance degradation as the conventional ESB methods. Some previous work considering the problem has been reported in [12], [13]. A cyclic least-square method has been presented in [12] for dealing with the direction-finding problem under coherent situation. However, it is observed that this method cannot tackle the problem efficiently. In order to improve the robustness of the DOA estimation, a cyclic algorithm based on the SS or improved SS technique has been presented in [13]. Making use of the multiple lag parameters and the optimal subarray size, the method presented by [13] provides satisfactory performance even in the presence of strong *signals not of interest* (SNOIs).

In this paper, the direction-finding problem of conjugate Cyclic MUSIC in coherent scene is extensively considered. Based on the spatial structure of the cyclic conjugate correlation matrix when narrow-band, far-field SOIs are considered for a ULA, we propose a Hankel approximation method (HAM) in conjunction with conjugate Cyclic MUSIC to cope with the coherent situation. Theoretical analysis is presented and shows that at least $2K$ sensor elements are required to decorrelate K coherent SOIs. Using the forward/backward (F/B) scheme of [6], we further develop a forward/backward Hankel Approximation Method (F/B HAM) which requires $[3K/2]$ sensor elements to achieve direction finding for K coherent SOIs. Simulation results show the effectiveness of the proposed methods.

This paper is organized as follows. In Section II, we briefly review the signal cyclostationarity. Then, the problem of direc-

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tion finding in the presence of coherent SOIs is described. Section III presents HAM and F/B HAM to deal with the ambiguity of direction finding due to coherent SOIs. Theoretical analysis of the proposed methods are then presented in Section IV. Section V provides several simulation examples for illustrating the effectiveness of the proposed methods. Finally, we conclude this paper in Section VI.

II. Background

A. Signal Cyclostationarity

For a signal $s(t)$, the cyclic correlation function and cyclic conjugate correlation function are defined as [8, pp. 41]

$$r_{\rm ss}(f,\tau) = \left\langle s\left(t+\frac{\tau}{2}\right)s^*\left(t-\frac{\tau}{2}\right)e^{-j2\pi ft}\right\rangle_\infty \tag{1}$$

and

$$r_{\rm ss^*}(f,\tau) = \left\langle s\left(t+\frac{\tau}{2}\right)s\left(t-\frac{\tau}{2}\right)e^{-j2\pi ft}\right\rangle_\infty \tag{2}$$

respectively, where the superscript "*" denotes the complex conjugate. Then, $s(t)$ is cyclostationary if $r_{\rm ss}(f,\tau)$ or $r_{\rm ss^*}(f,\tau)$ does not equal zero at some time delay τ and cycle frequency $f=\alpha\neq 0$. Many modulated signals exhibit cyclostationarity with cycle frequency equal to twice the carrier frequency or multiples of the baud rate or combinations of these. Moreover, when (1) and (2) do not equal zero for $f\neq 0, s(t)$ is addressed as possessing the selfcoherent and conjugate selfcoherent properties, respectively.

Let $\mathbf{x}(t)$ be the data vector received by an antenna array. The cyclic correlation matrix and cyclic conjugate correlation matrix are given by

$$\mathbf{R}_{xx}(f,\tau) = \left\langle \mathbf{x}\left(t+\frac{\tau}{2}\right)\mathbf{x}^H\left(t-\frac{\tau}{2}\right)e^{-j2\pi ft}\right\rangle_\infty \tag{3}$$

and

$$\mathbf{R}_{xx^*}(f,\tau) = \left\langle \mathbf{x}\left(t+\frac{\tau}{2}\right)\mathbf{x}^T\left(t-\frac{\tau}{2}\right)e^{-j2\pi ft}\right\rangle_\infty \tag{4}$$

respectively, where the superscript "H" denotes the conjugate transpose and "T" the transpose.

B. The Problem

Consider an M-element ULA excited by K narrow-band, far-field SOIs which have cycle frequency equal to α. The received data vector can be expressed as

$$\mathbf{x}(t) = \sum_{n=1}^{K} s_n(t)\mathbf{a}(\theta_n) + \mathbf{i}(t) \tag{5}$$

where $s_n(t)$ is the nth SOI, $\mathbf{i}(t)$ includes all SNOIs and noise, and $\mathbf{a}(\theta_n)$ denotes the direction vector of $s_n(t)$ impinging on the array from θ_n off broadside.

Let the phase origin be the qth sensor element. Then, we have

$$\mathbf{a}(\theta_n) = [e^{j(1-q)\varphi_n}, e^{j(2-q)\varphi_n}, \ldots, e^{j(M-q)\varphi_n}]^T \tag{6}$$

where $\varphi_n = (2\pi d\sin(\theta_n))/(\lambda)$ is the electrical angle, d denotes the interelement spacing, and λ is the wavelength of SOI. Substituting (5) into (4) yields

$$\begin{aligned}\mathbf{R}_{xx^*}(f,\tau) &= \sum_{n=1}^{K}\sum_{m=1}^{K} r_{s_n s_m^*}(f,\tau)\mathbf{a}(\theta_n)\mathbf{a}^T(\theta_m)\\ &= \mathbf{A}\mathbf{S}^*(f,\tau)\mathbf{A}^T\end{aligned} \tag{7}$$

where

$$\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_K)]$$
$$\mathbf{S}^*(f,\tau) = \langle \mathbf{s}(t+(\tau/2))\mathbf{s}^T(t-(\tau/2))e^{-j2\pi ft}\rangle_\infty$$

and $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T$. If the K SOIs are all uncorrelated, $\mathbf{S}^*(f,\tau)$ is diagonal and nonsingular. We note from (6) and (7) that when the SOIs are all uncorrelated, the corresponding $\mathbf{R}_{xx^*}(f,\tau)$ is a Hankel matrix. However, $\mathbf{S}^*(f,\tau)$ becomes block diagonal and singular in the presence of coherent SOIs. Hence, the Hankel structure for $\mathbf{R}_{xx^*}(f,\tau)$ is destroyed. This leads to the result that the conventional conjugate Cyclic MUSIC [8, pp. 214–215], cannot successfully resolve the DOAs of the SOIs.

III. The Proposed Methods

Let an M-element ULA be divided into overlapping subarrays of size N, with sensor elements $\{m, \ldots, m+N-1\}$ forming the mth subarray. From (5), we obtain

$$\begin{aligned}\mathbf{x}_k(t) &= \sum_{n=1}^{K} s_n(t)\mathbf{a}_k(\theta_n) + \mathbf{i}_k(t)\\ &= \sum_{n=1}^{K} s_n(t)e^{j(k-1)\varphi_n}\mathbf{a}_1(\theta_n) + \mathbf{i}_k(t)\end{aligned} \tag{8}$$

where $\mathbf{x}_k(t)$ and $\mathbf{a}_k(\theta_n)$ represent the received data and direction vectors of the kth subarray, respectively. Next, we present a HAM to decorrelate the coherent SOIs based on the spatial smoothing (SS) scheme of [5].

A. HAM

Since the cyclic conjugate correlation matrix possesses the Hankel property instead of the Toeplitz property when all the SOIs are uncorrelated, the methods proposed by [13] are not appropriate for solving the problem. Therefore, we define a new cyclic conjugate correlation matrix of the kth subarray as follows to alleviate the difficulty of conjugate Cyclic MUSIC for direction finding under coherent SOIs

$$\begin{aligned}&\mathbf{R}_{xx^*}(f,\tau,k)\\ &\quad= \left\langle \mathbf{x}_k\left(t+\frac{\tau}{2}\right)\mathbf{x}_{L-k+1}^T\left(t-\frac{\tau}{2}\right)e^{-j2\pi ft}\right\rangle_\infty\\ &\quad= \sum_{n=1}^{K}\sum_{m=1}^{K} r_{s_n s_m^*}(f,\tau)e^{j\{(k-1)\theta_n+(L-k)\theta_m\}}\mathbf{a}_1(\theta_n)\mathbf{a}_1^T(\theta_m)\end{aligned} \tag{9}$$

where $L = M-N+1$. We note from (9) that $\mathbf{R}_{xx^*}(f,\tau,k)$ is the kth submatrix along the backward main diagonal of the cyclic conjugate correlation matrix of the full array. The corresponding spatial smoothed cyclic conjugate correlation matrix for the conjugate selfcoherent case is given by

$$\bar{\mathbf{R}}_{xx^*}(f,\tau) = \frac{1}{L}\sum_{k=1}^{L}\mathbf{R}_{xx^*}(f,\tau,k). \tag{10}$$

From (10), we have

$$\bar{r}^*(i,j) = \frac{1}{L}\sum_{k=1}^{L} r^*(i+L-k, j+k-1) \tag{11}$$

where $\bar{r}^*(i,j)$ and $r^*(i,j)$ denote the (i,j)th elements of $\bar{\mathbf{R}}_{xx^*}(f,\tau)$ and $\mathbf{R}_{xx^*}(f,\tau)$, respectively. We observe from (11) that constructing $\bar{\mathbf{R}}_{xx^*}(f,\tau)$ is equivalent to performing the Hankel approximation on the cyclic conjugate correlation matrices of the subarrays. Hence, we term the proposed scheme as HAM.

B. F/B HAM

Based on the F/B SS scheme of [6], a F/B HAM scheme is proposed to obtain the F/B spatial smoothed cyclic conjugate correlation matrix as

$$\begin{aligned}\bar{\mathbf{R}}^{fb}_{xx^*}(f,\tau) &= \frac{1}{L}\sum_{k=1}^{L}\{\nu\mathbf{R}_{xx^*}(f,\tau,k) + (1-\nu) \\ &\quad\times \mathbf{I}_r\mathbf{R}^H_{xx^*}(f,\tau,k)\mathbf{I}_r\} \\ &= \frac{1}{L}\sum_{k=1}^{L}\mathbf{R}^{fb}_{xx^*}(f,\tau,k)\end{aligned} \tag{12}$$

where $\mathbf{I}_r$ denotes the reflection matrix given by

$$\mathbf{I}_r = \begin{bmatrix} \mathbf{0} & & & 1 \\ & & 1 & \\ & \dots & & \\ 1 & & & \mathbf{0}\end{bmatrix} \tag{13}$$

$0 < \nu < 1$, and $\mathbf{R}^{fb}_{xx^*}(f,\tau,k) = \nu\mathbf{R}_{xx^*}(f,\tau,k) + (1-\nu)\mathbf{I}_r\mathbf{R}^H_{xx^*}(f,\tau,k)\mathbf{I}_r$. Due to that $\mathbf{S}^*(f,\tau)$ may be a complex matrix, the factor ν is employed to avoid the situation where $\bar{\mathbf{R}}^{fb}_{xx}(f,\tau)$ becomes singular.

C. The Algorithm

Let $\bar{\mathbf{R}}_{xx^*}(\alpha,\tau,n)$ and $\bar{\mathbf{R}}^{fb}_{xx^*}(\alpha,\tau,n)$ represent the results of $\bar{\mathbf{R}}_{xx^*}(f,\tau)$ and $\bar{\mathbf{R}}^{fb}_{xx^*}(f,\tau)$ computed by using n data snapshots when $f=\alpha$. Then, the estimated DOAs of the K SOIs are obtained step-by-step as follows:

Step 1) Compute $\bar{\mathbf{R}}_{xx^*}(\alpha,\tau,n)$ and $\bar{\mathbf{R}}^{fb}_{xx^*}(\alpha,\tau,n)$ by (9), (10), and (12). Set $\hat{\mathbf{R}}_{xx^*}(n)$ equal to $\bar{\mathbf{R}}_{xx^*}(\alpha,\tau,n)$ and $\bar{\mathbf{R}}^{fb}_{xx^*}(\alpha,\tau,n)$ when HAM and F/B HAM are used, respectively.

Step 2) Perform the singular value decomposition (SVD) on $\hat{\mathbf{R}}_{xx^*}(n)$ and obtain

$$\begin{aligned}\hat{\mathbf{R}}_{xx^*}(n) &= [\hat{\mathbf{S}}(n)\hat{\mathbf{G}}(n)]\begin{bmatrix}\hat{\mathbf{\Sigma}}_S(n) & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{\Sigma}}_G(n)\end{bmatrix} \\ &\quad\times [\hat{\mathbf{V}}_S(n)\hat{\mathbf{V}}_G(n)]^H\end{aligned} \tag{14}$$

where $[\hat{\mathbf{S}}(n)\hat{\mathbf{G}}(n)]$ and $[\hat{\mathbf{V}}_S(n)\hat{\mathbf{V}}_G(n)]$ are two unitary matrices, $\hat{\mathbf{S}}(n)$ and $\hat{\mathbf{V}}_S(n)$ have size $M\times K$, and $\hat{\mathbf{G}}(n)$ and $\hat{\mathbf{V}}_G(n)$ have size $M\times(M-K)$. Moreover, the diagonal elements of $\hat{\mathbf{\Sigma}}_S(n)$ and $\hat{\mathbf{\Sigma}}_G(n)$ are nonnegative and appear in decreasing order with $\hat{\mathbf{\Sigma}}_G(n)\to 0$ as $n\to\infty$.

Step 3) Find the K minima of $\|\hat{\mathbf{G}}^H(n)\mathbf{a}(\phi)\|^2$ or the K maxima of $\|\hat{\mathbf{S}}^H(n)\mathbf{a}(\phi)\|^2$, where $\mathbf{a}(\phi)$ is the direction vector corresponding to impinging angle ϕ and $\|\cdot\|$ denotes the norm operator. The K found values of ϕ are then viewed as the DOAs of SOIs.

IV. Theoretical Analyses

For the K SOIs impinging on the array, we assume that the first P SOIs are coherent and the last $(K-P)$ SOIs are incoherent. Let *rank*$(\mathbf{A})$ represent the rank of matrix $\mathbf{A}$.

Theorem 1: If the number of subarrays is greater than or equal to the number of coherent signals, i.e., if $L \geq P$, then *rank* $(\bar{\mathbf{R}}_{xx^*}(f,\tau))$ is equal to K.

Proof: First, we rewrite (8) as

$$\begin{aligned}\mathbf{x}_k(t) &= \left\{\sum_{m=1}^{P} c_m e^{j(k-1)\varphi_m}\mathbf{a}_1(\theta_m)\right\}s_1(t) \\ &\quad + \sum_{n=P+1}^{K} s_n(t)e^{j(k-1)\varphi_n}\mathbf{a}_1(\theta_n) + \mathbf{i}_k(t)\end{aligned} \tag{15}$$

where $c_m s_1(t)$ and φ_m denote the mth wavefront and electrical angle, respectively. From (9) and (15), we have

$$\begin{aligned}&\mathbf{R}_{xx^*}(f,\tau,k) \\ &= r_{s_1 s_1^*}(f,\tau)\left\{\sum_{m=1}^{P}\sum_{n=1}^{P} c_m c_n e^{j\{(k-1)\varphi_m + (L-k)\varphi_n\}}\right. \\ &\quad\left.\times\, \mathbf{a}_1(\theta_m)\mathbf{a}_1^T(\theta_n)\right\} + \sum_{n=P+1}^{K} r_{s_n s_n^*}(f,\tau)\mathbf{a}_1(\theta_n)\mathbf{a}_1^T(\theta_n).\end{aligned} \tag{16}$$

The spatial smoothed cyclic conjugate correlation matrix is thus given by

$$\begin{aligned}&\bar{\mathbf{R}}_{xx^*}(f,\tau) \\ &= \frac{1}{L}\sum_{k=1}^{L}\mathbf{R}_{xx^*}(f,\tau,k) \\ &= r_{s_1 s_1^*}(f,\tau)\left\{\sum_{m=1}^{P}\sum_{n=1}^{P} c_m c_n\right. \\ &\quad\left.\times\left[\frac{1}{L}\sum_{k=1}^{L} e^{j\{(k-1)\varphi_m + (L-k)\varphi_n\}}\right]\mathbf{a}_1(\theta_m)\mathbf{a}_1^T(\theta_n)\right\} \\ &\quad + \sum_{n=P+1}^{K} r_{s_n s_n^*}(f,\tau)\mathbf{a}_1(\theta_n)\mathbf{a}_1^T(\theta_n).\end{aligned} \tag{17}$$

It is easy to show that $\mathrm{rank}(\mathbf{R}_{xx^*}(f,\tau,k)) = K-P+1$. Thus, $\mathrm{rank}(\bar{\mathbf{R}}_{xx^*}(f,\tau))$ is not more than $\min\{K, K-P+L\}$, where $\min\{a,b\}=a$ if $a\leq b$. Accordingly, we note that $\mathrm{rank}(\bar{\mathbf{R}}_{xx^*}(f,\tau)) = K$ is possible when $L\geq P$. Moreover, the component in (17) regarding the ith and jth coherent SOIs is given by

$$h_{i,i}\mathbf{a}_1(\theta_i)\mathbf{a}_1^T(\theta_i)+h_{j,j}\mathbf{a}_1(\theta_j)\mathbf{a}_1^T(\theta_j) + h_{i,j}\left\{\mathbf{a}_1(\theta_i)\mathbf{a}_1^T(\theta_j)+\mathbf{a}_1(\theta_j)\mathbf{a}_1^T(\theta_i)\right\} \quad (18)$$

where

$$h_{i,i} = r_{s_1 s_1^*}(f,\tau)c_i^2 e^{j(L-1)\varphi_i}$$
$$h_{j,j} = r_{s_1 s_1^*}(f,\tau)c_j^2 e^{j(L-1)\varphi_j}$$

$$\begin{aligned} h_{i,j} &= r_{s_1 s_1^*}(f,\tau)c_i c_j \frac{1}{L}\sum_{k=1}^{L} e^{j\{(k-1)\varphi_i+(L-k)\varphi_j\}} \\ &= r_{s_1 s_1^*}(f,\tau)c_i c_j \frac{e^{jL\varphi_j}-e^{jL\varphi_i}}{L(e^{j\varphi_j}-e^{j\varphi_i})} \end{aligned} \quad (19)$$

respectively. Since

$$\begin{aligned} &|(e^{jL\varphi_j}-e^{jL\varphi_i})/(L(e^{j\varphi_j}-e^{j\varphi_i}))| \\ &= (|e^{jL\varphi_j}-e^{jL\varphi_i}|)/(L|e^{j\varphi_j}-e^{j\varphi_i}|) \\ &\neq 1 \end{aligned}$$

and $|e^{j(L-1)(\varphi_i+\varphi_j)}| = 1$ when $\varphi_i \neq \varphi_j$, we have

$$\begin{aligned} h_{i,j}^2 - h_{i,i}h_{j,j} &= r_{s_1 s_1^*}^2(f,\tau)c_i^2 c_j^2 \left\{\left(\frac{e^{jL\varphi_j}-e^{jL\varphi_i}}{L(e^{j\varphi_j}-e^{j\varphi_i})}\right)^2 - e^{j(L-1)(\varphi_i+\varphi_j)}\right\}. \\ &\neq 0 \end{aligned} \quad (20)$$

Hence, the resulting matrix of (18) has rank 2. (20) reveals that the ith and jth coherent SOIs impinging from different angles can therefore be decorrelated. As a result, any two coherent SOIs can be decorrelated by using the spatial smoothed cyclic conjugate correlation matrix. Under the condition that $L \geq P$, the proof is complete.

Theorem 2: If the number of subarrays is greater than or equal to half the number of coherent signals, i.e., if $2L \geq P$, then $\mathrm{rank}(\bar{\mathbf{R}}_{xx^*}^{fb}(f,\tau))$ is equal to K.

Proof: We first consider the effect of using the forward/backward technique. Performing some necessary algebraic manipulations with $\mathbf{I}_r\mathbf{a}_1^*(\theta_m) = e^{-j(M+1-2q)\varphi_m}\mathbf{a}_1(\theta_m)$, where $\mathbf{a}_1(\theta_m)$ is the direction vector of the mth SOI at the first subarray, the component in $\mathbf{R}_{xx^*}^{fb}(f,\tau,k)$ regarding the ith and jth coherent SOIs is expressed as

$$\Omega_{xx^*}(i,j) = v_{i,i}\mathbf{a}_1(\theta_i)\mathbf{a}_1^T(\theta_i)+v_{j,j}\mathbf{a}_1(\theta_j)\mathbf{a}_1^T(\theta_j) + v_{i,j}\mathbf{a}_1(\theta_i)\mathbf{a}_1^T(\theta_j)+v_{j,i}\mathbf{a}_1(\theta_j)\mathbf{a}_1^T(\theta_i) \quad (21)$$

where

$$\begin{aligned} v_{a,b} &= \nu r_{s_1 s_1^*}(f,\tau)c_a c_b e^{j\{(k-1)\varphi_a+(L-k)\varphi_b\}} \\ &+ (1-\nu)r_{s_1 s_1^*}^*(f,\tau)c_a^* c_b^* e^{-j\{(k-1)\varphi_a+(L-k)\varphi_b\}} \\ &\times e^{-j(M+1-2q)(\varphi_a+\varphi_b)} \end{aligned} \quad (22)$$

$a \in \{i,j\}$ and $b \in \{i,j\}$. Thus, we obtain

$$\begin{aligned} \sigma &= \frac{v_{i,i}v_{j,j}-v_{i,j}v_{j,i}}{\nu(1-\nu)|r_{s_1 s_1^*}(f,\tau)|^2} \\ &= (c_i c_j^*)^2 e^{-j2(M+1-2q)\varphi_j}e^{j(L-1)\phi} \\ &\quad + (c_i^* c_j)^2 e^{-j2(M+1-2q)\varphi_i}e^{-j(L-1)\phi} \\ &\quad - |c_i|^2|c_j|^2 e^{-j(M+1-2q)(\varphi_i+\varphi_j)} \\ &\quad \times \left\{e^{j(2k-1-L)\phi}+e^{-j(2k-1-L)\phi}\right\} \\ &= \Big(c_i c_j^* e^{-j(M+1-2q)\varphi_j}e^{j\frac{L-1}{2}\phi} \\ &\quad - c_i^* c_j e^{-j(M+1-2q)\varphi_i}e^{-j\frac{L-1}{2}\phi}\Big)^2 \\ &\quad + |c_i c_j|^2 e^{-j(M+1-2q)(\varphi_i+\varphi_j)} \\ &\quad \times \left(2-e^{j(2k-1-L)\phi}-e^{-j(2k-1-L)\phi}\right)^2 \end{aligned} \quad (23)$$

where $\phi = \varphi_i - \varphi_j$. We note that c_i and c_j are the complex amplitude gains of the ith and jth SOIs, respectively. Moreover, $e^{j(M+1-2q)\varphi_i}$, $e^{j(M+1-2q)\varphi_j}$, and $e^{j((L-1)/2)\phi}$ are the phases determined by the array geometry structure. According to [6], we usually have $(c_i c_j^* e^{-j(M+1-2q)\varphi_j}e^{j((L-1)/2)\phi} - c_i^* c_j e^{-j(M+1-2q)\varphi_i}e^{-j((L-1)/2)\phi}) \neq 0$ and then $|\sigma| \neq 0$ from (23). As a result, the ith and jth coherent SOIs can be decorrelated by the F/B technique. From *Theorem 1*, $2L = P$ is then enough to decorrelate all coherent SOIs and thus the proof is complete.

Since the number of subarrays, given by $L = M - N + 1$ and the size of each subarray N must be at least $K+1$, it follows that the minimum numbers of sensor elements needed for applying HAM and F/B HAM are $(P+K)$ and $[2K+P/2]$, respectively, and thus $2K$ and $[3K/2]$, respectively, under the worst situation where the K SOIs are all coherent, i.e., $P = K$. Moreover, the HAM and F/B HAM are developed based on the SS [5] and F/B SS [6], respectively, the conclusions obtained from the above theorems are similar to those presented in [5] and [6].

V. Computer Simulations

In this section, two computer simulation examples performed on a PC with Pentium II-350 CPU using MATLAB programming language are presented for showing the effectiveness of the proposed methods.

Example 1: Here, a ULA with number of sensor elements $M = 5$ and interelement spacing = half wavelength of SOI is considered. The array is grouped into two overlapping subarrays with sensor elements $\{m, \cdots, 3+m\}$ for the mth subarray, $m = 1, 2$. The DOAs for three SOIs are 5°, 10°, and $-10°$ off array broadside, respectively. The first two SOIs are coherent, while the third SOI is uncorrelated with the others. The ensemble cyclic conjugate correlation matrix is given by $\mathbf{a}_{\mathrm{eq}}\mathbf{a}_{\mathrm{eq}}^T + \mathbf{a}(\theta_3)\mathbf{a}^T(\theta_3)$, where $\mathbf{a}_{\mathrm{eq}} = \mathbf{a}(\theta_1)+\mathbf{a}(\theta_2)$ and $\mathbf{a}(\theta_i)$ denotes the direction vector of the ith SOIs. We note from the theoretical analysis that the minimum numbers of sensor elements needed for HAM and F/B HAM are $P+K = 2+3 = 5$ and $[2K+P/2] = [2\times 3+2/2] = 4$, respectively. Thus, the first and second subarrays are used in HAM to decorrelate the coherent SOIs, while only the first subarray is used in F/B HAM. Fig. 1 shows the simulation results of conjugate Cyclic MUSIC

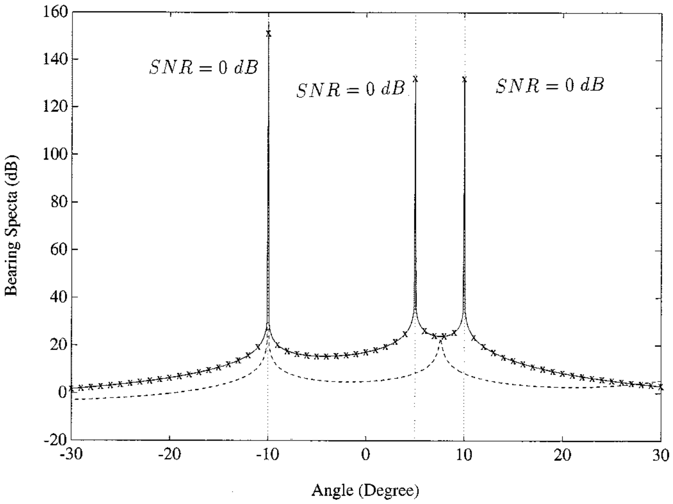

Fig. 1. Direction finding in coherent scene for *Example 1*. Two coherent SOIs from 5° and 10°, one incoherent SOI from -10°.——: Conjugate Cyclic MUSIC in conjunction with HAM. $- \times -$: Conjugate Cyclic MUSIC in conjunction with F/B HAM.– – – –: Conjugate Cyclic MUSIC.

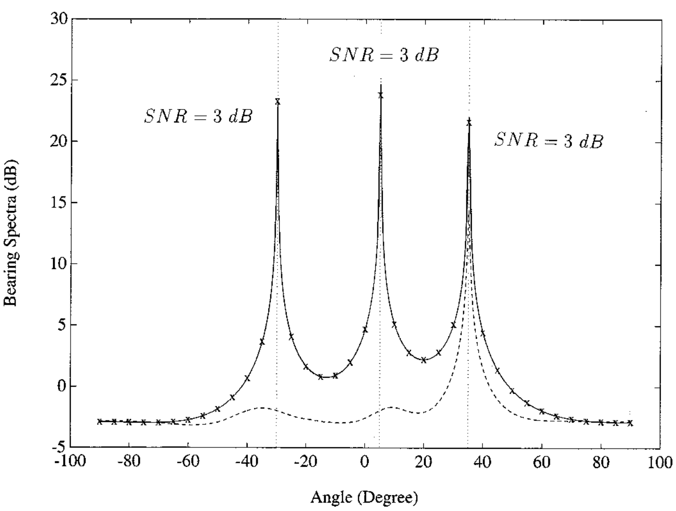

Fig. 2. Direction finding in coherent scene for *Example 2*. Two coherent SOIs from 5° and -30°, one incoherent SOI from 35°. —: Conjugate Cyclic MUSIC in conjunction with HAM. $- \times -$: Conjugate Cyclic MUSIC in conjunction with F/B HAM. – – – –: Conjugate Cyclic MUSIC.

in conjunction with HAM or F/B HAM. The result obtained by conventional conjugate Cyclic MUSIC is also provided for comparison. From Fig. 1, we observe that the coherent SOIs are successfully decorrelated by HAM and F/B HAM and almost unbiased estimation of DOA is obtained even in the coherent scene.

Example 2: Here, we consider a ULA with number of sensor elements $M = 6$ and interelement spacing = half wavelength of SOI. Moreover, the array is grouped into three overlapping subarrays with sensor elements $\{m, \cdots, 3+m\}$ for the mth subarray, $m = 1, 2, 3$. Three SOIs impinging from 5°, –30°, and 35° off broadside are BPSK baseband signals having rectangular pulse shape, carrier frequency = 1, baud rate = 1, and SNR = 3 dB. The number of snapshots used for simulation is 1000, the sampling frequency is 5, and the cycle frequency is set to $f = 2$. Moreover, the noise is spatially white Gaussian with mean zero and variance one. First, we consider the case that the first two SOIs are coherent, while the third SOI is uncorrelated with the others. We use the three subarrays and only the first subarray in HAM and F/B HAM, respectively. Fig. 2 depicts the simulation results obtained by averaging 100 independent runs. We note from this figure that HAM and F/B HAM successfully decorrelate the coherent SOIs as compared to the conjugate Cyclic MUSIC. Next, we consider the case that the three SOIs are all coherent. The three subarrays are used in HAM. However, we only use the first and second subarrays in F/B HAM to decorrelate the three coherent SOIs. Simulation results are obtained by averaging 100 independent runs and shown in Fig. 3. From this figure, we observe that conjugate Cyclic MUSIC in conjunction with HAM or F/B HAM can successfully resolve the coherent SOIs. Finally, the ambiguity of using conjugate Cyclic MUSIC in coherent scene is clearly observed from Figs. 1–3.

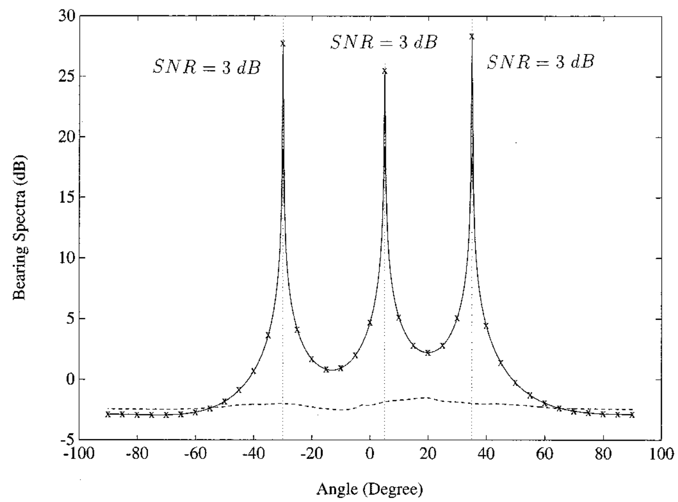

Fig. 3. Direction finding in coherent scene for *Example 2*. Three coherent SOIs from $5^\circ, -30^\circ$, and 35°. —: Conjugate Cyclic MUSIC in conjunction with HAM. $- \times -$: Conjugate Cyclic MUSIC in conjunction with F/B HAM. – – – –: Conjugate Cyclic MUSIC.

VI. CONCLUSION

This paper has presented HAM to decorrelate the SOIs in direction finding for a uniform linear array. In conjunction with conjugate Cyclic MUSIC, the direction of arrival (DOAs) of all SOIs can be successfully resolved in a coherent signal scenario. Theoretical analysis has been presented and shown that at least $2K$ sensor elements are required for decorrelating K coherent SOIs. Utilizing the F/B concept, we have also presented F/B HAM which requires $[3K/2]$ sensor elements for decorrelating K coherent SOIs. The effectiveness of the proposed methods for performing direction finding in a coherent scenario have also been demonstrated by simulation results.

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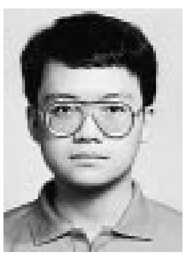

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