行政院國家科學委員會專題研究計畫成果報告

一些非微擾物理現象之探索
（三年期計畫）

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中 文 摘 要

本三年計畫對一些特定之非微擾物理現象從事理論研究以及其彼此間相關性之深入探索，研究題材雖以粒子物理與中能物理之範疇為主體，對其跨領域之特性，如宇宙學與統計物理，也希望能有重點性的突破。

嘗試深入探討之第一類題材係強作用物理或強子物理（含一些原子核物理）;雖然強作用應可由量子色動力學（Quantum Chormodynamics，即 QCD）加以描述，且於高能微擾預測得到佐證，強子之特性卻每每為其非微擾性質所主宰；如何定量處理 QCD 非微擾性質，以對強子之結構與反應（包括其非輕子弱衰變）建立第一原則出發之預測，計畫主持人希望藉著多年來物理界（以及自己）獲取之經驗，尋找重要突破點，以協助打開僵局。這方面之工作與成果，集中在 QCD 求和法則之各項應用及其 conceptual foundation 之探討。

相變（Phase Transitions）為擬從事探討之第二類題材，包括 QCD 中之手徵對稱破壞，產生質量之希格機制（Higgs Mechanisms），中子物質轉化為奇異夸克物質之相變，以及玻士愛因斯坦凝聚等，未來希望可以涵蓋這些相變之理論描述，以及其在其他領域，如早期宇宙、星球演化之末期等相關題材所扮演之角色能進一步釐清。

ABSTRACT

In this three-year project, we have attempted to conduct theoretical investigations of a few specfic nonperturbative physics phenomena, including possible correlations among them. Although the primary focus was on the topics in the area of particle and medium physics, it was hoped that some major progresses, or even some breakthrough, could be made in the related cross-disciplinary areas of cosmology and statistical physics.

The first major research topic had to do with strong interaction physics or hadron physics (including some nuclear physics problems). Although it is known, as already substantiated by perturbative QCD tests with high energy experiments, that strong interactions can be described by quantum chromodynamics or QCD, properties of hadrons or nuclei are dictated by the nonperturbative aspects of QCD. We had hoped to suitably exploit the experiences accumulated over years by our community (and by ourselves as well), to seek clues for breakthroughs, and to help unlock the present difficult situation, hoping that reliable methods to treat nonperturbative aspects of QCD can be obtained and first-principle predictions on hadron structure and properties (such as nonleptonic weak decays of hadrons) may be formulated accordingly.

The second major research topic has been focused on the general problem of phase transitions, including chiral symmetry breaking in QCD, Higgs mechanisms for mass generation in the standard model, phase transition from neutron matter to strange quark matter, and the recently discovered Bose-Einstein condensation. Our investigation of these phase transitions was also aimed to help clarify the role of phase transitions in the early universe, the end stage of stellar evolution, and other relevant cross-disciplinary topics. More progresses have yet to be made.

綜 合 說 明

非線性或非微擾物理現象已是許多物理不同領域研究之重要主題。強作用理論 QCD 係眾所周知之重要範例。

QCD 求和法則係針對難以正確解決之強作用理論 QCD 之一項攻堅方式；在執行本三年計畫前，主持人已與合作者及研究生等多方著墨，因此 1997 及 1998 仍屬 QCD 求和法則各項應用研究之量產階段。到 1997，主持人開始廣泛檢討 QCD 求和法則之觀念基礎，所寫之「QCD Sum Rules and Chiral Symmetry Breaking」，已開始對其觀念基礎（conceptual foundation）提出強烈質疑。緊接著，本人也開始針對 Large N_c QCD 進行類似求和法則之研究，發現求和法則與 Large N_c 之性質合併，其實可以將 QCD 解決到相當完整的地步。可惜主持人礙於身兼系主任之職，以及教育部卓越計畫總主持人兩項繁重職務，這方面研究成果之量產，也因之必須延後。

以 QCD 作為範例，進行非微擾物理現象之研究，大方向大致正確，也應該堅持下去。目前，我們也開始嚴謹地考慮 QCD 相變在早期宇宙演化中之確切角色。亦即夸克膠子相轉為強子相之相變，預期宇宙產生後 10^{-5} sec 至 10^{-4} sec 之間發生；相變如何發生，發生之際是否引致其他未曾預期之現象（如黑洞或拓樸弦之大量產生），以及是否留下可供偵測之痕跡，這些皆係亟待深入思考、分析以至解決之重要課題。

因此，我們這些研究將繼續深入，至少達到研究論文量產之收尾階段，同時也將開拓重要新方向，以期作出重大突破。

附錄：

1. 主持人 1997 至 2000 年之研究論文目錄。
2. 主持人一些研究論文之抽印本。

Research Publications During 1997-2000

Woei-Yann Pauchy Hwang

A. Refereed Journal Papers:

1. The QCD Sum Rule Approach for the Semileptonic Decay of the D or B Meson into a Light Meson and Leptons, Kwei-Chou Yang and W-Y. P. Hwang, Z. Phys. C - Particles and Fields **73**, 275 (1997).

2. QCD Sum Rules and Chiral Symmetry Breaking, W-Y. P. Hwang, Z. Phys. **C75**, 701 (1997), hep-ph/9601219.

3. Semi-inclusive Λ Production and Generalized Sullivan Processes, W-Y. P. Hwang and Chih-Yi Wen, Z. Phys. **A358**, 415 (1997).

4. QCD and Nuclear Physics, W-Y. P. Hwang, Chinese J. Phys. (Taipei) **35**, 444 (1997).

5. The Structure of Solitonic Quark Stars, J.-W. Chen, H.-Y. Chiu, and W-Y. P. Hwang, Chinese J. Phys. (Taipei) **35**, 543 (1997).

6. ρNN and ωNN Couplings in the External-Field QCD Sum Rule Method, Chih-Yi Wen and W-Y. P. Hwang, Phys. Rev. **C 56**, 3346 (1997).

7. Study of Fundamental Symmetries in Nuclei, W-Y. P. Hwang, Chinese J. Physics (Taipei) **35**, 847 (1997).

8. Σ_c and Λ_c Magnetic Moments from QCD Spectral Sum Rules, Shi-lin Zhu, W-Y. P. Hwang, and Ze-sen Yang, Phys. Rev. **D56**, 7273 (1997).

9. $D^* \rightarrow D\gamma$ and $B^* \rightarrow B\gamma$ as derived from QCD Sum Rules, Shi-lin Zhu, W-Y. P. Hwang, and Ze-sen Yang, Mod. Phys. Lett. **B12**, 3027 (1997).

10. Possible $\Sigma^0 - \Lambda$ Mixing in the QCD Sum Rule, Shi-lin Zhu, W-Y. P. Hwang, and Ze-sen Yang, Phys. Rev. **D57**, 1524 (1998).

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13. Electromagnetic Decay of Vector Mesons as derived from QCD Sum Rules, Shi-lin Zhu, W-Y. P. Hwang, and Ze-sen Yang, Phys. Lett. **B420**, 8 (1998).

14. Addendum: The Weak Parity-Violating Pion-Nucleon Coupling, E. M. Henley, W-Y. P. Hwang, and L. S. Kisslinger, Phys. Lett. **B 440**, 449-450 (1998).

15. Nucleon Spin Structure Functions in the Effective Chiral Quark Theory, Chun-Khiang Chua and W-Y. P. Hwang, Chinese J. Phys. (Taipei) **36**, 588 (1998).

16. Muon Capture in Deuterium, W-Y. P. Hwang and B.-J. Lin, International J. Mod. Phys. **E 8**, 101 (1999).

17. The Neutrino Magnetic Moment Induced by Leptoquarks, Chun-Khiang Chua and W-Y. P. Hwang, Phys. Rev. **D 60**, 073002 (1999).

18. Nucleon Spin Structure Functions in the Chiral Quark Model, Chun-Khiang Chua and W-Y. P. Hwang, Phys. Lett. **B451**, 283 (1999).

19. Neutrino Mass Induced Radiatively by Supersymmetric Leptoquarks, Chun-Khiang Chua, Xiao-Gang He, and W-Y. P. Hwang, Phys. Lett. **B479**, 224 (2000).

20. Nonleptonic Hyperon Decays with QCD Sum Rules, E.M. Heley, W-Y. P. Hwang, and L. S. Kisslinger, Phys. Lett. **B**, *resubmitted for publication* (2000).

21. Parity-violating Nuclear Force as derived from QCD Sum Rules, Chih-Yi Wen and W-Y. P. Hwang, Chinese J. Phys., *submitted for publication* (2000).

22. QCD Sum Rules and the Pseudoscalar Coupling, E.M. Henley, W-Y. P. Hwang, and L.S. Kisslinger, Phys. Lett. **B**, *re-submitted for publication* (2000).

23. Quark Propagator in Large N_c QCD, W-Y. P. Hwang, Euro. Phys. Journal **C**, *submitted for publication* (2000).

24. Green Functions in Large N_c QCD, W-Y. P. Hwang, Phys. Lett. **B**, *submitted for publication* (2000).

B. Conference Papers:

1. QCD Sum Rules and Parity-violating Nuclear Force, Proceedings of the First International Conference on Frontiers of Physics (OCPA), Shantou, China, August 5-9, 1995, Eds. L.-F. Li, K. K. Phua, S.S.M. Wong, and B.-L. Young (World Scientific, Singapore, 1997), p. 813.

2. QCD Sum Rules and Nonleptonic Weak Interactions, Proceedings of International Nuclear Physics Conference, Beijing, China, August 21-26, 1995, Eds. Sun Zuxun and Xu Jincheng (World Scientific, Singapore, 1996), p. 292.

3. Isospin Symmetry Breakings as from QCD Sum Rules, Proceedings of the 15th International Conference on Few-Body Problems in Physics, Groningen, Netherland, July 22-26, 1997, Nucl. Phys. **A631**, 487c (1998).

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7. Few-Nucleon Systems and QCD Sum Rules, Proceedings of the 16th International Conference on Few-Body Problems in Physics, Taipei, Taiwan, March 6-10, 2000, *in press*.

8. Large N_c QCD Sum Rules, Proceedings of the 3rd International Symposium on Symmetries in Subatomic Physics, Adelaide, Australia, March 13-17, 2000, (A.I.P. Conference Proceedings No. 539, Eds. X.-H. Gao, A.W. Thomas, and A.G. Williams), p. 160 (2000).

Invited talks delivered at International Physics Conferences/Workshops:

1. "Chiral Symmetry Breaking and QCD Sum Rules", *The Second International Symposium on Symmetries in Subatomic Physics*, June 25-28, 1997, Seattle, Washington, U.S.A.

2. "Isospin Symmetry Breakings as from QCD Sum Rules", *Talk at the 15th International Conference on Few Body Problems in Physics*, July 22-26, 1997, Groningen, Netherlands.

3. "Nonlocal Condensates in QCD", *Talk at the Second Joint Meeting of Chinese Physical Societies*, August 11-15, 1997, Taipei, Taiwan.

4. Nuclei as the Laboratory to Study Fundamental Symmetries, *Talk at the International Conference on Physics Since Parity Symmetry Breaking – in Memory of Professor C.S. Wu*, August 16-19, 1997, Nanjing, China.

5. Condensates in QCD, *The 7th Asia Pacific Physics Conference*, August 19-23, 1997, Beijing, China.

6. The JHF Opportunities and Taiwan, *The OECD Megascience Forum: Woking Group Meeting on Hadron Physics*, March 3-8, 1998, KEK, Tsukuba, Japan.

7. Nonperturbative Effects in Parton Distributions, International Workshop on JHF Science (JHF98), March 4-7, 1998, KEK, Tsukuba, Japan.

8. Few-Nucleon Systems and QCD Sum Rules, *The 16th International Conference on Few-Body Problems in Physics*, March 6-10, 2000, Taipei, Taiwan.

9. Large N_c QCD Sum Rules, *The 3rd International Symposium on Symmetries in Subatomic Physics*, Adelaide, Australia, March 13-17, 2000.

10. The Early Universe and Taiwan COSPA Project, *The 3rd Cross-Strait Symposium on High and Medium Energy Physics*, Jinan, Shantong, China, October 23-27, 2000.

11. XS3: Summary and Concluding Remarks, *The 3rd Cross-Strait Symposium on High and Medium Energy Physics*, Jinan, Shantong, China, October 23-27, 2000.

Z. Phys. C 73, 275–292 (1997)

ZEITSCHRIFT FÜR PHYSIK C

The QCD sum rule approach for the semileptonic decay of the *D* or *B* meson into a light meson and leptons

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Abstract. We use the method of QCD sum rules to treat the semileptonic weak decay of the D or B meson into a light meson and leptons. To obtain the transition form factors, we adopt the two-point Green's function in the presence of an external vector or axial-vector field. We find that this method can be related approximately to the traditional three-point Green's function in the heavy quark limit ($m_Q \to \infty$). Unlike some existing QCD sum rule calculations, our results indicate that the form factors have simple dipole or monopole behavior. We obtain results on the various form factors of the semileptonic decay of D and B mesons into a light meson and investigate various decay processes such as $\bar{B}^0 \to \pi^+ \tau^- \bar{\nu}_\tau$ and $\bar{B}^0 \to \rho^+ \tau^- \bar{\nu}_\tau$. The method allows us to take into account nonperturbative strong interaction effects, thereby providing a more reliable determination of the Cabibbo-Kobayashi-Maskawa matrix elements from the experimental data.

I Introduction

Semileptonic decays are particularly important in exploring the V-A structure of weak interactions. In these decays, strong interaction effects are contained in the transition form factors as used for defining the hadronic matrix elements. Reliable determination of these transition form factors helps us to extract the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements from the relevant experimental data.

There exist several methods for studying the semileptonic decay of a heavy meson (D or B) into a light meson and leptons, in the framework of relativistic or nonrelativistic quark models [1–4]. Another method [5] has also been employed recently by incorporating chiral symmetry into the effective heavy quark theory to compute form factors near the maximum momentum transfer squared, q_m^2. Nevertheless, it remains highly desirable, if possible, to tackle these problems using directly QCD.

In contradistinction with the lattice QCD treatment [6–8] of heavy meson decays, the method of QCD sum rules [9] provides another powerful tool in studying hadron physics within the framework of QCD. In the method of QCD sum rules we use perturbative QCD augmented with non-perturbative effects, such as through introduction of quark and gluon condensates, to describe both short- and large-distance physics. In the context of the three-point Green's function approach, the pion formfactor was solved at intermediate momentum transfer squared $0.5\,\mathrm{GeV}^2 < -q^2 < 1.5\,\mathrm{GeV}^2$ [10, 11], but the approach may fail at smaller values of q^2 – since in general the three-point Green's function approach works well only in the deep Euclidean region $p^2, p'^2, q^2 \ll 0$ and it may fail due to the presence of the singular terms $1/q^2$, $1/q^4$, etc., which are not small for sufficiently small q^2. In an extension of the original QCD sum rule approach [9], Ioffe and Smilga [12] and, independently, Balitsky and Yung [13] considered that the quarks move in a constant external electromagnetic field. Instead of the traditional three-point Green's function approach, they evaluated a two-point Green's function in the presence of an external field and successfully obtained the nucleon magnetic moments at *zero* momentum transfer.

Some authors [14–21] treated weak semileptonic decays of charmed or bottomed mesons using the three-point Green's function approach. In our attempt to employ the two-point Green's function approach, we try to replace, by some suitable induced condensates, certain diagrams which appear in the three-point Green's function approach and for which the usual operator product expansion (OPE) approach may fail when $q^2 \approx 0$. Furthermore, we adopt axial vector currents for heavy mesons since usage of the pseudoscalar currents (such as $c\gamma_5\bar{d}$ for D^+) often leads to very large contributions from the quark condensate, sometime even larger than the leading perturbative contribution, and the convergence of the series would be in doubt. These aspects will be addressed in some detail later in Sects. II and III.

Specifically, we treat the weak semileptonic decay of D or B meson into a light meson and leptons. To ensure that the method of QCD sum rules can be used in the small momentum transfer region, we consider the two-point Green's function in a varying external field. A similar idea was proposed first by Eletsky and Kogan [22] in evaluating the electromagnetic decays of D mesons at $q^2 = 0$. We also generalize the method to the region $q^2 \neq 0$, since, as discussed in [18], the analysis of the relevant QCD sum rules can also be performed for positive values of q^2 (the time-like region). Adopting suitable axial-vector currents to describe heavy mesons, we find that the series converges sufficiently well at the quark level and that the approach using the two-point Green's function in the external field yields almost the same results as the three-point Green's function approach. Our numerical results suggest that the form factors $f_i(q^2)$ (which describe the weak matrix elements) possess the pole dependence:

$$f_i(q^2) = \frac{f_i(0)}{(1 - q^2/m^2_{\text{pole}})^n},$$

where $n = 1$ or 2. Note that these results differ somewhat from some existing QCD sum rule calculations [18, 19]. For instance, in [18, 19], Ball et al. obtained unusual q^2-behaviors for both the form factors, $A_1(q^2)$ and $A_2(q^2)$ and the results are also quite sensitive to the chosen Borel range. (See below for the definition of these form factors.) Instead, we find that adoption of suitable axial-vector currents, instead of pseudoscalar currents, for describing heavy mesons leads to QCD sum rules which seem much better behaved.

The rest of this paper is organized as follows: In Sect. II, we derive the QCD sum rules, which allow an investigation of the various decay form factors of heavy mesons. In Sect. III, numerical results are described and discussed. Section IV contains a brief summary.

II The QCD sum rules

A. Form factors for the decay of a D or B meson into a light pseudoscalar meson and leptons

To treat the form factors for the semileptonic decay of a D or B meson into a light pseudoscalar meson and leptons, namely $M \to L + \ell + \nu_\ell$ [such as $\bar{B}^0 \to \pi^+ e^- \bar{\nu}_e$], we choose to consider the following Green's function in the external vector field ($\mathscr{V}$):

$$i(2\pi)^4 \delta^4(p - p' - q)\Pi^{\mathscr{V}}_{\mu\nu}(p, p'; q^2) = i^2 \iint d^4x d^4y\, e^{i(p'x - py)} \langle 0 | T\{j^{5(L)}_\mu(x), j^5_\nu(y)\} | 0 \rangle_{\mathscr{V}}, \quad (1)$$

where $j^{5(L)}_\mu = \bar{q}_1 \gamma_\mu \gamma_5 q_2$ and $j^5_\nu = \bar{Q}\gamma_\nu\gamma_5 q_1$ are axial-vector currents whose divergences carry the same quantum numbers as the pseudoscalar L (π or K) and M (D or B) mesons, respectively. We use Q as a generic notation for a heavy quark field operator (such as c or b), while q_1 or q_2 for a light quark field operator. In the following calculation, we neglect u and d quark masses, but keep terms linear in the s quark mass. The interaction with the external vector field is described by the additional term, $\Delta\mathscr{L}$, in the Lagrangian:

$$\Delta\mathscr{L}(x) = -\mathscr{V}^\alpha e^{iqx} J_\alpha(x), \quad (2)$$

where

$$J_\alpha = \bar{q}_2 \gamma_\alpha Q. \quad (3)$$

Here $\mathscr{V}^\alpha$ and q^α are the amplitude and momentum of the external vector field, respectively. Note that in the following calculation we consider only the amplitude of $\Pi^{\mathscr{V}}_{\mu\nu}$ linear in the external field $\mathscr{V}$. Generally speaking, the Green's function can be written via the dispersion relation as follows

$$\Pi_{\mu\nu}(p, p'; q^2) = \int ds \int ds' \frac{\rho_{\mu\nu}(p, p'; q^2)}{(s - p^2)(s' - p'^2)} + \text{subtraction terms}, \quad (4)$$

where the subtraction terms have the form

$$\text{subtraction terms} = P_{1\mu\nu}(p^2) \int \frac{\Delta(s')ds'}{s' - p'^2} + P_{2\mu\nu}(p'^2) \int \frac{\Delta(s)ds}{s - p^2} + P_{3\mu\nu}(p^2, p'^2). \quad (5)$$

One finds that the contribution from the subtraction terms are of no importance since, after performing the Borel transform, all of the subtraction terms, $P_{1\mu\nu}$, $P_{2\mu\nu}$, and $P_{3\mu\nu}$, disappear. (For further discussions, see [23].)

At the hadron level (on the rhs), the imaginary part of $\Pi^{\mathscr{V}}_{\mu\nu}$ is

$$\begin{aligned}\rho^{\mathscr{V}}_{\mu\nu}(p, p'; q^2) &= -\mathscr{V}^\alpha \langle 0 | j^{5(L)}_\mu(0) | L(p') \rangle \langle L(p') | J_\alpha(0) | M(p) \rangle \\ &\quad \times \langle M(p) | j^5_\nu(0) | 0 \rangle \delta(s' - m_L^2)\delta(s - m_M^2) + \ldots \\ &= -\mathscr{V}^\alpha f_L f_M [F_+(q^2) p'_\mu (p + p')_\alpha p_\nu \\ &\quad + F_-(q^2) p'_\mu (p - p')_\alpha p_\nu] \delta(s' - m_L^2)\delta(s - m_M^2) + \ldots, \end{aligned} \quad (6)$$

where we have adopted the conventional definitions:

$$\langle 0 | j^{5(L)}_\mu(0) | L(p') \rangle = i f_L p'_\mu,$$

$$\langle M(p) | j^5_\nu(0) | 0 \rangle = -i f_M p_\nu,$$

$$\langle L(p') | J_\alpha(0) | M(p) \rangle = [F_+(q^2)(p + p')_\alpha + F_-(q^2)(p - p')_\alpha]. \quad (7)$$

The ellipses in (6) stand for the contributions from axial-vector mesons or from higher excited states (coming from either the nondiagonal transitions from the lowest state to some excited state, or diagonal transitions from an excited state to itself). These contributions give rise to additional Lorentz structures (and thus new QCD sum rules for other transitions), which are not relevant for the purpose of the present paper. They also modify the coefficients of the two relevant Lorentz structures but these nonleading terms can presumably be approximated as a continuum starting from some threshold (s_0, s'_0), making use of the quark level calculation. Note that the contributions from low-lying axial-vector mesons do not pose specific threat since their masses are in general fairly large and lie somewhere (deep) in the continuum. We have also investigated the additional Lorentz structures to obtain the QCD sum rules relevant for the determination of the transition of the heavy meson into the low-lying axial vector meson and such results, being peripheral to our present purpose, are to be reported elsewhere.

Note that the Green's function, in (1), contains 14 different tensor structures:

$$\Pi^{\mathscr{V}}_{\mu\nu} = \mathscr{V}^{\alpha}(\Pi_+ p'_\mu (p+p')_\alpha p_\nu + \Pi_- p'_\mu (p-p')_\alpha p_\nu + \ldots), \tag{8}$$

but, for the purpose of determining the form factors F_+ and F_-, we need only the two amplitudes Π_+, Π_-; and, in fact, no other Lorentz structures can arise if only the form factors $F_\pm(q^2)$ are included, i.e., if only the $M \to L$ transition (7) is included. To obtain the relevant QCD sum rules, we try to equate the coefficients, obtained at the quark level (on the lhs) and parametrized at the hadron level (rhs), of the same Lorentz structure $-p'_\mu(p+p')_\alpha p_\nu$ and $p'_\mu(p-p')_\alpha p_\nu$ for the determination of the form factors F_+ and F_-.

Equation (6) gives us the expression on the rhs (the result at the hadron level). By performing the double Borel transform [9, 24],

$$\mathbf{B}[f(p^2, p'^2)] = \lim_{\substack{m\to\infty \\ -p'^2\to\infty \\ \frac{-p'^2}{mM'^2}\ \text{fixed}}} \lim_{\substack{n\to\infty \\ -p^2\to\infty \\ \frac{-p^2}{nM^2}\ \text{fixed}}} \frac{1}{n!m!}(-p'^2)^{m+1} \times \left[\frac{d}{dp'^2}\right]^m (-p^2)^{n+1}\left[\frac{d}{dp^2}\right]^n f(p^2, p'^2), \tag{9}$$

we then obtain the r.h.s of the QCD sum rules:

$$\text{For } F_+(q^2)\text{:} \quad -f_L f_M F_+(q^2) e^{-(m_M^2/M^2)} e^{-(m_L^2/M'^2)},$$
$$\text{For } F_-(q^2)\text{:} \quad -f_L f_M F_-(q^2) e^{-(m_M^2/M^2)} e^{-(m_L^2/M'^2)}. \tag{10}$$

As indicated earlier, the contributions of higher excited states may be transferred to the left-hand side of the sum rules and, to a sufficient approximation, they may be represented by the perturbative continuum (evaluated at the quark level), which begins at certain thresholds, s_0 and s'_0.

At the quark level, on the other hand, the Green's function in the deep Euclidean region $[p^2, p'^2, q^2 \leqslant 0]$ can be evaluated as a sum of the perturbative and non-perturbative contributions (including nonperturbative contributions from induced condensates) by means of the (short-distance) operator product expansion (OPE). For the perturbative part, we write down the Feynman integral of the bare loop:

$$\Pi^{\mathscr{V}}_{\mu\nu}(p, p'; q^2) = 3i\mathscr{V}^{\alpha}\int \frac{d^4k}{(2\pi)^4}\frac{Tr[(\hat{p}'+\hat{k})\gamma_\alpha(\hat{p}+\hat{k})\gamma_\nu\hat{k}\gamma_\mu]}{k^2(k+p')^2[(k+p)^2 - m_Q^2]}. \tag{11}$$

The relevant diagram is shown in Fig. 1a [the very first diagram in Figs. 1], in which the heavy quark propagator is represented by the heavy line. According to Cutkosky's rule [25], the integral can be solved easily by taking the imaginary part of the quark propagators: $1/(p^2 - m_Q^2 + i\varepsilon) \to -2\pi i\delta(p^2 - m_Q^2)$. In this case we have ignored subtraction terms, (5). Thus this integral may be rewritten as a double dispersion relation

$$\Pi^{\mathscr{V}}_{\mu\nu}(p, p'; q^2) = -\frac{\mathscr{V}^{\alpha}}{4\pi^2}\int\int_\Omega ds\,ds' \frac{\rho^{\mathscr{V}}_{\mu\nu\alpha}(s, s'; q^2)}{(s-p^2)(s'-p'^2)}, \tag{12}$$

where

$$\rho^{\mathscr{V}}_{\mu\nu\alpha} = 3i\int\frac{d^4k}{(2\pi)^4}(-2\pi i)^3\delta(k^2)\delta[(k+p')^2]\delta[(k+p)^2 - m_Q^2]\ Tr[(\hat{p}'+\hat{k})\gamma_\alpha(\hat{p}+\hat{k})\gamma_\nu\hat{k}\gamma_\mu]. \tag{13}$$

Making use of the integration formulas listed in Appendix A, we obtain

$$\Pi^{\text{pert}}_+(p, p'; q^2) = -\frac{1}{4\pi^2}\int\int_\Omega ds\,ds' \frac{\rho_+(s, s'; q^2)}{(s-p^2)(s'-p'^2)}, \tag{14}$$

for the coefficient of the structure $\mathscr{V}^\alpha p'_\mu(p+p')_\alpha p_\nu$, and

$$\Pi^{\text{pert}}_-(p, p'; q^2) = -\frac{1}{4\pi^2}\int\int_\Omega ds\,ds' \frac{\rho_-(s, s'; q^2)}{(s-p^2)(s'-p'^2)}, \tag{15}$$

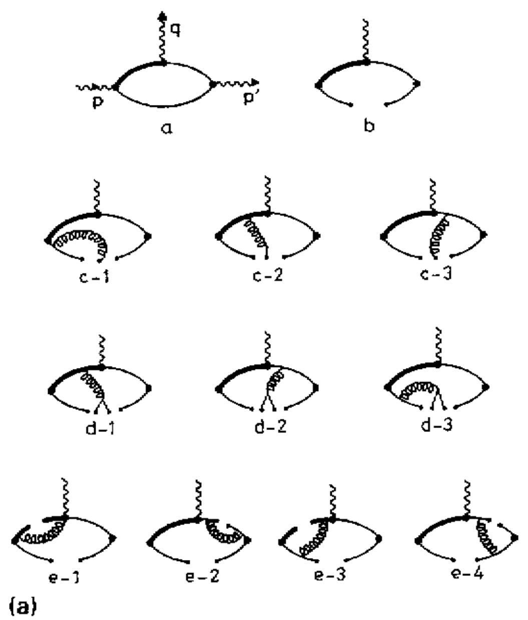
for the coefficient of the structure $\mathscr{V}^\alpha p'_\mu(p-p')_\alpha p_\nu$, where

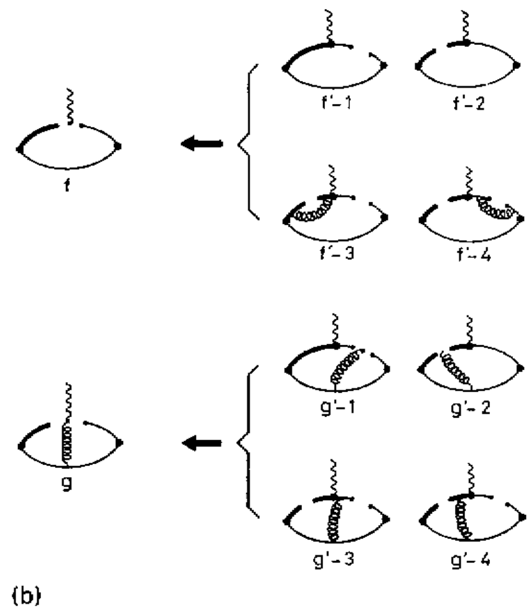
$$\begin{aligned}\rho_+(s, s'; q^2) &= -3\Bigg\{-\frac{40s'^2}{\lambda^{7/2}}[(s-m_Q^2)(m_Q^2-q^2) - s'm_Q^2] \times\left(m_Q^2\frac{s'-s-q^2}{2} + sq^2\right) \\ &\quad + \frac{2s'}{\lambda^{5/2}}[3[(s-m_Q^2)(m_Q^2-q^2) - s'm_Q^2](m_Q^2 - 2q^2) \\ &\quad + [(s-m_Q^2)^2 - ss'](s - s' - m_Q^2)] \\ &\quad + \frac{1}{\lambda^{3/2}}[m_Q^2(m_Q^2 - q^2 + 3s' - s) \\ &\quad - (2ss' - sq^2 - s'q^2)]\Bigg\},\end{aligned} \tag{16}$$

and

$$\begin{aligned}\rho_-(s, s'; q^2) &= -3\Bigg\{-\frac{40s'^2}{\lambda^{7/2}}[(s-m_Q^2)(m_Q^2-q^2) - s'm_Q^2] \times\left[m_Q^2\frac{3s+s'-q^2}{2} + s(s'-s)\right] \\ &\quad + \frac{2s'}{\lambda^{5/2}}[3[(s-m_Q^2)(m_Q^2-q^2) - s'm_Q^2] \\ &\quad \times(2s - 2s' - m_Q^2) - [(s-m_Q^2)^2 - ss'](s + s' - m_Q^2)] \\ &\quad + \frac{1}{\lambda^{3/2}}[(s'-s)q^2 - m_Q^2(m_Q^2 - q^2 + 3s' - s)]\Bigg\},\end{aligned} \tag{17}$$

with $\lambda \equiv (s + s' - q^2)^2 - 4ss'$. Note that in this approach the integration region Ω is obtained via the Landau equation [26]. Namely, the corresponding integration region





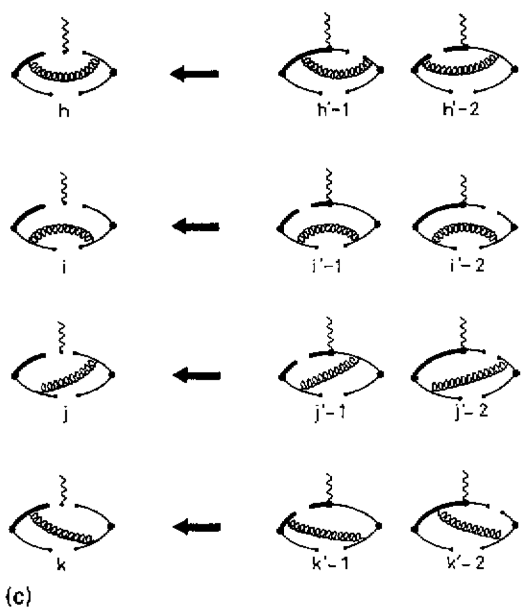


Fig. 1a–c. Diagrams for the two-point Green's function of (1) [or (33)] in the external vector field (or axial-vector field). The heavy quark propagator is represented by the heavy line. In this approach the right hand side of the heavy left-arrow, which corresponds to the three-point Green's function approach, is replaced by the left hand side of the heavy left-arrow, which corresponds to the two-point Green's function in the external field, respectively

Ω, which may be depicted as the shaded region in Fig. 2, is specified by the inequalities

$$\Omega:\quad s_0' > s' > 0, \quad s_0 > s > m_Q^2 + \frac{m_Q^2}{m_Q^2 - q^2}\, s', \tag{18}$$

where s_0 and s_0' are the thresholds of higher excited states, which can be decided by means of the relevant two-point sum rules [9, 27]. In Fig. 2, the region I corresponds to diagonal higher excited state transitions, while the region II corresponds to nondiagonal transitions.

Next, we consider the contributions involving the quark and quark-gluon condensates which are represented pictorially by Fig. 1(b)–1(e), which can be evaluated

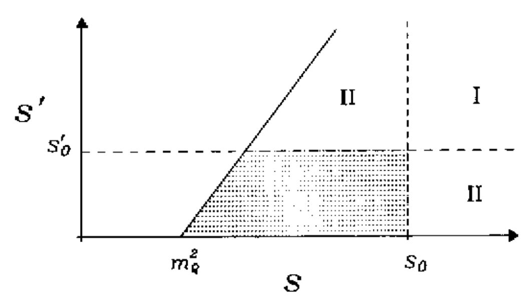


Fig. 2. Integration region in (18). The shaded region is denoted by Ω. The region I corresponds to diagonal higher excited state transitions, while the region II corresponds to nondiagonal transitions

in the standard manner. We only list the final results explicitly immediately below:

$$\Pi_{+}^{\text{nonpert}}(p, p'; q^2)$$
$$= m_0^2\langle\bar{q}_1q_1\rangle \frac{m_Q + m_{q_2}}{6p'^4(p^2 - m_Q^2)^2}$$
$$- 2\frac{g_c^2\langle\bar{q}_1q_1\rangle^2}{81}\left[\frac{1}{p'^6(p^2 - m_Q^2)} + \frac{4}{p'^4(p^2 - m_Q^2)^2}\right.$$
$$+ \frac{1}{p'^2(p^2 - m_Q^2)^3} - \frac{2m_Q^2 - q^2}{p'^4(p^2 - m_Q^2)^3}$$
$$\left. - \frac{3m_Q^2}{p'^2(p^2 - m_Q^2)^4} - \frac{m_Q^2 - q^2}{p'^6(p^2 - m_Q^2)^2}\right]$$
$$- \frac{2g_c^2\langle\bar{q}_1q_1\rangle\langle\bar{Q}Q\rangle}{9m_Q^4}\left[\frac{1}{p'^2(p^2 - m_Q^2)}\right.$$
$$\left. - \frac{1}{p'^2p^2} - \frac{m_Q^2}{p'^2p^4}\right]$$
$$- \frac{2g_c^2\langle\bar{q}_1q_1\rangle\langle\bar{q}_2q_2\rangle}{9}\frac{1}{p'^6(p^2 - m_Q^2)}, \tag{19}$$

and

$$\Pi_{-}^{\text{nonpert}}(p, p'; q^2)$$
$$= - m_0^2\langle\bar{q}_1q_1\rangle \frac{m_Q - m_{q_2}}{6p'^4(p^2 - m_Q^2)^2}$$
$$+ 2\frac{g_c^2\langle\bar{q}_1q_1\rangle^2}{81}\left[\frac{1}{p'^6(p^2 - m_Q^2)} + \frac{6}{p'^4(p^2 - m_Q^2)^2}\right.$$
$$- \frac{1}{p'^2(p^2 - m_Q^2)^3} - \frac{q^2}{p'^4(p^2 - m_Q^2)^3}$$
$$\left. + \frac{3m_Q^2}{p'^2(p^2 - m_Q^2)^4} - \frac{m_Q^2 - q^2}{p'^6(p^2 - m_Q^2)^2}\right]$$
$$- \frac{2g_c^2\langle\bar{q}_1q_1\rangle\langle\bar{Q}Q\rangle}{9m_Q^4}\left[\frac{1}{p'^2(p^2 - m_Q^2)} - \frac{1}{p'^2p^2} - \frac{m_Q^2}{p'^2p^4}\right]$$
$$+ \frac{2g_c^2\langle\bar{q}_1q_1\rangle\langle\bar{q}_2q_2\rangle}{9}\frac{1}{p'^6(p^2 - m_Q^2)}, \tag{20}$$

where we have adopted the definition $\langle g_c\bar{q}\sigma Gq\rangle = - m_0^2\langle\bar{q}q\rangle$. Note that Figs. 1(c) have been evaluated with the aid of the heavy-quark propagator in an external field:

$$iS^A(x_2 - x_1) = \int\frac{d^4p}{(2\pi)^4}e^{-ip(x_2 - x_1)}\left[\frac{i}{4}\frac{\lambda^a}{2}g_cG^a_{\mu\nu}(x_0)\right.$$
$$\times\frac{\sigma^{\mu\nu}(\hat{p} + m_Q) + (\hat{p} + m_Q)\sigma^{\mu\nu}}{(p^2 - m_Q^2)^2}$$
$$- \frac{iG^a_{\mu\nu}(x_0)\lambda^a}{4}g_c(x_1 - x_0)^\nu$$
$$\left.\times\left(\frac{1}{\hat{p} - m_Q}\gamma^\mu\frac{1}{\hat{p} - m_Q}\right)\right]. \tag{21}$$

Here we have used the fixed-point gauge, $(x - x_0)^\mu A^a_\mu = 0$, [9,24] for the gluon field, and adopted $\hat{p} \equiv \gamma_\mu p^\mu$. Note also that because both the contributions of the gluon condensate and the heavy quark condensates [9,28] are in fact rather small in the sum rules, we may neglect them here.

Finally, we need to consider the contributions arising from the induced condensates. To this end, we first caution that the short distance expansion (i.e., the usual OPE approach) is in fact not applicable in the evaluation of the contributions for the diagrams illustrated by Figs. 1(f')–(k') (albeit as done in the three-point Green's function approach), since the space-time point with q^2 entering is in general far from the other two points when $q^2 \approx 0$. To circumvent the problem, we choose to treat these diagrams by means of considering induced condensates in the presence of a suitable external field (as depicted in Figs. 1(f)–(k)). Such replacements are indicated by arrows in the figure.

To obtain the contributions corresponding to Figs. 1(f)–(k), we first write down the identity:

$$4\bar{Q}_\alpha q_{2\beta} = (1(\bar{Q}q_2) + \gamma_5(\bar{Q}\gamma_5q_2) + \gamma^\rho(\bar{Q}\gamma_\rho q_2)$$
$$- \gamma^\rho\gamma_5(\bar{Q}\gamma_\rho\gamma_5q_2) + \tfrac{1}{2}\sigma^{\rho\tau}(\bar{Q}\sigma_{\rho\tau}q_2))_{\alpha\beta}$$
$$= (1(\bar{Q}q_2) + \gamma_5(\bar{Q}\gamma_5q_2) + \gamma^\rho(\bar{Q}\gamma_\rho q_2)$$
$$- \gamma^\rho\gamma_5(\bar{Q}\gamma_\rho\gamma_5q_2) + \tfrac{1}{2}\sigma^{\rho\tau}\gamma_5(\bar{Q}\sigma_{\rho\tau}\gamma_5q_2))_{\alpha\beta}. \tag{22}$$

Thus we may expand the quark condensate $\langle q^a_{2i}(x)\bar{Q}^b_j(y)\rangle_{\mathscr{V}}$ in the approximation linear in $\mathscr{V}^\mu$ (i.e. the weak-field approximation):

$$\langle q^a_{2i}(x)\bar{Q}^b_j(y)\rangle_{\mathscr{V}} \approx - \frac{1}{12}\langle\bar{Q}(y)q_2(y)\rangle_{\mathscr{V}}\delta^{ab}(\mathbf{1})_{ij}$$
$$- \frac{1}{12}\langle\bar{Q}(y)\gamma_\mu q_2(y)\rangle_{\mathscr{V}}\delta^{ab}(\gamma^\mu)_{ij}$$
$$- \frac{1}{24}\langle\bar{Q}(y)\sigma_{\alpha\beta}q_2(y)\rangle_{\mathscr{V}}\delta^{ab}(\sigma^{\alpha\beta})_{ij} + \dots, \tag{23}$$

where a and b are color indices while i and j the Dirac indices. The induced condensates may be written as

$$\langle\bar{Q}(y)q_2(y)\rangle_{\mathscr{V}}$$
$$= - i\mathscr{V}^\mu\int d^4xe^{iqx}:\langle 0|\mathrm{T}\{\bar{Q}(y)q_2(y), \bar{q}_2(x)\gamma_\mu Q(x)\}|0\rangle:, \tag{24a}$$

$$\langle\bar{Q}(y)\gamma_\alpha q_2(y)\rangle_{\mathscr{V}}$$
$$= - i\mathscr{V}^\mu\int d^4xe^{iqx}:\langle 0|\mathrm{T}\{\bar{Q}(y)\gamma_\alpha q_2(y), \bar{q}_2(x)\gamma_\mu Q(x)\}|0\rangle:, \tag{24b}$$

$$\langle\bar{Q}(y)\sigma_{\alpha\beta}q_2(y)\rangle_{\mathscr{V}}$$
$$= - i\mathscr{V}^\mu\int d^4xe^{iqx}:\langle 0|\mathrm{T}\{\bar{Q}(y)\sigma_{\alpha\beta}q_2(y), \bar{q}_2(x)\gamma_\mu Q(x)\}|0\rangle:. \tag{24c}$$

Here the notation ": :" has been used to denote subtraction of the perturbative contributions:

$$:\pi(q^2): = \pi(q^2) - \pi^{\text{pert}}(q^2)$$
$$= \frac{1}{\pi}\int_0^\infty\frac{ds}{s - q^2}(\operatorname{Im}\pi(s) - \operatorname{Im}\pi^{\text{pert}}(s)). \tag{25}$$

$B \to \rho\, e\, \bar{\nu}_e$, we follow a similar line and consider the following Green's function in external vector ($\mathscr{V}$) and

with λ denoting the helicity state of the spin-1 particle (L^*).

Note that the calculation of perturbative parts in (1) with the aid of (23) and (24) will lead to (11), the bare loop of the three-point Green's function. This has led to the subtractions in (24a)–(24c). Note also that one can perform the expansion of (23), being a Taylor-series or short-distance expansion, only when the variable q^2 is in the deep Euclidean region.

To evaluate (25) (i.e., (24)), we may adopt the simple model

$$\operatorname{Im}\pi(s) = f\delta(s - m_f^2) + \operatorname{Im}\pi^{\text{pert}}(s)\theta(s - s_0). \tag{26}$$

The various values of f could be determined by means of relevant 2-point Green's functions via the method of QCD sum rules. This is described briefly in Appendix B. (See [29, 30] for similar discussions.) Note that the contribution of $\langle \bar{Q}(y) q_2(y)\rangle_{\mathscr{V}}$ to the relevant sum rules (F_+ and F_-) vanishes identically while the total contribution of $\langle \bar{Q}(y)\gamma_\mu q_2(y)\rangle_{\mathscr{V}}$ is proportional to $m_{q_2}\langle \bar{q}_2 q_2\rangle$, or to the small light quark mass. Therefore it is sufficient to consider only the induced condensate $\langle \bar{Q}(y)\sigma_{\alpha\beta} q_2(y)\rangle_{\mathscr{V}}$, which may be characterized by

$$\langle \bar{Q}(y)\sigma_{\alpha\beta} q_2(y)\rangle_{\mathscr{V}} = -i\mathscr{V}^{\nu} e^{iqy} \chi^{\mathscr{V}}_{Qq_2}(q^2)(g_{\nu\alpha} q_\beta - g_{\beta\nu} q_\alpha). \tag{27}$$

The calculation described in Appendix B yields

$$\chi^{\mathscr{V}}_{Qq_2} = \frac{f^{\mathscr{V}}}{m_{f^{\mathscr{V}}}^2 - q^2} - \frac{3}{8\pi}\int_{m_Q^2}^{s_0} ds \frac{m_Q}{s - q^2} \times \left[1 - 2\left(\frac{m_Q^2}{s}\right) + \left(\frac{m_Q^2}{s}\right)^2\right]. \tag{28}$$

In particular, we have calculated the induced condensates, $\chi^{\mathscr{V}}$, in the limit of the $SU(3)$ symmetry, i.e., m_u, m_d, and $m_s \to 0$. The results of $f^{\mathscr{V}}$ are listed in Table 1. It is of interest to note that, in the heavy quark limit, the following approximation appears to hold reasonably well:

$$\chi^{\mathscr{V}}_{Qq_2} = \frac{\langle \bar{q}_2 q_2\rangle}{q^2 - m_Q^2}.$$

Noting that the contribution of the induced condensate shown in Fig. 1(*g*) is proportional to the heavy quark condensate (so that we may neglect it here), the contributions of induced condensates are given by

$$\Pi_+^{\text{induced}}(p, p'; q^2) = -\frac{2g_c^2\langle \bar{q}_1 q_1\rangle}{9}\chi^{\mathscr{V}}_{Qq_2}(q^2)\frac{1}{(p^2 - m_Q^2)p'^4},$$

$$\Pi_-^{\text{induced}}(p, p'; q^2) = \frac{2g_c^2\langle \bar{q}_1 q_1\rangle}{9}\chi^{\mathscr{V}}_{Qq_2}(q^2)\frac{1}{(p^2 - m_Q^2)p'^4}. \tag{29}$$

Note that the results shown in Tables 1 and 2, together with some discussion in Appendix B, suggest that it is consistent to adopt, (1), axial-vector currents, instead of pseudoscalar currents, in the method with the two-point Green's function in the external field. The approach appears to yield, in the deep Euclidean region, the same results as in the three-point Green's function approach.

Table 1. Results of $f^{\mathscr{V}}$ and $s_0^{\mathscr{V}}$ obtained from relevant sum rules, (B.4), in Appendix B. The lowest-lying meson mass is used as an input parameter in the sum rule

Qq_2	$f^{\mathscr{V}}_{Qq_2}$(GeV^3)	$m_{f_{Qq_2}}$(GeV)	s_0 (GeV^2)
cd	0.22	2.01	6.0
bu	0.28	5.33	33

Table 2. Results of $f^{\mathscr{A}}$ and $s_0^{\mathscr{A}}$ obtained from relevant sum rules, (B.4), in Appendix B. The lowest-lying meson mass is used as an input parameter in the sum rule

Qq_2	$f^{\mathscr{A}}_{Qq_2}$(GeV^3)	$m_{f_{Qq_2}}$(GeV)	s_0 (GeV^2)
cd	0.48	2.42	9.3
bu	0.67	5.71	38.2

Carrying out the integrations in (16)–(17) and combining the results with (19), (20) and (29), we finally obtain

$$\begin{aligned}
&\Pi_+^{\text{lhs}}(p, p'; q^2)\\
&= \Pi_+^{\text{pert}}(p, p'; q^2) + \Pi_+^{\text{induced}}(p, p'; q^2)\\
&\quad + m_0^2\langle \bar{q}_1 q_1\rangle \frac{m_Q + m_{q_2}}{6p'^4(p^2 - m_Q^2)^2}\\
&\quad - \frac{2g_c^2\langle \bar{q}_1 q_1\rangle^2}{81}\left[\frac{1}{p'^6(p^2 - m_Q^2)} + \frac{4}{p'^4(p^2 - m_Q^2)^2}\right.\\
&\quad + \frac{1}{p'^2(p^2 - m_Q^2)^3} - \frac{2m_Q^2 - q^2}{p'^4(p^2 - m_Q^2)^3}\\
&\quad \left. - \frac{3m_Q^2}{p'^2(p^2 - m_Q^2)^4} - \frac{m_Q^2 - q^2}{p'^6(p^2 - m_Q^2)^2}\right]\\
&\quad - \frac{2g_c^2\langle \bar{q}_1 q_1\rangle\langle \bar{Q}Q\rangle}{9m_Q^4}\left[\frac{1}{p'^2(p^2 - m_Q^2)} - \frac{1}{p'^2 p^2} - \frac{m_Q^2}{p'^2 p^4}\right]\\
&\quad - \frac{2g_c^2\langle \bar{q}_1 q_1\rangle\langle \bar{q}_2 q_2\rangle}{9}\frac{1}{p'^6(p^2 - m_Q^2)},
\end{aligned} \tag{30}$$

and

$$\begin{aligned}
&\Pi_-^{\text{lhs}}(p, p'; q^2)\\
&= \Pi_-^{\text{pert}}(p, p'; q^2) + \Pi_-^{\text{induced}}(p, p'; q^2)\\
&\quad - m_0^2\langle \bar{q}_1 q_1\rangle \frac{m_Q - m_{q_2}}{6p'^4(p^2 - m_Q^2)^2}\\
&\quad + \frac{2g_c^2\langle \bar{q}_1 q_1\rangle^2}{81}\left[\frac{1}{p'^6(p^2 - m_Q^2)} + \frac{6}{p'^4(p^2 - m_Q^2)^2}\right.\\
&\quad - \frac{1}{p'^2(p^2 - m_Q^2)^3} - \frac{q^2}{p'^4(p^2 - m_Q^2)^3}
\end{aligned}$$

$$+\frac{3m_Q^2}{p'^2(p^2-m_Q^2)^4}-\frac{m_Q^2-q^2}{p'^6(p^2-m_Q^2)^2}\Bigg]$$
$$-\frac{2g_c^2\langle\bar{q}_1q_1\rangle\langle\bar{Q}Q\rangle}{9m_Q^4}\left[\frac{1}{p'^2(p^2-m_Q^2)}-\frac{1}{p'^2p^2}-\frac{m_Q^2}{p'^2p^4}\right]$$
$$+\frac{2g_c^2\langle\bar{q}_1q_1\rangle\langle\bar{q}_2q_2\rangle}{9}\frac{1}{p'^6(p^2-m_Q^2)}. \tag{31}$$

After performing the Borel transform and comparing the r.h.s of QCD sum rules (10), we obtain the QCD sum rules for the determination of the form factors $F_\pm(q^2)$:

$$F_+(q^2)=-f_L^{-1}f_M^{-1}e^{(m_M^2/M^2)}e^{(m_L^2/M'^2)}\mathbf{B}[\Pi_+^{\text{lhs}}]$$

and

$$F_-(q^2)=-f_L^{-1}f_M^{-1}e^{(m_M^2/M^2)}e^{(m_L^2/M'^2)}\mathbf{B}[\Pi_-^{\text{lhs}}]. \tag{32}$$

B. Form factors for the decay of the D or B meson into a light vector meson and leptons

To treat the form factors for semileptonic decays of D and B mesons into a light vector meson and leptons such as $\bar{B}^0\to\rho^+e^-\bar{\nu}_e$, we follow a similar line and consider the following Green's function in external vector ($\mathscr{V}$) and external axial-vector ($\mathscr{A}$) fields:

$$i(2\pi)^4\delta^4(p-p'-q)\,\tilde{\Pi}_{\mu\nu}^{\mathscr{A}(\mathscr{V})}(p,p';q^2)$$
$$=i^2\iint d^4x d^4y\, e^{i(p'x-py)}\langle 0|T\{j_\mu(x),j_\nu^5(y)\}|0\rangle_{\mathscr{A}(\mathscr{V})}, \tag{33}$$

with $j_\mu=\bar{q}_1\gamma_\mu q_2$ and $j_\nu^5=\bar{Q}\gamma_\nu\gamma_5 q_1$. Here the definitions of the notations are the same as before.

The interaction with the external field is described by the additional term in the Lagrangian:

$$\Delta\mathscr{L}^A(x)=-\mathscr{A}^\alpha e^{iqx}J_\alpha^5(x), \tag{34}$$

for the axial-vector field or

$$\Delta\mathscr{L}^V(x)=-\mathscr{V}^\alpha e^{iqx}J_\alpha(x), \tag{35}$$

for the vector field, where

$$J_\alpha^5=\bar{q}_2\gamma_\alpha\gamma_5 Q,$$
$$J_\alpha=\bar{q}_2\gamma_\alpha Q. \tag{36}$$

Here $\mathscr{A}(\mathscr{V})$ and q^α are the amplitude and momentum of the external axial-vector (or vector) field, respectively. Note that in the following calculation we consider only the amplitude of $\tilde{\Pi}_{\mu\nu}^{\mathscr{A}(\mathscr{V})}$ linear in the external field $\mathscr{A}(\mathscr{V})$.

At the hadron level (r.h.s.), the imaginary parts of $\tilde{\Pi}_{\mu\nu}^{\mathscr{A}(\mathscr{V})}$ are

$$\rho_{\mu\nu}^{\mathscr{A}}(p,p';q^2)$$
$$=-\mathscr{A}^\alpha\langle 0|j_\mu(0)|L^*(p',\lambda)\rangle\langle L^*(p',\lambda)|J_\alpha^5(0)|M(p)\rangle$$
$$\times\langle M(p)|j_\nu^5(0)|0\rangle\delta(s'-m_{L^*}^2)\delta(s-m_M^2)+\ \ldots, \tag{37}$$

and

$$\rho_{\mu\nu}^{\mathscr{V}}(p,p';q^2)$$
$$=-\mathscr{V}^\alpha\langle 0|j_\mu(0)|L^*(p',\lambda)\rangle\langle L^*(p',\lambda)|J_\alpha(0)|M(p)\rangle$$
$$\times\langle M(p)|j_\nu^5(0)|0\rangle\delta(s'-m_{L^*}^2)\delta(s-m_M^2)+\ \ldots, \tag{38}$$

respectively. In the calculation we adopt the definitions and Wirbel-Stech-Bauer (WSB) parametrization [1]:

$$\langle 0|j_\mu(0)|L^*(p',\lambda)\rangle=i\frac{m_{L^*}^2}{g_{L^*}}\varepsilon_{\mu,\lambda},$$

$$\langle L^*(p',\lambda)|J_\alpha^5(0)|M(p)\rangle$$
$$=(m_{L^*}+m_M)A_1(q^2)\varepsilon_{\alpha,\lambda}^*-\frac{(p+p')_\alpha}{m_{L^*}+m_M}(\varepsilon_\lambda^*\cdot p)A_2(q^2)$$
$$+(\varepsilon_\lambda^*\cdot p)2m_{L^*}A(q^2)q_\alpha,$$

$$\sum_{\lambda=1}^{3}\varepsilon_{\gamma,\lambda}\varepsilon_{\mu,\lambda}^*=-\left(g_{\gamma\mu}-\frac{p'_\gamma p'_\mu}{m_{L^*}^2}\right),$$

$$\langle L^*(p',\lambda)|J_\alpha(0)|M(p)\rangle=i\varepsilon_{\alpha\mu\nu\delta}p'^\mu p^\nu\varepsilon_\lambda^{*\delta}\frac{2V(q^2)}{m_M+m_{L^*}},$$

$$A(q^2)=\frac{A_0(q^2)-A_3(q^2)}{q^2},$$

$$A_3(q^2)=\frac{m_M+m_{L^*}}{2m_{L^*}}A_1(q^2)-\frac{m_M-m_{L^*}}{2m_{L^*}}A_2(q^2), \tag{39}$$

with λ denoting the helicity state of the spin-1 particle (L^*).

The Lorentz structures of $\tilde{\Pi}_{\mu\nu}^{\mathscr{A}(\mathscr{V})}$, in (33), may be written as follows,

$$\tilde{\Pi}_{\mu\nu}^{\mathscr{A}}=\mathscr{A}^\alpha(\tilde{\Pi}_{A_1}g_{\mu\alpha}p_\nu+\tilde{\Pi}_{A_2}p_\mu(p+p')_\alpha p_\nu$$
$$+\tilde{\Pi}_A p_\mu(p-p')_\alpha p_\nu+\cdots),$$
$$\tilde{\Pi}_{\mu\nu}^{\mathscr{V}}=i\mathscr{V}^\alpha(\tilde{\Pi}_V\varepsilon_{\alpha\mu\beta\rho}p_\nu p'^\beta p^\rho+\cdots), \tag{40}$$

where we obtain, at the hadronic level (rhs),

$$\tilde{\Pi}_{A_1}=\frac{f_M m_{L^*}^2}{g_{L^*}}\frac{1}{(m_M^2-p^2)(m_{L^*}^2-p'^2)}(m_M+m_{L^*})A_1(q^2),$$
$$\tilde{\Pi}_{A_2}=-\frac{f_M m_{L^*}^2}{g_{L^*}}\frac{1}{(m_M^2-p^2)(m_{L^*}^2-p'^2)}\frac{A_2(q^2)}{(m_M+m_{L^*})},$$
$$\tilde{\Pi}_A=\frac{f_M m_{L^*}^2}{g_{L^*}}\frac{1}{(m_M^2-p^2)(m_{L^*}^2-p'^2)}2m_{L^*}A(q^2),$$
$$\tilde{\Pi}_V=\frac{f_M m_{L^*}^2}{g_{L^*}}\frac{1}{(m_M^2-p^2)(m_{L^*}^2-p'^2)}\frac{2V(q^2)}{(m_M+m_{L^*})}. \tag{41}$$

Note that, as mentioned before, the contributions of higher excited states are transferred to the left hand side of the sum rules (via the continuum approximation). Note also that the amplitude $\tilde{\Pi}_{\mu\nu}^{\mathscr{V}}$ can in general be represented as

$$\tilde{\Pi}_{\mu\nu}^{\mathscr{V}}=i\mathscr{V}^\alpha(f_1p_\nu T_{\alpha\mu}+f_2p_\mu T_{\nu\alpha}+f_3p_\alpha T_{\mu\nu}+f_4S_{\alpha\mu\nu}$$
$$+f'_1p'_\nu T_{\alpha\mu}+f'_2p'_\mu T_{\nu\alpha}+f'_3p'_\alpha T_{\mu\nu}+f'_4S'_{\alpha\mu\nu}), \tag{42}$$

where $T_{\alpha\mu}=\varepsilon_{\alpha\mu\beta\rho}p'^\beta p^\rho$, $S_{\alpha\mu\nu}=\varepsilon_{\alpha\mu\nu\beta}p^\beta$, and $S'_{\alpha\mu\nu}=\varepsilon_{\alpha\mu\nu\beta}p'^\beta$. Among these tensors, there exist two relations [31, 32]

$$p_\nu T_{\alpha\mu}+p_\mu T_{\nu\alpha}+p_\alpha T_{\mu\nu}=(p\cdot p')S_{\mu\nu\alpha}-p^2S'_{\mu\nu\alpha},$$
$$p'_\nu T_{\alpha\mu}+p'_\mu T_{\nu\alpha}+p'_\alpha T_{\mu\nu}=-(p\cdot p')S'_{\mu\nu\alpha}+p'^2S_{\mu\nu\alpha}, \tag{43}$$

thereby reducing to only six linearly independent Lorentz structures. (For a proof of (43), consult Appendix C.) In this work, we choose $S_{\alpha\mu\nu}$, $S'_{\alpha\mu\nu}$, $p_\mu T_{\nu\alpha}-p_\alpha T_{\mu\nu}$,

$p'_\mu T_{\nu\alpha} - p'_\alpha T_{\mu\nu}$, $p_\nu T_{\alpha\mu}$, and $p'_\nu T_{\alpha\mu}$ as independent structures. We emphasize that the issue concerning linear independence does affect, to a minor extent, the final QCD sum rules and should be treated with care.

At the quark level (lhs), the derivation of the relevant QCD sum rules is carried out along the same line as before and it is sufficient to simply record the results. For the perturbative part, we obtain

$$\tilde{\Pi}^{\rm pert}_{A_1}(p,p';q^2) = -\frac{1}{4\pi^2}\int\int_\Omega ds\,ds'\,\frac{\rho_{A_1}(s,s';q^2)}{(s-p^2)(s'-p'^2)}, \qquad (44)$$

$$\tilde{\Pi}^{\rm pert}_{A_2}(p,p';q^2) = -\frac{1}{4\pi^2}\int\int_\Omega ds\,ds'\,\frac{\rho_{A_2}(s,s';q^2)}{(s-p^2)(s'-p'^2)}, \qquad (45)$$

$$\tilde{\Pi}^{\rm pert}_{A}(p,p';q^2) = -\frac{1}{4\pi^2}\int\int_\Omega ds\,ds'\,\frac{\rho_{A}(s,s';q^2)}{(s-p^2)(s'-p'^2)}, \qquad (46)$$

$$\tilde{\Pi}^{\rm pert}_{V}(p,p';q^2) = -\frac{1}{4\pi^2}\int\int_\Omega ds\,ds'\,\frac{\rho_{V}(s,s';q^2)}{(s-p^2)(s'-p'^2)}, \qquad (47)$$

where

$$\begin{aligned}\rho_{A_1}(s,s';q^2) &= 3\left\{\frac{4s'^2}{\lambda^{5/2}}\left(s-m_Q^2-\frac{s+s'-q^2}{2}\right)[(s-m_Q^2)(m_Q^2-q^2)\right.\\ &\quad - s'm_Q^2] - \frac{s'}{\lambda^{3/2}}[(m_Q^2-q^2)^2-s'q^2]\\ &\quad \left.+\frac{s'}{\lambda^{3/2}}(s-s'+q^2-2m_Q^2)m_Q m_{q_2}\right\}, \qquad (48)\end{aligned}$$

$$\begin{aligned}\rho_{A_2}(s,s';q^2) &= -3\left\{\frac{40s'^2}{\lambda^{7/2}}[(s-m_Q^2)(m_Q^2-q^2)-s'm_Q^2]\right.\\ &\quad \times\left[\left(\frac{s'+s-q^2}{2}\right)(s+s'-m_Q^2)-s'(2s-m_Q^2)\right]\\ &\quad + \frac{s'}{\lambda^{5/2}}[3(s-7s'-q^2)[(s-m_Q^2)(m_Q^2-q^2)\\ &\quad - s'm_Q^2] - 2s'[(s-s'-m_Q^2)^2-s'q^2]]\\ &\quad \left.+\frac{s'}{\lambda^{3/2}}(2s'+q^2-m_Q^2)\right\}, \qquad (49)\end{aligned}$$

$$\begin{aligned}\rho_{A}(s,s';q^2) &= -3\left\{\frac{40s'^2}{\lambda^{7/2}}[(s-m_Q^2)(m_Q^2-q^2)-s'm_Q^2]\right.\\ &\quad \times\left[s'm_Q^2-\left(\frac{s'+s-q^2}{2}\right)(s-s'-m_Q^2)\right]\\ &\quad + \frac{s'}{\lambda^{5/2}}[-3(s+s'-q^2)[(s-m_Q^2)(m_Q^2-q^2)-s'm_Q^2]\\ &\quad + 2s'[(s-m_Q^2)^2-s'(s'+2m_Q^2-q^2)]]\\ &\quad \left.+\frac{s'}{\lambda^{3/2}}(2s'-q^2+m_Q^2)\right\}, \qquad (50)\end{aligned}$$

and

$$\begin{aligned}\rho_V(s,s';q^2) &= -3\frac{s'}{\lambda^{5/2}}[\lambda(3m_Q^2-2q^2-s)+3(3s'-s+q^2)\\ &\quad \times[(s-m_Q^2)(m_Q^2-q^2)-s'm_Q^2]]. \qquad (51)\end{aligned}$$

For the induced condensates, with the aid of (22), we could easily expand the quark condensate $\langle q^a_{2i}(x)\bar{Q}^b_j(y)\rangle_{\mathscr{A}}$ in the approximation linear in $\mathscr{A}^\mu$:

$$\begin{aligned}\langle q^a_{2i}(x)\bar{Q}^b_j(y)\rangle_{\mathscr{A}} &\approx -\frac{1}{12}\langle\bar{Q}(y)\gamma_5 q_2(y)\rangle_{\mathscr{A}}\delta^{ab}(\gamma_5)_{ij}\\ &\quad + \frac{1}{12}\langle\bar{Q}(y)\gamma_\mu\gamma_5 q_2(y)\rangle_{\mathscr{A}}\delta^{ab}(\gamma^\mu\gamma_5)_{ij}\\ &\quad - \frac{1}{24}\langle\bar{Q}(y)\sigma_{\alpha\beta}\gamma_5 q_2(y)\rangle_{\mathscr{A}}\delta^{ab}(\sigma^{\alpha\beta}\gamma_5)_{ij}\\ &\quad + \ldots . \qquad (52)\end{aligned}$$

By the same token [as for (23)–(28)], we need to consider only the induced condensate $\langle\bar{Q}(y)\sigma_{\alpha\beta}\gamma_5 q_2(y)\rangle_{\mathscr{A}}$:

$$\langle\bar{Q}(y)\sigma^{\alpha\beta}\gamma_5 q(y)\rangle_{\mathscr{A}} = -i\mathscr{A}^\nu e^{iqy}\chi^{\mathscr{A}}_{Qq_2}(q^2)(g_{\nu\alpha}q_\beta - g_{\beta\nu}q_\alpha). \quad (53)$$

The result is

$$\chi^{\mathscr{A}}_{Qq_2} = \frac{f^{\mathscr{A}}}{m^2_{f^{\mathscr{A}}}-q^2} - \frac{3}{8\pi}\int_{m_Q^2}^{s_0} ds\,\frac{m_Q}{s-q^2}\left[1-2\left(\frac{m_Q^2}{s}\right)+\left(\frac{m_Q^2}{s}\right)^2\right], \qquad (54)$$

and the numerical values of $f^{\mathscr{A}}$ are listed in Table 2, in which the lowest-lying meson mass is used as the input parameter in the relevant sum rule. The detailed calculations of $f^{\mathscr{A}}$ are contained in Appendix B.

The contributions of induced condensates are then given by

$$\begin{aligned}\tilde{\Pi}^{\rm induced}_{A_1}(p,p';q^2) &= -\frac{2g_c^2\langle\bar{q}_1q_1\rangle}{9}\chi^{\mathscr{A}}_{Qq_2}(q^2)\frac{p^2-p'^2-q^2}{(p^2-m_Q^2)p'^4},\\ \tilde{\Pi}^{\rm induced}_{A_2}(p,p';q^2) &= \frac{2g_c^2\langle\bar{q}_1q_1\rangle}{9}\chi^{\mathscr{A}}_{Qq_2}(q^2)\frac{1}{(p^2-m_Q^2)p'^4},\\ \tilde{\Pi}^{\rm induced}_{A}(p,p';q^2) &= \frac{-2g_c^2\langle\bar{q}_1q_1\rangle}{9}\chi^{\mathscr{A}}_{Qq_2}(q^2)\frac{1}{(p^2-m_Q^2)p'^4},\\ \tilde{\Pi}^{\rm induced}_{V}(p,p';q^2) &= \frac{4g_c^2\langle\bar{q}_1q_1\rangle}{9}\chi^{\mathscr{V}}_{Qq_2}(q^2)\frac{1}{(p^2-m_Q^2)p'^4}. \end{aligned} \qquad (55)$$

Combining (44)–(51) and (55) and considering non-perturbative contributions from quark condensates and quark-gluon condensates, we may record the results at the quark level:

$$\begin{aligned}\tilde{\Pi}^{\rm lhs}_{A_1}&(p,p';q^2)\\ &= \tilde{\Pi}^{\rm pert}_{A_1}(p,p';q^2) + \tilde{\Pi}^{\rm induced}_{A_1}(p,p';q^2) - \frac{m_{q_2}\langle\bar{q}_1q_1\rangle}{p'^2(p^2-m_Q^2)}\\ &\quad + \frac{m_0^2\langle\bar{q}_1q_1\rangle}{6}\left[\frac{m_{q_2}-2m_Q}{p'^2(p^2-m_Q^2)^2}+\frac{m_{q_2}}{p'^4(p^2-m_Q^2)}\right.\end{aligned}$$

$$+\frac{m_{q_2}(m_Q-q^2)}{p'^4(p^2-m_Q^2)^2}+\frac{3m_{q_2}m_Q^2}{p'^2(p^2-m_Q^2)^3}\Bigg]$$
$$+\frac{g_c^2\langle\bar{q}_1q_1\rangle^2}{81}\Bigg[\frac{3}{p'^4(p^2-m_Q^2)}+\frac{2}{p'^2(p^2-m_Q^2)^2}$$
$$-\frac{2}{(p^2-m_Q^2)^3}-\frac{10m_Q^2+28m_Qm_{q_2}-2q^2}{p'^2(p^2-m_Q^2)^3}$$
$$+\frac{6(m_Q^2-q^2)+7m_Qm_{q_2}}{p'^4(p^2-m_Q^2)^2}+\frac{6m_Q^2}{(p^2-m_Q^2)^4}$$
$$+\frac{2(m_Q^2-q^2)(m_Q^2+m_Qm_{q_2}-q^2)}{p'^4(p^2-m_Q^2)^3}$$
$$+\frac{6m_Q^2(m_Q^2+m_Qm_{q_2}-q^2)}{p'^2(p^2-m_Q^2)^4}\Bigg]$$
$$+\frac{2g_c^2\langle\bar{q}_1q_1\rangle\langle\bar{Q}Q\rangle}{9}\Bigg[\frac{1}{p^4(p^2-m_Q^2)}+\frac{1}{p'^4p^2}$$
$$+\frac{2m_{q_2}m_Q^2+m_Q^2-q^2}{p^4(p^2-m_Q^2)p'^2}-\frac{4m_{q_2}m_Q^2}{p^2(p^2-m_Q^2)^2p'^2}\Bigg]$$
$$-\frac{4g_c^2\langle\bar{q}_1q_1\rangle\langle\bar{q}_2q_2\rangle}{9}\frac{1}{p'^4(p^2-m_Q^2)},\tag{56}$$

$$\tilde{\Pi}_{A_2}^{\text{lhs}}(p,p';q^2)$$
$$=\tilde{\Pi}_{A_2}^{\text{pert}}(p,p';q^2)+\tilde{\Pi}_{A_2}^{\text{induced}}(p,p';q^2)$$
$$+m_0^2\langle\bar{q}_1q_1\rangle\frac{m_{q_2}}{6p'^4(p^2-m_Q^2)^2}$$
$$-\frac{g_c^2\langle\bar{q}_1q_1\rangle^2}{27}\Bigg[\frac{2}{p'^2(p^2-m_Q^2)^3}+\frac{1}{p'^4(p^2-m_Q^2)^2}\Bigg]$$
$$-\frac{2g_c^2\langle\bar{q}_1q_1\rangle\langle\bar{Q}Q\rangle}{9m_Q^4}\Bigg[\frac{1}{p'^2(p^2-m_Q^2)}-\frac{1}{p'^2p^2}-\frac{m_Q^2}{p'^2p^4}\Bigg],\tag{57}$$

$$\tilde{\Pi}_{A}^{\text{lhs}}(p,p';q^2)$$
$$=\tilde{\Pi}_{A}^{\text{pert}}(p,p';q^2)+\tilde{\Pi}_{A}^{\text{induced}}(p,p';q^2)$$
$$-m_0^2\langle\bar{q}_1q_1\rangle\frac{m_{q_2}}{6p'^4(p^2-m_Q^2)^2}$$
$$-\frac{g_c^2\langle\bar{q}_1q_1\rangle^2}{81}\Bigg[\frac{2}{p'^2(p^2-m_Q^2)^3}-\frac{3}{p'^4(p^2-m_Q^2)^2}\Bigg]$$
$$+\frac{2g_c^2\langle\bar{q}_1q_1\rangle\langle\bar{Q}Q\rangle}{9m_Q^4}\Bigg[\frac{1}{p'^2(p^2-m_Q^2)}-\frac{1}{p'^2p^2}-\frac{m_Q^2}{p'^2p^4}\Bigg],\tag{58}$$

$$\tilde{\Pi}_{V}(p,p';q^2)$$
$$=\tilde{\Pi}_{V}^{\text{pert}}(p,p';q^2)+\tilde{\Pi}_{V}^{\text{induced}}(p,p';q^2)$$
$$-m_0^2\langle\bar{q}_1q_1\rangle\frac{m_{q_2}}{3p'^4(p^2-m_Q^2)^2}$$
$$+\frac{g_c^2\langle\bar{q}_1q_1\rangle^2}{81}\Bigg[\frac{4}{p'^2(p^2-m_Q^2)^3}-\frac{4}{p'^4(p^2-m_Q^2)^2}$$
$$-\frac{6m_Q^2}{p'^2(p^2-m_Q^2)^4}-\frac{2(m_Q^2-q^2)}{p'^4(p^2-m_Q^2)^3}\Bigg]+\frac{4g_c^2\langle\bar{q}_1q_1\rangle\langle\bar{Q}Q\rangle}{9m_Q^4}$$
$$\times\Bigg[\frac{1}{p'^2(p^2-m_Q^2)}-\frac{1}{p'^2p^2}-\frac{m_Q^2}{p'^2p^4}\Bigg]\tag{59}$$

Equating the coefficients of the identical Lorentz structure obtained both at the hadron level [rhs; (41)] and at the quark level [lhs; (56)–(59)], we obtain the QCD sum rules, after performing the Borel transform,

$$A_1(q^2)=\frac{g_{L^*}}{f_Mm_{L^*}^2}\frac{1}{m_M+m_{L^*}}e^{(m_M^2/M^2)}e^{(m_{L^*}^2/M'^2)}\mathbf{B}[\tilde{\Pi}_{A_1}^{\text{lhs}}],$$

$$A_2(q^2)=-\frac{g_{L^*}}{f_Mm_{L^*}^2}(m_M+m_{L^*})e^{(m_M^2/M^2)}e^{(m_{L^*}^2/M'^2)}\mathbf{B}[\tilde{\Pi}_{A_2}^{\text{lhs}}],$$

$$A(q^2)=\frac{g_{L^*}}{f_Mm_{L^*}^2}\frac{1}{2m_{L^*}^2}e^{(m_M^2/M^2)}e^{(m_{L^*}^2/M'^2)}\mathbf{B}[\tilde{\Pi}_{A}^{\text{lhs}}],$$

$$V(q^2)=\frac{g_{L^*}}{f_Mm_{L^*}^2}\frac{m_M+m_{L^*}}{2}e^{(m_M^2/M^2)}e^{(m_{L^*}^2/M'^2)}\mathbf{B}[\tilde{\Pi}_{V}^{\text{lhs}}].\tag{60}$$

As a parenthetical remark, we wish to point out that our simultaneous adoption of (1) and (33), as based on the axial vector and polar vector currents, would in general yield results naturally in line with chiral algebra, namely, current algebra, CVC, and PCAC, as these currents (with pseudoscalar currents as derived concept) enter as the cornerstone of chiral algebra. Accordingly, the predictions based upon the resultant QCD sum rules, (32) and (60), would be much in line with chiral symmetry, although such feature is by no means apparent.

III Numerical results and discussion

In the numerical analysis, we take the observed masses of mesons as from the Particle Data Group (PDG) [33], and use the following parameters [9, 24, 27, 34, 35]:

$$s_0^K=1.2\,\text{GeV}^2\quad s_0^D=6\,\text{GeV}^2\quad s_0^B=33\,\text{GeV}^2$$
$$s_0^\pi=0.8\,\text{GeV}^2\quad s_0^\rho=1.5\,\text{GeV}^2\quad s_0^{K^*}=1.7\,\text{GeV}^2,$$
$$f_K=160\,\text{MeV},\quad f_D=160\,\text{MeV},\quad f_B=160\,\text{MeV},$$
$$f_\pi=130\,\text{MeV},\quad g_{K^*}=4.25,\quad g_\rho=3.84,$$
$$m_c=1.30\,\text{GeV},\quad m_b=4.7\,\text{GeV},$$
$$m_u=m_d=0,\quad m_s=175\,\text{MeV},$$
$$\langle\bar{u}u\rangle=\langle\bar{d}d\rangle=-(240\,\text{MeV})^3,\quad\langle\bar{s}s\rangle=\langle\bar{u}u\rangle\times0.8,$$
$$m_0^2=0.8\,\text{GeV}^2,\tag{61}$$

where m_c and m_b are physical pole masses; the normalization point of m_s, m_0^2, and $\langle\bar{q}q\rangle$ refers to a lower normalization point $\mu\sim500$ MeV and $\Lambda=100$ MeV. To improve further the validity of the derived QCD sum rules, we adjust the condensates to be evaluated at the scale of $(M'^2M^2)^{1/2}$, i.e., perform the following replacements [24]:

$$m_s \to m_s L^{\gamma_m},$$

$$\langle\bar{q}q\rangle m_0^2 \to \langle\bar{q}q\rangle m_0^2 L^{\gamma\langle g_c\bar{q}\sigma Gq\rangle},$$

$$\langle\bar{q}q\rangle \to \langle\bar{q}q\rangle L^{\gamma\langle\bar{q}q\rangle}, \tag{62a}$$

with the anomalous dimensions given by $\gamma_{m_q} = -4/b$, $\gamma_{\langle g_c\bar{q}\sigma Gq\rangle} = -2/3b$, and $\gamma_{\langle\bar{q}q\rangle} = 4/b$. L is given by

$$L = \frac{\ln((M'^2M^2)^{1/2}/\Lambda^2)}{\ln(\mu^2/\Lambda^2)}, \tag{62b}$$

where $\Lambda = 100$ MeV, $\mu = 500$ MeV, and $b = 11 - 2n_f/3$ and n_f is the number of *unfrozen* quark flavors. To study the stability of our sum rules, we choose possible masses existing in the literature [36, 27] and let pole masses vary in the relevant range, m_c: 1.3–1.35 GeV, m_b: 4.7–4.8 GeV. In the case of m_c, when $m_c = 1.3$ GeV the sum rules are more stable than other choices. In the case of m_b, if we choose a larger value of m_b, we obtain a lower value of f_b; these two effects are compensated by each other in the analyses of form factors.

Note that in [27] the sum rule for f_B was to be evaluated at the scale of m_b, and yielded a lower value $f_B \approx 130$ MeV. However it has been pointed out by Neubert [34] that the large logarithms $(\alpha_s \ln m_b)^n$ to all orders in perturbation theory should be evaluated in the sum rule. Therefore according to the existing literature [34, 35] we use $f_B \approx f_D$. Note also that in the region $R^{-2} \ll Q^2 \ll m_M^2$ with R the confinement radius there exists an anomalous dimension $2/b$ for both the current operators $\bar{Q}\gamma_\nu\gamma_5 q$ and $\bar{Q}\gamma_\nu q$ [37–39]. In our numerical analysis of B-meson decay form factors, the sum rules are studied at the low scale of $Q^2 = \sqrt{M'^2M^2} \approx 2.7\ \text{GeV}^2$ [See below, (63)]. Therefore, to obtain physical form factors, which are independent of the subtraction scale, we incorporate the factor,

$$\left[\frac{\ln(m_b^2/\Lambda^2)}{\ln((M'^2M^2)^{1/2}/\Lambda^2)}\right]^{12/25}, \tag{62c}$$

into the relevant QCD sum rules.

The range of the Borel mass may be fixed by assuming that both the continuum and higher dimension corrections are well controlled. Combining the analysis of relevant mass sum rules [9, 27, 34], we therefore extract the suitable Borel mass ranges for the numerical analyses and list them below:

$$\text{For } \pi \quad 0.8\ \text{GeV}^2 < M^2 < 1.0\ \text{GeV}^2,$$

$$\text{For } K \quad 0.8\ \text{GeV}^2 < M^2 < 1.0\ \text{GeV}^2,$$

$$\text{For } \rho \quad 0.9\ \text{GeV}^2 < M^2 < 1.3\ \text{GeV}^2,$$

$$\text{For } K^* \quad 0.9\ \text{GeV}^2 < M^2 < 1.3\ \text{GeV}^2,$$

$$\text{For } D \quad 1.6\ \text{GeV}^2 < M^2 < 2.4\ \text{GeV}^2,$$

$$\text{For } B \quad 7.0\ \text{GeV}^2 < M^2 < 8.0\ \text{GeV}^2. \tag{63}$$

In these Borel ranges, the form factors are dominated by the perturbative bare loop, Fig 1(a), while the contribution of the highest dimension proportional to $\langle\bar{q}q\rangle^2$ is less than 15%; for the A_1 form factor the contribution of the $\langle g_c\bar{q}\sigma Gq\rangle$ is less than 50% of the bare loop. Such criteria for analyzing our sum rules are in fact different from those obtained by Ball et al. [18]. In the sum rules obtained by [18] the contribution from the quark condensate $\langle\bar{q}q\rangle$ may even be larger than that from the bare perturbative loop, while in our sum rules the contributions from quark condensate are numerically under better control. This aspect in fact explains why our form factors seem better behaved. It is similar to the situation when we determine the q^2-dependence of the pseudoscalar form factor g_P in weak interactions — the known axial form factor together with PCAC implies a simple pole behavior but some direct calculation, being spoiled mostly by higher order terms, without proper cancellation may lead to predictions at variance with PCAC.

As examples, in Fig. 3a and 3b, we plot the A_1 and A_2 form factors of $D^0 \to K^{*-}e^+\nu$, respectively, as a function of M'^2 and M^2. All results on the various form factors at $q^2 = 0$ are collected in Tables 3–4. Note that the values of F_- and A, which are also calculated here and are needed when the lepton mass becomes of importance for the decay processes [such as when there is a τ lepton in the final state; consult (D.1) and (D.2) in Appendix D], have not been calculated by means of QCD sum rules in the existing literature.

In Tables 5 and 6 we show various model predictions (including our results), together with the experimental

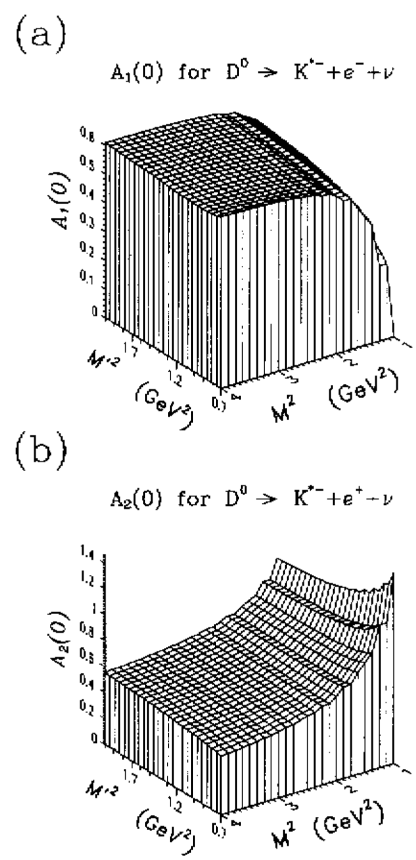


Fig. 3. The A_1 and A_2 form factors of $D^0 \to K^{*-}e^+\nu$ are plotted as a function of the square of the Borel masses, M^2 and M'^2

Table 3. Estimates of various weak semileptonic form factors for $D \to \pi$, K and for $B \to \pi$ in this work

	$F_+(0)$	$F_-(0)$
$D^0 \to \pi^- e^- \nu$	0.65 ± 0.10	-0.59 ± 0.08
$D^0 \to K^- e^- \nu$	0.75 ± 0.12	-0.59 ± 0.06
$\bar{B}^0 \to \pi^+ e^- \bar{\nu}$	0.29 ± 0.04	-0.28 ± 0.04

Table 4. Estimates of various weak semileptonic form factors for $D \to \rho$, K^* and for $B \to \rho$ in this work

	$A_1(0)$	$A_2(0)$	$A(0)$	$V(0)$
$D^0 \to \rho^- e^+ \nu$	0.34 ± 0.08	0.57 ± 0.08	0.30 ± 0.07	0.98 ± 0.11
$D^0 \to K^{*-} e^- \nu$	0.54 ± 0.04	0.67 ± 0.08	0.22 ± 0.03	1.10 ± 0.10
$\bar{B}^0 \to \rho^- e^- \bar{\nu}$	0.07 ± 0.01	0.16 ± 0.01	0.018 ± 0.002	0.19 ± 0.01

Table 5. Various semileptonic form factors for $D \to \pi$, K and for $B \to \pi$ from theoretical calculations and experiments

	$F_-(0)$	$F_-(0)$	Reference
$D^0 \to \pi^-$	0.65 ± 0.10	-0.59 ± 0.08	This work
	0.7 ± 0.2		AOS [21][a]
	0.5 ± 0.2		B [19][a]
	0.69		WSB [1][b]
	0.51		ISGW [3][b]
	$0.84 \pm 0.12 \pm 0.35$		BES [6][c]
	0.58 ± 0.09		LMMS [8][c]
	0.79		CDBGFN [5][d]
	$0.80^{+0.21}_{-0.14}$		PDG [33][e]
$D^0 \to K^-$	0.75 ± 0.12	0.59 ± 0.06	This work
	0.6 ± 0.1		AEK [14][a]
	0.8 ± 0.2		AOS [21]
	$0.6^{+0.15}_{-0.10}$		BBD [8][a]
	0.76		WSB [1]
	0.78		SI [4]
	$0.90 \pm 0.08 \pm 0.12$		BES [6]
	0.63 ± 0.08		LMMS [8]
	0.67		CDBGFN [5]
	0.71 ± 0.06		E691 [45][e]
$\bar{B}^0 \to \pi^+$	0.29 ± 0.04	-0.28 ± 0.04	This work
	0.26 ± 0.02		B [19]
	0.29		BKR [46][a]
	0.33		WSB [1]
	0.09		ISGW [3]
	0.89		CDBGFN [5]

[a]QCD sum rules
[b]Quark model
[c]Lattice calculation
[d]HQET in combining with the chiral perturbation theory
[e]experiment

results. For the form factor F_+, our results in Table 5 agree with various model calculations as well as experimental results except for the HQET calculation on $F_+^{B\to\pi}$ where the value of $F_+^{B\to\pi}$ obtained in [5] is almost three times as large as ours. An interesting quantity related to the SU(3) symmetry breaking effect is the ratio $F_+^{D\to\pi}/F_+^{D\to K}$. Using the values listed in Table 3, we obtain $F_+^{D\to\pi}/F_+^{D\to K} \approx 0.87$, which is somewhat smaller than, but is consistent with, the recent CLEO experimental [40] result $F_+^{D\to\pi}/F_+^{D\to K} = 1.29 \pm 0.21 \pm 0.11$.

Let us next turn our attention to Table 6 for the semileptonic decays into vector mesons. For the decay $D^0 \to K^{*-} e^+ \nu_e$ our value $A_2(0)/A_1(0) = 1.24^{+0.26}_{-0.22}$ is different from the earlier E691 [41] experiment $A_2(0)/A_1(0) = 0.0 \pm 0.5 \pm 0.2$, but is quite close to the recent measurement both from the group of E653 [42] $(A_2(0)/A_1(0) = 0.82^{+0.22}_{-0.23})$ and E687 [43] $(A_2(0)/A_1(0) = 0.78 \pm 0.18 \pm 0.10)$. For the decay $D^0 \to \rho^- e^+ \nu_e$ our value $A_2(0)/A_1(0) = 1.68^{+0.82}_{-0.66}$ is quite different from the other theory calculations. For the decay $\bar{B}^0 \to \rho^+ e^- \bar{\nu}_e$ we obtain very small form factor values: $A_1 = 0.07 \pm 0.01$, $A_2 = 0.16 \pm 0.01$, $A = 0.018 \pm 0.002$, $V = 0.19 \pm 0.01$. Once again, the value of $A_2(0)/A_1(0)$ is approximately 2. Recently, Blasi et al. [44] have used the same approach as in [18, 19] to evaluate the form factors for $\bar{B}_s^0 \to K^{*+} e^- \bar{\nu}_e$, and have obtained small values $A_1 = 0.3 \pm 0.1$, $A_2 \approx 0$, and $V = 0.12 \pm 0.02$. It would be difficult to understand the unusually large SU(3) symmetry breaking if the results reported in [18] and [44] would be correct. Measurements on this ratio in future experiments would be most welcome.

Next we consider the q^2 dependence of the various form factors. As the discussion of the property of discontinuity in [18], the method of QCD sum rules works well in the region $(m_Q^2 - q^2)^2/q^2 > s_0'$. Therefore, we could obtain the q^2 dependence of form factors in a quite large range of q^2 and then extrapolate to all possible ranges of q^2. In Figs. 4–9, we plot various form factors as a function of q^2. Note that the q^2 range chosen for these plots satisfies $(m_Q^2 - q^2)^2/q^2 > s_0'$ with m_Q the heavy quark mass and s_0' characterized by (18) and (61). This is the range which we make predictions based on QCD sum rules and which differs slightly from the physical range in which the processes take place.

All of the form factors are determined at the central values of Borel masses in (63). It turns out that the results are very insensitive to what Borel masses are chosen in the ranges listed in (63). Moreover, our results seem to indicate simple pole behaviors:

$$
\begin{aligned}
F_+(q^2) &= \frac{F_+(0)}{1 - q^2/m_{F_+}^2}, \\
F_-(q^2) &= \frac{F_-(0)}{1 - q^2/m_{F_-}^2}, \\
A_1(q^2) &= \frac{A_1(0)}{1 - q^2/m_{A_1}^2}, \\
A_2(q^2) &= \frac{A_2(0)}{(1 - q^2/m_{A_2}^2)^2}, \\
A(q^2) &= \frac{A(0)}{(1 - q^2/m_A^2)^2}, \\
V(q^2) &= \frac{V(0)}{(1 - q^2/m_V^2)^2},
\end{aligned}
\tag{64}
$$

where m_{F_+}, m_{F_-}, m_{A_1}, m_{A_2}, m_A, and m_V are fitted pole masses (parameters). In Table 7 we list the pole masses

Table 6. Various semileptonic form factors for $D \to \rho$, K^* and for $B \to \rho$ from theoretical calculations and experiments

	$A_1(0)$	$A_2(0)$	$A(0)$	$V(0)$	Reference
$D^0 \to \rho^-$	0.34 ± 0.08	0.57 ± 0.08	0.30 ± 0.07	0.98 ± 0.11	This work
	0.5 ± 0.2	0.4 ± 0.1		1.0 ± 0.2	B [19][a]
	0.78	0.92		1.23	WSB [1][b]
	$0.65 \pm 0.15^{+0.24}_{-0.23}$	$0.59 \pm 0.31^{+0.22}_{-0.23}$		$1.07 \pm 0.49 \pm 0.35$	BES [7][c]
	0.45 ± 0.04	0.02 ± 0.26		0.78 ± 0.12	LMMS [8][c]
	0.55	0.28		1.01	CDBGFN [5][d]
$D^0 \to K^{*-}$	0.54 ± 0.04	0.67 ± 0.08	0.22 ± 0.03	1.1 ± 0.1	This work
	0.9 ± 0.2	0.8 ± 0.3		1.7 ± 0.6	AOS [21][a]
	0.50 ± 0.15	0.60 ± 0.15		1.1 ± 0.25	BBD [8][a]
	0.88	1.15		1.27	WSB [1][b]
	0.8	0.8		1.1	SI [4]
	0.83 ± 0.42	0.59 ± 0.38		1.43 ± 0.94	BES [7][c]
	0.53 ± 0.03	0.19 ± 0.21		0.86 ± 0.10	LMMS [8]
	$0.46 \pm 0.05 \pm 0.05$	$0.0 \pm 0.2 \pm 0.1$		$0.9 \pm 0.3 \pm 0.1$	E691 [41][e]
$\bar{B}^0 \to \rho^+$	0.07 ± 0.01	0.16 ± 0.01	0.018 ± 0.002	0.19 ± 0.01	This work
	0.5 ± 0.1	0.4 ± 0.2		0.6 ± 0.2	B [19][a]
	0.28	0.28		0.33	WSB [1][b]
	0.21	0.20		1.04	CDBGFN [5]

[a]QCD sum rules
[b]Quark model
[c]Lattice calculation
[d]HQET in combining with the chiral perturbation theory
[e]experiment

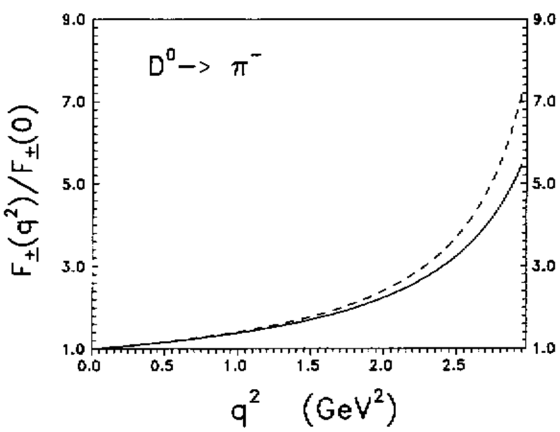


Fig. 4. The ratios $F_+(q^2)/F_+(0)$ and $F_-(q^2)/F_-(0)$ for $D^0 \to \pi^-$ as a function of q^2. The solid curve is the ratio $F_+(q^2)/F_-(0)$, while dashed curve for $F_-(q^2)/F_-(0)$

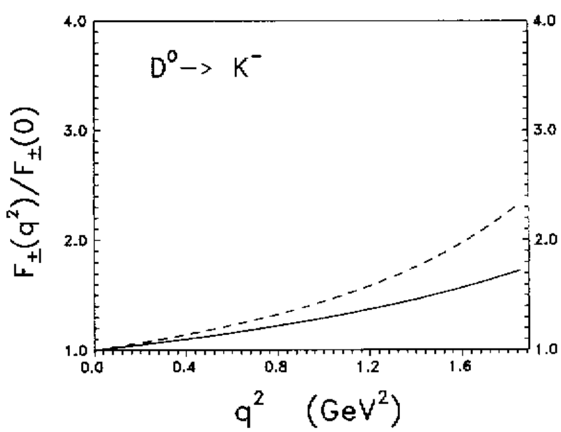


Fig. 5. The ratios $F_+(q^2)/F_+(0)$ and $F_-(q^2)/F_-(0)$ for $D^0 \to K^-$ as a function of q^2. The solid curve is the ratio $F_+(q^2)/F_-(0)$,while dashed curve for $F_-(q^2)/F_-(0)$

fitted from our numerical analyses. Our results are consistent with the existing experimental results [47, 48]. For instance, our fitted pole mass for $F_+^{D\to K}$ is 2.10 GeV, to be compared to the experimental results: $2.1^{+0.4}_{-0.2}$ GeV (E691 [47]), $1.8^{+0.5}_{-0.2}{}^{+0.3}_{-0.2}$ (Mark III [48]).

Recently Grinstein and Mende [49] demonstrated that the form factors F_+ and F_- have the monopole behaviors in the triple limit of large N_c, heavy quarks, and chiral symmetry. To compare the difference between our result and that shown by Grinstein and Mende, in Fig. 6, we plot our results of F_+ and F_- represented by the solid and dashed curve, respectively, for the process $\bar{B}^0 \to \pi^+ e^- \bar{\nu}$ as well as the result [1, 49] from the monopole dominance model represented by the dotted curve. In the dotted curve, the pole mass is 5.32 GeV, which is the lowest-lying meson mass with the quantum number $J^P = 1^-$ in the t-channel. Therefore, from Fig. 6, corrections to the triple limit (large N_c, heavy quarks, and chiral symmetry) appear to be quite small and can be understood reasonably well.

On the other hand [18] obtained the monopole behavior both for F_+ and V and, however, unusual behaviors for the form factors, A_1 and A_2, and their results are sensitive to what Borel range is chosen. We feel that the unusual behavior may be caused by the fact that more higher order terms must first be taken into account to improve the stability and the well-behaved nature of the resultant sum rules. As indicated earlier, our consistent adoption of the axial vector and polar vector currents in (1) and (33) is more in line with chiral symmetry and does give rise to better behaved QCD sum rules.

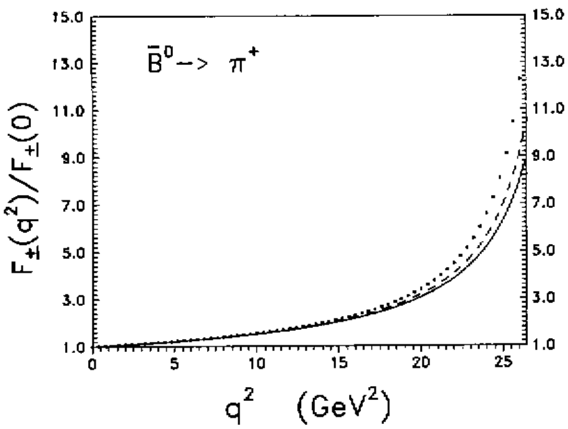


Fig. 6. The ratios $F_+(q^2)/F_+(0)$ and $F_-(q^2)/F_-(0)$ for $\bar{B}^0 \to \pi^+$ as a function of q^2. The solid curve is the ratio $F_+(q^2)/F_+(0)$, while dashed curve for $F_-(q^2)/F_-(0)$

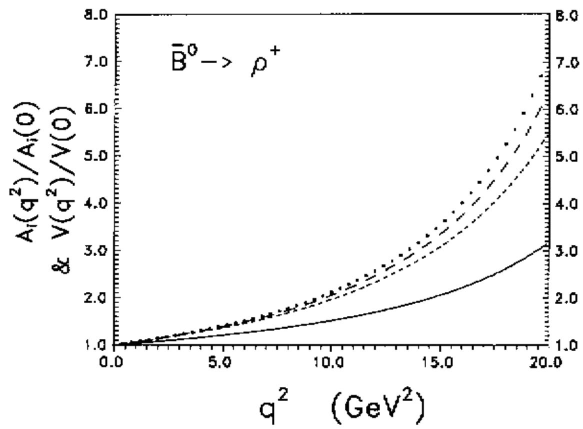


Fig. 9. The ratios $A_i(q^2)/A_i(0)$ and $V(q^2)/V(0)$ for $\bar{B}^0 \to \rho^+$ as a function of q^2. The solid curve is the ratio $A_1(q^2)/A_1(0)$, the short-dashed curve is the ratio $A_2(q^2)/A_2(0)$, the long-dashed curve is the ratio $A(q^2)/A(0)$, and the dotted curve is the ratio $V(q^2)/V(0)$

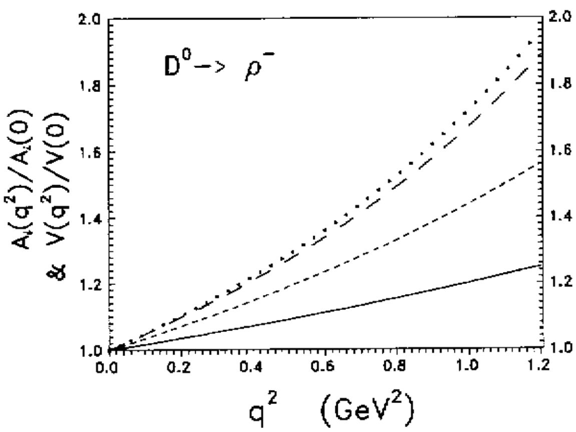


Fig. 7. The ratios $A_i(q^2)/A_i(0)$ and $V(q^2)/V(0)$ for $D^0 \to \rho^-$ as a function of q^2. The solid curve is the ratio $A_1(q^2)/A_1(0)$, the short-dashed curve is the ratio $A_2(q^2)/A_2(0)$, the long-dashed curve is the ratio $A(q^2)/A(0)$, and the dotted curve is the ratio $V(q^2)/V(0)$

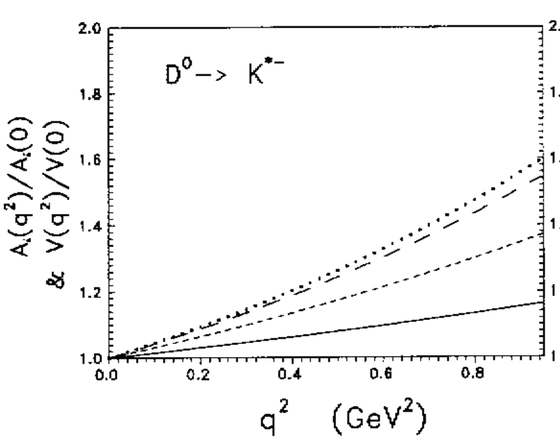


Fig. 8. The ratios $A_i(q^2)/A_i(0)$ and $V(q^2)/V(0)$ for $D^0 \to K^{*-}$ as a function of q^2. The solid curve is the ratio $A_1(q^2)/A_1(0)$, the short-dashed curve is the ratio $A_2(q^2)/A_2(0)$, the long-dashed curve is the ratio $A(q^2)/A(0)$, and the dotted curve is the ratio $V(q^2)/V(0)$

Table 7. Pole masses fitted from numerical analysis. Units are GeV

$\bar{Q}q_2$	m_{F_+}	m_{F_-}	m_{A_1}	m_{A_2}	m_A	m_V
$\bar{c}d$	1.95	1.8	2.45	2.45	2.10	2.05
$\bar{c}s$	2.1	1.8	2.6	2.55	2.2	2.13
$\bar{b}u$	5.45	5.4	5.4	5.9	5.75	5.65

Table 8. Partial decay rates for semileptonic D_- and B_- decays into π or K

	$\Gamma(10^{11}s^{-1})$
$\Gamma(D^0 \to \pi^- e^- \nu_e)$	$(1.34^{+0.44}_{-0.38})\lvert V_{cd}\rvert^2$
$\Gamma(D^0 \to K^- e^+ \nu_e)$	$(0.86^{+0.30}_{-0.25})\lvert V_{cs}\rvert^2$
$\Gamma(\bar{B}^0 \to \pi^+ e^- \bar{\nu})$	$(0.54^{+0.16}_{-0.14})\lvert V_{ub}\rvert^2 10^2$

In Table 8 we list our results for the decay rates: $D^0 \to \pi^- e^- \nu$, $D^0 \to K^- e^+ \nu$, and $\bar{B}^0 \to \pi^+ e^- \bar{\nu}$, while in Figs. 10–12 we plot relevant electron spectra as a function of the electron energy (in the laboratory frame, i.e. in the frame in which the initial heavy meson is at rest). Note that the formulas listed in Appendix D are used for the calculations. Our result for the decay rate $\Gamma(D^0 \to K^- e^+ \nu) = (0.86^{+0.30}_{-0.25})|V_{cs}|^2 \times 10^{11}\,s^{-1}$ is consistent with the experimental data $(0.77 \pm 0.13) \times (0.975)^2\ 10^{11}\ s^{-1}$ measured by the group of E691 [45], who have taken a weighted average for $\Gamma(D \to Ke\nu)$. Our result for the decay rate $\Gamma(D^0 \to \pi^- e^- \nu) = (1.34^{+0.44}_{-0.38})|V_{cd}|^2 \times 10^{11}\,s^{-1}$ is a little lower than, but still consistent with, the experimental value $(1.9^{+1.1}_{-0.6}) \times (0.22)^2\ 10^{11}\ s^{-1}$ (PDG [33]). So far there is no reliable experimental measurement on $\Gamma(\bar{B}^0 \to \pi^+ e^- \bar{\nu})$, and our result is consistent with those reported in the other calculations except for that in Ref. [5].

In Table 9 we list our results for the decay rates: $D^0 \to \rho^- e^+ \nu$, $D^0 \to K^{*-} e^+ \nu$, and $\bar{B}^0 \to \rho^+ e^- \bar{\nu}$, while in Figs. 13–15 we plot relevant electron spectra as a function

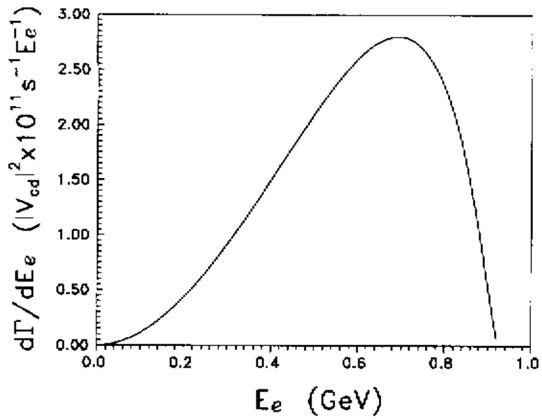


Fig. 10. The electron spectra $d\Gamma/dE_e$ is plotted as a function of the electron energy E_e for $D^0 \to \pi^- e^- \nu_e$

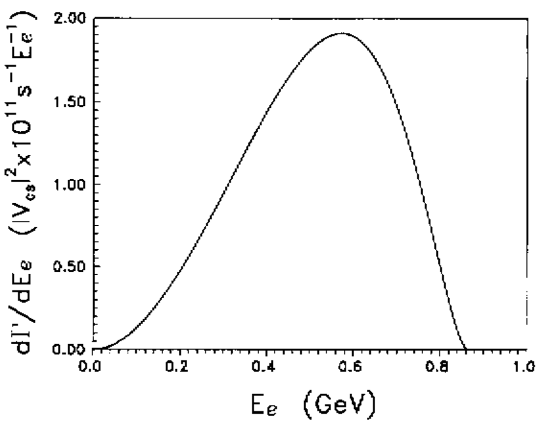


Fig. 11. The electron spectra $d\Gamma/dE_e$ is plotted as a function of the electron energy E_e for $D^0 \to K^- e^+ \nu_e$

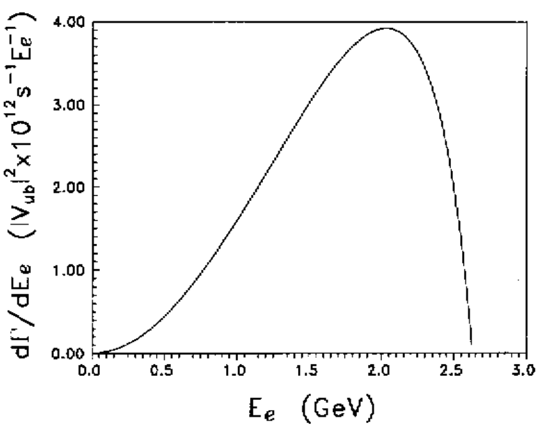


Fig. 12. The electron spectra $d\Gamma/dE_e$ is plotted as a function of the electron energy E_e for $\bar{B}^0 \to \pi^+ e^- \bar{\nu}_e$

of the electron energy. Our results for the decay rate $\Gamma(D^+ \to \bar{K}^{*0} e^+ \nu_e)$ are consistent with the experimental results obtained by the E691 [41] experiment, except the value Γ_L/Γ_T; our value is $\Gamma_L/\Gamma_T = 0.82 \pm 0.26$, while the result from the E691 experiment $\Gamma_L/\Gamma_T = 1.8^{+0.6}_{-0.4} \pm 0.3$. However, our result is quite close to the recent measure-

Table 9. Partial decay rates for semileptonic D_- and B_- decays into ρ or K^*. Γ_L stands for the portion of the rate with a longitudinal polarized meson L, Γ_+ with a positive helicity meson L, Γ_- with a negative helicity meson L. $\Gamma_T = \Gamma_- + \Gamma_-$

	$\Gamma(10^{11}s^{-1})$	Γ_L/Γ_T	Γ_+/Γ_-
$\Gamma(D^0 \to K^{*-} e^+ \nu_e)$	$(0.40 \pm 0.08)\lvert V_{cs}\rvert^2$	0.87 ± 0.20	0.16 ± 0.05
$\Gamma(D^0 \to \rho^- e^- \nu_e)$	$(0.28^{+0.15}_{-0.08})\lvert V_{cd}\rvert^2$	$0.35^{+0.37}_{-0.24}$	$0.015^{+0.043}_{-0.002}$
$\Gamma(\bar{B}^0 \to \rho^+ e^- \nu_e)$	$(0.31^{+0.09}_{-0.05})\lvert V_{ub}\rvert^2 10^2$	$0.48^{+0.46}_{-0.26}$	$0.049^{+0.043}_{-0.022}$

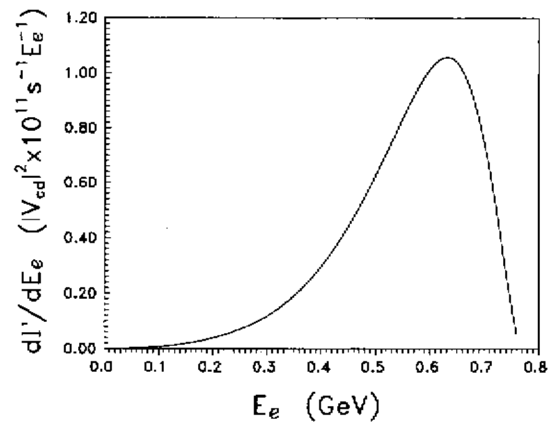


Fig. 13. The electron spectra $d\Gamma/dE_e$ is plotted as a function of the electron energy E_e for $D^0 \to \rho^- e^+ \nu_e$

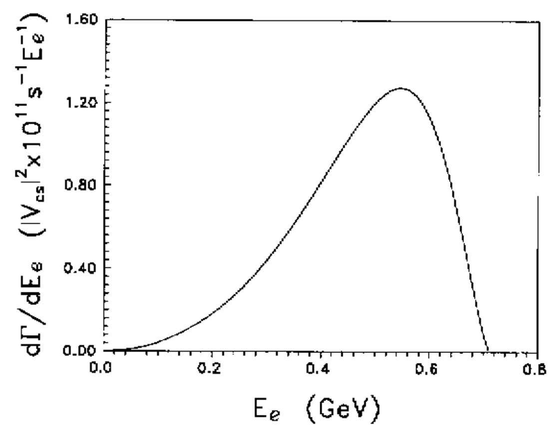


Fig. 14. The electron spectra $d\Gamma/dE_e$ is plotted as a function of the electron energy E_e for $D^0 \to K^{*-} e^+ \nu_e$

ments both from the group of E653 [42] and E687 [43]:

$$\Gamma_L/\Gamma_T = 1.18 \pm 0.18 \pm 0.08, \quad \text{E653},$$

$$\Gamma_L/\Gamma_T = 1.20 \pm 0.13 \pm 0.13, \quad \text{E687}. \tag{65}$$

The decay $D \to \rho l \nu$ has recently been measured by the E653 experiment [50]. They find

$$\frac{\Gamma(D^+ \to \rho^0 \mu^+ \nu)}{\Gamma(D^+ \to \bar{K}^{*0} \mu^+ \nu)} = 0.044^{+0.031}_{-0.025} \pm 0.014, \tag{66}$$

while our result is

$$\frac{\Gamma(D^+ \to \rho^0 e^+ \nu)}{\Gamma(D^+ \to \bar{K}^{*0} e^+ \nu)} = (0.35 \pm 0.10) \times \frac{|V_{cd}|^2}{|V_{cs}|^2}. \tag{67}$$

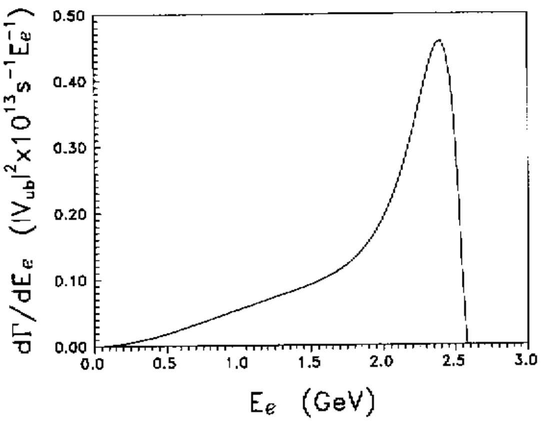


Fig. 15. The electron spectra $d\Gamma/dE_e$ is plotted as a function of the electron energy E_e for $\bar{B}^0 \to \rho^- e^- \bar{\nu}_e$

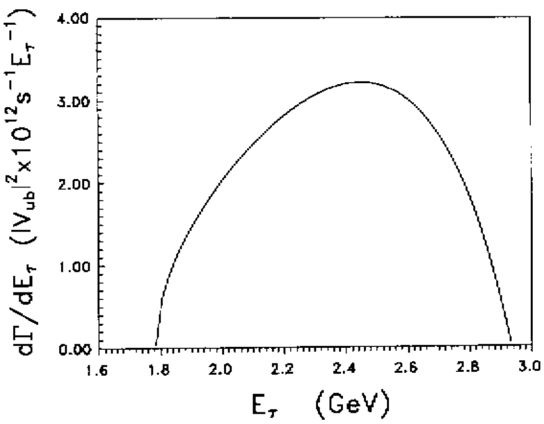


Fig. 16. The energy spectra $d\Gamma/dE_\tau$ as a function of the τ energy E_τ for $\bar{B}^0 \to \pi^+ \tau^- \bar{\nu}_\tau$

If taking $|V_{cd}|^2/|V_{cs}|^2 \approx 0.05$, we obtain our value $\Gamma(D^+ \to \rho^0 e^+ \nu)/\Gamma(D^+ \to \bar{K}^{*0} e^+ \nu) \approx 0.018 \pm 0.005$, which is somewhat lower than, but still consistent with, the experimental value. [Note that in our analysis we have used the isospin relations $\Gamma(D^0 \to K^{*-} e^- \nu) = \Gamma(D^+ \to \bar{K}^{*0} e^+ \nu)$ and $\Gamma(D^0 \to \rho^- e^+ \nu) = 2\Gamma(D^- \to \rho^0 e^+ \nu)$.]

For the decay $\bar{B}^0 \to \rho^+ e^- \nu_e$, so far there is no reliable experimental result, except the upper limit on the branch ratio. For instance, CLEO [51] gave $B(B^- \to \rho^0 e^- \nu) < (1.6 - 2.7) \times 10^{-4}$, while our result, $\Gamma(\bar{B}^0 \to \rho^+ e^- \nu) = (0.31 {}^{+0.09}_{-0.05}) |V_{ub}|^2 10^{13}\,\mathrm{s}^{-1}$, is smaller than all existing theoretical predictions.

In closing this section, we may mention briefly the semileptonic B-meson decays including a nonzero τ mass. These processes have not been considered by means of QCD sum rules so far, since the form factors F_- and A should also be determined. The relevant formulas [52] are contained in appendix D. In Figs. 16 and 17, we plot the energy spectra $d\Gamma/dE_\tau$ a function of the τ energy E for $\bar{B}^0 \to \pi^+ \tau^- \bar{\nu}_\tau$ and $\bar{B}^0 \to \rho^- \tau^- \bar{\nu}_\tau$, respectively. The decay rates are given by

$$\Gamma(\bar{B}^0 \to \pi^- \tau^- \bar{\nu}_\tau) = (0.27 \pm 0.07) \times |V_{ub}|^2 \times 10^{13}\,\mathrm{s}^{-1},$$
$$\Gamma(\bar{B}^0 \to \rho^+ \tau^- \bar{\nu}_\tau) = (0.15 \pm 0.01) \times |V_{ub}|^2 \times 10^{13}\,\mathrm{s}^{-1}, \quad (68)$$

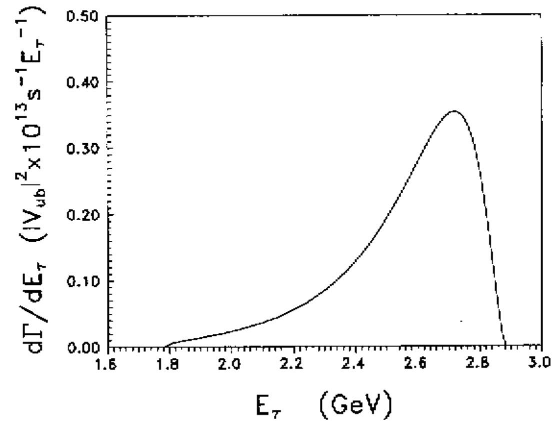


Fig. 17. The energy spectra $d\Gamma/dE_\tau$ as a function of the τ energy E_τ for $\bar{B}^0 \to \rho^+ \tau^- \bar{\nu}_\tau$

which are to be compared with data from future experiments.

IV Summary

In this paper, we have used the method of QCD sum rules to treat the semileptonic weak decay of the D or B meson into a light meson and leptons. To obtain the transition form factors, we have adopted suitable axial-vector currents, instead of pseudoscalar currents, in the description of the D and B mesons and evaluated the two-point Green's function in the presence of an external vector or axial-vector field. We find that this method can be related approximately to the traditional three-point Green's function in the heavy quark limit ($m_Q \to \infty$). [See Sect. II and Appendix B.] Our results indicate that the form factors have simple dipole or monopole behavior, which is compatible with the assertion of [49]. We have obtained results on the various form factors for the semileptonic decays of the D and B mesons into light mesons and studied various decay processes such as $\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}_\ell$ and $\bar{B}^0 \to \rho^+ \ell^- \bar{\nu}_\ell$. The method allows us to disentangle strong interaction effects, thereby providing a better determination of the Cabibbo-Kobayashi-Maskawa matrix elements from the relevant experimental data.

Acknowledgement. We would like to thank Dr. Hai-Yang Cheng for valuable discussions. This work was supported by the National Science Council of the Republic of China through the grant NSC84-2112-M002-021Y.

Appendix A: some useful integrals

In this appendix, we list some useful formulae related to using Cutkosky's rule [25] to treat the dispersive part of the multi-loop integral (such as (11) in the text).

$$I = \int d^4k \delta(k^2) \delta[(k+p')^2] \delta[(k+p)^2 - m_Q^2]$$
$$= \frac{\pi}{2\lambda^{1/2}}, \quad (A.1)$$

$$I_\mu = \int d^4k\delta(k^2)\delta[(k+p')^2]\delta[(k+p)^2 - m_Q^2]k_\mu$$

$$= \frac{\pi}{2\lambda^{3/2}}[s'(2\Delta - u)p_\mu + (2ss' - \Delta u)p'_\mu], \tag{A.2}$$

$$I_{\mu\nu} = \int d^4k\delta(k^2)\delta[(k+p')^2]\delta[(k+p)^2 - m_Q^2]k_\mu k_\nu$$

$$= -\frac{\pi}{4\lambda^{3/2}}s'[\Delta(m_Q^2 - q^2) - s'm_Q^2]g_{\mu\nu}$$

$$+ \frac{\pi}{2\lambda^{5/2}}s'^2[\lambda - 6(\Delta(m_Q^2 - q^2) - s'm_Q^2)]p_\mu p_\nu$$

$$+ \frac{\pi}{2\lambda^{5/2}}[\lambda\Delta^2 - 6ss'(\Delta(m_Q^2 - q^2) - s'm_Q^2)]p'_\mu p'_\nu$$

$$- \frac{\pi}{2\lambda^{5/2}}s'[\lambda\Delta - 3u(\Delta(m_Q^2 - q^2) - s'm_Q^2)]$$

$$\times(p'_\mu p_\nu + p_\mu p'_\nu), \tag{A.3}$$

$$I_{\mu\nu\lambda} = \int d^4k\delta(k^2)\delta[(k+p')^2]\delta[(k+p)^2 - m_Q^2]k_\mu k_\nu k_\lambda$$

$$= \frac{\pi}{\lambda^{7/2}}s'^3\left(\Delta - \frac{u}{2}\right)$$

$$\times[\lambda - 10(\Delta(m_Q^2 - q^2) - s'm_Q^2)]p_\mu p_\nu p_\lambda$$

$$+ \frac{\pi}{\lambda^{7/2}}\left(s's - \frac{\Delta u}{2}\right)$$

$$\times[\lambda\Delta^2 - 10ss'(\Delta(m_Q^2 - q^2) - s'm_Q^2)]p'_\mu p'_\nu p'_\lambda$$

$$- \frac{\pi}{2\lambda^{7/2}}s'^2[\lambda(\Delta^2 - ss') + (\Delta(m_Q^2 - q^2) - s'm_Q^2)$$

$$\times(3\lambda - 10\Delta u + 20ss')]$$

$$\times(p'_\mu p_\nu p_\lambda + p_\mu p'_\nu p_\lambda + p_\mu p_\nu p'_\lambda)$$

$$- \frac{\pi}{2\lambda^{7/2}}s'[\lambda\Delta(ss' - \Delta^2) + (\Delta(m_Q^2 - q^2) - s'm_Q^2)$$

$$\times(3\lambda\Delta - 10uss' + 20\Delta ss')]$$

$$\times(p'_\mu p'_\nu p_\lambda + p'_\mu p_\nu p'_\lambda + p_\mu p'_\nu p'_\lambda)$$

$$- \frac{\pi}{2\lambda^{5/2}}s'^2\left(\Delta - \frac{u}{2}\right)(\Delta(m_Q^2 - q^2) - s'm_Q^2)$$

$$\times(g_{\mu\nu}p_\lambda + g_{\mu\lambda}p_\nu + g_{\nu\lambda}p_\mu)$$

$$- \frac{\pi}{2\lambda^{5/2}}s'\left(ss' - \frac{\Delta u}{2}\right)(\Delta(m_Q^2 - q^2) - s'm_Q^2)$$

$$\times(g_{\mu\nu}p'_\lambda + g_{\mu\lambda}p'_\nu + g_{\nu\lambda}p'_\mu), \tag{A.4}$$

where $s = p^2$, $s' = p'^2$, $\Delta = s - m_Q^2$, $u = s + s' - q^2$, and $\lambda = u^2 - 4ss'$.

Appendix B

To determine the induced condensates $\chi_{Qq_2}^{\mathscr{V}(\mathscr{A})}$, we first need to know both the values of $s_0^{\mathscr{V}(\mathscr{A})}$ and $f^{\mathscr{V}(\mathscr{A})}$. Therefore we consider the following two-point Green's functions:

$$\Pi_{\alpha\beta\nu}^{\mathscr{V}} = \int d^4xe^{iqx}\langle 0|\mathrm{T}\{\bar{Q}(y)\sigma_{\alpha\beta}q_2(y), \bar{q}_2(x)\gamma_\mu Q(x)\}|0\rangle \tag{B.1a}$$

$$\Pi_{\alpha\beta\nu}^{A} = \int d^4xe^{iqx}\langle 0|\mathrm{T}\{\bar{Q}(y)\sigma_{\alpha\beta}\gamma_5 q_2(y), \bar{q}_2(x)\gamma_\mu\gamma_5 Q(x)\}|0\rangle \tag{B.1b}$$

At the quark level (rhs), the results of (B.1) are

$$\Pi_{\alpha\beta\nu}^{\mathscr{V}} = (g_{\nu\alpha}q_\beta - g_{\beta\nu}q_\alpha)\left(\frac{3}{8\pi}\int_{m_Q^2}^{s_0} ds\frac{m_Q}{s - q^2}\right.$$

$$\times\left[1 - 2\left(\frac{m_Q^2}{s}\right) + \left(\frac{m_Q^2}{s}\right)^2\right] + \frac{\langle\bar{q}_2q_2\rangle}{q^2 - m_Q^2}$$

$$\left. + \langle g_c\bar{q}_2\sigma Gq_2\rangle\left[\frac{-1}{3(q^2 - m_Q^2)^2} + \frac{m_Q^2}{2(q^2 - m_Q^2)^3}\right]\right), \tag{B.2a}$$

$$\Pi_{\alpha\beta\nu}^{\mathscr{A}} = (g_{\nu\alpha}q_\beta - g_{\beta\nu}q_\alpha)\left(\frac{3}{8\pi}\int_{m_Q^2}^{s_0} ds\frac{m_Q}{s - q^2}\right.$$

$$\times\left[1 - 2\left(\frac{m_Q^2}{s}\right) + \left(\frac{m_Q^2}{s}\right)^2\right] - \frac{\langle\bar{q}_2q_2\rangle}{q^2 - m_Q^2}$$

$$\left. - \langle g_c\bar{q}_2\sigma Gq_2\rangle\left[\frac{-1}{3(q^2 - m_Q^2)^2} + \frac{m_Q^2}{2(q^2 - m_Q^2)^3}\right]\right). \tag{B.2b}$$

At the hadron level, we write the form as

$$\Pi_{\alpha\beta\nu}^{\mathscr{V}(\mathscr{A})} = (g_{\nu\alpha}q_\beta - g_{\beta\nu}q_\alpha)\frac{f^{\mathscr{V}(\mathscr{A})}}{m_{f^{\mathscr{V}(\mathscr{A})}}^2 - q^2}. \tag{B.3}$$

Note that the contributions of higher excited states are transferred to the left hand side of the sum rules and approximated by the perturbative continuum, which begin from a certain threshold, $s_0^{\mathscr{V}(\mathscr{A})}$. After performing the Borel transform, we obtain the QCD sum rules for $f^{\mathscr{V}(\mathscr{A})}$:

$$f^{\mathscr{V}(\mathscr{A})} = \left\{\frac{3}{8\pi}\left[M^2m_Q(e^{-(m_Q^2/M^2)} - e^{-(s_0/M^2)})\right.\right.$$

$$- m_Q^5\left[\frac{e^{-(s_0/M^2)}}{s_0} - \frac{e^{-(m_Q^2/M^2)}}{m_Q^2}\right.$$

$$\left. + \frac{1}{M^2}E\left(\frac{m_Q^2}{M^2}\right) - \frac{1}{M^2}E\left(\frac{s_0}{M^2}\right)\right]$$

$$\left. - 2m_Q^3\left[E\left(\frac{m_Q^2}{M^2}\right) - E\left(\frac{s_0}{M^2}\right)\right]\right]$$

$$\left. \pm\langle\bar{q}_2q_2\rangle e^{-(m_Q^2/M^2)}\left(1 - \frac{m_Q^2}{3M^2} - \frac{m_0^2m_Q^2}{4M^4}\right)\right\}e^{(m_{\mathscr{V}}^2/M^2)}, \tag{B.4}$$

where $E(x) = \int_x^\infty t^{-1}e^{-t}dt$. In the numerical analysis, we choose to use the standard parameter set ($m_c = 1.3$ GeV and $m_b = 4.7$ GeV), as shown in Sect. III, and also consider the corrections from anomalous dimensions. (For example see 62(a)–(c).) One should note that there is a constraint between s_0 and f:

$$\chi^{\mathscr{V}(\mathscr{A})} = \pm\frac{\langle\bar{q}q\rangle}{q^2 - m_Q^2}, \tag{B.5}$$

when $q^2 \to -\infty$. In Figs. 18 and 19, we plot the resultant $f^{\mathscr{V}}$ and $f^{\mathscr{A}}$, respectively, as a function of the Borel mass

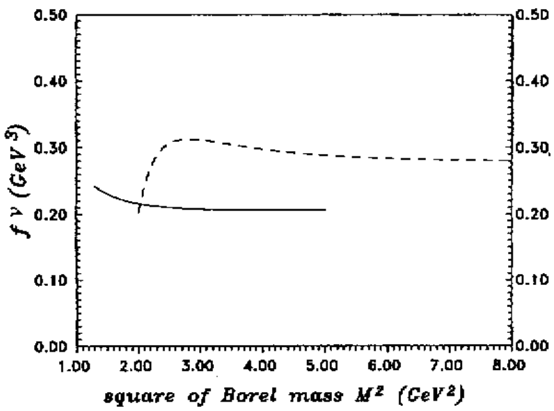


Fig. 18. The $f^{\mathscr{V}}$ is plotted as a function of the Borel mass squared M^2. The solid curve is the result for $f^{\mathscr{V}}_{cd}$, while the dashed curve for $f^{\mathscr{V}}_{bu}$

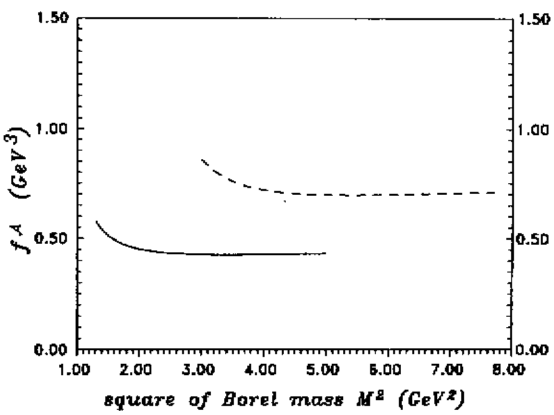


Fig. 19. The $f^{\mathscr{A}}$ is plotted as a function of the Borel mass squared M^2. The solid curve is the result for $f^{\mathscr{A}}_{cd}$, while the dashed curve for $f^{\mathscr{A}}_{bu}$

squared. To obtain reliable values of $f^{\mathscr{V}(\mathscr{A})}$, we perform the analyses in a suitable Borel mass ranges, in which both the contributions from the continuum and the higher dimension correction (at the quark level) are controllable. Numerically, the final results are shown both in Table 1 and 2. As a matter of fact, in the large m_Q limit, the contributions of the induced condensates are of marginal importance, when we discuss weak form factors (See the text in this paper). That is, it is reasonable to substitute (28) (or (54)) by (B.5), and the condition is then the same as that in the three-point approach.

Appendix C

In this appendix, we give a proof of (43) in the text. We begin by considering the following calculation about the trace of the γ-matrix,

$$\mathrm{Tr}(\hat{p}'\gamma_\mu\hat{p}\hat{p}\gamma_\nu\gamma_\lambda\gamma_5). \tag{C.1}$$

Or, we have

$$\mathrm{Tr}(\hat{p}'\gamma_\mu\hat{p}\hat{p}\gamma_\nu\gamma_\lambda\gamma_5) = 4ip^2\varepsilon_{\alpha\mu\nu\lambda}p'^\alpha. \tag{C.2}$$

On the other hand, the trace can be rewritten as

$$\begin{aligned}\mathrm{Tr}(\hat{p}'\gamma_\mu\hat{p}\hat{p}\gamma_\nu\gamma_\lambda\gamma_5) &= -\mathrm{Tr}(\hat{p}'\hat{p}\gamma_\mu\hat{p}\gamma_\nu\gamma_\lambda\gamma_5) + 2p_\mu\mathrm{Tr}(\hat{p}'\hat{p}\gamma_\nu\gamma_\lambda\gamma_5)\\ &= 4i(-p_\nu\varepsilon_{\alpha\beta\mu\lambda}p'^\alpha p^\beta + p_\lambda\varepsilon_{\alpha\beta\mu\nu}p'^\alpha p^\beta\\ &\quad + (p'\cdot p)\varepsilon_{\alpha\nu\lambda\mu}p^\alpha + p_\nu\varepsilon_{\alpha\beta\nu\lambda}p'^\alpha p^\beta).\end{aligned} \tag{C.3}$$

Comparing (C.2) and (C.3), we obtain

$$p_\nu T_{\lambda\mu} + p_\mu T_{\nu\lambda} + p_\lambda T_{\mu\nu} = -(p\cdot p')S_{\mu\nu\lambda} + p^2 S'_{\mu\nu\lambda}. \tag{C.4}$$

Letting $p \leftrightarrow p'$, we may also obtain

$$p'_\nu T_{\lambda\mu} + p'_\mu T_{\nu\lambda} + p'_\lambda T_{\mu\nu} = (p'\cdot p)S'_{\mu\nu\lambda} - p'^2 S_{\mu\nu\lambda}. \tag{C.5}$$

Appendix D

The differential decay rates for $M \to L^{(*)}l^-\bar{\nu}_l$ are given [52] by

$$\frac{\mathrm{d}\Gamma(M\to L\,l^-\bar{\nu}_l)}{\mathrm{d}E_l\,\mathrm{d}q^2} = \frac{G_F^2}{64\pi^3}|V_{Qq_2}|^2\frac{q^2-\mu^2}{m_M^2}\left[(1-\cos^2\theta)\mathscr{H}_0^2 + \frac{\mu^2}{q^2}(\mathscr{H}_0\cos\theta + \mathscr{H}_t)^2\right]$$

and

$$\begin{aligned}&\frac{\mathrm{d}\Gamma(M\to L^*\,l^-\bar{\nu}_l)}{\mathrm{d}E_l\,\mathrm{d}q^2}\\ &= \frac{G_F^2}{128\pi^3}|V_{Qq_2}|^2\frac{q^2-\mu^2}{m_M^2}[(1-\cos\theta)^2H_-^2\\ &\quad + (1+\cos\theta)^2H_+^2 + 2(1-\cos^2\theta)H_0^2]\\ &\quad + \frac{\mu^2}{q^2}(\sin^2\theta(H_+^2+H_-^2) + 2H_0^2\cos^2\theta + 2H_t^2\\ &\quad + 4H_t\cdot H_0\cos\theta)],\end{aligned} \tag{D.1}$$

respectively, with the helicity amplitudes

$$\mathscr{H}_0 = \frac{\tilde{\lambda}^{1/2}}{(q^2)^{1/2}}F_-(q^2),$$

$$\mathscr{H}_t = \frac{1}{(q^2)^{1/2}}[(m_M^2 - m_L^2)F_+(q^2) + q^2F_-(q^2)],$$

$$H_\pm = (m_M + m_{L^*})A_1(q^2) \mp \frac{\tilde{\lambda}^{1/2}}{m_M + m_{L^*}}V(q^2),$$

$$H_0 = \frac{1}{2m_{L^*}(q^2)^{1/2}}\left[(m_M^2 - m_{L^*}^2 - q^2)(m_M + m_{L^*})A_1(q^2) - \frac{\tilde{\lambda}}{m_M + m_{L^*}}A_2(q^2)\right],$$

$$\begin{aligned}H_t = \frac{\tilde{\lambda}^{1/2}}{2m_{L^*}(q^2)^{1/2}}&[(m_M + m_{L^*})A_1(q^2) - (m_M - m_{L^*})A_2(q^2)\\ &+ 2m_{L^*}q^2A(q^2)].\end{aligned} \tag{D.2}$$

Here E_l is the lepton energy in the M rest system and μ is the lepton mass. θ is the polar angle between the meson $L^{(*)}$ and the lepton l^- in the $(l^-\bar{\nu}_l)$ system and is given by

$$\cos\theta = \frac{(m_M^2 - m_{L^{(*)}}^2 + q^2)(q^2 + \mu^2) - 4q^2 m_M E_l}{\tilde{\lambda}^{1/2}(q^2 - \mu^2)}, \qquad \text{(D.3)}$$

where $\tilde{\lambda} = (m_M^2 + m_{L^{(*)}}^2 - q^2)^2 - 4m_M^2 m_{L^{(*)}}^2$. For a fixed electron energy, q^2 varies over the region

$$q_-^2 \le q^2 \le q_+^2, \qquad \text{(D.4)}$$

where

$$q_\pm^2 = \frac{1}{a}(b \pm \sqrt{b^2 - ac}),$$

$$a = m_M^2 + \mu^2 - 2m_M E_l,$$

$$b = m_M E_l(m_M^2 - m_L^2 + \mu^2 - 2m_M E_l) + \mu^2 m_L^2,$$

$$c = \mu^2[(m_M^2 - m_L^2)^2 + \mu^2 m_M^2 - (m_M^2 - m_L^2)2m_M E_l], \qquad \text{(D.5)}$$

and the related range of E_l is: $\mu < E_l < (m_M^2 - m_{L^{(*)}}^2 + \mu^2)/(2m_M)$.

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QCD sum rules and chiral symmetry breaking

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Abstract. We investigate, in the context of QCD sum rules, the role of Goldstone bosons arising from chiral symmetry breaking, especially on the need of matching the world of hadrons to that of some effective chiral quark theory. We describe how such matching yields predictions on induced condensates which reproduce the observed value of the strong πNN coupling constant. Our result indicate that the observed large πNN coupling comes primarily from the nonperturbative induced-condensate effect. On the other hand, the prediction on the parity-violating weak πNN coupling, $f_{\pi NN} \sim (3.0 \pm 0.5) \times 10^{-7}$ is in good agreement with the value ($\sim 2 \times 10^{-7}$) obtained by Adelberger and Haxton from an overall fit to the existing data, but may appear to be slightly too large (but not yet of statistical significance) as compared to the most recent null experiment. Altogether, we wish to argue that the proposal of matching onto a chirally broken effective theory should enable us to treat, in a quantitative and consistent manner, the various nonleptonic strong and weak processes involving Goldstone bosons.

I Introduction

The problem of strong interaction physics has been with us for more than half a century, despite the fact that the very nature of the problem has been changing with the so-called "underlying theory" which nowadays is taken universally to be quantum chromodynamics or QCD. Although the asymptotically free nature of QCD allows us to test the candidate theory at high energies, the nonperturbative feature dominates at low energies so that it remains almost impossible to solve problems related to hadrons or nuclei, except perhaps through lattice simulations but it is unlikely that, through the present-day computation power which is still not quite adequate, simulations would give us most of what we wish to know. As a result, it would be useful to evaluate the merits of using the method of QCD sum rules [1] to bridge between the description of the hadron properties and what we may expect from QCD.

The ground state, or the vacuum, of QCD is known to be nontrivial, in the sense that there are non-zero condensates, including gluon condensates, quark condensates, and perhaps infinitely many higher-order condensates. In such a theory, propagators, i.e. causal Green's functions, such as the quark propagator

$$iS_{ij}^{ab}(x) \equiv < 0 \mid T(q_i^a(x)\bar{q}_j^b(0)) \mid 0 >, \tag{1}$$

carry all the difficulties inherent in the theory. Higher-order condensates, such as a four-quark condensate,

$$< 0 \mid T(\bar{\psi}(z)\gamma_\mu\psi(z)q_i^a(x)\bar{q}_j^b(0)) \mid 0 >,$$

with $\psi(z)$ also labeling a quark field, represent an infinite series of unknowns unless some useful ways for reduction can be obtained. As the vacuum, $\mid 0 >$, is highly nontrivial, there is little reason to expect that Wick's theorem (of factorization), as obtained for free quantum field theories, is still of validity. Thus, we must look for alternative routes in order to obtain useful results or relations.

Accordingly, it might be useful, before embarking on the subject of chiral symmetry breaking in relation to QCD sum rules, to detour slightly by considering the feasibility of working directly with the various matrix elements (rather than the operators). To this end, we note that, at the classical level, we may start with the QCD lagrangian [2],

$$\begin{aligned}\mathcal{L}_{QCD} = &\frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^{\mu a})^2 \\ &-\bar{\eta}^a\partial_\mu(\partial^\mu\delta^{ac} + gf^{abc}A^{\mu b})\eta^c \\ &+\bar{\psi}\{i\gamma^\mu(\partial_\mu + ig\frac{\lambda^a}{2}A_\mu^a) - m\}\psi,\end{aligned} \tag{2}$$

with the gauge-fixing and ghost terms (i.e., the second and third terms) shown explicitly. We then have the equations for interacting fields,

$$\{i\gamma^\mu(\partial_\mu + ig\frac{\lambda^a}{2}A_\mu^a) - m\}\psi = 0; \tag{3}$$

$$\partial^\nu G_{\mu\nu}^a - 2gf^{abc}G_{\mu\nu}^b G_\nu^c + g\bar{\psi}\frac{\lambda^a}{2}\gamma_\mu\psi = 0, \tag{4}$$

which may be re-interpreted as the equations of motion for *quantized* interacting fields. As the (standard) rule for quantization, the equal-time (anti-)commutators among these quantized interating fields are identical to those among non-interacting quantized fields.

condensates.

In light of the above

$+\frac{1}{2}(1-\frac{\pi}{4f_\pi^2})\partial^\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}$

Allowing the operator $\{i\gamma^\mu\partial_\mu - m\}$ to act on the matrix element defined by (1), we obtain, making use of what we have just stated about interacting field equations and (canonical) quantization rules,

$$\{i\gamma^\mu\partial_\mu - m\}_{ik} iS_{kj}^{ab}(x) = i\delta^4(x)\delta^{ab}\delta_{ij}$$
$$+ < 0 \mid T(\{g\frac{\lambda^n}{2}A_\mu^n\gamma^\mu q(x)\}_i^a \bar{q}_j^b(0)) \mid 0 > . \tag{5}$$

This equation can be solved by splitting the propagator into a singular, perturbative part and a nonperturbative part:

$$iS_{ij}^{ab}(x) = iS_{ij}^{(0)ab}(x) + i\tilde{S}_{ij}^{ab}(x), \tag{6}$$

with

$$iS_{ij}^{(0)ab}(x) \equiv \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} iS_{ij}^{(0)ab}(p), \tag{7a}$$

$$iS_{ij}^{(0)ab}(p) = \delta^{ab}\frac{i(\not p + m)_{ij}}{p^2 - m^2 + i\epsilon}. \tag{7b}$$

Accordingly, we have

$$\{i\gamma^\mu\partial_\mu - m\}_{ik} i\tilde{S}_{kj}^{ab}(x)$$
$$=< 0 \mid T(\{g\frac{\lambda^n}{2}A_\mu^n\gamma^\mu q(x)\}_i^a \bar{q}_j^b(0)) \mid 0 > . \tag{8}$$

With the fixed-point gauge,

$$A_\mu^n(x) = -\frac{1}{2}G_{\mu\nu}^n x^\nu + \cdots, \tag{9}$$

we may then solve the nonperturbative part $i\tilde{S}_{ij}^{ab}(x)$ as a power series in x^μ,

$$i\tilde{S}_{ij}^{ab}(x) = -\frac{1}{12}\delta^{ab}\delta_{ij} < \bar{q}q > + \frac{i}{48}m(\gamma_\mu x^\mu)_{ij}\delta^{ab} < \bar{q}q >$$
$$+ \frac{1}{192} < \bar{q}g\sigma\cdot Gq > \delta^{ab}x^2\delta_{ij} + \cdots, \tag{10}$$

The first term is the integration constant which defines the so-called "quark condensate", while the mixed quark-gluon condensate appearing in the third term arises because of (8) and (9). It is obvious that the series (10) is a short-distance expansion, which converges for small enough x_μ.

A practitioner would recognize immediately that (10) is just the standard quark propagator cited in most papers in QCD sum rules. The approach which we suggest here is based upon two key elements, namely, the set of interacting field equations *plus* the rule of canonical quantization (for interacting fields). The equations which we obtain, such as (5), are much the same as the set of Schwinger-Dyson equations (for the matrix elements). An important aspect in our derivation is that the nontriviality of the vacuum $\mid 0 >$ is observed at every step – a central issue in relation to QCD. Although (10) seems to be standard (and natural), our approach makes it intuitive and relatively straightforward to work out the problem to an arbitrary order as well as to treat many other (propagator-like) matrix elements such as those to be used later in the paper.

In light of the fact that the vacuum $\mid 0 >$ is nontrivial (a fact which renders useless many lessons from perturbative quantum field theory), the route of starting directly from the interacting field equations, augmented by the canonical quantization rule, may be pursued seriously. The method, in which renormalizations need to be defined order by order in a consistent manner, may be contrasted with the usual path-integral formulation in which Green's functions may be derived, in an elegant manner, from the generating functional. The method should be seriously exploited mainly because, just like what we have done in the above, it might offer some new insights for the problem at hand, i.e. QCD.

As another important exercise, we may split the gluon propagator into the singular, perturbative part and the nonperturbative part. For the nonperturbative part, our interacting-field-equation-based approach yields

$$g^2 < 0 \mid: G_{\mu\nu}^n(x)G_{\alpha\beta}^m(0): \mid 0 >$$
$$= \frac{\delta^{nm}}{96} < g^2G^2 > (g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})$$
$$-\frac{\delta^{nm}}{192} < g^3G^3 > \{x^2(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}) - g_{\mu\alpha}x_\nu x_\beta$$
$$+g_{\mu\beta}x_\nu x_\alpha - g_{\nu\beta}x_\mu x_\alpha + g_{\nu\alpha}x_\mu x_\beta\} + O(x^4), \tag{11}$$

with

$$< g^2G^2 > \equiv < 0 \mid: g^2G_{\mu\nu}^n(0)G^{n\mu\nu}(0): \mid 0 >, \tag{12a}$$

$$< g^3G^3 > \equiv < 0 \mid: g^3 f^{abc}g^{\mu\nu}G_{\mu\alpha}^a(0)$$
$$\times G^{b\alpha\beta}(0)G_{\beta\nu}^c(0): \mid 0 > . \tag{12b}$$

Again, the first term in (11) is the integration constant for the differential equation satisfied by the gluon propagator. Note that inclusion of the gluon condensate $< g^2G^2 >$ in (11) is still standard but the triple gluon condensate $< g^3G^3 >$ is a new entry required by the interacting field equation (4). Our approach indicates when condensates of entirely new types should be introduced as we try to perform operator-product expansions to higher dimensions.

The method of QCD sum rules may now be regarded as the various approaches in which we try to exploit the roles played by the quark and gluon condensates which enter (10) and (11) for problems involving hadrons. To this end, we wish to set up a quantum field theory in which the vacuum is nontrivial, while leaving some condensates as parameters to be determined *via* first principle, such as from lattice simulations of QCD. The aim is such that, once the fundamental condensate parameters are given, one can make predictions on the various physical quantities associated with hadrons. Of course, the method of QCD sum rules in its present broad context has its orgin in [1].

II Induced condensates and the effective chiral quark theory

The quark condensate which enters the various QCD sum rules, such as the Belyaev-Ioffe nucleon mass sum rule [3] and the sum rules [4] for the nucleon axial couplings, is perhaps the best known parameter among all condensates:

$$a \equiv -(2\pi)^2 < \bar{q}q > \simeq 0.546\, GeV^3, \tag{13}$$

which is the order parameter in the chirally broken phase of QCD. It is related to the pion decay constant f_π with $4\pi f_\pi$ being the scale governing the convergence property of the expansion in chiral perturbation theory. One point which is worth mentioning at this juncture is that, as we vary Q^2 in probing the hadron substructure, the nonzero condensate as listed in (13) does not disappear suddenly although the effects may become negligibly small for sufficiently large Q^2 – unlike the situation when one varies the temperature T and sees some phase transition at some critical temperature T_c. As long as we submit to the picture where the condensate (13) differs from zero, it is important to recognize that we are working with a condensated phase of QCD – which already differs from the naive QCD (without condensation). It is also natural to ask what other effects such condensation (phase transition) brings to light. For instance, it is believed that the spontaneous symmetry breaking leading to sizable condensates also gives rise to the dynamically-generated or constituent mass to a quark. Such spontaneous symmetry breaking refers to the breaking of $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry for QCD of N_f flavors of almost massless quarks into the $SU(N_f)_V$ symmetry which, according to Goldstone theorem, must give rise to $N_f^2 - 1$ almost massless Goldstone bosons, pseudoscalar bosons in the present case (with pions and other pseudoscalar mesons as the natural candidate). If we restrict ourselves to the world of up and down quarks (as we shall do in the rest of this paper), there is little reason to reject the role played by Goldstone pions while accepting quite large empirical values for the various condensates.

In light of the above consideration, we are led to investigate, in the context of QCD sum rules, the role of Goldstone bosons arising from chiral symmetry breaking, especially on the need of matching the world of hadrons to that of some effective chiral quark theory. We immediately recognize that such proposal has many raminifications for the method of QCD sum rules. In any event, the proposal should first be checked out to leading order in the Goldstone fieldss – such as very small Goldstone pion fields. A natural problem for such investigation is the determination of the strong πNN coupling, for which we shall show that the proposal indeed yields predictions on induced condensates which are just needed to reproduce the observed value and that such large πNN coupling comes primarily from the nonperturbative induced-condensate effect. A closely related question is the parity-violating (p.v.) weak πNN coupling which is considerably more complicated since it involves calculation (and regularization) of three-loop diagrams. In any event, it is of importance to use the determination of the πNN couplings as the first testing ground for the proposal. This is what we wish to do in the present paper, while having to put off investigations of some other serious issues for the future.

What should we use while still insisting on using quarks and gluons as the effective degrees of freedom (DOF's)? A plausible choice is to accept the chiral lagrangian of Weinberg [5] and Georgi [6] as the candidate effective theory, in which Goldstone bosons arising from chiral symmetry breaking couple to quarks (and indirectly to gluons) but such couplings should vanish in the long-wavelength limit. The choice may not be unique as far as identification of Goldstone pions (with physical pions) is concerned, but many different choices may turn out to be equivalent (as some people believe to be the case).

On the other hand, it is often said that the long-distance realization, or the low-energy effective theory, of QCD is chiral perturbation theory (χPT) [7], in which Goldstone bosons are the only effective DOF's. As we go up in the energy scale, we must eventually end up in having to deal directly with QCD itself. Thus, it is also natural to anticipate that, at some intermediate energy scale, the two languages, namely χPT and QCD, can be matched onto each other or, in somewhat loose terms, the DOF's of the two theories couple in a way dictated by symmetries, yielding again the effective chiral quark theory [5, 6]. The resultant theory is unique if symmetries dictates completely the interactions, as suspected to be so in the present case.

Therefore, there leaves little room for what we may choose for the effective chiral quark theory [5, 6], onto which we shall try to map the world of hadrons. As the primary example of this paper, we consider the breaking of the $SU(2)_L \times SU(2)_R$ chiral symmetry into the vector $SU(2)$ symmetry. The effective chiral quark theory is then specified, to dimension six, by [5]

$$\begin{aligned}\mathscr{L} = \; & \bar{\psi}\{i\gamma^\mu\partial_\mu - \frac{1}{4f_\pi^2}\gamma^\mu(\partial_\mu\boldsymbol{\pi})\times\boldsymbol{\pi}\cdot\boldsymbol{\tau}\}\psi \\ & -\frac{g_A}{2f_\pi}\bar{\psi}\gamma^\mu\gamma_5\boldsymbol{\tau}\psi\cdot\partial_\mu\boldsymbol{\pi} - m\bar{\psi}\psi \\ & +\frac{1}{2}(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2})\partial^\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi} \\ & -\frac{\mu^2}{2}(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2})\boldsymbol{\pi}^2 - \bar{\psi}\Gamma_\alpha\psi\bar{\psi}\Gamma^\alpha\psi, \end{aligned} \tag{14}$$

to which the gluon DOF's should be added as in (2), but with considerably weaker coupling constant.

The method of QCD sum rules starts with a choice of some correlation function which may be evaluated at the quark level on the one hand (the so-called "left-hand side" or LHS) while may also be interpreted at the hadron level (the so-called "right-hand side" or RHS). To study the nucleon properties, for instance, we may choose

$$\Pi(p) = i\int d^4x e^{ip\cdot x} < 0|T[\eta_p(x)\bar{\eta}_p(0)]|0 > , \tag{15a}$$

with the usual choice [8] of the current $\eta(x)$

$$\eta_p(x) = \epsilon^{abc}[u^{aT}(x)C\gamma_\mu u^b(x)]\gamma^5\gamma^\mu d^c(x). \tag{15b}$$

Here C is the charge conjugation operator and $< 0 \mid \eta(0) \mid N(p) >= \lambda_N u_N(p)$ with λ_N the amplitude for overlap and $u_N(p)$ the nucleon Dirac spinor (normalized such that $\bar{u}_N(p)u_N(p) = 2m_N$). And we may work out both the LHS and RHS sides and obtain the now well-known QCD sum rules [3, 4].

This is "duality" in the sense that some quantity assumes suitable meaning both at the quark level and at the hadron level. As elucidated in detail so far, we wish to investigate the proposal that, in such duality picture, the language to be used at the quark level is the effective chiral quark theory, rather than QCD in the chirally symmetric phase (in which Goldstone bosons do not enter as relevant DOF's). Since chiral symmetry breaking has already taken place with the order parameter as given by (13), it appears quite natural that we should be trying to match the world of hadrons to a world in which we could talk about not only quarks and gluons but also sizable basic condensates.

To proceed further, we note [9] that, on an empirical ground, the quark propagator is modified in the presence of sufficiently small pion fields:

$$i\delta S^{ab}(x) = -\frac{i\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{4\pi^2x^2}\,g_{\pi q}\gamma^5\delta^{ab} + \frac{i}{24}\,\boldsymbol{\tau}\cdot\boldsymbol{\pi}\,g_{\pi q}\chi_\pi <\bar{q}q> \delta^{ab}\gamma^5 - \frac{i}{3\cdot 2^7}\,m_0^\pi <\bar{q}q> g_{\pi q}\boldsymbol{\tau}\cdot\boldsymbol{\pi}x^2\gamma^5, \tag{16}$$

with

$$<\bar{q}i\tau_j\gamma_5 q>_\pi \equiv \chi_\pi g_{\pi q}\pi_j <\bar{q}q> \tag{17a}$$

$$<\bar{q}\,i\gamma_5\tau_j\sigma\cdot Gq>_\pi \equiv m_0^\pi g_{\pi q}\pi_j <\bar{q}q>. \tag{17b}$$

Here $g_{\pi q}$ is the pion-quark coupling, which has been introduced on a purely empirical ground. The susceptibility χ_π enters in the evaluation of both strong and weak pion-nucleon coupling constants, while m_0^π enters only for the weak one.

We may begin by working in the limit of massless Goldstone pions ($\mu^2 = 0$). In this limit, all the terms involving Goldstone bosons must contain the derivative $\partial_\mu\boldsymbol{\pi}$, as shown by the lagrangian of (14). Denote these terms altogether by $\delta\mathscr{L}$. For the primary purpose of the present paper, it is sufficent to treat such pion fields as some small classical fields. Thus, we obtain the modification to the quark propagator as defined by (1).

$$\begin{aligned} i\delta S_{ij}^{ab}(x) &=< 0 \mid T(i\int d^4z\,\delta\mathscr{L}(z)q_i^a(x)\bar{q}_j^b(0)) \mid 0> \\ &= -i\int d^4z\,\partial^\mu\boldsymbol{\pi}(z)\cdot <0 \mid T(\boldsymbol{\alpha}_\mu(z)q_i^a(x)\bar{q}_j^b(0)) \mid 0> \\ &= i\frac{g_A}{2f_\pi}\int d^4z\,\boldsymbol{\pi}(z)\cdot\partial^\mu <0 \mid T(\mathbf{A}_\mu(z)q_i^a(x)\bar{q}_j^b(0)) \mid 0>, \end{aligned} \tag{18}$$

with $\mathbf{A}_\mu(x)$ the isovector axial vector current as given by

$$\mathbf{A}_\mu = \bar{\psi}\gamma^\mu\gamma_5\boldsymbol{\tau}\psi - \frac{2f_\pi}{g_A}(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2})\partial^\mu\boldsymbol{\pi} - \frac{1}{2g_Af_\pi}\bar{\psi}\gamma^\mu\boldsymbol{\tau}\times\boldsymbol{\pi}\psi. \tag{19}$$

Note that the second equality in (18) follows since we shall treat pions fields $\boldsymbol{\pi}(x)$ as some small classical field. It is straightforward to show that PCAC holds:

$$\partial_\mu\mathbf{A}^\mu = \frac{2f_\pi}{g_A}\mu^2\boldsymbol{\pi} + O(\frac{1}{f_\pi}). \tag{20}$$

A little algebra yields, in the limit with $\mu^2 = 0$,

$$\begin{aligned} i\delta S_{ij}^{ab}(x) &= -i\frac{g_A}{2f_\pi}\boldsymbol{\pi}(x)\cdot <0 \mid T(\{\gamma_5\boldsymbol{\tau}q(x)\}_i^a\bar{q}_j^b(0)) \mid 0> \\ &\quad -i\frac{g_A}{2f_\pi}\boldsymbol{\pi}(0)\cdot <0 \mid T(q_i^a(x)\{\bar{q}(0)\gamma_5\boldsymbol{\tau}\}_j^b \mid 0>. \end{aligned} \tag{21}$$

This is in fact a notable case in which the expectation value of the product of some four-quark operators can be reduced to an expression involving only two-quark operators – via the application of PCAC.

The two propagators appearing in (21) can be solved in a way introduced in the previous section for the propagator $iS_{ij}^{ab}(x)$. Namely, we use (3) and equal-time anti-commutators and obtain

$$\begin{aligned} &\{i\gamma^\mu\partial_\mu + m\}_{ik} <0 \mid T(\{\gamma_5\boldsymbol{\tau}q(x)\}_k^a\bar{q}_j^b(0)) \mid 0> \\ &= -i\delta^4(x)\delta^{ab}\gamma^5\boldsymbol{\tau} \\ &\quad - <0 \mid T(\{\gamma^5\boldsymbol{\tau}g\frac{\lambda^n}{2}A_\mu^n\gamma^\mu q(x)\}_i^a\bar{q}_j^b(0)) \mid 0>, \end{aligned} \tag{22a}$$

$$\begin{aligned} &\{i\gamma^\mu\partial_\mu - m\}_{ik} <0 \mid T(q_k^a(x)\{\bar{q}(0)\gamma_5\boldsymbol{\tau})\}_j^b \mid 0> \\ &= i\delta^4(x)\delta^{ab}\gamma^5\boldsymbol{\tau} \\ &\quad + <0 \mid T(\{g\frac{\lambda^n}{2}A_\mu^n\gamma^\mu q(x)\}_i^a\{\bar{q}(0)\gamma^5\boldsymbol{\tau}\}_j^b) \mid 0>. \end{aligned} \tag{22b}$$

The solutions to (22a) and (22b) are

$$\begin{aligned} &<0 \mid T(\{\gamma_5\boldsymbol{\tau}q(x)\}_i^a\bar{q}_j^b(0)) \mid 0> \\ &= \int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot x}\delta^{ab}\frac{i[(-\not{p}+m)\gamma^5\boldsymbol{\tau}]_{ij;mn}}{p^2-m^2+i\epsilon} \\ &\quad -\frac{1}{12}\delta^{ab}\gamma^5\boldsymbol{\tau} <\bar{q}q> - \frac{i}{48}m\gamma_\mu x^\mu\gamma^5\boldsymbol{\tau}\delta^{ab} <\bar{q}q> \\ &\quad +\frac{1}{192} <\bar{q}g\sigma\cdot Gq> \delta^{ab}\gamma^5\boldsymbol{\tau}x^2 + \cdots, \end{aligned} \tag{23a}$$

$$\begin{aligned} &<0 \mid T(q_i^a(x)\{\bar{q}(0)\gamma_5\boldsymbol{\tau})\}_j^b \mid 0> \\ &= \int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot x}\delta^{ab}\frac{i[(\not{p}+m)\gamma^5\boldsymbol{\tau}]_{ij;mn}}{p^2-m^2+i\epsilon} \\ &\quad -\frac{1}{12}\delta^{ab}\gamma^5\boldsymbol{\tau} <\bar{q}q> + \frac{i}{48}m\gamma_\mu x^\mu\gamma^5\boldsymbol{\tau}\delta^{ab} <\bar{q}q> \\ &\quad +\frac{1}{192} <\bar{q}g\sigma\cdot Gq> \delta^{ab}\gamma^5\boldsymbol{\tau}x^2 + \cdots, \end{aligned} \tag{23b}$$

We caution that the quark mass m in these equations is the one appearing in the effective chiral quark theory, (14), rather than that in the bare QCD, (2) [the chirally symmetric phase of QCD].

Substituting (23a) and (23b) back into (21) and treating $\boldsymbol{\pi}$ as a small constant field, we obtain

$$\begin{aligned} i\delta S^{ab}(x) &= +\frac{g_Am}{f_\pi}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot x}\frac{\delta^{ab}\gamma_5}{p^2-m^2+i\epsilon} \\ &\quad +i\frac{g_A}{f_\pi}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\{\frac{1}{12} <\bar{q}q> \delta^{ab}\gamma_5 \\ &\quad -\frac{1}{192} <\bar{q}g\sigma\cdot Gq> \delta^{ab}x^2\gamma_5\}. \end{aligned} \tag{24}$$

In the effective chiral quark theory [6], we use $m \approx 0.35\,GeV$. As we usually take $p^2 \sim 1\,GeV^2$, we may neglect m^2 as compared to p^2 in the first term of (24) so that the expression (24) reduces to (16) with

$$g_{\pi q} = -\frac{g_Am}{f_\pi}, \tag{25a}$$

$$\chi_\pi g_{\pi q} = \frac{2g_A}{f_\pi}, \tag{25b}$$

$$m_0^\pi <\bar{q}q> g_{\pi q} = \frac{2g_A}{f_\pi} <\bar{q}g\sigma\cdot Gq>. \tag{25c}$$

Accordingly, the pion-quark coupling $g_{\pi q}$ and the induced condensates as characterized by the susceptibilities χ_π and m_0^π are completely determined when the connection with the effective chiral quark theory is made. Numerically, we may use [6]

$$f_\pi = 93\,MeV, \qquad g_A = 0.7524, \tag{26a}$$

so that

$$g_{\pi q} \sim -2.83, \qquad \chi_\pi a = -\frac{2a}{m} \approx -3.14\, GeV^2. \tag{26b}$$

Note that we also have $m_0^\pi = \frac{2m_0^2}{m}$ with $< \bar{q}g\sigma \cdot Gq > \equiv -m_0^2 < \bar{q}q >$ but we shall discuss the parameter m_0^2 later in the paper. We re-iterate again that, since the whole derivation is made by making connection with an effective chiral quark theory, it is natural to adopt the constituent quark mass in (26).

III Pion-nucleon couplings

To investigate the proposal that the matching is done between the hadron world and the effective chiral quark theory [5, 6], we wish to consider the evaluation of both the strong and weak πNN coupling constants. Such evaluation, by itself being of fundamental importance, can readily be generalized to treat a large number of physical processes involving Goldstone bosons, such as nonleptonic decays of hyperons or heavy mesons ($\Lambda \to N\pi$, $D \to K\pi$, etc.).

To begin, it is useful to recall the Belyaev-Ioffe nucleon mass sum rule, taking into account the quark mass terms [3, 10],

$$\begin{aligned} &\frac{1}{8}M_B^6 L^{-4/9} E_2 + \frac{1}{32} < g_c^2 G^2 > M_B^2 L^{-4/9} E_0 \\ &+\frac{1}{6}a^2 L^{4/9} - \frac{a^2 m_0^2}{24 M_B^2} L^{-2/27} \\ &-\frac{maM_B^2}{4} L^{-4/9} E_0 - \frac{mam_0^2}{8} L^{-26/27} = \beta_N^2 e^{-M^2/M_B^2}, \end{aligned} \tag{27}$$

with $\beta_N^2 \equiv \frac{1}{4}(2\pi)^4 \mid \lambda_N \mid^2$. The factors containing L, $L = 0.621 \ln(10 M_B)$, give the evolution in Q^2 arising from the anamolous dimensions, and the $E_i(M_B^2)$ functions take into account excited states to ensure the proper large-M_B^2 behavior [3, 4].

By analyzing the observed mass splittings in heavy quarkonia, Narison [11] extracted an updated average value on the gluon condensate:

$$< g_c^2 G^2 > = (0.89 \pm 0.11) GeV^4, \tag{28}$$

which, albeit somewhat larger than the early value obtained by SVZ [1], appears to be more appropriate for future QCD sum rule analyses. It is unlikely that the gluon condensate suitable for light quark systems would be smaller than this value (as extracted primarily from charmonium systems). On the other hand, the quoted value on the quark condensate (as an order parameter of the phase transition), namely (13), may have an uncertainty as large as 20%, should we use the instanton picture of the QCD vacuum [12] as a guide. Making use of the Ward-Takahashi identities associated with a specific nonlinear chiral transformation, Jacquot and Richert [13] obtain

$$m_{dyn}(\mu) < \bar{q}q > = -\frac{1}{48\pi^2} < g^2(\mu) G^2 >, \tag{29}$$

where $m_{dyn}(\mu)$ is the dynamically generated quark mass at the scale μ. This relation also follows from a dilute-gas approximation of the instanton QCD vacuum. Substituting the current values of the condensates into this relation, it is seen that the dynamically generated quark mass may be interpreted as the constituent quark mass. Since the relation (29) would fail badly with the current quark mass as the input, this result may be regarded as one of the reasons why the constituent or dynamically generated mass might be more appropriate for QCD sum rule studies.

It is possible to adopt (29) as a constraint among the quark mass, and the quark and gluon condensates. Such constraint leads to the gluon condensate which is a few times larger than the original SVZ value [1] but which is pretty much in line with the results from lattice simulations.

As indicated earlier, it is likely that the mixed quark-gluon condensate is smaller than the current value $m_0^2 \sim 0.8\, GeV^2$ especially when the effective chiral quark theory is used. Lacking any further guideline on the matter, we recall the reduction factor in α_s [6] and arbitrarily take $m_0^2 \sim 0.4\, GeV^2$, but caution that further justification is needed for coming up with a better value of m_0^2. The convergence property of the mass sum rule, (27), would be a suspect for $m_0^2 \approx 0.8\, GeV^2$ but becomes very stable when a smaller value is assumed. In a sense we can make use of the nucleon mass sum rule to constrain the mixed quark-gluon condensate but the sum rule becomes so well behaved for sufficiently small m_0^2 that the constraint loses its efficiency.

The logarithmic derivative of the LHS of (27) gives the prediction on the nucleon mass M, a prediction which turns out to be quite stable with respect to the Borel mass M_B. We obtain $M = (0.94 \pm 0.02)\, GeV$ for $M_B = (1.10 \pm 0.05)\, GeV$. Substituting this value back into (27), we obtain $\beta_N^2 = (0.177 \pm 0.005) GeV^6$. An additional error of at least $\pm 0.4\, GeV$ for the predicted nucleon mass should be understood if we consider the uncertainty in the various condensates.

We turn our attention to the QCD sum rule for the strong πNN coupling [14, 9].

$$\begin{aligned} &\frac{1}{4}M_B^6 L^{-4/9} E_2 - \frac{\chi_\pi a}{8} M_B^2 L^{2/9} E_1 - \frac{11}{96} < g_c^2 G^2 > M_B^2 E_0 \\ &+\frac{1}{3}a^2 L^{4/9} + mam_0^2 L^{-26/27}\{\frac{3}{8}\ln\frac{M_B^2}{\mu^2} + \frac{49}{48}\} \\ &+\frac{1}{12} m_0^2 a^2 \frac{L^{-2/27}}{M_B^2} = g_{\pi q}^{-1}\{g_{\pi NN} + BM_B^2\}\beta_N^2\, e^{-M^2/M_B^2}. \end{aligned} \tag{30}$$

Note that the term $B\, M_B^2$ is introduced to absorb some effect from excited states (with less singular pole behavior).

Numerically, we obtain $g_{\pi NN} = -(14.8 \pm 0.7)$ for $M_B = (1.10 \pm 0.05)\, GeV$, again a fairly stable result with respect to the Borel mass. An additional error of at least ± 2.0 should be understood in view of the uncertainties involved in the various condensates.

We wish to point out that about 50% of the contribution in fact comes from the $\chi_\pi a$ term, a nonperturbative effect. This result confirms the long-standing conviction that pion physics associated with nucleons is non-perturbative and can only be obtained by taking into account the effects related to the nontrivial vacuum. Experimentally, we have $\mid g_{\pi NN} \mid \approx 13.5$, which is in general agreement with the sum rule prediction.

It is indeed remarkable that the observed strong πNN coupling has been reproduced in QCD sum rules primarily because of the inclusion of the nonperturbative terms – the first term on the LHS is the leading perturbative contribution but it is not of much numerical importance. In other words, we have essentially shown that the large πNN coupling is nonperturbative in origin.

Finally, we may also turn our attention to the weak p.v. pion-nucleon coupling, which is defined as follows [15–17].

$$\mathscr{L}^{p.v.}_{\pi NN} = \frac{f_{\pi NN}}{\sqrt{2}} \bar{\psi}(\boldsymbol{\tau} \times \boldsymbol{\pi})_3 \psi. \tag{31}$$

Only charged pions can be emitted or absorbed. Here we modify the QCD sum rule which we obtained earlier [9] (and which has some pathological behavior) by multiplying both sides (LHS and RHS) by the factor $(p^2 - M^2)$ immediately before the Borel transform. This has helped to produce a QCD sum rule which is very well behaved.

$$\begin{aligned} &\frac{G_F \sin^2\theta_W(\frac{17}{3} - \gamma)}{96\pi^2} \left[(4M_B^{10} - M^2 M_B^8) L^{-4/9} E_3 \right. \\ &-(2M_B^8 - \frac{2}{3} M^2 M_B^6)\chi_\pi \, a L^{-4/9} E_2 \\ &\left. -(M_B^6 - \frac{1}{2} M^2 M_B^4) m_0^\pi \, a \, E_1 L^{-4/9}\right] \\ &= g_{\pi q}^{-1}\{f_{\pi NN} + B' M_B^2\}\beta_N^2 \, 2M^2 \, e^{-M^2/M_B^2}. \end{aligned} \tag{33}$$

This sum rule still suffers from the fact that the contribution from the gluon condensate diagrams is yet to be included; these diagrams are much more complicated to evaluate although they are of the same order or smaller than uncertainties of our calculation.

Numerically, we obtain $f_{\pi NN} = (3.04 \pm 0.01) \times 10^{-7}$ for $M_B = (1.10 \pm 0.05) GeV$, a prediction which is even more stable than the previous two sum rules (on the nucleon mass and the strong pion-nucleon coupling). The uncertainties in the condensates and in the terms which have been neglected amount to at least $\pm 0.5 \times 10^{-7}$. About 50% of the contribution comes from the nonperturbative $\chi_\pi a$ term.

Our prediction of $f_{\pi NN} \sim (3.0 \pm 0.5) \times 10^{-7}$ (at $M_B \sim 1.0\, GeV$) is in good agreement with the value ($\sim 2 \times 10^{-7}$) obtained by Adelberger and Haxton [15] from an overall fit to the existing data, but may appear to be slightly too large (but not yet of statistical significance) as compared to the most recent experiment [18]. Once again, we see that the non-perturbative physics dictates in the prediction of the parity-violating weak πNN coupling, a typical case of non-leptonic weak interactions. Our results suggest the need to incorporate such non-perturbative effects into any systematic study of nonleptonic weak interactions. We suspect that many important issues, such as the $\Delta I = \frac{1}{2}$ rule, in relation to nonleptonic weak interactions cannot be addressed in any meaningful manner without suitable incorporation of non-perturbative condensate effects.

IV Discussion and summary

In light of the very encouraging results which we have obtained for the nucleon mass and the strong and weak πNN couplings, it is tempting to take seriously the proposal that the matching in QCD sum rules is done between the hadron world and the effective chiral quark theory [5, 6]. In particular, non-perturbative QCD effects in a large number of physical processes involving Goldstone bosons may now be treated on the same footing.

The uncertainties involved in the various condensates pose some problem in the analysis of QCD sum rules. As a numerical indicator, we note that an increase of 10% in the gluon condensate produces little change in the predicted nucleon mass M (-0.4%) and the p.v. pion-nucleon coupling $f_{\pi NN}$ (-1.5%) but reduces the value of the strong pion-nucleon coupling $\mid g_{\pi NN} \mid$ by 5.7%. A reduction of m_0^2 by 20% (on the mixed quark-gluon condensate) produces the changes of -2.1%, -12.6%, and $+15.1\%$ respectively for M, $\mid g_{\pi NN} \mid$, and $f_{\pi NN}$. On the other hand, a reduction of the quark condensate by 10% increases M by 4%, decreases $\mid g_{\pi NN} \mid$ by 2.2%, and reduces $f_{\pi NN}$ by 12.4%. These results enable us to assign some errors to our predictions cited in the previous section. Nevertheless, some room remains for an entirely different set of parameters. For instance, we may remark that a relativistic version of the effective chiral quark theory based on the Bethe-Salpeter equation (BSE) and Schwinger-Dyson equation (SDE), such as [19], could also lead to similar final numerical results. Therefore it is premature to take the present numerics as the credence towards only the nonrelativistic chiral quark model [6].

There are also many other important issues related to the proposal. For example, the adoption of the constituent quark mass could modify the existing QCD sum rule calculations, perhaps quite severely in some cases. Furthermore, pions may also be treated as "quantized fields", leading to the concept of "pion condensation" in addition to the better known pion-loop corrections, should we take the effective chiral quark theory to the next level of sophistication. Finally, there are issues concerning the importance of the instantons [12] when pions are involved.

First, we discuss briefly how Goldstone pions can be treated as "quantized fields". This involves integration of the renormalization procedure as now widely adopted for chiral perturbation theory with the standard renormalization for quantized gauge field theories. This is not a trivial task which requires further studies. To a given order (in terms of, e.g., the number of pion loops), it might be possible to obtain the "renormalized" effective chiral quark theory which can then be used for mapping onto the world of hadrons. For the purpose of the present paper, however, we note that the problems which involve such renormalizations involve terms at least bi-linear in pion fields and thus are always of higher order in nature.

Next, we may discuss the changes when the constituent quark mass instead of the very small current quark mass is adopted. As a qualitative estimate, we approximate the first term of (24) by the first term of (16) – we neglect a term of order m^2/p^2 (with $p^2 \sim 1\, GeV^2$), a (10-20)% effect. It affects most of the existing QCD sum rule predictions only in quantitative detail, but not qualitatively. This point is also exemplified in detail in our analysis of the sum rules on the nucleon mass and the strong pion-nucleon coupling.

It is clear that the proposal of matching the world of hadrons to that of the effective chiral quark theory raises many interesting questions related to fundamentals in QCD sum rules. It is also clear that it would be well beyond the scope of the present paper to address many such questions in detail. It is hoped that the encouraging results which we and z defined in the standard manner), the contribution from

photon strikes and smashes the nucleon itself. In other words, ... a finite amount to cross sections

have obtained in the present paper would help to motivate further investigations along this direction.

To sum up, we have investigated, in the context of QCD sum rules, the role of Goldstone bosons arising from chiral symmetry breaking, especially on the need of matching the world of hadrons to that of some effective chiral quark theory. We have shown how the proposal yields predictions on induced condensates which reproduce the observed value of the strong πNN coupling constant. Our result indicate that the observed large πNN coupling comes primarily from the nonperturbative induced-condensate effect. On the other hand, the prediction on the parity-violating weak πNN coupling, $f_{\pi NN} \sim (3.0 \pm 0.5) \times 10^{-7}$, is in good agreement with the value ($\sim 2 \times 10^{-7}$) obtained by Adelberger and Haxton [15] from anoverall fit to the existing data, but may appear to be slightly too large (but not yet of statistical significance) as compared to the most recent null experiment [18]. Altogether, we wish to argue that the proposal of matching onto a chirally broken effective theory should enable us to treat, in a quantitative and consistent manner, the various nonleptonic strong and weak processes involving Goldstone bosons.

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ZEITSCHRIFT FÜR PHYSIK A

Semi-inclusive Λ production and generalized sullivan processes

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Abstract. In this paper, we suggest that semi-inclusive Λ production with a high energy electron or muon beam may be employed as an effective way to pin down the possible importance of the generalized Sullivan process, in which the virtual photon may strike and smash the meson in the cloud surrounding the target proton. The role of such generalized Sullivan processes in deep inelastic scattering (DIS) has been exploited recently in connection with the observed violation of the Gottfried sum rule as well as with the so-called "proton spin crisis". Our present numerical results indicate that, in the kinematic region characterized typically by small x, large z, and a small specific range of y (with the parton variables x, y, and z defined in the standard manner), the contribution from Sullivan processes may indeed be of numerical importance as compared to that due to the fragmentation process.

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1 Introduction

Nowadays there is little doubt that strong interactions are described by quantum chromodynamics [QCD]. QCD is asymptotically free so that the theory has been tested, successfully to some extent, in high energy processes. To use QCD to describe hadron properties at low and intermediate energies, however, it has become of increasing importance (and also of increasing complication) to be able to separate clearly the nonperturbative aspects of QCD from its perturbative parts.

One of the potentially very important nonperturbative aspects has to do with pions and other low-lying pseudoscalar mesons, often recognized as the Goldstone bosons arising from breaking of $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry in QCD for N_f flavors of almost massless quarks into the $SU(N_f)_V$ symmetry. These (Goldstone) particles couple to, e.g., nucleons nonperturbatively and it has become clear that such couplings cannot be reproduced in any existing perturbative QCD calculations. Thus, effects caused by the pion clouds associated with nucleons (or with other hadrons) are likely nonperturbative in nature and would have to be carefully identified in any effort of isolating nonperturbative aspects from QCD. One imperative question is whether it is possible to go over to high energy limits so as to avoid dealing directly with such nonperturbative pion exchange effect. As we shall see, the answer to such important question may be highly nontrivial.

An important observation has in fact been made by Sullivan [1] in 1972 that in deep-inelastic-scattering (DIS) of a nucleon by leptons, the process in which the virtual photon strikes the pion emitted by the nucleon and smashes the pion into debris, *scales* like the original process where the virtual photon strikes and smashes the nucleon itself. In other words, the process will contribute by a finite amount to cross sections in the Bjorken scaling limit, i.e., the standard high energy limit as characterized by $Q^2 \to \infty$ and $\nu \equiv p \cdot q/m_p \to \infty$ with $x \equiv Q^2/(2m_p\nu)$ held fixed. Following Sullivan's derivation [1], we have, for the process depicted pictorially by Fig. 1a but with K^+ and Λ replaced by π and N, respectively,

$$\delta F_{2N}^{\pi}(x,Q^2) = \int_x^1 dy f_\pi(y) F_{2\pi}(\frac{x}{y},Q^2) \tag{1}$$

$$f_\pi(y) = \frac{3}{16\pi^2}(\frac{f_{\pi NN}2m_N}{\mu})^2 y \int_{-\infty}^{t^m} dt \frac{(-t)\mid F_\pi(t)\mid^2}{(-t+\mu^2)^2} \tag{2}$$

with $t^m = -\frac{m_N^2 y^2}{(1-y)}$. Here m_N and μ are, respectively, the nucleon mass and the pion mass. $F_{2\pi}(x)$ is the pion structure function, as would be measured in DIS with the pion as the target. $\delta F_{2N}(x)$ is the correction to the nucleon structure function due to the specific Sullivan process. $f_\pi(y)$ is the probability of finding a pion with momentum fraction y of the nucleon's momentum. $F_\pi(t)$ is the form factor which characterizes the t-dependence of the πNN coupling $f_{\pi NN}$.

It is of great interest to look into the structure of (1) and (2) since it indicates that the correction $\delta F_{2N}(x)$ does not vanish in the Bjorken limit as long as the pion structure function $F_{2\pi}(x,Q^2)$ does not vanish much more rapidly compared to the nucleon structure function $F_{2N}(x)$ (which would be naturally the case). As we shall see later in the paper, (1) and (2) may also be verified using the light-cone perturbation theory so that little doubt could be cast into the validity of (1) and (2). Nevertheless, this result has the severe implication, an implication largely ignored by the high energy physics community so far, that the pionic effect addressed here appears to be one of the nonperturbative QCD effects which cannot be thrown away by simply going over to the high energy limit.

The pion exchange effect indicated by (1) and (2) has been employed by many authors in a variety of contexts. For instance, Thomas and Frankfurt et al. [2] have attempted to attribute the observed asymmetry $\frac{1}{2}[\bar{u}(x)+\bar{d}(x)]-\bar{s}(x)$ to Sullivan processes and obtained a nucleon bag radius of about 0.8 fermi or a monopole πNN form factor of about 500 MeV (an unusally soft form factor). On the other hand, the so-called EMC effect, observed originally by the EMC collaboration [3] and later confirmed by others [4-6], may be explained by the pion-excess picture [7-10] which is based upon a treatment of the Sullivan process in nuclear medium (as contrasted with that in free space). Recently, Henley and Miller and several other authors [11-13] have related Sullivan processes to the violation of the Gottfried sum rule [14], as observed by the NMC group [15]. In addition, much interest has been generated in the community by the experiments [16] to pin down the nucleon spin structure functions – leading to some puzzle often referred to as the "proton spin crisis". It was pointed out [17] that such puzzle can also be understood as depolarization of the valent quark spins by the presence of pseudoscalar Goldstone bosons (thus forcing the core to reverse its spin).

Of course, it may be argued that in all the experiments cited above the relevant Q^2 is typically in the range of a few GeV^2 up to a little over 10 GeV^2 which may not be high energy enough and thus cannot easily deny the meson-exchange interpretations. However, the fact that such meson-exchange interpretations work so well should already be quite disturbing to most high energy theorists, especially those who has ignored completely the nonperturbative pion exchange effects until now – not mentioning that the contribtions from such processes may actualy persist in the very high energy limit. For this sake, it is therefore of importance to examine in the near future if there are other important manifestations of the nonperturbative meson-exchange effect in high energy processes which can be tested experimentally and should be isolated carefully in future theoretical works. To this end, we choose in this paper to consider, as an important example, the semi-inclusive Λ production in deep inelastic scattering (DIS) by a charged lepton beam (e or μ):

$$l+p\longrightarrow l'+\Lambda+X, \tag{3}$$

Here for the sake of definiteness the semi-inclusive Λ production with the proton target is considered. At this juncture, we wish to mention that some authors [18] have also investigated the pion contribution to semi-inclusive deep inelastic scattering with slow baryons (N's or Δ's) in the final states, an investigation related but complimentary to the present work which we shall mention briefly later in the paper.

In contrast with the idea of a parton model in which a hadron consists of partons of different kinds, the fragmentation hypothesis of Field and Feynman [19] states that a high-energy parton of a specific kind (a quark, an antiquark, or a gluon) has different probabilities of fragmenting into different hadrons. In such fragmentation picture, the fragmentation function $D_q^h(z)$ is introduced for a given parton of flavor q and the hadron

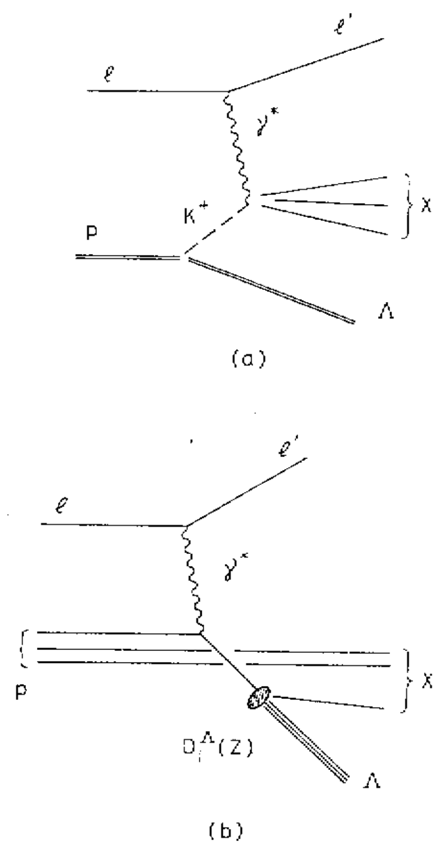


Fig. 1.a,b. Semi-inclusive production of Λ in the deep inelastic scattering of the proton by a charged lepton: **a** production mechanism through the Sullivan process and **b** production mechanism in the fragmentation picture

h, with z the fraction of the parton's longitudinal momentum carried away by the hadron. The fragmentation picture has been used in many high energy processes, including semi-inclusive hadron production in deep inelastic scattering (DIS), a process as illustrated by Fig. 1b.

It is clear that the Sullivan process as illustrated by Fig. 1a also provides a mechanism for semi-inclusive production of Λ in DIS. In light of the potentially important role played by the Sullivan process in connection with the parton model, it is a natural question to investigate whether the generalized Sullivan process as shown by Fig. 1a may give rise to some important contribution as compared to that given in the fragmentation picture (Fig. 1b). This will be the primary focus of the present paper.

2 Formulation

In this section, we wish to derive the basic formulae for describing the Sullivan process as illustrated by Fig. 1a, first using the procedure adopted by Sullivan [1] and then obtaining the same result making use of the alternative method developed by Drell, Levy, and Yan [20]. Following Sullivan [1], we introduce the following variables,

$$\ell(\ell)+p(p)\rightarrow\ell'(\ell')+\Lambda(p')+X(p_n);$$
$$q\equiv\ell-\ell',\qquad \nu\equiv p\cdot q/m,$$

$$\begin{aligned}
\omega &= \frac{2p\cdot q}{Q^2} = \frac{2m\nu}{Q^2},\\
s &= (p+q)^2 \approx Q^2(\omega-1);\\
k &\equiv p-p',\\
\omega' &= \frac{2k\cdot q}{Q^2} = \frac{2\mu\nu'}{Q^2},\\
s' &= (k+q)^2 \approx Q^2(\omega'-1),\\
t &\equiv k^2 = m^2+m_\Lambda^2-2mE',
\end{aligned} \tag{4}$$

with ℓ, ℓ', p, p', and p_n the four-momentum variables and m, m_Λ, and μ the mass of the proton, Λ, and the kaon, respectively. E' is the energy of Λ.

The amplitude which corresponds to Fig. 1(a) may be written as follows,

$$A_\mu = g_{KN\Lambda}\frac{\overline{u}(p')i\gamma_5 u(p)}{t-\mu^2} < X(p_n)|J_\mu^{em}(0)|K(k)>, \tag{5}$$

which is to be used in the calculation of the hadronic tensor.

$$W_{\mu\nu}(p,q) = \frac{1}{2M}\sum_{spin\ av}\sum_X\int\frac{d^3p'}{(2\pi)^3 2p'_0}\times \\ \times\prod_{i=1}^{n}[\frac{d^3p_i}{(2\pi)^3 2p_{i0}}]A_\mu^* A_\nu (2\pi)^3\delta^4(p_n+p'-p-q). \tag{6}$$

The spin average part may be calculated easily,

$$\begin{aligned}
&\sum_{spin\ av} A_\mu^* A_\nu\\
&= \frac{1}{2}g_{KN\Lambda}^2\frac{-Tr(\hat{p}'+m_\Lambda)\gamma_5(\hat{p}+m)\gamma_5}{(t-\mu^2)^2}\\
&\quad < K|J_\mu^{em}(0)|X>< X|J_\nu^{em}(0)|K>\\
&= g_{KN\Lambda}^2\frac{(m_\Lambda-m)^2-t}{(t-\mu^2)^2}\\
&\quad < K|J_\mu^{em}(0)|X>< X|J_\nu^{em}(0)|K>.
\end{aligned}$$

The DIS with the kaon as the target would be described by the hadronic tensor,

$$\begin{aligned}
W_{\mu\nu}^K(k,q) = \frac{1}{2\mu}\sum_X\int\prod_{i=1}^{n}[\frac{d^3p_i}{(2\pi)^3 2p_{i0}}]\ \cdot\\
\cdot < K|J_\mu^{em}(0)|X>< X|J_\nu^{em}(0)|K>\ \cdot\\
\cdot (2\pi)^3\delta^4(p_n-k-q),
\end{aligned} \tag{7}$$

so that we may rewrite the hadronic tensor as

$$W_{\mu\nu}(p,q) = \frac{\mu}{M}\int\frac{d^3p'}{(2\pi)^3 2p'_0}g_{KN\Lambda}^2\ \cdot \\ \cdot\frac{(m_\Lambda-m)^2-t}{(t-\mu^2)^2}W_{\mu\nu}^K(k,q). \tag{8}$$

Using the relation

$$d^3p' = \frac{\pi E'}{\omega}dt\,d\omega',$$

we obtain

$$W_{\mu\nu}(p,q) = \frac{\mu}{m}\int dt\,d\omega' g_{KN\Lambda}^2\frac{\pi}{2\omega}\ \cdot \\ \cdot\frac{(m_\Lambda-m)^2-t}{(t-\mu^2)^2}W_{\mu\nu}^K(k,q). \tag{9}$$

Using the relation between the hadronic tensor $W_{\mu\nu}$ and the structure function F_2,

$$W_\mu^{K\,\mu}(k,q) = -\frac{\omega'}{\mu}F_2^K(\omega'),\quad W_\mu^\mu(p,q) = -\frac{\omega}{m}F_2(\omega). \tag{10}$$

we obtain

$$\delta F_2(\omega) = \frac{1}{16\pi^2}\int\frac{d\omega'\,\omega'}{\omega^2}\ \cdot \\ \cdot\int dt\frac{(m_\Lambda-m)^2-t}{(t-\mu^2)^2}F_2^K(\omega')g_{KN\Lambda}^2(t). \tag{11}$$

With suitable substitutions, (11) is just what we have cited as (1) and (2) at the beginning of this paper.

In order to compare this result with what may be obtained from the fragmentation picture, we change the variables t to z (defined as the momentum fraction carried away by the hadron from the parental quark) and express ω as x. We have

$$\begin{aligned}
&x = \frac{1}{\omega},\qquad z' = \frac{\omega'}{\omega}\quad \text{(kaon's momentun fraction)};\\
&y = \frac{\nu}{E},\qquad z = \frac{E'}{\nu};\\
&t = m^2+m_\Lambda^2-2myzE;
\end{aligned} \tag{12}$$

so that

$$\begin{aligned}
dt\,d\omega' &= \left|\frac{\partial(t,\omega')}{\partial(z',z)}\right|dz\,dz' = \begin{vmatrix}-2myE & 0\\ 0 & \frac{1}{x}\end{vmatrix}dz\,dz'\\
&= \frac{2myE}{x}dz\,dz'.
\end{aligned} \tag{13}$$

We thus obtain from (11)

$$\delta F_2(x) = \frac{m^2yE}{4\pi^2}\int dz'\,dz\frac{z'(yzE-m_\Lambda)}{(t-\mu^2)^2}\ \cdot \\ \cdot F_2^K(\frac{x}{z'})g_{KN\Lambda}^2(t). \tag{14}$$

Using the standard relation [6],

$$\begin{aligned}
\frac{d^2\sigma}{dx\,dy} &= \frac{\pi\alpha^2}{mEx^2y^2}[1+(1-y)^2-\frac{m}{E}xy]F_2(x)\\
&\approx \frac{2\pi\alpha^2}{Q^2xy}[1+(1-y)^2]F_2(x),
\end{aligned} \tag{15}$$

we obtain the differential cross section due to the Sullivan process of Fig. 1a,

$$\begin{aligned}
\frac{d\sigma}{dx\,dy\,dz} = \frac{\pi\alpha^2}{mEx^2y^2}[1+(1-y)^2]\frac{m^2yE}{4\pi^2}\ \cdot\\
\cdot\int_x^{z'_{max}}dz'\frac{z'(yzE-m_\Lambda)}{(t-\mu^2)^2}\ \cdot\\
\cdot F_2^K(\frac{x}{z'})g_{KN\Lambda}^2(t).
\end{aligned} \tag{16}$$

The range of the variables $(x,\ y,\ z,\ z')$ is to be discussed later in the section.

Note that, using the expression for $F_2(x)$, we may deduce the probability of finding a kaon within a proton; i.e. $F_2(x)$ can be expressed as the probability function convoluted with $F_2^K(x)$. Denote such probability function as $f_K^p(z')$.

$$f_K^p(z') \equiv \int G_K^p(z', z)\, dz$$
$$= \int \frac{g_{KNA}^2}{4\pi^2} m^2 y E \frac{z'(yzE - m_A)}{(t-\mu^2)^2}\, dz, \qquad (17)$$

which is just (2) but expressed in different variables.

There are different ways to treat Sullivan processes. In addition to the above covariant method originally adopted by Sullivan [1], we proceed to use the alternative method developed by Drell, Levy, and Yan [20], which is essentially a perturbation theory in the infinite momentum frame, or better known as the light-cone perturbation theory. We shall see that the alternative method reproduces the same results (at least in the leading approximation). In light of the potential importance of (14) and (16), it is of interest to make certain that the same result can indeed be derived in such completely different formulation [20].

In the formulation developed by Drell, Levy, and Yan [20], one writes down the S-matrix, for a given interaction hamiltonian H_I,

$$S = Te^{-i\int_{-\infty}^{\infty} dt\, H_I(t)}, \qquad (18)$$

with the matrix element given by

$$< b|S|a >= -2\pi i \delta(E_b - E_a)\{< b|H_I|a >$$
$$+ \sum_n \frac{< b|H_I|n >< n|H_I|a >}{E_a - E_n + i\epsilon}$$
$$+ \sum_{n_1,n_2} \frac{< b|H_I|n_2 >< n_2|H_I|n_1 >< n_1|H_I|a >}{(E_a - E_{n1} + i\epsilon)(E_a - E_{n2} + i\epsilon)}$$
$$+ \cdots\}. \qquad (19)$$

Some general characteristics of this formulation are [20]: Although each term is not covariant, the complete answer to a given order in perturbation series must be. The particle content of each term is clear, offering intuitive interpretations. At each vertex the three momenta are conserved, although the energy is not. A virtual particle is always on its mass shell, but the intermediate state may not have the same energy as the initial or final state.

The physical or dressed state $|A' >$ of a particle can be expressed in terms of the bare state $|A >$ as follows [20],

$$|A' >= U|A >, \qquad U = Te^{-i\int_{-\infty}^{0} dt\, H_I(t)}.$$

Expanding U to lowest order, we obtain

$$|A' > = \sqrt{Z}|A > + \sum_{B,C} \int d^3p_B d^3p_C |B, C > \cdot$$
$$\cdot < B, C|(-i)\int_{-\infty}^{0} dt\, H_I(t)|A >$$
$$= \sqrt{Z}|A > +$$
$$+ \sum_{B,C} \int d^3p_B d^3p_C \frac{1}{E_A - E_B - E_C} |B, C > \cdot$$
$$\cdot < B, C|H_I|A > . \qquad (20)$$

It is convenient to express the matrix element of H_I as

$$< B, C|H_I|A > =$$
$$\frac{g V_{A\to BC}}{(2\pi)^{\frac{3}{2}} \sqrt{2E_A 2E_B 2E_C}} \delta^3(p_A - p_B - p_C). \qquad (21)$$

If $B \neq C$, the probability of finding a particle in the final state $|B, C >$ is

$$dP_{A\to BC} = \frac{|V_{A\to BC}|^2}{(E_A - E_B - E_C)^2} \frac{g^2}{(2\pi)^3} \frac{d^3p_B}{2E_A 2E_B 2E_C}. \qquad (22)$$

Note that a factor of $1/2$ must be included here, if B, C are identical particles.

Simple expressions arise as we go over to the infinite momentum frame, i.e., by taking the following variables in the limit $P \to \infty$.

$$p_A = (P + \frac{m_A^2}{2P}, P, \mathbf{0}),$$
$$p_B = (|z|P + \frac{p_\perp^2 + m_B^2}{2|z|P}, zP, \mathbf{p}_\perp),$$
$$p_C = (|1-z|P + \frac{p_\perp^2 + m_C^2}{2|1-z|P}, (1-z)P, -\mathbf{p}_\perp).$$

Here we see that at each vertex the three momentum is conserved while the energy is not, each particle is on mass shell, and p_B, p_C carry the momenta of fractions z and $1-z$ of the parent particle, respectively. The energy denominator in, e.g., (20) or (22) is

$$E_A - E_B - E_C$$
$$= \frac{-p_\perp^2 + z(1-z)m_A^2 - (1-z)m_B^2 - zm_C^2}{2z(1-z)P}, \quad (0 < z < 1)$$
$$= -2(z-1)P, \qquad (z > 1)$$
$$= 2zP, \qquad (z < 0)$$

so that the relevant region for z is between 0 and 1. Substituting these expressions back into (22), we find

$$dP_{A\to BC} = \frac{g^2}{16\pi^2} \cdot$$
$$\cdot \frac{z(1-z)|V_{A\to BC}|^2}{[p_\perp^2 - m_A^2 z(1-z) + m_B^2(1-z) + m_C^2 z]^2} dz\, dp_\perp^2. \qquad (23)$$

Using the form for the KNA vertex,

$$V_{A\to BC} = i\overline{u}(p_A)\gamma_5 u(p_N),$$

we obtain the spin average

$$\sum_{spin\ av} |V_{A\to BC}|^2 = -\frac{1}{2} Tr(\hat{p}_A + m_A)\gamma_5(\hat{p}_N + m_N)\gamma_5$$
$$= \frac{1}{1-z'}\{p_\perp^2 + [m_A - (1-z')m_N]^2\}, \qquad (24)$$

with z' and $1-z'$ the momentum fractions carried by K and A, respectively. (Here we use z' in order not to confuse with $z = E_A/\nu$.) We thus obtain the probability of finding a kaon with longitudinal momentum fraction z' and transverse momentum $\mathbf{p}_\perp$.

$$dP_{p\to KA} \equiv G_K^p(z', p_\perp^2) dz'\, dp_\perp^2$$
$$= \frac{g_{KNA}^2}{16\pi^2} \cdot$$
$$\cdot \frac{z'(p_\perp^2 + (m_A - (1-z')m_p)^2)}{(p_\perp^2 - m_p^2 z'(1-z') + m_K^2(1-z') + m_A^2 z')^2} dz'\, dp_\perp^2. \qquad (25)$$

In the light cone language, we use P^+ to denote $E + P_3$. We note that the ratio of any two P^+ is invariant under Lorentz boost in the z direction. Thus, we have

$$1 - z' = \frac{P_\Lambda^+}{P_p^+} = \frac{1}{m_p}(E_\Lambda - \sqrt{E_\Lambda^2 - m_\Lambda^2 - p_\perp^2}). \tag{26}$$

The last equality is obtained by going over to the rest frame. In order to compare the result with (14) and (16), we change the variables $(z', p_\perp^2) \to (z', z)$.

$$p_\perp^2 = 2m_p \nu z(1-z') - m_\Lambda^2 - m_p^2(1-z')^2,$$

$$\frac{\partial(z', p_\perp^2)}{\partial(z', z)} = \begin{vmatrix} 1 & 0 \\ \cdots & 2m_p\nu(1-z') \end{vmatrix} = 2m_p\nu(1-z'). \tag{27}$$

$$G_K^p(z', z) = G_K^p(z', p_\perp^2)\left|\frac{\partial(z', p_\perp^2)}{\partial(z', z)}\right|$$
$$= \frac{g_{KN\Lambda}^2}{16\pi^2}\frac{4m_p^2 \nu z'(\nu z - m_\Lambda)}{(2m_p\nu z - m_p^2 + m_K^2 - m_\Lambda^2)^2}. \tag{28}$$

It is straightforward to verify that this last result is in fact identical to (17) which is obtained in Sullivan's original approach. The cross section may be obtained as follows,

$$d\sigma = G_K^p(z', z) dz'\, dz\, d\sigma(eK \to eX),$$

with $d\sigma(eK \to eX)$ the cross section that the electron hits the kaon with momentum fraction z' of the proton. We may use the formulae obtained earlier for $eN \to eX$ to describe $eK \to eX$ by using suitable definition of x and y. Denoting them as x' and y', we have

$$x' \equiv \frac{Q^2}{2p_K \cdot q} = \frac{Q^2}{2z'p\cdot q} = \frac{x}{z'}$$

$$y' \equiv \frac{p_K \cdot q}{p_K \cdot p_e} = \frac{p \cdot q}{p \cdot p_e} = y$$

Here p_K, p_e, p represent the momenta of the kaon, the electron and the proton, respectively. We have

$$d\sigma(eK \to eX) = \frac{2\pi\alpha^2}{Q^2 x' y'}[1 + (1-y')^2]F_2^K(x')dx'\, dy'$$
$$= \frac{\pi\alpha^2}{mEx^2y^2}[1 + (1-y)^2]F_2^K(\frac{x}{z'})dx\, dy. \tag{29}$$

To sum up, we obtain

$$\frac{d\sigma}{dx\,dy\,dz\,dz'} = G_K^p(z', z)\frac{\pi\alpha^2}{mEx^2y^2}[1 + (1-y)^2]F_2^K(\frac{x}{z'}), \tag{30}$$

with $G_K^p(z', z)$ specified by (28). Note that the ranges for the variables x, y, z, and z' may be obtained from the constraints

$$x \le z' \le 1,$$
$$0 \le p_\perp^2 \le E_\Lambda^2 - m_\Lambda^2.$$

Upon changing the variables from $(z', p_\perp^2)$ to (z', z), the constraints become

$$x \le z' \le 1,$$
$$1 \ge z \ge \frac{1}{2m_pEy}[\frac{m_\Lambda^2}{1-z'} + m_p^2(1-z')] \equiv f(z').$$

The minimum of $f(z')$ occurs at $z' = 1 - m_\Lambda/m_p < 0$. To integrate the cross section of (30) in the order of z', z, y, and x, the corresponding integration region is specified by

$$x \le z' \le 1 - \frac{1}{m_p}(Eyz - \sqrt{E^2y^2z^2 - m_\Lambda^2}) \tag{31a}$$

$$1 \ge z \ge \frac{1}{2m_pEy}[\frac{m_\Lambda^2}{1-x} + m_p^2(1-x)] = f(x), \tag{31b}$$

$$1 \ge y \ge \frac{1}{2m_pE}[\frac{m_\Lambda^2}{1-x} + m_p^2(1-x)], \tag{31c}$$

$$0 \le x \le 1 - \frac{1}{m_p}(E - \sqrt{E^2 - m_\Lambda^2}). \tag{31d}$$

Note that (30) is the same as (16), except being expressed in terms of slightly different set of variables. In other words, the alternative formulation yields the same expression for the cross section as in the original Sullivan approach. In the present approach, we find that, by choosing the arguments to be $(z', p_\perp^2)$, the probability functions of the kaon and Λ are the same. That is, $G_K^p(1 - z', p_\perp^2) = G_\Lambda^p(z', p_\perp^2)$. Such aspect is by no means transparent in the original Sullivan approach.

To close this section, we turn to the standard fragmentation picture as illustrated by Fig. 1b. Denote by $D_q^\Lambda(z)dz$ the probability of quark q fragmenting into Λ of momentum fraction between z and $z + dz$ [16]. The formula for $d\sigma/dxdydz$ in the fragmentation picture is given by

$$\frac{d^3\sigma}{dx\,dy\,dz} = \frac{\pi\alpha^2}{mEx^2y^2}[1 + (1-y)^2 - \frac{m}{E}xy]\sum_q e_q^2 x f_q(x) D_q^\Lambda(z)$$
$$\approx \frac{2\pi\alpha^2}{Q^2xy}[1 + (1-y)^2]\sum_q e_q^2 x f_q(x) D_q^\Lambda(z). \tag{31}$$

3 Sample Numerical Results

To describe the numerical results, we write the cross section of (32) as $d\sigma^{(2)}$ for the fragmentation picture (Fig. 1b) in contradistinction with $d\sigma^{(1)}$, (30) as from the Sullivan process (Fig. 1a). The ratio $d\sigma^{(1)}/d\sigma^{(2)}$ is then given by

$$\frac{\frac{d\sigma^{(1)}}{dx\,dy\,dz}}{\frac{d\sigma^{(2)}}{dx\,dy\,dz}} = \frac{\int_x^{z'_{max}} dz'\, F_2^K(\frac{x}{z'})G_K^p(z', z)}{\sum_q e_q^2 x f_q^p(x) D_q^\Lambda(z)}. \tag{32}$$

We choose to present the numerical results on this ratio as a gauge of the relative importance between these two processes, mainly because the current crude status regarding the fragmentation functions [6, 16, 21, 22] does not give us enough confidence in the calculated absolute magnitudes of these cross sections.

To obtain sample numerical results, it is sufficient to use the standard simple ansatz [16, 6, 21,22]:

$$\begin{Bmatrix} xf_u^{K^+}(x) = xf_{\bar s}^{K^+}(x) = v^{K^+}(x) + s^{K^+}(x) \\ xf_{\bar u}^{K^+}(x) = xf_d^{K^+}(x) = xf_s^{K^+}(x) = xf_{\bar d}^{K^+}(x) = s^{K^+}(x) \end{Bmatrix} \tag{33}$$

$$\begin{Bmatrix} xf_u^p(x) = u^p(x) + s^p(x), xf_d^p(x) = d^p(x) + s^p(x) \\ xf_{\bar u}^p(x) = xf_{\bar d}^p(x) = 2xf_s^p(x) = 2xf_{\bar s}^p(x) = s^p(x) \end{Bmatrix} \tag{34}$$

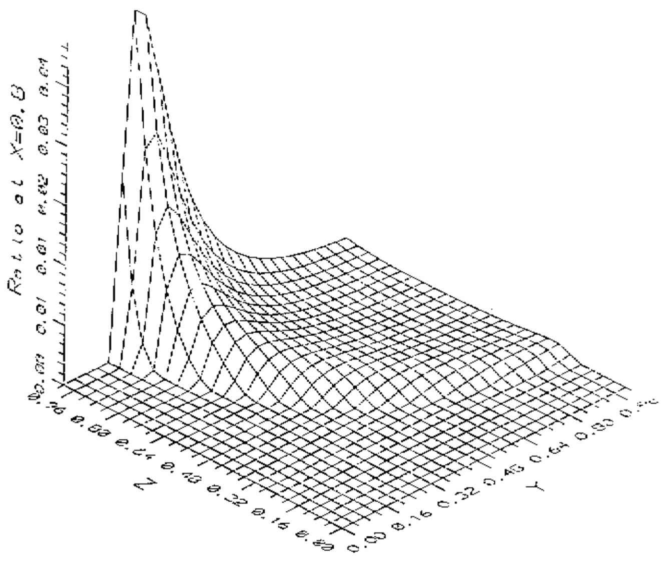


Fig. 2. The ratio given by (33) is shown as a function of y and z for a fixed value of x ($x = 0.8$). The ridge is specified by the curve $yz = \frac{1}{2m_p E}[\frac{m_\Lambda^2}{1-x} + m_p^2(1-x)]$. On one side of this curve, the ratio drops to zero because it is out of the range mentioned above (31). Note that the maximum occurs at $z = 1$

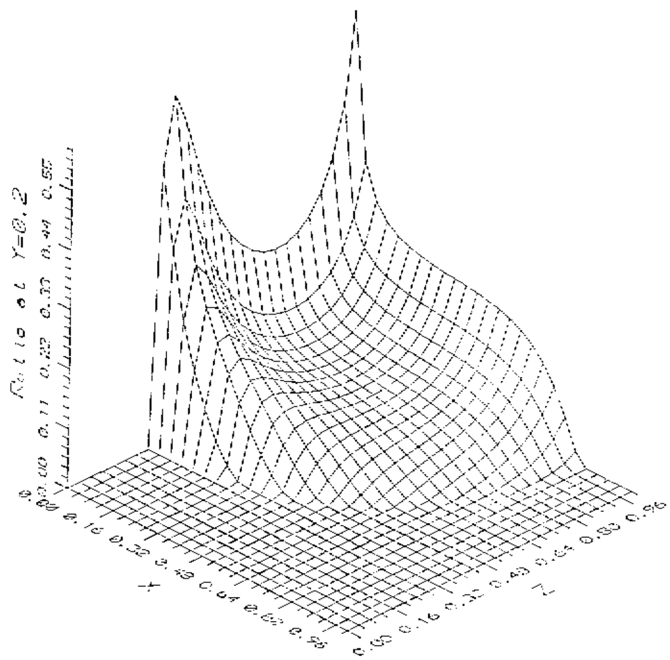


Fig. 3. The ratio given by (33) is shown as a function of x and z for a fixed value of y ($y = 0.2$)

$$u^p(x) = A_u^p x^{\alpha_u^p}(1-x)^{\beta_u^p}, \qquad d^p(x) = A_d^p x^{\alpha_d^p}(1-x)^{\beta_d^p},$$
$$s^p(x) = A_s^p(1-x)^{\gamma^p};$$
$$v^{K^+}(x) = A^{K^+} x^{\alpha^{K^+}}(1-x)^{\beta^{K^+}},$$
$$s^{K^+}(x) = A_s^{K^+}(1-x)^{\gamma^{K^+}}.$$

The various parameters are fitted from experiments and found to be [21]

$$A^{K^+} = 0.4942, \qquad A_s^{K^+} = 0.027;$$
$$\alpha^{K^+} = 0.33, \qquad \beta^{K^+} = 1.17, \qquad \gamma^{K^+} = 9.29;$$
$$\alpha_u^p = \alpha_d^p = 0.57, \qquad \beta_u^p = \beta_d^p - 1 = 3.28, \qquad \gamma^p = 9.33;$$
$$A_u^p = 2.906, \qquad A_d^p = 1.651, \qquad A_s^p = 0.04617.$$

As for the fragmentation functions, we use the simple form (adapted from [22]):

$$zD_u^\Lambda = zD_d^\Lambda = zD_s^\Lambda = B_s(1-z)^4 + B_v z(c-z)^3,$$
$$zD_{\bar{u}}^\Lambda = zD_{\bar{d}}^\Lambda = zD_{\bar{s}}^\Lambda = B_s(1-z)^4. \tag{35}$$

$$B_s = 0.1, \qquad B_v = 0.2, \qquad c = \frac{11}{9}.$$

We take the dipole form for the $KN\Lambda$ coupling [12].

$$g_{KN\Lambda}(t) = g_{KN\Lambda}^0 \left(\frac{\Lambda^2 - m_K^2}{\Lambda^2 - t}\right)^2, \tag{36}$$

with the cutoff Λ taken to be 1400 MeV.

The sample numerical results are shown in Figs. 2-5. For the sake of illustration, we take the incident electron energy E to be 20 GeV.

In Fig. 2, the ratio is shown as a function of y and z for a fixed value of x ($x = 0.8$). It is seen that there is a ridge along the curve

$$yz = \frac{1}{2m_p E}[\frac{m_\Lambda^2}{1-x} + m_p^2(1-x)].$$

On one side of this curve, the ratio drops to zero because it is out of the range mentioned above (31). Note that the maximum occurs at $z = 1$.

In Fig. 3, the ratio is shown as a function of x and z for a fixed value of y ($y = 0.2$). Here the parameters are chosen such that the ratio is clearly of numerical importance in a visible part of the (x, z) parameter space.

In Fig. 4, the ratio is shown as a function of x and y for a fixed value of z ($z = 0.1$). It is seen that, when z is fixed, the maximum occurs at $x = 0$. The global maximum may be located by plotting the curve with $x = 0$ and $z = 1$ and is found to be located at $y = 0.075$. In general, we find that the global maximum occur at $x = 0$, $z = 1$ and $yE = 1.5$ GeV when we change the electron energy E. The maximum value for the ratio is approximately 24 (which is rather large) and is independent of E. However, such point is mostly academic (and likely of little experimental interest) since the peak is fairly sharp and narrow. As the incident electron energy E increases, the peak becomes sharper; that is, the ratio at the points other than the peak decreases rapidly. This is because the coupling goes to zero as E becomes infinity except for the region with small yzE.

In Fig. 5a, the double differential cross section $\frac{d\sigma}{dxdy}$ for the Sullivan process, as obtained from integrating (30) subject to (31), is plotted as a function of x and y. The unit is $10^{-15}\,\text{MeV}^{-2}$. For a given value of x, the cross section vanishes at small y (where it is kinematically forbidden), rises rapidly once y crosses the threshold, reaches at some peak value and then falls off as $y \to 1$. In Fig. 5b, we display the same differential cross section at a specific value of x (= 0.01) as a function of y. The dots indicate the points which we have

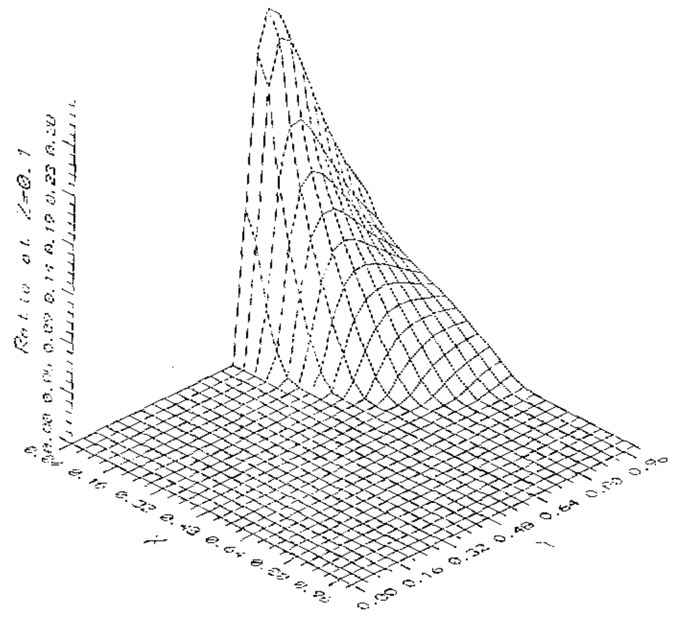
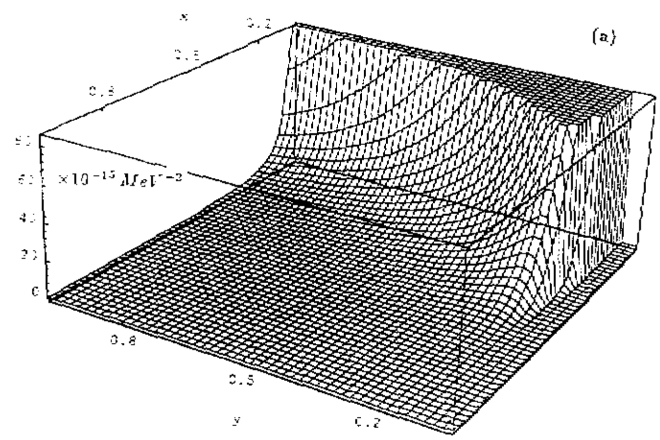


Fig. 4. The ratio given by (33) is shown as a function of x and y for a fixed value of z (z = 0.1). Note that, when z is fixed, the maximum occurs at x = 0



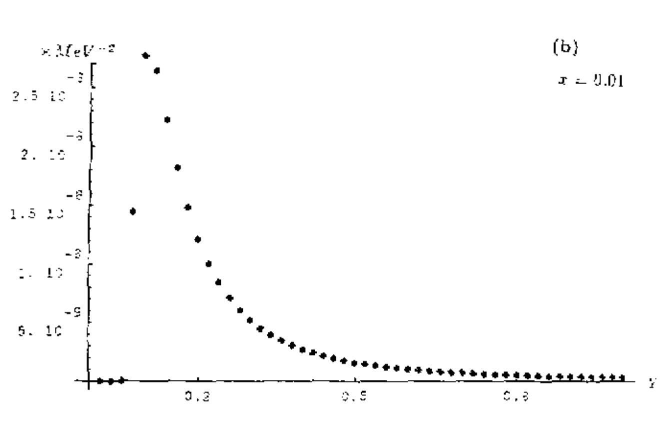


Fig. 5.a,b. The double differential cross section $\frac{d\sigma}{dxdy}$ for the Sullivan process, as obtained from integrating (30) subject to (31), is shown: **a** as a function of x and y (in units of $10^{-15}\,\mathrm{MeV}^{-2}$) and **b** shown as a function of y for x = 0,01

performed the integrations (over z' and z) to obtain the predicted cross sections.

Unlike Figs. 2-4 for which the ratio of the cross sections is shown, Figs. 5a and 5b gives the cross section itself. Yet, it should be regarded as semi-quantitative owing to the uncertainties in connection with (34)-(37). Nevertheless, it may serve as the basis for analyzing the feasibility of the proposed experiment. Assuming a luminosity of $10^{32}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ (and with 100 % detection efficiency), it takes a day of running time to see about three events for a cross section of $10^{-15}\,\mathrm{MeV}^{-2}$. The estimates shown in Figs. 5a and 5b are clearly quite encouraging in terms of the experimental feasibility.

The sample numerical results shown by Figs. 2-5 are very suggestive. To confirm the potential importance of the generalized Sullivan process, Fig. 1a, in the semi-inclusive Λ production or, equivalently, to identify the kinematic region where the standard fragmentation picture, Fig. 1b, may not be adequate, it is essential to identify those events which have relatively small y (say, $0.05 \leq y \leq 0.2$ with $y \equiv \nu/E$ the fraction of the energy transfer), large z (say, $z \geq 0.6$ - most of the energy transferred to the proton is again carried away by Λ), and small x (say $0.05 \leq x \leq 0.4$). It is expected from Figs. 2-5 that events in this region arise partially from the mechanism associated with the generalized Sullivan process, Fig. 1a. An appropriate analysis of the data in this region must take into account both the mechainsms - the contribution from the generalized Sullivan process Fig. 1a and that from the standard fragmentation picture Fig. 1b. Failing to include the contribution from the generalized Sullivan process, one would obtain fragmentation functions which would in general over-predict cross sections elsewhere.

The proposed experiment to confirm the major conclusion of the present paper is not an easy one but should be feasible. Unlike the case of non-strange hadrons [18], semi-inclusive production of Λ in general has smaller cross sections in the fragmentation picture but has much larger signal-to-background ratios as far as the generalized Sullivan process is concerned. Detection of Λ through its $N\pi$ decay modes also offers an unambiguous experimental signal. In fact, polarization of Λ can also be measured without much difficulty, although predictions on the Λ spin is left as a subject for future investigation. In our opinion, more efforts should be dedicated to measurements of semi-inclusive hadron production cross sections so that a clear picture towards the various fragmentation functions, a concept closely related to parton distributions, may be obtained.

4 Summary

In this paper, we have investigated the proposal of using semi-inclusive Λ production with a high energy electron or muon beam as a means of testing the suggestion that the generalized Sullivan process, in which the virtual photon may strike and smash the meson in the cloud surrounding the target proton, may play an important role in deep inelastic scattering (DIS) by charged leptons. We have considered two different ways to formulate the Sullivan process as illustrated in Fig. 1a and obtain the same result. Our sample numerical results, Figs. 2-5,indicate that, in the kinematic region characterized typi-

cally by small x, large z, and a small specific range of y, the contribution from Sullivan processes Fig. 1a is of numerical importance as compared to that due to the fragmentation process Fig. 1b. It is clear that semi-inclusive Λ production with a high energy electron or muon beam may offer an alternative means of testing the importance of the generalized Sullivan process.

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QCD and Nuclear Physics

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Upon a brief explanation of what nuclear physics and QCD is all about, I would like to discuss how the imminent future of nuclear physics hinges on progresses in QCD. In addition, I wish to describe my recent work that a closed set of coupled differential equations for nonlocal condensates in QCD (which characterize the nontrivial feature of the QCD ground state or vacuum) may be derived in the large N_c limit (with N_c the number of color). As a specific example, the leading-order equations for the nonlocal condensates appearing in the quark propagator are derived and explicit solutions are obtained.

PACS. 12.38.Aw – General properties of QCD.
PACS. 12.38.Lg – Other nonperturbative calculations.
PACS. 95.30.–k – Fundamental aspects of astrophysics.

I. Introduction

One common characterization of "nuclear physics" is the "physics of strong interactions" and in this sense the problem of strong interaction physics has been around for more than half a century (since early thirties when the question regarding the origin of nuclear force was first raised in a serious manner). The very nature of the problem varies considerably with the so-called "underlying theory", especially when quantum chromodynamics or QCD is taken universally to be such underlying theory. Thanks to its asymptotically free nature, QCD as the candidate theory of strong interactions has been tested to an accuracy of about 10 % at high energies.

QCD as the foundation of nuclear physics presents us a challenge of some unusual kind, as the basic degrees of freedom (DOF's) of QCD are quarks and gluons, remarkably different from nuclei and hadrons – the building blocks of nuclear physics. QCD is a nonlinear quantum field theory, the so-called nonabelian gauge theory or Yang-Mills theory, and it is the quantum version of such theory which matters. QCD at low energies is dictated by the various nonperturbative aspects associated with the theory.

Perhaps because of our inability to deal with nuclear waste coming out of nuclear power plants or due to the horror picture that an atomic bomb has the potential to incur, some nuclear physicists have for some time tried to do away possible associations which the name "nuclear physics" may bring about and in some instances do not want to be called "nuclear physicists" any longer. Indeed, I think that there is some need to deal with the social impact or the animosity which a "poor" or "bad" name may have caused. Unfortunately or fortunately, the Sun will continue to shine on us for billions of years to come because the interior of the Sun is a gigantic nuclear power reactor, a reactor which is far from being free of the nuclear waste problem. Nuclear physics is the most central piece of knowledge if we can ever understand the Sun in some of its entirety. To say it in fair terms, a good portion of progresses in astrophysics depends on nuclear physics. On the other hand, quarks, leptons, and many other advanced objects of less animosity would not have been with us, should we have been deprived of the ability of using nuclei as a laboratory to produce or access them.

As of today, it remains almost impossible to solve problems related to hadrons or nuclei, except perhaps through lattice simulations of QCD but it is unlikely that, through the present-day computation power which is still not quite adequate, simulations would give us any good part of what we wish to know. Nevertheless, the need of solving QCD directly has gained tremendous importance over the last decade and, to this end, any fruitful attempt in confronting directly with QCD should be given due attention.

As I see it, the benefit to be gained from basing nuclear physics on QCD is really immense. First of all, it is a quantized nonlinear phenomenon which we are trying to grapple with. The lesson will help us to tackle many other nonlinear phenomena, either quantum or classical. I think that the modern physics era, with relativity and quantum principle as two pillars, is closing its glory chapter at the turn of the century – one of the new directions is, in my opinion, something which has to do with nonlinear phenomena. Second, it is "nuclear physics in its new form and new life" which we are trying to come up with. Any science that has life of some sort must, from time to time, be endowed with some drastically new elements, either experimental or purely conceptual. Finally, perhaps as a wishful thinking, deeper understanding of the subject may someday enable us to deal with the nuclear waste problem, or other problem of similar magnitude, in a technically efficient manner and the positive picture towards the term "nuclear physics" may thus be evolved from there.

In the rest of my talk (this paper), I wish to describe my recent research on QCD, especially in explaining how a closed set of coupled differential equations for nonlocal condensates in QCD (which characterize the nontrivial feature of the QCD ground state or vacuum) may be derived in the large N_c limit (with N_c the number of color). As a specific example, the leading-order equations for the nonlocal condensates appearing in the quark propagator are derived and explicit solutions are obtained. In choosing to do so, I am inclined more as a conscientious effort to avoid talking about QCD without specifics while paying little attention, perhaps I should have done so, to doing proper justice to what numerous other nuclear physicists (better than I am) have accomplished in the recent past. Well, at this 50th Anniversary Celebration of our department, I may have the uniquue excuse in doing what I am trying to do here.

II. Introduction to QCD

The ground state, or the vacuum, of QCD is known to be nontrivial, in the sense that there are non-zero condensates, including gluon condensates, quark condensates, and perhaps infinitely many higher-order condensates. In such a theory, propagators, i.e. causal Green's functions, such as the quark propagator

$$iS_{ij}^{ab}(x) \equiv < 0 \mid T(q_i^a(x)\bar{q}_j^b(0)) \mid 0 >, \tag{1}$$

carry all the difficulties inherent in the theory. Higher-order condensates, such as a four-quark condensate,

$$< 0 \mid T(\bar{\psi}(z)\gamma_\mu\psi(z)q_i^a(x)\bar{q}_j^b(0)) \mid 0 >,$$

with $\psi(z)$ also labeling a quark field, represent an infinite series of unknowns unless some useful ways for reduction can be obtained. As the vacuum, $\mid 0 >$, is highly nontrivial, there is little reason to expect that Wick's theorem (of factorization), as obtained for free quantum field theories, is still of validity. Thus, we must look for alternative methods in order to obtain useful results.

It is known that the equations for Green functions up to a certain order usually involve Green functions of even higher order, thereby making such hierarchy of equations often useless in practice. In what follows, however, we wish to show that, provided that we may use the large N_c approximation to treat condensates of much higher order, there is in fact a natural way of setting up closed sets of differential equations which govern the inter-related Green functions to a given order. We consider this as an important accomplishment, both because we can always go over to the next level of sophistication in order to improve the approximation and because the large N_c expansion has been shown to yield desirable results for describing hadron physics.

In light of the nontrivial QCD vacuum, we shall begin by considering the feasibility of working directly with the various matrix elements such as the quark propagator of Eq. (1). Useful relations may be derived if we regard the equations for interacting fields [1],

$$\left\{ i\gamma^\mu \left(\partial_\mu + ig\frac{\lambda^a}{2}A_\mu^a \right) - m \right\} \psi = 0; \tag{2}$$

$$\partial^\nu G_{\mu\nu}^a - 2gf^{abc}G_{\mu\nu}^b A_\nu^c + g\bar{\psi}\frac{\lambda^a}{2}\gamma_\mu\psi = 0, \tag{3}$$

as the equations of motion for *quantized* interacting fields, subject to the standard rule for quantization that the equal-time (anti-)commutators among these quantized interating fields are identical to those among non-interacting quantized fields. As our basic example, we allow the operator $\{i\gamma^\mu\partial_\mu - m\}$ to act on the matrix element defined by Eq. (1) and obtain

$$\{i\gamma^\mu\partial_\mu - m\}_{ik} iS_{kj}^{ab}(x) = i\delta^4(x)\delta^{ab}\delta_{ij} + < 0 \mid T\left(\left\{ g\frac{\lambda^n}{2}A_\mu^n\gamma^\mu q(x) \right\}_i^a \bar{q}_j^b(0) \right) \mid 0 > . \tag{4}$$

We should always keep in mind that the QCD vacuum $\mid 0 >$ is a nontrivial ground state which is in general not annihilated by operating on it the annihilation operators.

Eq. (4) can be solved by splitting the propagator into a singular, perturbative part and a nonperturbative part:

$$iS_{ij}^{ab}(x) = iS_{ij}^{(0)ab}(x) + i\tilde{S}_{ij}^{ab}(x), \tag{5}$$

where

$$iS_{ij}^{(0)ab}(x) \equiv \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} iS_{ij}^{(0)ab}(p), \tag{6a}$$

$$iS_{ij}^{(0)ab}(p) = \delta^{ab} \frac{i(\hat{p}+m)_{ij}}{p^2 - m^2 + i\epsilon}, \tag{6b}$$

with $\hat{a} \equiv \gamma^\mu a_\mu$ for a four-vector a_μ. The nonperturbative part then satisfies the equation:

$$\{i\gamma^\mu \partial_\mu - m\}_{ik} i\tilde{S}_{kj}^{ab}(x) = \left\langle 0 \mid T\left(\left\{g\frac{\lambda^n}{2} A_\mu^n \gamma^\mu q(x)\right\}_i^a \bar{q}_j^b(0)\right) \mid 0\right\rangle. \tag{7}$$

This would be pretty much the end of the story unless we could find some way to proceed. We should note that Eq. (4) may also be derived by making use of, e.g., the path-integral formulation, and the issue of how to define renormalized composite operators, i.e. products of field operators, is by no means trivial (and fortunately we need not worry about such problem for the sake of this paper).

As a useful benchmark, we note that, with the fixed-point gauge,

$$A_\mu^n(x) = -\frac{1}{2} G_{\mu\nu}^n x^\nu + \cdots, \tag{8}$$

we may solve the nonperturbative part $i\tilde{S}_{ij}^{ab}(x)$ as a power series in x^μ,

$$\begin{aligned} i\tilde{S}_{ij}^{ab}(x) = & -\frac{1}{12}\delta^{ab}\delta_{ij} < \bar{q}q > + \frac{i}{48} m\hat{x}_{ij}\delta^{ab} < \bar{q}q > \\ & + \frac{1}{192} < \bar{q}g\sigma\cdot Gq > \delta^{ab} x^2 \delta_{ij} + \cdots. \end{aligned} \tag{9}$$

The first term is the integration constant which defines the so-called "quark condensate", while the mixed quark-gluon condensate appearing in the third term arises because of Eqs. (7) and (8). It is obvious that the series (9) is a short-distance expansion, which converges for small enough x_μ. We note that it is just the standard quark propagator cited in most papers in QCD sum rules [2].

The approach which we suggest here [3] is based upon two key elements, namely, the set of interacting field equations *plus* the rule of canonical quantization (for interacting fields). The equations which we obtain, such as Eq. (5), are much the same as the set of Schwinger-Dyson equations (for the matrix elements). An important aspect in our derivation is that the nontriviality of the vacuum $\mid 0 >$ is observed at every step $-$ a central issue in relation to QCD.

As another important exercise, we may split the gluon propagator into the singular, perturbative part and the nonperturbative part. The nonperturbative part is given by [3]

$$\begin{aligned} g^2 < 0 \,|\!: G^n_{\mu\nu}(x) G^m_{\alpha\beta}(0)\!:\!|\, 0 >= & \frac{\delta^{nm}}{96} < g^2G^2 > (g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}) \\ & -\frac{\delta^{nm}}{192} < g^3G^3 > \{x^2(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}) \\ & -g_{\mu\alpha}x_\nu x_\beta + g_{\mu\beta}x_\nu x_\alpha - g_{\nu\beta}x_\mu x_\alpha + g_{\nu\alpha}x_\mu x_\beta\} \\ & +O(x^4), \end{aligned} \tag{10}$$

with

$$< g^2G^2 > \equiv < 0 \,|\!: g^2 G^n_{\mu\nu}(0) G^{n\mu\nu}(0)\!:\!|\, 0 >, \tag{11a}$$

$$< g^3G^3 > \equiv < 0 \,|\!: g^3 f^{abc} g^{\mu\nu} G^a_{\mu\alpha}(0) G^{b\alpha\beta}(0) G^c_{\beta\nu}(0)\!:\!|\, 0 > . \tag{11b}$$

Again, the first term in Eq. (10) is the integration constant for the differential equation satisfied by the gluon propagator. Note that inclusion of the gluon condensate $< g^2G^2 >$ in Eq. (10) is standard but the triple gluon condensate $< g^3G^3 >$ is a new entry required by the interacting field equation (3). Our approach indicates when condensates of entirely new types should be introduced as we try to perform operator-product expansions to higher dimensions.

III. Condensates in QCD

To give a pedagogical and specific example, we wish to focus on the quark propagator, as specified by Eqs. (1) and (4)-(7). We write

$$i\tilde{S}^{ab}_{ij}(x) = \delta^{ab}\{\delta_{ij} f(x^2) + i\hat{x}_{ij} g(x^2)\}, \tag{12}$$

with $x^2 \equiv x_0^2 - \mathbf{x}^2$. $f(x^2)$ and $g(x^2)$ are what we refer to as "nonlocal condensates" in connection with the quark propagator. We note that

$$<: \bar{q}(x)\, q(0) :> = -12 f(x^2), \tag{13}$$

which is the nonlocal quark condensate in the standard sense. To proceed further, we work only with the leading term in the fixed-point gauge and introduce

$$\begin{aligned} <: \{gG_{\mu\nu}q(x)\}^a_i \bar{q}^b_j(0) :> = & \delta^{ab}\{(\gamma_\mu x_\nu - \gamma_\nu x_\mu)A(x^2) \\ & + i\sigma_{\mu\nu}B(x^2) + (\gamma_\mu x_\nu - \gamma_\nu x_\mu)\hat{x}C(x^2) + i\sigma_{\mu\nu}\hat{x}D(x^2)\}, \end{aligned} \tag{14}$$

with $G_{\mu\nu} \equiv (\lambda^n/2)G^n_{\mu\nu}$, an antisymmetric operator. The invariant functions $A(x^2)$, $B(x^2)$, $C(x^2)$, and $D(x^2)$ are additional nonlocal condensates which we deal with explicitly in this paper.

Under the assumption that we keep only the leading term in the fixed-point gauge (a simplifying assumption which can be removed whenever necessary), we have

$$\{i\gamma^\alpha\partial_\alpha - m\}_{ik} <: \{gG_{\mu\nu}q(x)\}^a_k\bar{q}^b_j(0) :>$$

$$= -\frac{1}{2}x^\beta <: \{g^2G_{\mu\nu}G_{\alpha\beta}\gamma^\alpha q(x)\}^a_i\bar{q}^b_j(0) :> \tag{15a}$$

$$= -\frac{1}{2}\cdot\frac{1}{96}\cdot\frac{4}{3}\cdot < g^2G^2 ><: \{(\gamma_\mu x_\nu - \gamma_\nu x_\mu)q(x)\}^a_i\bar{q}^b_j(0) :> \tag{15b}$$

$$= \frac{1}{144} < g^2G^2 > \delta^{ab}(\gamma_\mu x_\nu - \gamma_\nu x_\mu)\{f(x^2) + i\hat{x}g(x^2)\}. \tag{15c}$$

Here the second line, (15a), follows from the field equation and the third line, (15b), is based on the large N_c approximation that the contribution in which $G_{\mu\nu}$ and $G_{\alpha\beta}$ do not couple to color-singlet is suppressed by a factor of $1/N_c^2$. Thus, the factorization in the present case is justified to order $1/N_c^2$, rather than just order $1/N_c$. We also note that we have used Eq. (10) to leading order in obtaining Eq. (15b), but this approximation can be relaxed if necessary.

Now, we may use Eqs. (7) and (15c) and obtain a closed set of equations:

$$2f'(x^2) - mg(x^2) = -\frac{3}{2}i(B - x^2C), \tag{16a}$$

$$2x^2g'(x^2) + 4g(x^2) + mf(x^2) = \frac{3}{2}x^2(A - D), \tag{16b}$$

$$4iB' - 2iC - 2ix^2C' - mA = -\frac{1}{144} < g^2G^2 > f(x^2), \tag{16c}$$

$$2iA + 2ix^2D' - mB = 0, \tag{16d}$$

$$-2iA' + 4iD' - mC = -\frac{i}{144} < g^2G^2 > g(x^2), \tag{16e}$$

$$2iB' + 2iC - mD = 0, \tag{16f}$$

where the derivatives are with respect to the variable x^2.

Treating m as an expansion parameter,

$$F(x^2) = \sum_{k=0}^{\infty} m^k F_k(x^2), \tag{17}$$

we may solve the coupled equations, (16a)-(16f), order by order in m. To leading order in m, we obtain

$$x^2f_0''' + 3f_0'' - \xi_0^2x^2f_0' - 2\xi_0^2f_0 = 0, \tag{18}$$

$$(x^2)^3g_0''' + 5(x^2)^2g_0'' + \{2x^2 - \xi_0^2(x^2)^3\}g_0' - \{2 + 2\xi_0^2(x^2)^2\}g_0 = 0, \tag{19}$$

with $\xi_0^2 \equiv < g^2G^2 > /384$. The equations for A_0, B_0, C_0, and D_0 can easily be solved once we obtain f_0 and g_0.

Eq. (19) can be simplified considerably by introducing

$$g_0(x^2) \equiv (x^2)^{-2}\tilde{g}_0(x^2), \tag{20a}$$

which leads to the equation:

$$x^2\tilde{g}_0''' - \tilde{g}_0'' - \xi_0^2 x^2 \tilde{g}_0' = 0. \tag{20b}$$

Eqs. (17) and (20) can be solved by iteration, leading to the result:

$$f_0(t) = a_0\left\{1 + \frac{1}{1\cdot 3}(\xi_0 t)^2 + \frac{1}{1\cdot 3^2\cdot 5}(\xi_0 t)^4 + \cdots\right\}$$
$$+a_1 t\left\{1 + \frac{1}{2\cdot 4}(\xi_0 t)^2 + \frac{1}{2\cdot 4^2\cdot 6}(\xi_0 t)^4 + \cdots\right\}, \tag{21}$$

$$\tilde{g}_0'(t) = c_2 t^2\left\{1 + \frac{1}{2\cdot 4}(\xi_0 t)^2 + \frac{1}{2\cdot 4^2\cdot 6}(\xi_0 t)^4 + \cdots\right\}, \tag{22}$$

with $t \equiv x^2$ and

$$a_0 = -\frac{1}{12} < \bar{q}q >, \qquad a_1 = \frac{1}{192} < \bar{q}g_c\sigma\cdot Gq >, \qquad c_2 = -\frac{g_c^2 < \bar{q}q >^2}{2^5\cdot 3^4}. \tag{23}$$

Eq. (23) is obtained by comparing to the well-known series expansion for the quark propagator (see, e.g., [4]). Note that there are two integration constants, a_0 and a_1, for $f(x^2)$ but there is only one permissible constant for $g(x^2)$ (and c_2 is in fact a four-quark condensate taken in the large N_c limit).

For a number of applications, it is useful to obtain analytic expressions for $f_0(t)$ and $g_0(t)$. This turns out to be possible by way of Laplace transforms.

$$\bar{f}_0(s) \equiv \int_0^\infty dse^{-st}f_0(t), \qquad \bar{\tilde{g}}_0'(s) \equiv \int_0^\infty dse^{-st}\tilde{g}_0'(t). \tag{24}$$

We obtain

$$\bar{f}_0(s) = -\frac{2a_1}{\xi_0^2} - \frac{a_0}{\xi_0}\frac{s}{\sqrt{s^2-\xi_0^2}}\sec^{-1}\frac{s}{\xi_0} + \frac{\gamma_0 s}{\sqrt{s^2-\xi_0^2}}, \tag{25}$$

$$\bar{\tilde{g}}_0'(s) = \frac{2c_2}{(s^2-\xi_0^2)^{3/2}}, \tag{26}$$

with $\gamma_0 = 2a_1/\xi_0^2$. Looking up the table for Laplace transforms, we find

$$\tilde{g}_0' = \frac{2c_2}{\xi_0^2}\cdot \xi_0 t\cdot I_1(\xi_0 t), \tag{27}$$

with $I_1(z)$ the modified Bessel function of the first kind, to order one. It is straightforward to show that Eq. (27) yields the series expansion in Eq. (22). Also, the function $I_1(\xi_0 t)$ enters the second series in $f_0(t)$ as in Eq. (21).

To close our presentation of the explicit solution to leading order in m, we note that Eqs. (16) yields

$$f_0'(t) = -i\tfrac{3}{4}(tB_0(t))', \tag{28a}$$

$$C_0(t) = -B_0'(t), \tag{28b}$$

$$tg_0'(t) + 2g_0(t) = -\frac{3}{4}t(tD_0(t))', \tag{28c}$$

$$A_0(t) = -tD_0'(t). \tag{28d}$$

Thus, the functions A_0, B_0, C_0, and D_0 can be solved explicitly once f_0 and g_0 are known.

On the other hand, we may go beyond the leading order in m and obtain, as example,

$$\begin{aligned} & tf_1''' + 3f_1'' - \xi_0^2 t f_1' - 2\xi_0^2 f_1 \\ = \; & \frac{1}{2}tg_0'' + \frac{3}{2}g_0' - \frac{3}{8}\{tA_0' + 2A_0 + t(tD_0)'' + 2(tD_0)'\}; \end{aligned} \tag{29a}$$

$$\begin{aligned} & t^3 g_1''' + 5tg_1'' + (2t - \xi_0^2 t^3)g_1' - (2 + 2\xi_0^2 t^2)g_1 \\ = \; & -\frac{1}{2}(t^2 f_0'' + tf_0' - f_0) + \frac{3t^2}{8}(-iB_0' + 2iC_0 + itC_0'). \end{aligned} \tag{29b}$$

These equations can also be solved by Laplace transforms. In other words, the nonlocal condensate functions $f(x^2)$ and $g(x^2)$ can be analytically solved order by order in m.

IV. Discussion and summary

Much of the material which I have described so far has been presented elsewhere in a preprint [5], in which I have also discussed a couple of problems where application of our results is most relevant. In the context of the standard QCD sum rules (such as the Belyaev-Ioffe nucleon mass sum rules [6,4]), our analytical results on nonlocal condensates may not be very useful if the resultant series converges rapidly, and in general our analytical expressions may be used to perform further analytical analysis of the problems as a way to improve the results obtained via short-distance expansions (in x_μ). In the case when one considers the response of the QCD vacuum to some external fields, such as the method of QCD sum rules in the presence of an external axial field $Z_\mu(x)$ [7], our analytical expressions for nonlocal condensates help to determine the induced condensates previously treated as new parameters, thereby making the external-field QCD sum rule method more powerful than what it used to be.

The specific way of obtaining a closed set of coupled differential equations for the nonlocal condensates in relation to the quark propagator may be generalized to other condensates, such as those involved in the gluon propagator. Such approach may be very useful for understanding, in terms of QCD, a large number of problems in hadron physics or nuclear physics.

To sum up, I wish to re-iterate that problems in nuclear physics must now be synthesized starting from QCD and in return QCD is giving a new life to nuclear physics.

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Study of Fundamental Symmetries in Nuclei

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Weak interactions in nuclei, with beta decays and muon capture reactions in particular, have been used, over the last forty years, as the unique laboratory for studying fundamental symmetries. Upon surveying briefly my past research efforts in relation to the hypothesis of conserved vector currents (CVC), partially conserved axial currents (PCAC), and the absence of second-class currents, I wish to identify some unique roles that weak interactions in nuclei are yet to play as we venture into the twenty-first century.

PACS. 11.30.Rd – Chiral symmetries.
PACS. 11.40.Ha – Partially conserved axial-vector currents.
PACS. 23.40.–s – beta decay; double beta decay; electron and muon capture.

I. Introduction

It is my great honour to be able to dedicate this contribution to Dr. Ta-You Wu on the occasion of his ninetieth birthday. My encounter with Dr. Wu did not actually begin until about ten years ago, shortly after I returned to Taiwan on a permanent basis and Dr. Edward Yen at Tsing Hua University brought me to pay him a brief visit. Since then, Dr. Wu has gradually assumed the role of being a teacher, a fatherly figure, as well as an intimate friend (when frustrations or feelings can be expressed without hesitation, nor reservation) – a role grown out of my frequent contacts with him, first through preparation of the manuscript on the textbook "Relativistic Quantum Mechanics and Quantum Fields" (World Scientific, Singapore, 1991), then on the arrangements of his regular classes at Taida, and lately even on regular visits whenever I could find time to do so. I truly admire Dr. Wu when he says that people respects him because he poses no threats to anyone else – his state of mind is way beyond what Confucius says about "knowing and feeling the Heaven's will"(知天命).

There are only a very small number of people who, through many years of hard work and tough thinking, luckily reach the state of truly understanding and deeply enjoying a certain subject. One day near the end of the last year, Dr. Wu showed me [1], during my say-hello visit at his residence, his note on the Lorentz invariance of *Ldt* in Hamilton's principle. He sort of entertained himself with the notion that for so many years of teaching on the subject he did not pinpoint the specific aspect now published in [1]. Over the last decade, I have learned from Dr. Wu quite a few aspects of similar nature. In his seventy years both as a physicist and as a teacher, Dr. Wu has recognized and accumulated a wealth of knowledge regarding subtleties in different areas of physics and revealed many of these points to me primarily through informal conversations. To this date, Dr. Wu often entertains himself and his visitors by discussing the subtle aspects of a specific physics subject with little concern whether the subject is currently fashionable or not. This is why he has kept me as one of his sincere admirers.

In a 1957 letter to Dr. Ta-You Wu immediately after he was awarded the Nobel prize, Professor Yang acknowledged Dr. Wu for initiating him "into the field of symmetry laws and group theory in the Spring of 1942". To my knowledge, Dr. Wu's keen interest in symmetry concepts remains strong as of today. It might therefore be appropriate for me to add to this celebration some of my work making use of nuclei as the laboratory to study fundamental symmetries.

An $A = (Z, N)$ nucleus is a system of Z protons and N neutrons stacking up closely in a volume with a radius of a few fm. Stable nuclei ranging from the deuteron ($A = 2$) to nuclei of more than 200 nucleons are found in nature. Admittingly, nuclei are unique systems consisting of strongly interacting elementary particles and account for the diverse variety of objects in our visual world. As most of the known elementary particles do not live long enough, experimentations with nuclei offer an important avenue for further understanding how the laws of nature work or testing if the perceived conjecture is indeed acceptable. For instance, the theory for weak interactions involving charged currents, including Fermi's current-current interactions and discovery of parity violation, began with observations associated with β decays in nuclei.

Therefore, it should not be a surprise that many of the underlying ideas or fundamental symmetries associated with the standard model may be tested in nuclei. The subject itself has grown to the point that it is no longer possible to attempt an overview of the field, which include many subject matters, methodologies, and occasionally variations in the choice of specific models for nuclei or currents. Instead, we wish in this contribution to describe a very specific example and methodology to demonstrate the theme that nuclei offers a unique laboratory for testing the validity of fundamental symmetries.

II. General aspects: theory versus experiment

We wish to use the famous $A = 12$ triad as our illustrative example. The ground states of the ^{12}B and ^{12}N nuclei and the $15.110\,MeV$ excited state of the ^{12}C nucleus constitute an $J = 1^+$ isospin triplet. There are many electroweak reactions involving the $A = 12$ triad, including:

$$
\begin{aligned}
\text{Beta decays:}\quad & {}^{12}B \to {}^{12}C(g.s.) + e^- + \bar{\nu}_e, \\
& {}^{12}N \to {}^{12}C(g.s.) + e^+ + \nu_e, \\
\text{Muon Capture:}\quad & \mu + {}^{12}C(g.s.) \to \nu_\mu + {}^{12}B, \\
\text{Gamma decay:}\quad & {}^{12}C^*(15.110) \to {}^{12}C(g.s.) + \gamma, \\
& e^- + {}^{12}C(g.s.) \to e^- + {}^{12}C^*(15.110), \\
\text{Neutrino excitations:}\quad & \nu_e + {}^{12}C(g.s.) \to e^- + {}^{12}N, \\
& \bar{\nu}_e + {}^{12}C(g.s.) \to e^+ + {}^{12}B, \\
& \nu(\bar{\nu}) + {}^{12}C(g.s.) \to \nu(\bar{\nu}) + {}^{12}C^*(15.110).
\end{aligned}
$$

With $V_\lambda(x)$ and $A_\lambda(x)$ the hadronic weak polar and axial vector currents, Lorentz invariance allows us to introduce the following covariant nuclear form factors:

$$
\begin{aligned}
& <{}^{12}B(p',\xi') \mid V_\lambda(0) \mid {}^{12}C(p)> \\
&= -\sqrt{2}\epsilon_{\lambda\kappa\rho\eta}\xi'^*_\kappa \frac{q_\rho}{2m_p}\frac{Q_\eta}{2M}F_M(q^2),
\end{aligned}
\tag{1a}
$$

$$
\begin{aligned}
& <{}^{12}B(p',\xi') \mid A_\lambda(0) \mid {}^{12}C(p)> \\
&= \sqrt{2}\left\{\xi'^*_\lambda F_A(q^2) + q_\lambda \frac{q\cdot\xi'^*}{m_\pi^2}F_P(q^2) - \frac{Q_\lambda}{2M}\frac{q\cdot\xi'^*}{2m_p}F_E(q^2)\right\}
\end{aligned}
\tag{1b}
$$

where

$$
\begin{aligned}
& q_\lambda \equiv (p'-p)_\lambda, \quad Q_\lambda \equiv (p'+p)_\lambda, \\
& M = \frac{1}{2}(M_i + M_f) = \frac{1}{2}[M({}^{12}C) + M({}^{12}B)], \\
& \xi'^* \equiv (\vec{\xi}'^*, i\xi_0'^*), \\
& \xi'^*\cdot\xi' \equiv \vec{\xi}'^*\cdot\vec{\xi}' - \xi_0'^*\xi_0' = 1, \\
& \xi'^*\cdot p' = 0.
\end{aligned}
$$

Here $F_{M,A,P,E}(q^2)$ are referred to as the "weak magnetism", "axial", "pseudoscalar", and "weak electricity" form factors. In terms of these form factors, the transition amplitude T for the muon capture reaction from ${}^{12}C(g.s.)$ to the ground state of ${}^{12}B$:

$$
\mu^-(p^{(\mu)}, s^{(\mu)}) + {}^{12}C(p) \to \nu_\mu(p^{(\nu)}, s^{(\nu)}) + {}^{12}B(p',\xi') \tag{2}
$$

is given by

$$\begin{aligned}\mathcal{T} = \frac{G}{\sqrt{2}} <^{12} B(p', \xi') \mid [V_\lambda(0) + A_\lambda(0)] \mid ^{12} C(p) > \\ \cdot i\bar{u}^{(\nu)}(p^{(\nu)}, s^{(\nu)})\gamma_\lambda(1+\gamma_5)u^{(\mu)}(p^{(\mu)}, s^{(\mu)}).\end{aligned} \tag{3}$$

We introduce the following combinations of the covariant form factors [2],

$$G_V = -F_M(q^2)\frac{E^{(\nu)}}{2m_p}. \tag{4a}$$

$$G_A = -F_A(q^2) - F_M(q^2)\frac{E^{(\nu)}}{2m_p}, \tag{4b}$$

$$G_P = F_P(q^2)\frac{m_\mu E^{(\nu)}}{m_\pi^2} - F_E(q^2)\frac{E^{(\nu)}}{2m_p} - F_M(q^2)\frac{E^{(\nu)}}{2m_p}, \tag{4c}$$

with

$$\begin{aligned} q^2 &= (p'-p)^2 = (p^{(\mu)} - p^{(\nu)})^2 = -m_\mu^2 + 2m_\mu E^{(\nu)} = 0.740 m_\mu^2, \\ E^{(\nu)} &= (m_\mu - \triangle) - \frac{(m_\mu - \triangle)^2}{2M(^{12}B)} = 91.41\, MeV, \\ \triangle &= M(^{12}B) - M(^{12}C) = 13.881\, MeV. \end{aligned} \tag{4d}$$

Denoting the direction of the muon spin by $\hat{n}$, we have

$$\begin{aligned} G^{-2}\int \frac{d^2\Omega^{(\nu)}}{4\pi}\sum_{s^{(\nu)}} \mid \mathcal{T}(s^{(\mu)}, s^{(\nu)}, \xi'^*) \mid^2 = \tfrac{1}{3}(3G_A^2 - 2G_AG_P + G_P^2) \\ + i\vec{\xi}' \times \vec{\xi}'^* \cdot \hat{n}\, \tfrac{1}{3}(3G_A^2 - 2G_AG_P), \end{aligned} \tag{5a}$$

$$G^{-2}\int \frac{d^2\Omega^{(\nu)}}{4\pi}\frac{1}{2}\sum_{s^{(\mu)}}\sum_{s^{(\nu)}}\sum_{\xi'} \mid \mathcal{T}(s^{(\mu)}, s^{(\nu)}, \xi'^* \mid^2 = 3G_A^2 - 2G_AG_p + G_P^2. \tag{5b}$$

Taking into account the effect of the initial-state Coulomb interaction factorized in a way analogous to the role of Fermi functions in beta decays, we cast the capture rate in the form:

$$\begin{aligned} &\Gamma\,(\mu^{-\,12}C(g.s.) \to \nu_\mu\,^{12}B(g.s.)) = \Gamma_0(3G_A^2 - 2G_AG_P + G_P^2), \\ &\Gamma_0 \equiv \left(\frac{GE^{(\nu)}}{\pi}\right)^2\left(1 + \frac{E^{(\nu)}}{m_\mu + M_f}\right)^{-1} C(^{12}C)\left(\frac{Z(^{12}C)}{137}\frac{m_\mu M_i}{m_\mu + M_i}\right)^3, \\ &Z(^{12}C) = 6, \qquad C(^{12}C) = 0.841. \end{aligned} \tag{6}$$

To obtain observables for the capture reaction with initially polarized muons, we choose the $\hat{z}$ axis for the quantization of the spin of the recoil ^{12}B so that $\vec{\xi}'(\pm) = (\mp 1/\sqrt{2})(\hat{x} \pm i\hat{y})$ and $\vec{\xi}'(0) = \hat{z}$ (with $\hat{x}$, $\hat{y}$, $\hat{z}$ three orthogonal unit vectors) may be used in connection with Eq. (5a). In this way, we obtain the average polarization P_{av} and the average alignment A_{av} as below [2]:

$$\begin{aligned} P_{av} &\equiv \frac{h_{+1} - h_{-1}}{h_{+1} + h_{-1} + h_0} \\ &= \frac{2}{3}\left(1 - \frac{G_P^2}{3G_A^2 - 2G_AG_P + G_P^2}\right), \end{aligned} \tag{7a}$$

$$A_{av} \equiv \frac{h_{+1} + h_{-1} - 2h_0}{h_{+1} + h_{-1} + h_0} = 0. \tag{7b}$$

In the case of unpolarized muon capture, we can also choose $\hat{z} = \hat{\nu}$ in Eqs. (4) (with all the $\hat{n}$ terms dropped out) and calculate the longitudinal polarization P_L and the longitudinal alignment A_L [2]:

$$P_L = \frac{2(2G_AG_V - G_V^2)}{3G_A^2 - 2G_AG_P + G_P^2}, \tag{8a}$$

$$A_L = \frac{2(2G_AG_P - G_P^2)}{3G_A^2 - 2G_AG_P + G_P^2}. \tag{8b}$$

We see that all physical observables associated with the muon capture reaction (2) can be expressed in terms of G_V, G_A, and G_P, or in terms of the covariant form factors $F_{M,A,P,E}(q^2)$.

Along the same line, the transition amplitudes for beta decays are given by

$$\begin{aligned} &\mathcal{T}(^{12}B \to {}^{12}C + e^- + \bar{\nu}_e) \\ =&\frac{G}{2} <^{12} C(p_1) \mid [V_\lambda^\dagger(0) + A_\lambda^\dagger(0)] \mid^{12} B(p_2, \xi) > \cdot i\bar{u}_e(p_e)\gamma_\lambda(1+\gamma_5)v_\nu(p_\nu), \end{aligned} \tag{9a}$$

$$\begin{aligned} &\mathcal{T}(^{12}N \to {}^{12}C + e^+ + \nu_e) \\ =&\frac{G}{2} <^{12} C(p_1) \mid [V_\lambda(0) + A_\lambda(0)] \mid^{12} N(p_2, \xi) > \cdot i\bar{v}_e(p_e)\gamma_\lambda(1+\gamma_5)u_\nu(p_\nu), \end{aligned} \tag{9b}$$

Accordingly, the $e^\mp$ decay energy and angular distributions and the corresponding asymmetry coefficients $\alpha_\pm$ are given by [3]:

$$\begin{aligned} &d^3\Gamma(e^\mp) \\ =&\frac{G^2}{8\pi^4} \mid \sqrt{2}F_A^\mp(0) \mid^2 F_\mp(Z, E_e)p_eE_e(\Delta^\mp - E_e)^2(1 + \eta_\mp + a_\mp E_e)dE_e d\Omega_e \\ &\times \left\{1 \mp (h_1 - h_{-1})(1 + \alpha_\mp E_e)\cos\vartheta_e + (1 - 3h_0)\alpha_\mp E_e\left(\frac{3}{2}\cos^2\vartheta_e - \frac{1}{2}\right)\right\}, \end{aligned} \tag{10}$$

where

$$F_{\mp}(Z, E_e) \equiv \text{Fermi function for the } e^{\mp} \text{ decays},$$

$$h_1, h_{-1}, h_0 \equiv \text{populations of the } J_z = 1, -1, 0 \text{ states of } {}^{12}B \text{ or } {}^{12}N,$$

$$h_1 + h_{-1} + h_0 = 1,$$

$$\cos\vartheta_e \equiv \hat{p}_e \cdot \hat{z},$$

$$\begin{aligned}
a_{\mp} &= \pm\frac{4}{3m_p}\frac{F_M^{\mp}(0)}{F_A^{\mp}(0)}, \\
\alpha_{\mp} &= \frac{1}{3m_p}\left(\pm\frac{F_M^{\mp}(0)}{F_A^{\mp}(0)} - \frac{F_E^{\mp}(0)}{F_A^{\mp}(0)}\right), \\
\eta_{\mp} &\equiv \frac{\Delta^{\mp}}{3m_p}\left(\mp 2\frac{F_M^{\mp}(0)}{F_A^{\mp}(0)} + \frac{F_E^{\mp}(0)}{F_A^{\mp}(0)}\right), \\
\Delta^{\mp} &\equiv M({}^{12}C, {}^{12}N) - M({}^{12}C).
\end{aligned} \tag{11}$$

The form factors $F^{\mp}_{A,M,E}(q^2)$ are defined in the sense of Eqs. (1). They are all related since suitable isospin rotations allow us to transform from one triad member to the other. Again, we see that all physical observables associated with beta dacays can be expressed in terms of the same set of covariant form factors $F_{M,A,P,E}(q^2)$.

We may also introduce, along the same line as Eqs. (1),

$$\begin{aligned}
&< {}^{12}C^*(p', \xi') \mid J_\lambda^{e.m.}(0) \mid {}^{12}C(p) > \\
&= -\epsilon_{\lambda\kappa\rho\eta}\xi_\kappa'^* \frac{q_\rho}{2m_p}\frac{Q_\eta}{2M}\mu(q^2),
\end{aligned} \tag{12}$$

with $\mu(q^2)$ the M1 (magnetic) transition form factor which can be measured via the electron scattering $e^- + {}^{12}C \to e^- + {}^{12}C^*(15.110)$. The γ decay rate is given by

$$\Gamma({}^{12}C^* \to {}^{12}C\,\gamma) = \frac{\alpha}{3}\frac{E_\gamma^3}{m_p^2} \mid \mu(0) \mid^2 . \tag{13a}$$

Using the experimental value of $37.0 \pm 1.1\,eV$, we obtain

$$\mu(0) = 1.97 \pm 0.02. \tag{13b}$$

On the other hand, the hadronic electromagnetic currents can be decomposed as follows,

$$J_\lambda^{e.m.}(x) = \frac{1}{2}I_\lambda^{(3)}(x) + Y_\lambda(x), \tag{14}$$

with $I_\lambda^{(3)}(x)$ and $Y_\lambda(x)$ the third component of the isovector current and the hypercharge current, respectively. The conserved vector current (CVC) hypothesis identify $I_\lambda^{(3)}(x)$ of Eq. (14), the charge-lowering and charge-raising weak currents $V_\lambda(x)$ and $V_\lambda^\dagger(x)$ as the members of the same conserved isovector currents $\vec{I}_\lambda(x)$. CVC leads to

$$F_M^{\mp}(0) = \mu(0) = 1.97 \pm 0.02. \tag{15}$$

It also leads to the prediction that $F_M(q^2)$ and $\mu(q^2)$ have the same q^2 dependence.

Now we turn our attention back to beta decays. With the appropriately calculated f values [4], Eq. (10) may be integrated to yield [3]:

$$\Gamma(e^-) = 124.51\, sec^{-1} [F_A^-(0)]^2 \left(1 + 0.005 \frac{F_E^-(0)}{F_A^-(0)}\right), \tag{16a}$$

$$\Gamma(e^+) = 251.28\, sec^{-1} [F_A^+(0)]^2 \left(1 + 0.006 \frac{F_E^+(0)}{F_A^+(0)}\right), \tag{16b}$$

which are to be compared with the experimental values:

$$\Gamma(e^-) = 32.98 \pm 0.10\, sec^{-1}, \tag{17a}$$

$$\Gamma(e^+) = 59.60 \pm 0.20\, sec^{-1}. \tag{17b}$$

In late 1970's, several beautiful experiments were performed to measure the variation with energy of the asymmetry coefficients $\alpha_\pm$ on the nuclear β decays: $^{12}B \to ^{12} Ce^- \nu_e$ and $^{12}N \to ^{12} Ce^+ \nu_e$. These measurements yield

$$\begin{aligned} \alpha_- &= -(0.07 \pm 0.20)/GeV (\text{Lebrun et al. [5]}), \\ &= +(0.24 \pm 0.44)/GeV (\text{Brandle et al. [6]}) \\ &= +(0.25 \pm 0.34)/GeV (\text{Masuda et al. [7]}), \\ \alpha_+ &= -(2.77 \pm 0.52)/GeV (\text{Masuda et al. [7]}) \\ &\quad -(2.73 \pm 0.39)/GeV (\text{Brandle et al. [8]}). \end{aligned} \tag{18a}$$

The standard picture, consisting of CVC, the absence of second-class currents (SCC), and the hypothesis of partially conserved axial currents (PCAC), leads to the prediction [3, 9]:

$$\begin{aligned} \alpha_- &= (0.88 \pm 0.03)/GeV, \\ \alpha_+ &= -(2.75 \pm 0.03)/GeV. \end{aligned} \tag{18b}$$

The remarkable agreement between experiment and theory has constituted the basis for our confidence towards the standard picture. Note that, in the discussion so far, there is little sensitivity toward the validity of PCAC, which relates the divergence of the charged weak axial currents to the pion source currents (thus resulting in the pion pole dominance for the pseudoscalar form factor).

Finally, we return to the muon capture reaction (2). In principle, P_{av}, A_{av}, P_L, and A_L can be measured by detecting the angular distribution of the electrons from the subsequent β decay of the recoil $^{12}B(g.s.)$ (with respect to the $\hat{z}$ axis defined by $\hat{n}$ or $\hat{\nu}$). In practice, the nonzero value of A_L enters the angular distribution by a multiptication factor of $\alpha_- E_e(\frac{3}{2}cos^2\vartheta_e - \frac{1}{2})$ and so might be too small to be of any interest. Nonetheless, the experimental determination of the capture rate [Eq.(6)], average polarization P_{av} [Eq. (7a)], and longitudinal polarization P_L [Eq.(8a)] already allow us to solve $G_{V,A,P}$; the solution is unique if the standard signs of $G_{V,A,P}$ are adopted.

Experimentally, we have [10]:

$$\left[\Gamma(\mu^{-12}C \to \nu_\mu{}^{12}B)\right]_{\rm exp} = (6.2 \pm 0.3) \times 10^{-3} sec^{-1}. \tag{19}$$

On the other hand, Possoz et al. [11] measured the "apparent" average polarization of the recoil ^{12}B(g.s.) from polarized-muon capture by ^{12}C(g.s.) and subtracted the contribution due to those recoil ^{12}B(g.s.) which were produced indirectly, e.g. $\mu^{-12}C(g.s.) \to \nu_\mu{}^{12}B^* \to \nu_\mu{}^{12}B(g.s.)\gamma$. More careful analysis on the subtraction of the background in their experiment were performed by Kobayashi *et al.* [12] and by Hwang [2]. The final model-independent result for the average polarization of $^{12}B(g.s.)$ produced directly form $\mu^{-12}(g.s.) \to \nu_\mu{}^{12}B(g.s.)$ is given by

$$(P_{av})_{\rm exp} = 0.47 \pm 0.05. \tag{20}$$

We note that, making use of the formulae (6) and (7a), we may extract G_A and G_P numerically from the data (19) and (20). The value of G_A in turn gives the ratio $F_M^-(q^2)/F_A^-(q^2)$ at $q^2 = 0.740m_\mu^2$. Taking, with justification, the assumption of similar q^2 dependence for $F_{M,A,E}(q^2)$, viz.:

$$\frac{F_M(q^2)}{F_M(0)} \approx \frac{F_A(q^2)}{F_A(0)} \approx \frac{F_E(q^2)}{F_E(0)}, \tag{21}$$

we obtain

$$F_M^-(0)/F_A^-(0) = 4.73 \pm 1.16. \tag{22a}$$

As indicated earlier, we have, granting validity of CVC,

$$F_M(0)/F_A(0) = 3.86 \pm 0.12. \tag{22b}$$

The agreement between (22a) and (22b) constitutes an impressive test of CVC.

Meanwhile, the asymmetry measurements [5, 6, 7, 8] yields

$$F_E(0)/F_A(0) = 3.67 \pm 0.44, \tag{23}$$

a value which can be understood in the nucleon-only impulse approximation (NOIA) [12] (without the presence of second-class currents). Thus, we deduce from the value of G_P at $q^2 = 0.740 m_\mu^2$

$$F_P(q^2)/F_A(q^2) = -(1.08 \pm 0.24). \tag{24a}$$

On the other hand, assuming PCAC for the nucleon pseudoscalar form factor and using the Cohen-Kurath wave functions of ${}^{12}C(g.s.)$ in the NOIA calculation, we obtain [2]:

$$\begin{aligned}[F_P(q^2)/F_A(q^2)]_{NOIA} &= -1.02(1 + q^2/m_\pi^2)^{-1} + \delta \\ &= -0.99.\end{aligned} \tag{24b}$$

The excellent agreement between Eqs. (24a) and (24b) constitutes a test of PCAC, another important ingredient in the standard picture [namely, CVC, PCAC, and the absence of second-class axial vector currents].

To sum up, we have seen in a model-independent fashion that, even without the CVC test by extracting the shape factors form the observed $e^\mp$ energy spectra [13] in the β decays of ${}^{12}B$ and ${}^{12}N$, the basic ingredients of the standard picture, namely, CVC, PCAC, and the absence of second-class axial vector currents, can be individually tested and confirmed by the combination of the muon capture data [10,11] and the results of the asymmetry measurements [5-8].

III. Discussion and summary

The standard picture, namely, CVC, PCAC, and the absence of second-class currents, is an overall statement of the various symmetries including chiral symmetry. It is of utmost importance to ask if the existing data have verified individually the basic ingredients of the standard picture. The analysis presented in the previous section demonstrates that there is a completely model-independent positive answer to the question, making use of the $A = 12$ nuclei as the laboratory for testing fundamental symmetries.

There are other nuclei which also deserve serious attention. For instance, the case with $A = 8$ looks quite promising [14,15]. The $A = 6$ [16] and $A = 3$ [17] nuclei are another noteworthy places. Nevertheless, it remains difficult to arrive at a completely model-independent result regarding the validity of the standard picture.

What may be of much interest is that, by going from beta decays ($q^2 \approx 0$) to the muon capture reaction ($q^2 \approx m_\mu^2$), we are able to better determine the weak magnetism form factor so as to test CVC in a decisive manner and we also begin to have a real chance to determine the pseudoscalar form factor in order to compare with the PCAC prediction. It is clear that we should attempt to employ reactions such as (μ, ν_μ) or (ν_μ, μ) at even larger q^2 so as to test the various symmetries to higher precision.

Furthermore, the recent measurement [17] of the muon capture reaction has reached a remarkable experimental accuracy:

$$\Gamma(\mu^- + {}^3He \to \nu_\mu + {}^3H) = (1496 \pm 4)\, sec^{-1}. \tag{25}$$

With the data of such precision, one may begin to use it to test specific symmetry to high precision, or equivalently, to constrain physics beyond the standard model [18]. Should the experimental error cited for Eq. (19) [of about 5 %] could be improved to a similar level [of about 0.3 % in Eq. (25)], the various conclusions drawn in the previous section would become highly precise. We could be heading for another era of precision physics using nuclei as the laboratory for studying physics beyond the standard model.

In summary, we have used $A = 12$ triad to show in a model-independent fashion that, even without the CVC test by extracting the shape factors form the observed $e^{\mp}$ energy spectra [13] in the β decays of ^{12}B and ^{12}N, the basic ingredients of the standard picture, namely, CVC, PCAC, and the absence of second-class axial vector currents, can be individually tested by the combination of the muon capture data [10,11] and the results of the asymmetry measurements [5-8]. The opportunity to employ nuclei to perform high-precision symmetry studies is genuine and should not be overlooked.

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ρNN and ωNN couplings in the external-field QCD sum rule method

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We extend the method of QCD sum rules in external fields to determine the ρNN and ωNN strong couplings, including both the vector and tensor couplings. Such information has been used for years in the construction of nuclear potentials, but our attempt represents the first direct contact with QCD in terms of the basic parameters associated with the nontrivial QCD vacuum. [S0556-2813(97)02812-4]

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I. INTRODUCTION

The strong ρNN and ωNN couplings are the basic input for the description of nuclear forces in terms of meson exchanges between the nucleons [1–4]. Although it is now generally believed that quantum chromodynamics (QCD) describes strong interactions among quarks, antiquarks, and gluons (building blocks of hadrons), the linkage between QCD and the phenomenological ρNN and ωNN couplings has so far yet to be made.

The method of QCD sum rules [5–9] was introduced to allow some linkage to be made between QCD and hadron physics, especially in addressing the role of the various quark and gluon condensates associated with the nontrivial QCD vacuum. As shown by Ioffe and Smilga [10], it is possible to determine coupling constants (at $q^2=0$) by considering the response of the QCD vacuum to some external fields in the evaluation of the two-point correlation function. In this paper, we wish to obtain ρNN and ωNN strong couplings making use of the external-field QCD sum rule method. To obtain simultaneously the vector and tensor couplings, we write the external vector meson field as

$$Z_\mu=-\tfrac{1}{2}Z_{\mu\nu}x^\nu, \tag{1}$$

where $Z_{\mu\nu}$ is to be regarded as a constant background field. For a vector meson field, $Z_{\mu\nu}$ is traceless, i.e., $Z^\mu_\mu=0$, to ensure the Lorentz condition $\partial_\mu Z^\mu=0$. However, we note that $Z_{\mu\nu}$ can in general be decomposed into symmetric and antisymmetric parts. Under the influence of $Z_{\mu\nu}$, the vacuum structure of QCD changes in different ways, resulting in five main induced condensate parameters introduced later in the paper. Two of these induced condensates correspond to the symmetric part of $Z_{\mu\nu}$, while the others correspond to the antisymmetric part. In deriving the QCD sum rules for ρNN and ωNN strong couplings, we note that the procedure adopted for the antisymmetric part of $Z_{\mu\nu}$ has much in common with that first introduced by Ioffe and Smilga [10] for calculating the magnetic moments of the nucleon, while we find that the symmetric part of $Z_{\mu\nu}$ gives rise to QCD sum rules which have some nice features, including a prediction such as

$$\frac{g_{\rho NN}}{g_{\omega NN}}=\frac{1}{3}, \tag{2}$$

in excellent agreement with the values adopted in nuclear force studies.

One purpose of the present study is to first understand whether the ρNN and ωNN strong couplings can be described adequately in the context of QCD sum rules and then to proceed to use the framework to undertake more difficult investigations such as the determination of the parity-violating ρNN and ωNN weak couplings (which involve regularization of three-loop diagrams and which will be described in some detail in our next publication [11]).

This paper is organized briefly as follows: In Sec. II, we outline the key ingredients associated with the QCD sum rules. In Sec. III, we describe the derivation of the QCD sum rules for the ρNN and ωNN strong couplings. A numerical analysis is given in Sec. IV, while Sec. V contains a brief summary.

II. METHOD OF QCD SUM RULES

Following the procedure [10] to derive QCD sum rules in the presence of an external field, we consider the correlation function as defined by

$$i\int d^4x\, e^{ip\cdot x}\langle 0|T\eta_N(x)\bar{\eta}_N(0)|0\rangle_Z=\Pi(p)+hZ_{\mu\nu}\Pi^{\mu\nu}(p), \tag{3}$$

where $\Pi(p)$ is the correlation function when the external field $Z_{\mu\nu}$ is absent. Here $\Pi(p)$ is just the function which we use to determine the nucleon mass [7]. The second term describes the linear response of the correlator to a small classical external field $Z_{\mu\nu}$, as characterized by Eq. (1), where Z_μ stands for either the isovector ρ meson or isoscalar ω meson field. Such linear response is the primary focus of this paper. The entity h describes the coupling between the designated external field and the up (u) quark field; more explicitly, the interaction between the external field ($Z=\rho,\omega$) and the quarks may be specified by

$$\mathcal{L}_{\rho qq}=-h\bar{\psi}\vec{\tau}\gamma_\mu\psi\cdot\vec{\rho}^{\,\mu}, \tag{4}$$

$$\mathcal{L}_{\omega qq}=-h\bar{\psi}\gamma_\mu\psi\omega^\mu, \tag{5}$$

with ψ the Dirac spinor consisting of the (u,d) isospin doublet. We note that we use the same coupling constant h for ρ and ω fields. We wish to also note that, at the quark level, there is not much room in the choice of the form for the ρqq and ωqq couplings, as dictated by both that the quarks are considered pointlike in the QCD sum rule method and that the Lorentz four-vector nature and the isospin structure of the external fields in question are already given. The similarity to the electromagnetic field $F_{\mu\nu}$ treated as an external field [8] should not be taken as to indicate that our sum rules are not independent or the vector-dominance model is automatically ensured, as it is important to recognize that $F_{\mu\nu}$ is *antisymmetric*, while we find it is useful to focus on the sum rules related to both the *symmetric* and *antisymmetric* parts of $Z_{\mu\nu}$.

The entity η_N $(=\eta_p,\eta_n)$ is some composite operator with the nonvanishing matrix element $\langle 0|\eta_N|N\rangle$, which may be chosen as

$$\eta_p(x)=\epsilon^{abc}[u^{aT}(x)C\gamma_\mu u^b(x)]\gamma_5\gamma_\mu d^c(x), \tag{6}$$

$$\eta_n(x)=\epsilon^{abc}[d^{aT}(x)C\gamma_\mu d^b(x)]\gamma_5\gamma_\mu u^c(x), \tag{7}$$

where $u^a(x)$ and $d^a(x)$ are the up and down quark fields, respectively. The choice of these composite operators is not unique [12], but the above choice has become fairly standard in the literature.

We need to characterize how the nontrivial QCD vacuum responds to the external filed $Z_{\mu\nu}$. To lowest order, we may introduce the following five condensation parameters $(\chi,\kappa,\xi,\zeta,\phi)$. Three of them (χ,κ,ξ) are related to the antisymmetric part of $Z_{\mu\nu}$, while the other two (ζ,ϕ) have to do with the symmetric part. With Z^S and Z^A describing, respectively, the symmetric and antisymmetric parts of the external field $Z_{\mu\nu}$, we have

$$\langle\bar{q}\sigma_{\alpha\beta}q\rangle_Z=\chi h_q Z^A_{\alpha\beta}\langle\bar{q}q\rangle, \tag{8}$$

$$g\langle\bar{q}G_{\alpha\beta}q\rangle_Z=\kappa h_q Z^A_{\alpha\beta}\langle\bar{q}q\rangle, \tag{9}$$

$$g\epsilon_{\mu\nu\lambda\sigma}\langle\bar{q}\gamma_5 G_{\lambda\sigma}q\rangle_Z=i\xi h_q Z^A_{\mu\nu}\langle\bar{q}q\rangle, \tag{10}$$

$$\langle\bar{q}\tfrac{1}{2}(\gamma_\mu\nabla_\nu+\gamma_\nu\nabla_\mu)q\rangle_Z=\zeta h_q Z^S_{\mu\nu}\langle\bar{q}q\rangle, \tag{11}$$

$$\langle\bar{q}\tfrac{1}{2}(\nabla_\mu\nabla_\nu+\nabla_\nu\nabla_\mu)q\rangle_Z$$
$$=-\frac{g}{8}\langle\bar{q}\sigma\cdot Gq\rangle g_{\mu\nu}+\frac{i}{2}h_q Z^S_{\mu\nu}\langle\bar{q}q\rangle+i\phi h_q Z^S_{\mu\nu}\langle\bar{q}q\rangle, \tag{12}$$

where $G_{\alpha\beta}\equiv G^n_{\alpha\beta}\lambda^n/2$ is the gluon field. The coupling constant h_q $(q=u$ or $d)$ depends on the external field Z; for instance, $h_u=-h_d=h$ for the ρ^0 meson field and $h_u=h_d=h$ for the ω meson field.

III. ρNN AND ωNN COUPLINGS

A. Quark propagator

To evaluate the correlation function at the quark level, we need to know the quark propagator in the nontrivial QCD vacuum under the influence of the external field Z_μ. We may write

$$\langle Tq^a(x)\bar{q}^b(0)\rangle_Z=iS^{ab}_Z(x)+\langle :q^a(x)\bar{q}^b(0):\rangle_Z. \tag{13}$$

The first term on the right-hand side is just the ordinary propagator in the background fields Z_μ. It can be calculated from the expression

$$iS^{ab}_Z(x)=iS^{ab}_0(x)-i\int d^4z\; iS^{ac}_0(x-z)\gamma_\mu \times\left[gA^\mu_n(z)\left(\frac{\lambda_n}{2}\right)^{cd}+hZ^\mu(z)\delta^{cd}\right]iS^{db}_0(z), \tag{14}$$

where $iS^{ab}_0(x)$ is the Fourier transform of $i\delta^{ab}/(\hat{p}-m)$.

The second term in Eq. (13) has to do with the nontrivial property of the QCD vacuum; that is, the vacuum expectation value of the normal product of operators in general differs from zero. We may obtain the explicit expression making use of the Taylor expansion together with the equation of motion

$$\langle :q(x)\bar{q}(0):\rangle=\langle :q\bar{q}:\rangle+x^\mu\langle :(\partial_\mu q)q:\rangle +\tfrac{1}{2}x^\mu x^\nu\langle :(\partial_\mu\partial_\nu q)q:\rangle+\cdots, \tag{15}$$

$$\langle :q^a\bar{q}^b:\rangle=-\tfrac{1}{12}\delta^{ab}[1+\tfrac{1}{2}h_q\chi\sigma_{\mu\nu}Z^A_{\mu\nu}]\langle\bar{q}q\rangle, \tag{16}$$

$$\langle :(\partial_\mu q^a)\bar{q}^b:\rangle=-\tfrac{1}{12}\delta^{ab}\zeta h_q\gamma^\nu Z^S_{\mu\nu}\langle\bar{q}q\rangle, \tag{17}$$

$$\begin{aligned}\langle :(\partial_\mu\partial_\nu q^a)\bar{q}^b:\rangle=&\frac{h_q}{144}\left(1+\kappa-\frac{\xi}{2}\right)\langle\bar{q}q\rangle Z^{\rho\sigma}_A(g_{\mu\rho}\sigma_{\nu\sigma}\\&+g_{\nu\rho}\sigma_{\mu\sigma})+\frac{g}{96}\langle\bar{q}\sigma\cdot Gq\rangle g_{\mu\nu}+\frac{h_q}{144}\\&\times(1+\kappa+\xi)\langle\bar{q}q\rangle Z^{\rho\sigma}_A g_{\mu\nu}\sigma_{\rho\sigma}-\frac{ih_q}{24}\\&\times(1+2\phi)\langle\bar{q}q\rangle Z^S_{\mu\nu}+\frac{h_q}{48}\phi\langle\bar{q}q\rangle Z^{\rho\sigma}_S\\&\times(g_{\mu\rho}\sigma_{\nu\sigma}+g_{\nu\rho}\sigma_{\mu\sigma}).\end{aligned} \tag{18}$$

Some formulas useful for simplification are listed below:

$$\langle q^a_i\bar{q}^b_j G^m_{\alpha\beta}\rangle_0=-\frac{1}{192}\langle\bar{q}\sigma\cdot Gq\rangle(\sigma_{\alpha\beta})_{ij}\left(\frac{\lambda_n}{2}\right)^{ab}, \tag{19}$$

$$g\langle q_i^a\overline{q_j^b}G^n_{\alpha\beta}\rangle_Z = -\frac{h_q}{16}\langle\bar{q}q\rangle\left(\frac{\lambda_n}{2}\right)^{ab}\left[\delta_{ij}\kappa Z_{\alpha\beta}-\frac{i}{4}\xi\epsilon_{\alpha\beta\lambda\sigma}Z^{\lambda\sigma}(\gamma_5)_{ij}\right]. \tag{20}$$

Here the subscript Z denotes the condensate values induced by the external field and the subscript 0 represents the case without the external field.

To sum up, we obtain the quark propagator as a short-distance expansion:

$$\begin{aligned}\langle Tq^a(x)\bar{q}^b(0)\rangle &= i\delta^{ab}\frac{\hat{x}}{2\pi^2x^4}+\frac{ig}{32\pi^2x^2}\left(\frac{\lambda^n}{2}\right)^{ab}G^n_{\alpha\beta}(\hat{x}\sigma^{\alpha\beta}+\sigma^{\alpha\beta}\hat{x})+\delta^{ab}\frac{ih_q}{32\pi^2x^2}Z_{\alpha\beta}(\hat{x}\sigma^{\alpha\beta}+\sigma^{\alpha\beta}\hat{x})\\ &\quad-\delta^{ab}\frac{h_q}{8\pi^2x^4}Z_{\alpha\beta}x^\alpha x^\beta\hat{x}-\frac{1}{12}\delta^{ab}\langle\bar{q}q\rangle-\frac{ih_q}{144}\delta^{ab}\left(1+\kappa-\frac{\xi}{2}\right)\langle\bar{q}q\rangle Z_A^{\rho\sigma}(x_\rho x_\sigma-\hat{x}\gamma_\sigma x_\rho)\\ &\quad-\frac{1}{12}\delta^{ab}\zeta h_q x^\mu\gamma^\nu Z^S_{\mu\nu}\langle\bar{q}q\rangle-\frac{1}{24}\delta^{ab}\chi h_q\sigma_{\mu\nu}Z_A^{\mu\nu}\langle\bar{q}q\rangle+\delta^{ab}\frac{g}{192}\langle\bar{q}\sigma\cdot Gq\rangle x^2+\delta^{ab}\frac{h_q}{288}(1+\kappa+\xi)\\ &\quad\times\langle\bar{q}q\rangle Z_A^{\rho\sigma}\sigma_{\rho\sigma}x^2-\delta^{ab}\frac{ih_q}{48}(1+\phi)\langle\bar{q}q\rangle Z^S_{\mu\nu}x^\mu x^\nu+\delta^{ab}\frac{ih_q}{48}\phi\langle\bar{q}q\rangle Z^S_{\mu\nu}\hat{x}x^\mu\gamma^\nu,\end{aligned} \tag{21}$$

with $\hat{x}\equiv\gamma^\mu x_\mu$. Note that it is an extension from [10] since $Z^S_{\mu\nu}$ has been explicitly included. The terms in the antisymmetric part also differ slightly from [8].

B. Correlation function evaluated at the quark level

To evaluate the correlation function at the quark level, we make use of the quark propagator together with the factorization approximation:

$$\langle T\eta_p(x)\overline{\eta_p}(0)\rangle = -2\epsilon^{abc}\epsilon^{a'b'c'}\mathrm{Tr}[iS_u^{bb'}(x)\gamma_\nu CiS_u^{aa'}(x)^TC\gamma_\mu]\gamma_5\gamma^\mu iS_d^{cc'}(x)\gamma^\nu\gamma_5, \tag{22}$$

where $iS_q(x)$ represents the quark propagator obtained earlier. The factorization approximation (via Wick's theorem for free quantum field theories) is justified in the large-N_c limit of QCD. The above expression is suitable for the background ρ^0 or ω fields. Slight modification is needed for the $\rho^\pm$ fields, but the results are consistent with the form of strong ρNN interactions.

Substituting the quark propagator into the above expression (and after carrying out the routine but tedious simplications), we obtain the correlation function in momentum space:

$$\begin{aligned}h\Pi_{\alpha\beta}(p) &= \frac{1}{16\pi^4}(\hat{p}\sigma_{\alpha\beta}+\sigma_{\alpha\beta}\hat{p})\left\{\frac{1}{2}h_up^2\ln(-p^2)-\frac{\chi}{3}h_u\frac{a^2}{p^2}+\frac{a^2}{6p^4}\left[h_d+\frac{2}{3}h_u-\frac{1}{3}h_u(\kappa-2\xi)\right]-\frac{\chi m_0^2}{24}h_u\frac{a^2}{p^4}-\frac{1}{48}(4h_u\right.\\ &\quad\left.+h_d)\frac{b}{p^2}\right\}-\frac{a}{96\pi^4}\sigma_{\alpha\beta}\left\{-h_d\chi p^2\ln(-p^2)-\frac{1}{p^2}h_um_0^2\ln\left(-\frac{p^2}{\mu^2}\right)-[6h_u-h_d-2h_d(2\kappa-\xi)]\ln(-p^2)\right\}\\ &\quad-\frac{ia}{16\pi^4}\hat{p}(p_\alpha\gamma_\beta-p_\beta\gamma_\alpha)\left\{\left(h_u+\frac{1}{2}h_d\right)\frac{1}{p^2}+\frac{1}{3}h_d\chi\ln(-p^2)+\frac{1}{4}h_um_0^2\frac{1}{p^4}\right\}+\frac{i}{4\pi^4}\hat{p}p_\alpha p_\beta\left\{\frac{1}{8}(2h_u+h_d)\ln(-p^2)\right.\\ &\quad\left.-\frac{a}{3p^2}\zeta(h_u+h_d)+\frac{a^2}{3p^6}(2h_u+h_d)\right\}+\frac{i}{8\pi^4}(p_\alpha\gamma_\beta+p_\beta\gamma_\alpha)\left\{\frac{1}{8}(2h_u+h_d)\ln(-p^2)+\frac{a}{3}\zeta(2h_u-h_d)\ln(-p^2)\right.\\ &\quad\left.-\frac{a^2}{6p^4}(2h_u+h_d)\right\}-\frac{i}{32\pi^4}\hat{p}(p_\alpha\gamma_\beta+p_\beta\gamma_\alpha)\left\{\frac{a}{p^2}h_d\phi\right\}+\frac{ia}{16\pi^4}p_\alpha p_\beta\left\{\frac{1}{p^2}(2h_u+h_d+\phi h_d)-\frac{8a}{3p^4}\xi h_u\right.\\ &\quad\left.+\frac{1}{2p^4}gh_um_0^2\right\},\end{aligned} \tag{23}$$

where $a \equiv -(2\pi)^2\langle q\rangle$, $b \equiv \langle g_c^2 G^2\rangle$, and $\langle g_c \sigma \cdot G_q\rangle \equiv -m_0^2\langle q\rangle$. The above result indicates that there are four different Lorentz-Dirac tensor structures for the symmetric part and three for the antisymmetric part, resulting in seven QCD sum rules.

C. Correlation function at the hadronic level

In general, the composite operator η_p connects the QCD vacuum to the proton state as well as to the excited baryons and the continuum (such as the $N\pi$ continuum). To consider the ground-state contribution, we introduce the ωNN couplings as follows:

$$\mathcal{L}_{Zpp} = -h\bar{\psi}_p\left(F_1^\omega \gamma_\mu + F_2^\omega \frac{i}{2m}\sigma_{\mu\nu}q^\nu\right)\psi_p \cdot \omega^\mu, \quad (24)$$

where ω^μ stands for the ω meson field and the coupling constant h at the quark level is factored out for the sake of convenience. The ρNN couplings are introduced in a similar way by substituting ω_μ by $\vec{\tau}\cdot\vec{\rho}_\mu$.

Accordingly, we write the correlation function in the background Z field as follows:

$$\Pi_{\mu\nu}(p) = -\frac{\lambda_N^2}{4}\frac{1}{(p^2-m^2)^2}\left\{F_1^Z\left[(\hat{p}\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat{p}) + 2m\sigma_{\mu\nu} + 2i(\hat{p}+m)g_{\mu\nu} + 2i(p_\mu\gamma_\nu + p_\nu\gamma_\mu) - 8ip_\mu p_\nu \frac{\hat{p}+m}{p^2-m^2}\right] + F_2^Z\left[\frac{p^2+m^2}{m}\sigma_{\mu\nu} - 2i\frac{\hat{p}}{m}(p_\mu\gamma_\nu - p_\nu\gamma_\mu) + (\hat{p}\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat{p})\right]\right\} + \cdots, \quad (25)$$

where λ_N is the overlap amplitude with $\langle 0|\eta_p(0)|P(p)\rangle = \lambda_N u(p)$ [with $u(p)$ the proton Dirac spinor]. The ellipsis in the above equation stands for the contributions from the excited baryons and the continuum.

D. QCD sum rules

Comparing the results of the correlation function evaluated at both the quark and hadron levels, we obtain QCD sum rules corresponding to different Lorentz-Dirac tensor structures. We follow the standard procedure by performing Borel transform to both sides in order to accentuate the contributions from the ground-state baryon (i.e., the proton). We obtain, for $p_\mu\gamma_\nu + p_\nu\gamma_\mu$,

$$\frac{\lambda_N^2 F_1^Z h}{2M^2} e^{-m^2/M^2} = \frac{M^4}{64\pi^4}(2h_u+h_d) + \frac{\zeta M^2 a}{24\pi^4}(2h_u - h_d) + \frac{a^2}{48\pi^4 M^2}(2h_u+h_d); \quad (26)$$

for $\hat{p}p_\mu p_\nu$,

$$\frac{\lambda_N^2 F_1^Z h}{M^4} e^{-m^2/M^2} = \frac{M^2}{32\pi^4}(2h_u+h_d) - \frac{\zeta a}{12\pi^4}(h_u+h_d) + \frac{a^2}{24\pi^4 M^4}(2h_u+h_d); \quad (27)$$

for $p_\mu p_\nu$,

$$\frac{m_N\lambda_N^2 F_1^Z h}{M^4} e^{-m^2/M^2} = \frac{a}{16\pi^4}(2h_u+h_d+\phi h_d) + \frac{\zeta a^2}{6\pi^4 M^2}h_u - \frac{g m_0^2 a}{32\pi^4 M^2}h_u; \quad (28)$$

for $\hat{p}(p_\mu\gamma_\nu + p_\nu\gamma_\mu)$,

$$0 = -\frac{a}{32\pi^4}\phi h_a: \quad (29)$$

for $\hat{p}\sigma_{\mu\nu} + \sigma_{\mu\nu}\hat{p}$,

$$-\frac{\lambda_N^2 h}{4M^2}(F_1^Z+F_2^Z)e^{-m^2/M^2} = \frac{1}{16\pi^4}\left[-\frac{1}{2}M^4 h_u + \frac{\chi a^2}{3}h_u - \frac{\chi m_0^2}{24}\frac{a^2}{M^2}h_u + \frac{1}{18}\frac{a^2}{M^2}[2h_u+3h_d-h_u(\kappa+2\xi)]\right]; \quad (30)$$

for $\hat{p}(p_\mu\gamma_\nu - p_\nu\gamma_\mu)$,

$$\frac{i\lambda_N^2 h}{2m_N M^2}F_2^Z e^{-m^2/M^2} = \frac{ia}{16\pi^4}\left[\frac{\chi}{3}M^2 h_d + \left(h_u + \frac{1}{2}h_d\right)\right]; \quad (31)$$

for $\sigma_{\mu\nu}$,

$$-\frac{m_N\lambda_N^2 h}{2M^2}\left[F_1^Z + \left(1-\frac{M^2}{2m_N^2}\right)F_2^Z\right]e^{-m^2/M^2} = -\frac{a}{96\pi^4}\left[\chi M^4 h_d + [6h_u - h_d - h_d(4\kappa-2\xi)]M^2 + m_0^2 h_u \ln\left(\frac{M^2}{\mu^2}\right)\right]. \quad (32)$$

It is clear that the first four sum rules have to do with the symmetric part of the external field, while the remaining three are from the antisymmetric part. It is to be understood that on the left-hand sides of these sum rules contributions from the excited baryons and continuum are yet to be included explicitly, while on the right-hand sides terms of higher dimensions may become of numerical importance (especially when cancellation occurs for terms which have been explicitly included).

Some general remarks are in order by looking at the above QCD sum rules. First of all, the first two sum rules are consistent with each other if the terms in ζa are negligible compared to the others. We assume that this is indeed the

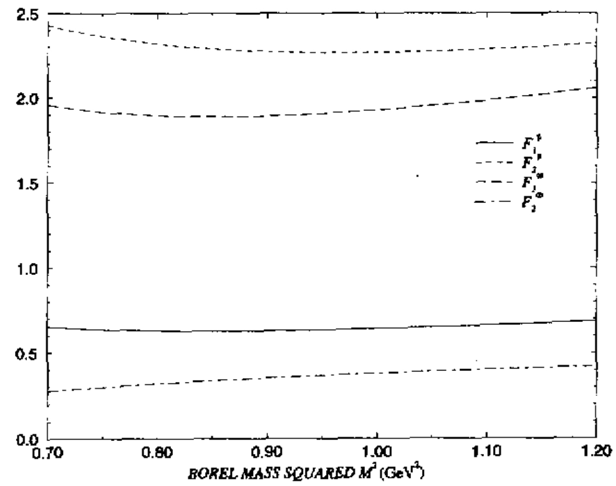

FIG. 1. Vector and tensor couplings F_1^ρ, F_1^ω, F_2^ρ, and F_2^ω plotted as functions of the Borel mass squared M^2 in the range of $0.7\ \mathrm{GeV}^2 \leqslant M^2 \leqslant 1.2\ \mathrm{GeV}^2$.

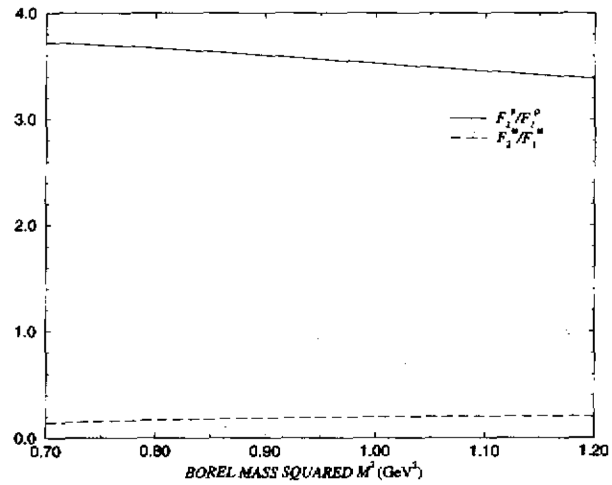

FIG. 2. Ratios F_2^ρ/F_1^ρ and F_2^ω/F_1^ω plotted as functions of the Borel mass squared M^2.

case and use the resultant equation to determine the vector meson-baryon coupling F_1. In particular, we have

$$\frac{\lambda_N^2 F_1^Z h}{2M^2}\, e^{-m^2/M^2} = \left(\frac{M^4}{64\pi^4} + \frac{a^2}{48\pi^4 M^2}\right)(2h_u + h_d), \quad (33)$$

which yields

$$\frac{F_1^\rho}{F_1^\omega} = \frac{g_{\rho NN}}{g_{\omega NN}} = \frac{1}{3}. \quad (34)$$

Such prediction is in excellent agreement with the nuclear force studies: $g_\rho = 2.79$ and $g_\omega = 8.37$ [4,3].

On the other hand, we see from the fourth sum rule that the condensate parameter ϕ is likely to be small unless the excited-state contributions or terms of much higher dimensions come into play for cancellation.

Assuming that the induced condensate parameters χ, κ, and ξ are identical for the ρ and ω (as they are determined primarily by the Lorentz-Dirac tensor structure), we may use F_1 obtained above in connection with the last three sum rules and eliminate all the induced condensate parameters altogether. In this way, we obtain

$$\frac{\lambda_N^2}{2M^2}(F_2^\rho - F_2^\omega)e^{-m^2/M^2} = \frac{1}{16\pi^4}\left(\frac{1}{2}M^4 + \frac{4}{3}\frac{a^2}{M^2}\right), \quad (35)$$

$$\frac{\lambda_N^2}{2M^2}(F_2^\rho + F_2^\omega)e^{-m^2/M^2} = \frac{m_N a}{8\pi^4}. \quad (36)$$

These sum rules allow us to determine the tensor couplings F_2^ρ and F_2^ω in a model-independent manner.

IV. NUMERICAL RESULTS

To analyze each individual sum rule (corresponding to the specific Lorentz-Dirac tensor structure), knowledge of the induced condensates is an important input. As shown at the end of the previous section, however, it is possible to eliminate these unknowns from the equations, resulting in four equations on F_1^ρ, F_1^ω, F_2^ρ, and F_2^ω. These equations involve only one well-known condensate parameter, i.e., the quark condensate a ($\approx 0.546\ \mathrm{GeV}^3$). Of course, derivation of these equations suffers from the fact that contributions from the excited states and continuum and terms of higher dimensions have been neglected. Granting that these higher-order contributions are of much less numerical significance, however, predictions from these four equations can be used with reasonable confidence.

There are additional "standard" modifications which may be implemented in the analysis without jeopardizing the possibility of obtaining model-independent results. First, the anomalous dimension associated with the composite operator can be taken into account for each term in the sum rules. Second, the contributions from the excited states and continuum may be approximated by the prediction at the quark level beyond a certain threshold, say, $p^2 \geqslant W^2$—thus, the need to consider explicitly these contributions can be avoided altogether. The resultant sum rules are recorded below:

$$F_1^\rho = \frac{1}{32\pi^4\lambda_N^2}\left(M^6 E_1 L^{-4/9} + \frac{4}{3}a^2 L^{4/9}\right)e^{m^2/M^2}, \quad (37)$$

$$F_1^\omega = 3F_1^\rho, \quad (38)$$

$$F_2^\rho = \frac{1}{32\pi^4\lambda_N^2}\left(M^6 E_1 L^{-4/9} + \frac{8}{3}a^2 L^{4/9} + 4maM^2\right)e^{m^2/M^2}, \quad (39)$$

$$F_2^\omega = \frac{1}{32\pi^4\lambda_N^2}\left(-M^6 E_1 L^{-4/9} - \frac{8}{3}a^2 L^{4/9} + 4maM^2\right)e^{m^2/M^2}, \quad (40)$$

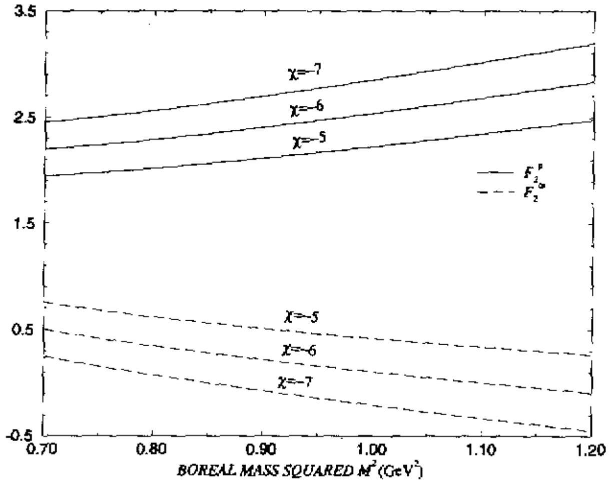

FIG. 3. Tensor couplings, as predicted by the sum rule (31), plotted for several reasonable values of χ (in units of GeV^{-2}). The value $\chi=-6$ reproduces very well our previous prediction on the tensor couplings.

with

$$E_1=1-\left(1+\frac{W^2}{M^2}\right)e^{-W^2/M^2}, \quad L=\frac{\ln(M/\Lambda_{\rm QCD})}{\ln(\mu/\Lambda_{\rm QCD})}.$$

To perform numerical analysis, we use the Belaeyav-Ioffe nucleon mass sum rule to fix the overlap parameter λ_N^2 ($\approx 1.2\times 10^{-3}$ GeV^6). We use the standard parameters: $a=0.546$ GeV, $\Lambda_{\rm QCD}=0.1$ GeV, $\mu=0.5$ GeV, and $W=1.45$ GeV [10]. The sample numerical results are presented in Figs. 1–4.

In Fig. 1, we plot the vector and tensor couplings F_1^ρ, F_1^ω, F_2^ρ, and F_2^ω as functions of the Borel mass squared M^2 in the range of 0.7 GeV$^2\leq M^2\leq 1.2$ GeV^2. It is seen from this figure that these functions are well behaved, giving rise to the predictions

$$F_1^\rho\approx 0.6, \quad F_2^\rho\approx 2.3,$$

$$F_1^\omega\approx 1.9, \quad F_2^\omega\approx 0.4.$$

As indicated earlier, $F_1^\omega/F_1^\rho\approx 3$ is in line with nuclear force studies [3,4]. However, F_2^ρ/F_1^ρ is a little too low in this regard. It appears rather difficult, in the context of the QCD sum rule method, to get a value for F_2^ρ which is larger than the above prediction by 50% (as required in nuclear force studies). Nevertheless, this should not be considered as a major drawback, since exchange of some heavier tensor mesons with suitable quantum numbers could easily help to jack up the number for F_2^ρ and such possibility is taken into account effectively in nuclear force studies by treating F_2^ρ as an adjustable parameter.

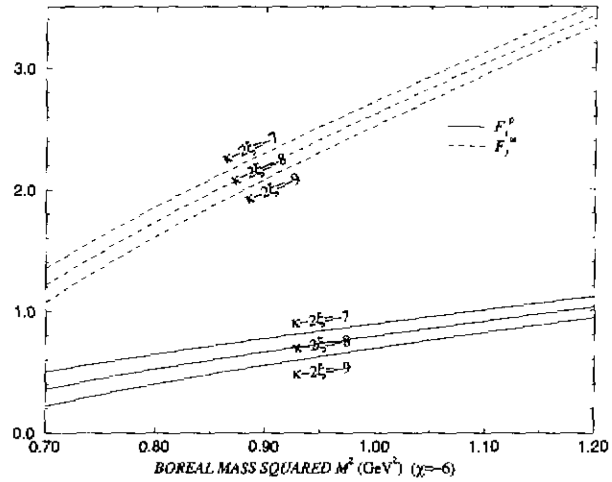

FIG. 4. Sum rules for the vector couplings analyzed for several different values of $\kappa-2\xi$ at $\chi=-6$ GeV^{-2}. It is seen that the sum rules are insensitive to $\kappa-2\xi$, leaving this parameter undetermined at this stage.

In Fig. 2, the ratios F_2^ρ/F_1^ρ and F_2^ω/F_1^ω are plotted as functions of the Borel mass squared M^2. The smooth curves in Fig. 2 correspond to

$$\frac{F_2^\rho}{F_1^\rho}\approx 3.6, \quad \frac{F_2^\omega}{F_1^\omega}\approx 0.2,$$

which may be compared to the values obtained in the vector-dominance model (VDM), namely, $\mu_V=3.7$ and $\mu_S=-0.12$. Thus the results obtained from QCD sum rules are in accord with the validity of the VDM. This result should not be considered as "automatic" even though the ρqq and ωqq couplings are very similar to the electromagnetic coupling. This is because in our analysis we have also employed the sum rules related to the symmetric part of $Z_{\mu\nu}$ (which are absent in the electromagnetic case).

From nuclear potential studies [3,4] which fit the nucleon-nucleon scattering data, we have $g_\rho=2.79$ and $g_\omega=8.37$. Making use of such values, we obtain the ρ quark or ω quark coupling $h\approx 4.4$.

We may now turn our attention to the original QCD sum rules which involve the induced condensation parameters χ. In particular, we may use the sum rule (31) to provide a determination of χ. The result is illustrated in Fig. 3, for several reasonable values of χ (in units of GeV^{-2}). The value $\chi=-6$ reproduces very well our previous prediction on the tensor couplings.

Using Eq. (31) to eliminate the unknown F_2^Z in Eq. (30), we obtain the sum rules for only the vector coupling F_1^Z. These sum rules are analyzed in Fig. 4, for several different values of $\kappa-2\xi$ at $\chi=-6$ GeV^{-2}. It is seen that the sum rules are insensitive to $\kappa-2\xi$, leaving this parameter undetermined at this stage.

V. SUMMARY

In this paper, we have extended the method of QCD sum rules in external fields to determine the ρNN and ωNN

strong couplings, including both the vector and tensor couplings. For the vector couplings, we obtain $g_{\rho NN}/g_{\omega NN}=1/3$, while for the tensor couplings the large ρNN tensor coupling F_2^ρ arises naturally. Such information has been used for years in construction of nuclear potentials, but the QCD sum rule method offers the first justification of such parameter values directly from QCD.

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Σ_c and Λ_c magnetic moments from QCD spectral sum rules

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The QCD spectral sum rules in the presence of the external electromagnetic field $F_{\mu\nu}$ are used to calculate the magnetic moments of Σ_c and Λ_c. Our results are $\mu_{\Sigma_c^{++}}=(2.1\pm0.3)\mu_N$, $\mu_{\Sigma_c^+}=(0.6\pm0.1)\mu_N$, $\mu_{\Sigma_c^0}=(-1.6\pm0.2)\mu_N$, and $\mu_{\Lambda_c}=(0.15\pm0.05)\mu_N$. [S0556-2821(97)00723-6]

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QCD spectral sum rules (QSSRs) [1] have been successful in extracting the masses and coupling constants of low-lying mesons and baryons. In this approach the nonperturbative effects are taken into account through various condensates in the QCD vacuum. As shown in [2,3] the light baryon masses are determined by the chiral symmetry-breaking quark condensate. In the infinite heavy quark mass limit the QSSR was first used to evaluate the heavy baryon mass [4]. The QSSRs with a finite heavy quark mass were treated in [5,6]. Recently the QSSR was employed in the framework of the heavy quark effective theory [7–11].

The baryon magnetic moment, like the baryon mass, is another important static quantity. Ioffe and Smilga [12] and, independently, Balitsky and Yung [13] extracted the nucleon magnetic moment treating the electromagnetic field $F_{\mu\nu}$ as an external field in the QSSR approach. They found that the nucleon magnetic moment is essentially related to the quark condensate and three susceptibilities χ, κ, and ξ. Later on the octet baryon magnetic moments [15] were obtained in a similar manner. In this work we shall employ the same approach to calculate the magnetic moments of Σ_c and Λ_c.

We shall consider the two-point correlator $\Pi_{\Sigma_c}(p)$ in the presence of an external electromagnetic field $F_{\alpha\beta}$:

$$\Pi_{\Sigma_c}(p)=i\int d^4x\langle 0|T\{\eta_{\Sigma_c}(x),\bar{\eta}_{\Sigma_c}(0)\}|0\rangle_{F_{\mu\nu}}e^{ip\cdot x}$$
$$=\Pi_0(p)+\Pi_{\mu\nu}(p)F^{\mu\nu}+\cdots \tag{1}$$

where $\Pi_0(p)$ is the polarization operator without the external field $F_{\mu\nu}$. The η_{Σ_c} in Eq. (1) is the interpolating current with Σ_c quantum numbers:

$$\eta_{\Sigma_c}(x)=\epsilon^{abc}\{[u^{aT}(x)Cc^b(x)]u^c(x)$$
$$-[u^{aT}(x)C\gamma_5c^b(x)]\gamma_5u^c(x)\}, \tag{2}$$

and

$$\langle 0|\eta_{\Sigma_c}(0)|\Sigma_c\rangle=\lambda_{\Sigma_c}\nu_{\Sigma_c}(p), \tag{3}$$

where $u^a(x)$ and $c^b(x)$ is the up and charm quark field, λ_{Σ_c} is the overlap amplitude of the interpolating current with the baryon state, and the ν_{Σ_c} is the Dirac spinor of the heavy baryon.

Three different tensor structures contribute to $\Pi_{\mu\nu}(p)$,

$$\Pi_{\mu\nu}(p)=\Pi(p)(\sigma_{\mu\nu}\hat{p}+\hat{p}\sigma_{\mu\nu})+\Pi_1(p)i(p_\mu\gamma_\nu-p_\nu\gamma_\mu)\hat{p}$$
$$+\Pi_2(p)\sigma_{\mu\nu}. \tag{4}$$

As in the original QSSR analysis of nucleon magnetic moment [12], we shall consider the first tensor structure $(\sigma_{\mu\nu}\hat{p}+\hat{p}\sigma_{\mu\nu})$.

The presence of the external field $F_{\mu\nu}$ will induce three new condensates in the QCD vacuum [12]:

$$\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_{F_{\mu\nu}}=e_q\chi F_{\mu\nu}\langle 0|\bar{q}q|0\rangle,$$

$$g_s\langle 0|\bar{q}\frac{\lambda^n}{2}G^n_{\mu\nu}q|0\rangle_{F_{\mu\nu}}=e_q\kappa F_{\mu\nu}\langle 0|\bar{q}q|0\rangle, \tag{5}$$

$$g_s\epsilon^{\mu\nu\lambda\sigma}\langle 0|\bar{q}\gamma_5\frac{\lambda^n}{2}G^n_{\lambda\sigma}q|0\rangle_{F_{\mu\nu}}=ie_q\xi F^{\mu\nu}\langle 0|\bar{q}q|0\rangle,$$

where q refers to the up and down quark, e_q is the quark charge. The χ, κ, and ξ in Eq. (5) are the quark condensate susceptibilities, which have been the subject of various studies [12–15]. Their values employed by different groups are consistent with each other. We shall adopt the values $\chi=-4.5\ \ \mathrm{GeV}^{-2}$, $\kappa=0.4$, $\xi=-0.8$.

At the phenomenological level we have

$$\mathrm{Im}\Pi(s)=\frac{1}{4}\mu_{\Sigma_c}\lambda_{\Sigma_c}^2\delta'(s-m_{\Sigma_c}^2)+C\delta(s-m_{\Sigma_c}^2)+\mathrm{Im}\Pi^{\mathrm{pert}}(s)\theta(s-t_{\Sigma_c}), \tag{6}$$

where the first term corresponds to the Σ_c magnetic moment and is of the double pole. The second term comes from the transition $\Sigma_c \to$ excited states and is of single pole. The third term is the usual continuum contribution and t_{Σ_c} is the continuum threshold. The single-pole transition term does not damp out after Borel transform. Therefore it should be explicitly included in the QSSR analysis.

Within an operator product expansion we obtain, to lowest order of α_s and for condensates up to dimension six at the quark level,

$$\mathrm{Im}\Pi(s)=\mathrm{Im}\Pi^{\mathrm{pert}}(s)+\mathrm{Im}\Pi^{\mathrm{np}}(s), \tag{7}$$

$$\mathrm{Im}\Pi^{\mathrm{pert}}(s)=\frac{e_u}{8}s\left(1-\frac{m_c^2}{s}\right)^3. \tag{8}$$

Making a Borel tranform of $\Pi(p)$ and transferring the continuum contribution to the left-hand side, we obtain

$$\begin{aligned}M_B^2\int_{m_c^2}^{t_{\Sigma_c}}&\mathrm{Im}\Pi^{\mathrm{pert}}(s)e^{-(s/M_B^2)}ds L^{-4/9}\\&+\Big\{-\frac{1}{12}\chi e_u a^2 M_B^2(1-e^{-(t_{\Sigma_c}-m_c^2/M_B^2)})L^{-(16/27)}\\&-\frac{1}{24}e_c a^2-\frac{1}{36}e_u a^2\left(\frac{m_c^2}{M_B^2}+1\right)\\&+\frac{1}{96}\chi m_0^2 e_u a^2\left(2\frac{m_c^2}{M_B^2}+1\right)L^{-(10/9)}-\frac{1}{72}\kappa e_u a^2\\&\times\left(2\frac{m_c^2}{M_B^2}-1\right)-\frac{1}{36}\xi e_u a^2\left(\frac{m_c^2}{M_B^2}+1\right)\Big\}e^{-(m_c^2/M_B^2)}L^{4/9}\\&=\frac{1}{4}(2\pi)^4\lambda_{\Sigma_c}^2 e^{-(m_{\Sigma_c}^2/M_B^2)}\mu_{\Sigma_c}(1+CM_B^2),\end{aligned} \tag{9}$$

where $a=-(2\pi)^2\langle 0|\bar{q}q|0\rangle=0.55\ \mathrm{GeV}^3$, $am_0^2=(2\pi)^2 g_s\langle 0|\bar{q}\sigma G q|0\rangle$, $m_0^2=0.8\ \mathrm{GeV}^2$, $q=u,d$, $L=\ln(10M_B)/\ln(5)$. C is the unknown constant to be determined from the sum rule, which parametrizes the transition contribution. We have checked that in the chiral limit $m_c\to 0$, our result reproduces the sum rule for nucleon magnetic moment [12].

For the charm quark mass we use $m_c=1.47\pm 0.1$ GeV. We adopt the estimated continuum threshold and overlapping amplititude in the QSSR analysis [5,6,9–11,16]. $t_{\Sigma_c}=10\ \mathrm{GeV}^2$ and $(2\pi)^4\lambda_{\Sigma_c}^2=0.8\pm 0.1\ \mathrm{GeV}^6$. For the Σ_c mass we may either use the predictions in the QSSR analysis or the experimental value [22], $m_{\Sigma_c}=2.455$ GeV.

We may further improve the numerical analysis by taking into account the renormalization group evolutions of the sum rule (9) through the anomalous dimensions of the condensates and currents. The working interval of the Borel mass M_B^2 for the sum rule (9) is $1.7\ \mathrm{GeV}^2\leq M_B^2\leq 2.5\ \mathrm{GeV}^2$ where both the continuum contribution and power corrections are controllable [5,6]. Moving the factor $\frac{1}{4}(2\pi)^4\lambda_{\Sigma_c}^2 e^{-(m_{\Sigma_c}^2/M_B^2)}$ on the right-hand side to the left and fitting the new sum rules with a straight-line approximation we may extract the Σ_c magnetic moment. We show the Borel mass dependence of the new sum rule and the fitting straight line in Fig. 1 for the continuum threshold $t_{\Sigma_c}=10\ \mathrm{GeV}^2$.

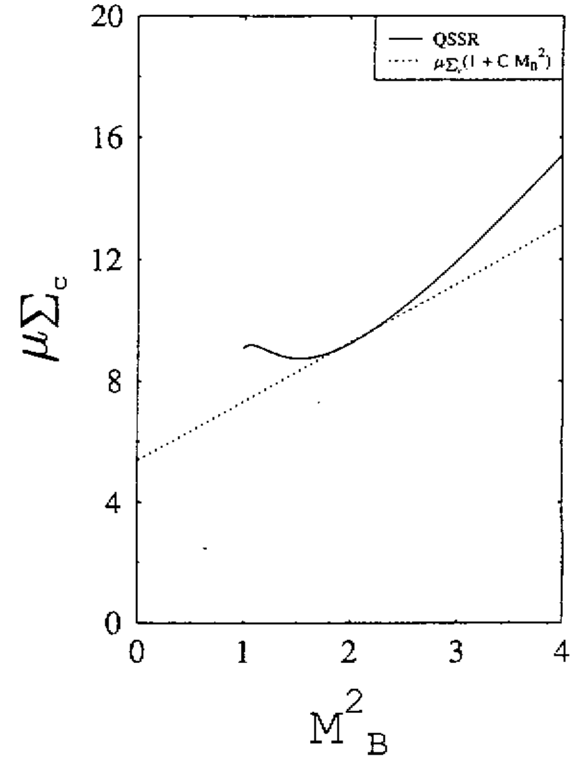

FIG. 1. The Borel mass dependence of the Σ_c^{++} magnetic moment for the continuum threshold $t_{\Sigma_c}=10\ \mathrm{GeV}^2$. The solid curve is the QCD sum rule prediction for $\mu_{\Sigma_c^{++}}$. The dotted line is a straight-line approximation. The intersect with Y axis is the Σ_c magnetic moment in unit of $e/2m_{\Sigma_c}$.

It can be seen in Fig. 1 that the nondiagonal transition contribution is important, though it is not dominant in the working interval of the Borel mass $1.7\ \mathrm{GeV}^2\leq M_B^2\leq 2.5\ \mathrm{GeV}^2$. The sum rule is insensitive to the susceptibilities κ and ξ due to their small values. Their contributions are less than 5%. When χ varies from $-4.5\ \mathrm{GeV}^{-2}$ to $-3.5\ \mathrm{GeV}^{-2}$ or to $-5.5\ \mathrm{GeV}^{-2}$, the sum rules change within 10%.

Our final result is $\mu_{\Sigma_c^{++}}=(5.4\pm 0.5)\,e/2m_{\Sigma_c}$, where $e/2m_B$ is a natural unit in QSSR analysis of the baryon magnetic moment. By replacing e_u in Eq. (9) with $(e_u+e_d)/2$ or e_d we arrive at the magnetic moments for the other Σ_c multiplets, $\mu_{\Sigma_c^+}=(0.6\pm 0.1)\,e/2m_{\Sigma_c}$ and $\mu_{\Sigma_c^0}=(-4.2\pm 0.4)\,e/2m_{\Sigma_c}$. In units of nuclear magneton $\mu_{\Sigma_c^{++}}=(2.1\pm 0.3)\mu_N$, $\mu_{\Sigma_c^+}=(0.23\pm 0.03)\mu_N$, and $\mu_{\Sigma_c^0}=(-1.6\pm 0.2)\mu_N$.

Similarly we can extract the magnetic moments of Λ_c with the following interpolating current:

$$\eta_{\Lambda_c}(x)=\epsilon^{abc}[u^{aT}(x)C\gamma_5 u^b(x)]c^c(x). \tag{10}$$

The final sum rule reads as follows:

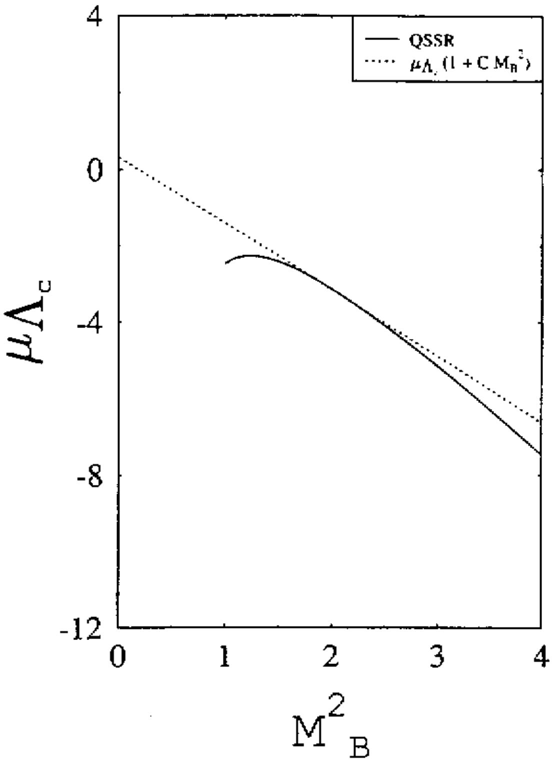

FIG. 2. The Borel mass dependence of μ_{Λ_c} and the fitting straight line.

$$\frac{3e_c}{16}M_B^2\int_{m_c^2}^{t_{\Lambda_c}}\left[\frac{m_c^2}{2}\left(1-\frac{m_c^4}{s^2}\right)-\frac{s}{3}\left(1-\frac{m_c^6}{s^3}\right)\right]e^{-(s/M_B^2)}ds L^{-(4/9)}$$
$$+\left\{-\frac{e_c}{24}a^2+\frac{e_u+e_d}{144}a^2\left(1-\frac{m_c^2}{M_B^2}\right)\right.$$
$$\left.+\frac{e_u+e_d}{576}\chi m_0^2a^2L^{-10/9}+\frac{e_u+e_d}{48}\kappa a^2\right\}e^{-(m_c^2/M_B^2)}L^{4/9}$$
$$=\frac{1}{4}(2\pi)^4\lambda_{\Lambda_c}^2e^{-(m_{\Lambda_c}^2/M_B^2)}\mu_{\Lambda_c}(1+CM_B^2), \qquad (11)$$

With the parameters [5,6,9–11,16] $t_{\Lambda_c}=10\ \mathrm{GeV}^2$, $(2\pi)^4\lambda_{\Lambda_c}^2=1.0\pm0.2\ \mathrm{GeV}^6$, and $m_{\Lambda_c}=2.285$ GeV, we get $\mu_{\Lambda_c}=(0.15\pm0.05)\mu_N$. The Borel dependence of μ_{Λ_c} and the fitting line is shown in Fig. 2. As in the analysis of the Σ_c magnetic moment, the straight line approximation is good.

It is not difficult to extend the same analysis to extract the magnetic moments of Σ_b and Λ_b. Yet the overlapping amplititudes λ_{Σ_b} and λ_{Λ_b} determined in the QSSR approach with finite bottom quark mass have large errors. Therefore we do not present numerical results here.

In summary, we have calculated the magnetic moment of Σ_c and Λ_c using the external field method in the QCD sum rules. There are no experimental data for the heavy baryon magnetic moments. Yet naive predictions have been made in the phenomenological models such as nonrelativistic quark model (NRQM) [17,18], bag model [19], and the Skyrme model [20,21]. Especially in the quark model the heavy baryon magnetic moments have a rather simple form. $\mu_{\Lambda_c}=\mu_c$, $\mu_{\Sigma_c^{++}}=\frac{8}{9}\mu_p-\frac{1}{3}\mu_c$, $\mu_{\Sigma_c^+}=\frac{2}{9}\mu_p-\frac{1}{3}\mu_c$, and $\mu_{\Sigma_c^0}=-\frac{4}{9}\mu_p-\frac{1}{3}\mu_c$. Our result of the Σ_c magnetic moment is in good agreement with the NRQM prediction. In the μ_{Λ_c} sum rule (11), the contribution from higher-dimension condensates is significant and comparable with the charm quark perturbative contribution numerically, since the light quark perturbative contribution and the induced light quark condensate vanishes. If we turn off all the higher-dimension nonperturbative contributions by setting the quark condensate $a=0$, we arrive at $\mu_{\Lambda_c}=0.35\mu_N$, which is very close to the NRQM prediction. Therefore the higher-dimension condensates lead to the possible deviation from the naive quark model result. It will be very interesting to measure μ_{Λ_c} experimentally.

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QCD sum rules: Isospin symmetry breakings in pion-nucleon couplings

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We use the method of QCD sum rules in the presence of an external point field to investigate isospin symmetry breakings in pion-nucleon couplings. Possible manifestations of such isospin symmetry breakings are examined in the context of low-energy nucleon-nucleon scattering. We discuss numerical results in relation to both the existing data and other theoretical predictions. [S0556-2813(98)04801-8]

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I. INTRODUCTION

Isospin symmetry and its possible violations constitute an integral part of nuclear physics [1], not only in the sense that good symmetry offers an effective classification of nuclear or hadronic structure but also in the proximity of the isospin symmetry concept to other fundamental ideas such as chiral symmetry. In general, isospin symmetry breaking effects observed in nuclear physics may arise from different sources such as, in addition to the well known electromagnetic interactions, strong interactions but augmented with hadron mass differences (such as the $n-p$ or $\pi^{\pm}-\pi^0$ mass difference) or with different meson-nucleon couplings (such as nonuniversal πNN couplings) or with isospin mixings (such as $\rho-\omega$ or $\pi-\eta$ mixing). Determination of isospin symmetry breaking effects requires information on quantities which may not be directly accessible from experiments, such as the meson-nucleon couplings or the isospin mixing parameters. Accordingly, reliable theoretical predictions become indispensible for advancing our understanding of the problem.

Unfortunately, it appears that a quantitative treatment of isospin symmetry breakings may involve the capability of treating strong interactions at a quantitative level or, in other words, the ability to handle the complications due to quantum chromodynamics (QCD) in a reliable manner. To some extent, the method of QCD sum rules, as proposed originally by Shifman, Vainshtein, and Zakharov [2] and adopted, or extended, by many others [3–5], incorporates the nonperturbative QCD effects through various condensates (associated with the nontrivial QCD vacuum), thereby offering some hope of being able to treat strong interactions in a quantitative manner.

In this work, we wish to focus on the determination of isospin symmetry breakings in pion-nucleon couplings, with some emphasis on possible manifestations of such symmetry breaking in nucleon-nucleon scattering. As the present effort is based primarily on the method of QCD sum rules, it differs much from the quark-model calculations [6,7]; as we shall see, this work is considerably more extensive than a recent work [8] also making use of the QCD sum rule method. Specifically, we wish to adopt a method in which the pion field π is treated as the external field. The idea of treating the pion as an external field was first suggested in [4] and has not been used in any extensive manner until recently [9,10].

II. ISOSPIN SYMMETRY BREAKINGS IN PION-NUCLEON COUPLINGS

In the method of QCD sum rules in the presence of an external field, we attempt to evaluate, both at the quark and hadron levels, the correlation function specified by

$$\Pi(p)\equiv i\int d^4x e^{ip\cdot x}\langle 0|T[\eta_p(x)\bar{\eta}_p(0)]|0\rangle|_{\pi}, \qquad (1)$$

where we have

$$\eta_p(x)=\epsilon^{abc}\{u^{aT}(x)C\gamma_\mu u^b(x)\}\gamma^5\gamma^\mu d^c(x),$$

$$\bar{\eta}_p(y)=\epsilon^{abc}\{\bar{u}^b(y)\gamma_\nu C\bar{u}^{aT}(y)\}\bar{d}^c(y)\gamma^\nu\gamma^5. \qquad (2)$$

This correlation function allows us to determine the π^0pp coupling. By evaluating the appropriate correlation function in the presence of the external pion field, we may determine the pion-nucleon couplings g_{π^0pp}, g_{π^0nn}, and g_{π^-pn}. It appears that the induced condensates involved in the external field method are now well understood and it becomes relatively straightforward to perform calculations to higher dimensions (as required for reliable predictions). This is what we choose to use in this paper and we use it to obtain isospin symmetry breakings in πNN couplings.

At this juncture, we wish to note that an alternative method is to evaluate correlation functions with the T product of currents sandwiched between the vacuum and the one-pion state, as first suggested by [11] and recently adopted by some authors [12–14]. Using the soft-pion theorem, we may rewrite such matrix element, between the vacua, of the commutator of the axial charge Q_5 and the original T product of the currents, thereby resulting in a $g_{\pi NN}$ sum rule which is related to the nucleon mass sum rules in some way. Unfortunately, a precise relation cannot be established unless the commutator between the axial charge and the nucleon current $\eta(x)$ is known. Note that the knowledge of such a commutator goes beyond current algebra and chiral dynamics.

On the other hand, it is far from trivial if any such relation can be established in the external-field QCD sum rule method in which it is not necessary to take the soft-pion limit. An important indication as to why we may be away from the soft-pion limit is that the quark masses (and some other chiral symmetry breaking parameters) are not set to zero throughout the derivation. Indeed, the sum rule which we obtain in what follows for $g_{\pi NN}$ cannot be obtained from the nucleon mass sum rules by some chiral rotation and thus represents independent information. Nevertheless, it appears that such an aspect is fairly subtle and deserves further investigation.

At the quark level, we have

$$\begin{aligned}\langle 0|T\eta_p(x)\bar{\eta}_p(0)|0\rangle_{\pi^0} &= 2i\epsilon^{abc}\epsilon^{a'b'c'}\ \mathrm{Tr}\ \{S_u^{bb'}(x)\gamma_\nu C[S_u^{aa'}(x)]^T C\gamma_\mu\}\\ &\quad\times\gamma_5\gamma_\mu S_d^{cc'}(x)\gamma_\nu\gamma_5 .\end{aligned} \tag{3}$$

Here the quark propagator is given, in configuration space, by

$$S(x)=S^{(0)}(x)+S^{(\pi)}(x), \tag{4}$$

with

$$\begin{aligned}iS^{(0)ab}(x) &= \frac{i\delta^{ab}}{2\pi^2x^4}\hat{x}+\frac{i}{32\pi^2x^2}\frac{\lambda^n_{ab}}{2}g_cG^n_{\mu\nu}(\sigma^{\mu\nu}\hat{x}+\hat{x}\sigma^{\mu\nu})\\ &\quad-\frac{\delta^{ab}}{12}\langle\bar{q}q\rangle+\frac{\delta^{ab}x^2}{192}\langle g_c\bar{q}\sigma\cdot Gq\rangle-\frac{m_q\delta^{ab}}{4\pi^2x^2}\\ &\quad+\frac{m_q}{32\pi^2}\frac{\lambda^n_{ab}}{2}G^n_{\mu\nu}\sigma^{\mu\nu}\ln(-x^2)\\ &\quad-\frac{\delta^{ab}\langle g_c^2G^2\rangle}{2^9\times 3\pi^2}m_qx^2\ln(-x^2)+\frac{i\delta^{ab}m_q}{48}\langle\bar{q}q\rangle\hat{x}\\ &\quad-\frac{1}{2^7\times 3^2}\,\mathrm{i}m_q\langle g_c\bar{q}\sigma\cdot Gq\rangle\delta^{ab}x^2,\end{aligned} \tag{5}$$

and

$$\begin{aligned}iS^{(\pi)ab}(x) &= -\frac{i\delta^{ab}}{4\pi^2x^2}g_q\vec{\tau}\cdot\vec{\pi}\gamma_5+\frac{i\delta^{ab}}{24}g_q\vec{\tau}\cdot\vec{\pi}\gamma_5\chi\langle\bar{q}q\rangle\\ &\quad-\frac{i\delta^{ab}}{384}m_0^\pi g_q\vec{\tau}\cdot\vec{\pi}\gamma_5\langle\bar{q}q\rangle x^2\\ &\quad+\frac{i}{2^5\pi^2}g_cg_q\vec{\tau}\cdot\vec{\pi}\gamma_5\frac{\lambda^n_{ab}}{2}G^n_{\mu\nu}\sigma^{\mu\nu}\ln(-x^2)\\ &\quad-\frac{i\delta^{ab}}{2^9\times 3\pi^2}g_q\vec{\tau}\cdot\vec{\pi}\gamma_5\langle g_c^2G^2\rangle x^2\ln(-x^2)\\ &\quad-\frac{\delta^{ab}}{48}g_q\vec{\tau}\cdot\vec{\pi}\gamma_5\langle\bar{q}q\rangle\hat{x},\end{aligned} \tag{6}$$

where $\hat{x}\equiv x_\mu\gamma^\mu$ and we have also introduced

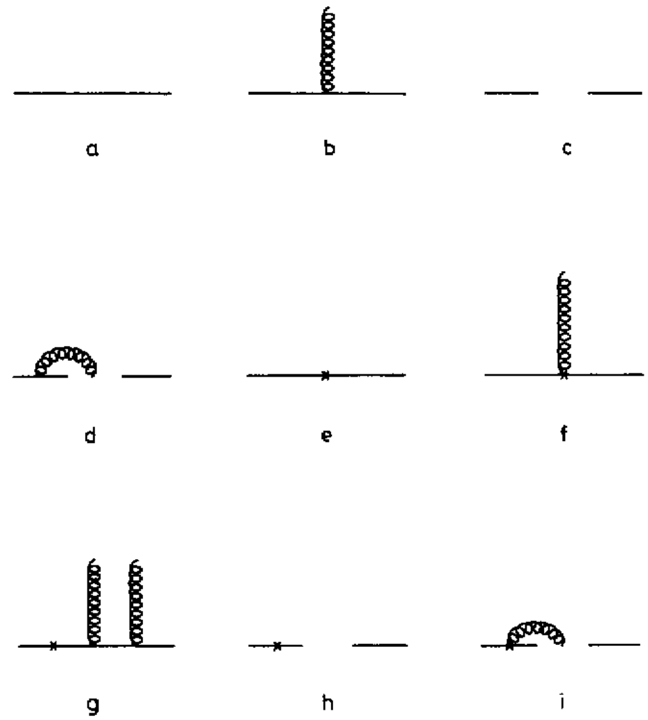

FIG. 1. Pictorial representation of the various terms in the quark propagator $S^{(0)ab}(x)$.

$$\langle\bar{\psi}(0)i\gamma^5\tau^j\psi(0)\rangle\equiv g_q\chi\pi^j\langle\bar{q}q\rangle$$

and

$$\langle\bar{\psi}(0)i\gamma^5\tau^j g_c\sigma\cdot G\psi(0)\rangle\equiv g_q m_0^\pi\tau^j\langle\bar{q}q\rangle$$

with ψ the isospin doublet consisting of u and d quarks. The various terms in the quark propagators $iS^{(0)ab}(x)$ and $iS^{(\pi)ab}(x)$ may be represented pictorially by the diagrams shown in Figs. 1 and 2, respectively. We wish to stress that we have in fact performed our calculations in momentum space especially when ambiguities arise and, in addition, special attention has been directed to adoption of the γ_5 in d dimensions in relation to dimensional regularization.

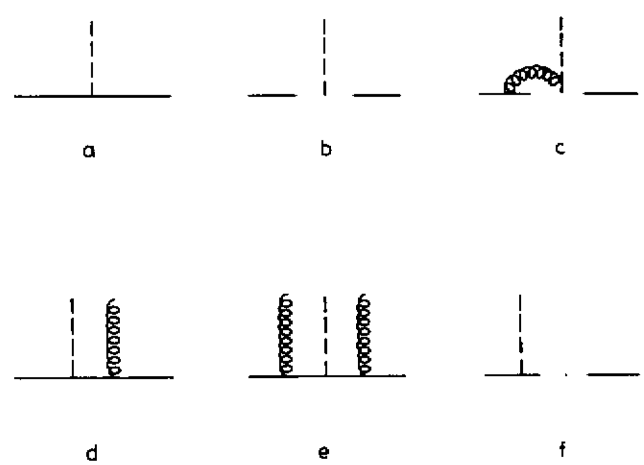

FIG. 2. Pictorial representation of the various terms in the quark propagator $S^{(\pi)ab}(x)$.

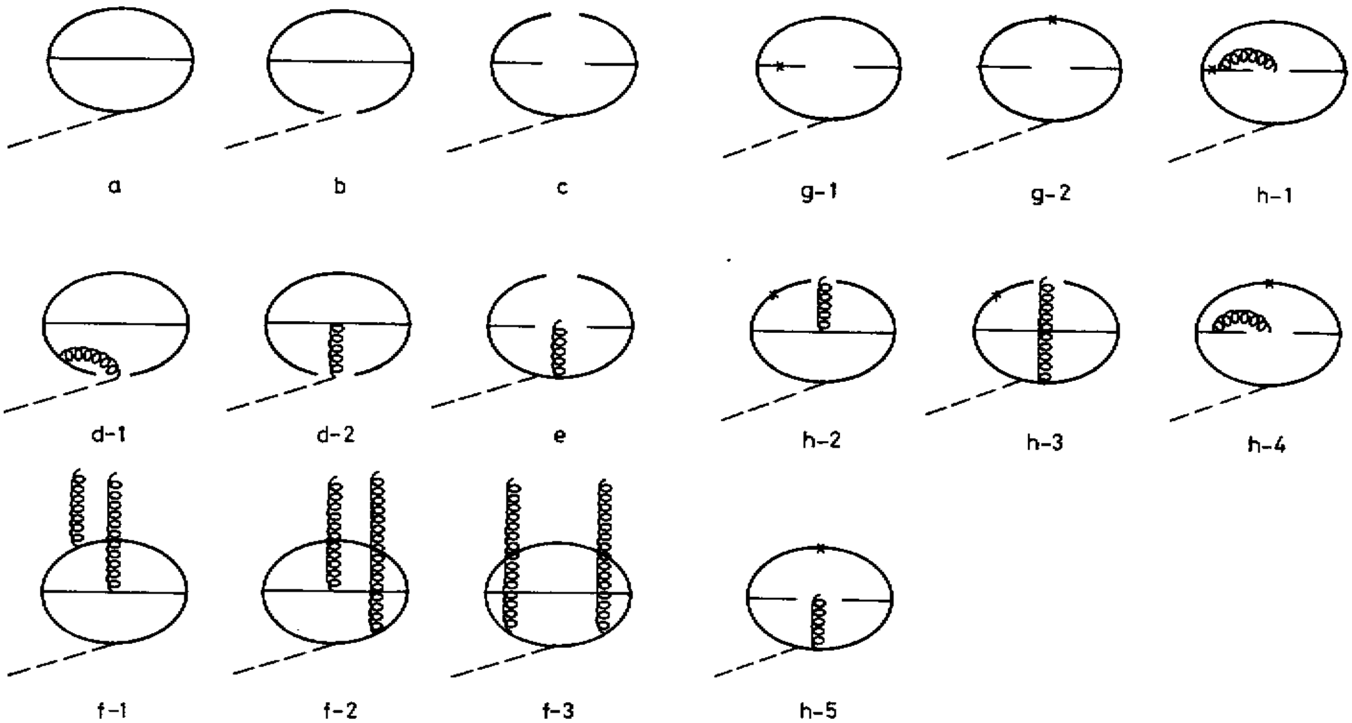

FIG. 3. The diagrams used to determine the QCD sum rule for $g_{\pi^0 pp}$.

To incorporate isospin symmetry violations, we note that $\langle\bar{q}q\rangle$ appearing in the quark propagator needs to be interpreted properly; in particular, it is to be understood as $\langle\bar{u}u\rangle$ in the (uu) channel, or as $\langle\bar{d}d\rangle$ in the (dd) channel, or as $(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)/2$ for $(d\rightarrow u)$ or $(u\rightarrow d)$ propagation. We also note that Fig. 2(f) has been drawn in a way to indicate that the choice of q in $\langle\bar{q}q\rangle$ should follow the flavor of the quark connecting to $x=0$ (on the right-hand side of the diagram)—a result following the usage of the equation of motion. No potential ambiguity should arise for all the other diagrams shown in Figs. 1 and 2. The assumption has also been made that, once we have made the suitable interpretation of $\langle\bar{q}q\rangle$, χ and m_0^π are considered to be universal. In other words, we refrain from introducing, at the quark level, further unknown sources for isospin symmetry violations; our assumption regarding the universal nature of the susceptibilities χ and m_0^π is further supported by the derivation of induced condensates as illustrated in [10]. This is a crucial point since the terms in $\chi\langle\bar{u}u\rangle$ and $\chi\langle\bar{d}d\rangle$ turn out to be one of the most important contributions which distinguish the coupling $g_{\pi^0 pp}$ from $g_{\pi^0 nn}$.

We note that, for the determination of $\pi^- pn$ or $\pi^+ np$ coupling, we find

$$\begin{aligned}\langle 0|T\eta_p(x)\bar{\eta}_n(0)|0\rangle_{\pi^-}\\ &=-4i\epsilon^{abc}\epsilon^{a'b'c'}\gamma_\mu\gamma_5 S_d^{aa'}(x)\gamma_\nu C[S^{(\pi)bb'}(x)]^T\\ &\times C\gamma_\mu S_u^{cc'}(x)\gamma_5\gamma_\nu .\end{aligned}\tag{7}$$

We note that the absence of $iS^{(0)bb'}(x)$ in the present case is due to a change in the electric charge. We also note that the structure (which does not involve the trace of some product) seems quite different from the $\pi^0 pp$ case, but we will see that isospin symmetry is exact should we assume $m_u=m_d$ and $\langle\bar{u}u\rangle=\langle\bar{d}d\rangle$. Finally, to obtain QCD sum rules up to dimension eight (as compared to the leading perturbative term), we need to enumerate diagrams which are proportional to 1, $\chi\langle\bar{q}q\rangle$, $\langle g_c^2G^2\rangle$, $m_0^\pi\langle\bar{q}q\rangle$, $\langle\bar{q}q\rangle^2$, $\langle\bar{q}q\rangle\langle g_c\bar{q}\sigma\cdot Gq\rangle$, $m_q\langle\bar{q}q\rangle$, and $m_q\langle\bar{q}g_c\sigma\cdot Gq\rangle$. Here it is clearly of importance to be able to distinguish between $\langle\bar{d}\hat{O}d\rangle$ and $\langle\bar{u}\hat{O}u\rangle$ in the quark propagators used to evaluate the correlation functions.

On the other hand, we may parametrize the correlation functions at the hadronic level in the standard manner:

$$\begin{aligned}&\int d^4x\langle 0|T\eta_p(x)\bar{\eta}_n(0)|0\rangle|_{\pi^-}\\ &=\lambda_n\lambda_p\frac{i}{\hat{p}-m_p}(-\gamma_5\sqrt{2}g_{\pi^- pn})\frac{i}{\hat{p}-m_n}+\cdots ,\end{aligned}\tag{8}$$

$$\begin{aligned}&\int d^4x\langle 0|T\eta_p(x)\bar{\eta}_p(0)|0\rangle|_{\pi^0}\\ &=\lambda_p^2\frac{i}{\hat{p}-m_p}+\lambda_p^2\frac{i}{\hat{p}-m_p}(-\gamma_5 g_{\pi^0 pp})\frac{i}{\hat{p}-m_p}+\cdots .\end{aligned}\tag{9}$$

We consider the various diagrams shown in Fig. 3 and obtain the quark-level expression [referred to as the left-hand side (LHS)] for the $\pi^0 pp$ coupling (directly from the momentum-space calculations),

$$\frac{1}{(2\pi)^4} g_{\pi^0 dd} \not{p}\gamma_5 \Bigg\{ \frac{p^4 \ln(-p^2)}{2} - \frac{\chi}{2} a_d p^2 \ln(-p^2) - \frac{11}{24} b \ln(-p^2) + \frac{4}{3} a_u^2 \frac{1}{p^2} - \frac{1}{3} a_u^2 m_0^2 \frac{1}{p^4} + \frac{\ln(-p^2)+\gamma_E}{p^2} m_0^2 \left[\frac{7}{8}(a_u m_d + a_d m_u) - \frac{1}{4} m_u a_u \right] + \frac{m_0^2}{p^2} \left[\frac{5}{12} m_u a_u - \frac{1}{8} m_d a_d + \frac{67}{48} m_d a_u + \frac{115}{48} m_u a_d \right] \Bigg\} \tag{10}$$

with $a_q \equiv -(2\pi)^2 \langle \bar{q}q \rangle$, $\langle \bar{q}\sigma \cdot Gq \rangle \equiv -m_0^2 \langle \bar{q}q \rangle$, and $b \equiv \langle g_c^2 G^2 \rangle$. Accordingly, we obtain, upon Borel transform, the following QCD sum rule for the $\pi^0 pp$ coupling:

$$M^6 - \frac{\chi}{2} a_d M^4 - \frac{11}{24} b M^2 + \frac{4}{3} a_u^2 + \frac{m_0^2}{3} a_u^2 \frac{1}{M^2} + m_0^2 \ln \frac{M^2}{\mu^2} \left[\frac{7}{8}(a_u m_d + a_d m_u) - \frac{1}{4} m_u a_u \right] + m_0^2 \left[\frac{5}{12} m_u a_u - \frac{1}{8} m_d a_d + \frac{67}{48} m_d a_u + \frac{115}{48} m_u a_d \right] = \frac{g_{\pi^0 pp}}{g_{\pi^0 dd}} \tilde{\lambda}_p^2 e^{-m_p^2/M^2} + \text{excited states continuum}, \tag{11}$$

where M is the Borel mass and $\tilde{\lambda}_p^2 = (2\pi)^4 \lambda_p^2$. A similar expression can be obtained for the $\pi^0 nn$ coupling through the replacement $u \leftrightarrow d$. It is of some interest to note that the contribution from Figs. 3(d1) is cancelled by that from Fig. 3(d2), so that the induced susceptibility m_0^π does not enter in the determination of strong πNN coupling, an aspect already observed in [9].

For the $\pi^+ np$ coupling, we have, with $\bar{a} \equiv (a_u + a_d)$,

$$M^6 - \frac{\chi}{2} \bar{a} M^4 - \frac{11}{24} b M^2 + \frac{4}{3} a_u a_d + \frac{1}{3}(a_d - a_u) a_d - (m_u - m_d) a_u M^2 + m_0^2 \left\{ -\frac{1}{2} a_u m_u - \frac{5}{24} a_d m_d + \frac{29}{12} a_u m_d + \frac{19}{8} a_d m_u \right\} + m_0^2 \ln \frac{M^2}{\mu^2} \left\{ a_u m_d + a_d m_u - \frac{1}{4} m_u (a_d + a_u) \right\} + \frac{m_0^2}{3} a_u a_d \frac{1}{M^2} = \frac{g_{\pi^+ np}}{g_{\pi^+ du}} (2\pi)^4 \lambda_n \lambda_p e^{-\bar{m}^2/M^2} + \text{excited states} + \text{continuum}. \tag{12}$$

As a standard practice, we may improve the applicability of these QCD sum rules by (i) taking into account the anomalous dimension of each term (operator) and (ii) invoking the continuum approximation in which contributions from excited states and the continuum are approximated by what we may obtain at the quark level above a certain threshold W. Results from such improvements can be duplicated easily from Yang *et al.* [15]. In this way, we obtain, for the $\pi^0 pp$ coupling [15,9],

$$M^6 L^{-4/9} E_2 - \frac{\chi}{2} a_d M^4 L^{2/9} E_1 - \frac{11}{24} b M^2 E_0 + \frac{4}{3} a_u^2 L^{4/9} + \frac{m_0^2}{3} a_u^2 \frac{1}{M^2} L^{-2/27} + m_0^2 \ln \frac{M^2}{\mu^2} L^{-26/27} \left[\frac{7}{8}(a_u m_d + a_d m_u) - \frac{1}{4} m_u a_u \right] + m_0^2 L^{-26/27} \left[\frac{5}{12} m_u a_u - \frac{1}{8} m_d a_d + \frac{67}{48} m_d a_u + \frac{115}{48} m_u a_d \right] = \frac{g_{\pi^0 pp}}{g_{\pi^0 dd}} \tilde{\lambda}_p^2 e^{-m_p^2/M^2}, \tag{13}$$

and, for the $\pi^+ np$ coupling,

$$M^6 L^{-4/9} E_2 - \frac{\chi}{2} \bar{a} M^4 L^{2/9} - \frac{11}{24} b M^2 E_0 + \left\{ \frac{4}{3} a_u a_d + \frac{1}{3}(a_d - a_u) a_d \right\} L^{4/9} - (m_u - m_d) a_u M^2 L^{-4/9} E_0 + m_0^2 L^{-26/27} \times \left\{ -\frac{1}{2} a_u m_u - \frac{5}{24} a_d m_d + \frac{29}{12} a_u m_d + \frac{19}{8} a_d m_u \right\} + m_0^2 \ln \frac{M^2}{\mu^2} L^{-26/27} \left\{ a_u m_d + a_d m_u - \frac{1}{4} m_u (a_d + a_u) \right\} + \frac{m_0^2}{3} a_u a_d L^{-2/27} \frac{1}{M^2} = \frac{g_{\pi^+ np}}{g_{\pi^+ du}} (2\pi)^4 \lambda_n \lambda_p e^{-\bar{m}^2/M^2}. \tag{14}$$

Here $E_0 = 1 - e^{-x}$, $E_1 = 1 - (1+x)e^{-x}$, and $E_2 = 1 - (1 + x + x^2/2)e^{-x}$ with $x = W/M^2$, and $L = 0.621 \ln(10M)$.

There are several sources for isospin symmetry breakings, viz., at the hadronic level (RHS), we have $m_n \neq m_p$, $\lambda_n \neq \lambda_p$, and $W_n \neq W_p$ (with W the threshold parameter in the continuum approximation for treating the excited states and the continuum). Apart from the well known neutron-proton mass difference, we use the parameters previously adopted by Yang *et al.* [15].

At the quark level (LHS), the isospin symmetry breaking caused by $\langle \bar{d}d \rangle \neq \langle \bar{u}u \rangle$ and $m_d \neq m_u$ have been made explicit in the formulas given above. Such effects have also been considered by other authors [8,15].

In addition, there is an effect due to the non-universal pion-quark couplings. Conceptually, we may start with a universal pion-quark coupling, such as in the effective chiral quark theory, but the vertex renormalizations due to electromagnetic interactions, as shown in Fig. 4 and also pointed out by Cao and Hwang [6], modify $\pi^0 uu$, $\pi^0 dd$, and $\pi^+ ud$ in a different manner. Here we have adopted the dimensional regularization scheme to work out such differences and conclude that the nonuniversal pion-quark couplings so obtained could be a reasonably important source for isospin symmetry

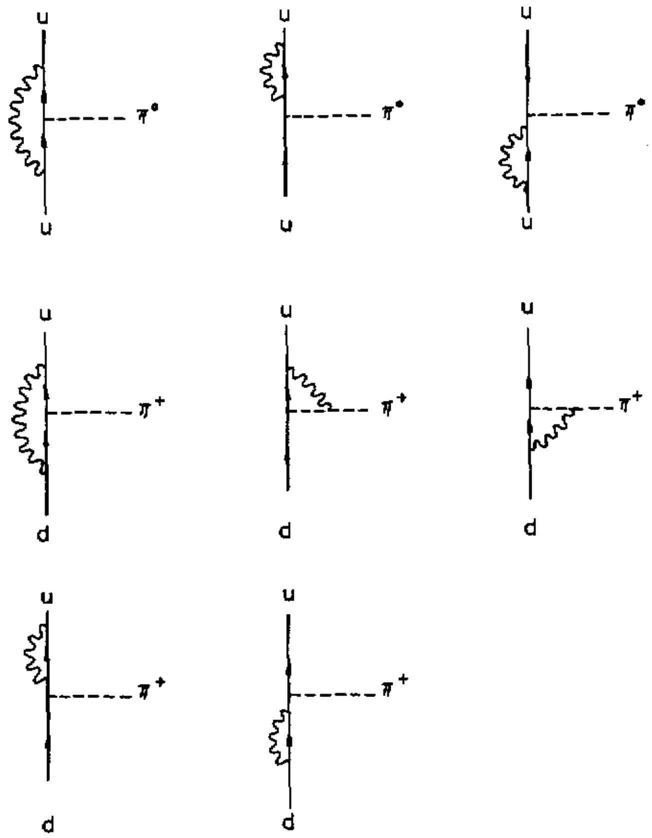

FIG. 4. The vertex renormalizations (including the self-energy contributions) leading to the nonuniversal pion-quark couplings, an important source for isospin symmetry breakings.

breaking. Specifically, we obtain, in the minimum subtraction scheme, the following expressions for the pion-quark couplings:

$$g_{\pi^+du} \to g_{\pi^+du}\left\{1+\frac{\alpha}{4\pi}\left(-\frac{43}{18}+\frac{1}{6}\gamma_E\right)\right\},$$

$$g_{\pi^0uu} \to g_{\pi^0uu}\left\{1+\frac{\alpha}{4\pi}\left(\frac{52}{9}-\frac{4}{3}\gamma_E\right)\right\},$$

$$g_{\pi^0dd} \to g_{\pi^0dd}\left\{1+\frac{\alpha}{4\pi}\left(\frac{13}{9}-\frac{1}{3}\gamma_E\right)\right\}. \tag{15}$$

Numerically, these expressions give rise to 2.18×10^{-3} to the charge symmetry breaking term $g_{\pi^0uu}-g_{\pi^0dd}$ and 4.2×10^{-3} for the charge dependent combination $g_{\pi^0uu}-g_{\pi^+du}$.

To perform numerical analyses, we introduce the following parameters:

$$\lambda\equiv\frac{\lambda_p+\lambda_n}{2},\quad \sigma\equiv\frac{\lambda_n-\lambda_p}{\lambda}$$

$$\bar{m}\equiv\frac{m_p+m_n}{2},\quad \epsilon\equiv\frac{m_n-m_p}{\bar{m}},$$

$$\frac{g_{\pi^0pp}}{g_{\pi^0dd}}\equiv K,$$

$$\frac{g_{\pi^0nn}}{g_{\pi^0uu}}\equiv K(1+\rho_1),$$

$$\frac{g_{\pi^-pn}}{g_{\pi^-ud}}\equiv K(1+\rho_2),$$

$$\frac{g_{\pi^+np}}{g_{\pi^+du}}\equiv K(1+\rho_3).$$

Moving the pion-quark coupling to the phenomenological side, we may write the RHS as follows:

$$\pi^0pp\to K\lambda^2e^{-m^2/M^2}\left(1-\sigma+\epsilon\frac{m^2}{M^2}\right),$$

$$\pi^0nn\to K\lambda^2e^{-m^2/M^2}\left(1+\sigma-\epsilon\frac{m^2}{M^2}+\rho_1\right),$$

$$\pi^-pn\to K\lambda^2e^{-m^2/M^2}(1+\rho_2),$$

$$\pi^+np\to K\lambda^2e^{-m^2/M^2}(1+\rho_3).$$

The various parameters which we adopt are $\sigma=-10^{-3}$ [15], $\epsilon=1.3\times 10^{-3}$, $\gamma=-6.57\times 10^{-3}$ [16], $a_q=0.546\ \mathrm{GeV}^3$, $m_u=5.1$ MeV, $m_d=8.9$ MeV, $b=0.474\ \mathrm{GeV}^4$, $m_0^2=0.8\ \mathrm{GeV}^2$, $\mu=0.5$ GeV, and $\chi a=-3.14\ \mathrm{GeV}^2$ [10]. This set of parameter values is to be referred to as the "standard set"in the rest of this paper. The working interval for analyzing the QCD sum rules for the various πNN couplings is $0.8\ \mathrm{GeV}^2\leq M_B^2\leq 1.5\ \mathrm{GeV}^2$, a standard choice for analyzing the various QCD sum rules associated with the nucleon.

The QCD sum rules (13) and (14) yield a prediction on the strong πNN coupling, which is numerically quite stable (as compared to, e.g., the nucleon mass sum rules). Figures 5(a)–5(c) display such numerical stability for a broad range of parameter values. In Fig. 5(a), the predicted strong πNN coupling is shown as a function of the Borel mass squared M_B^2 (in units of GeV^2) for three different values of the continuum threshold W. Note that in [15] a standard value of $W=2.25\ \mathrm{GeV}^2$ on the continuum threshold is adopted to ensure numerical stability of in the working interval $0.8\ \mathrm{GeV}^2\leq M_B^2\leq 1.5\ \mathrm{GeV}^2$. In Fig. 5(b), similar plots are given for three different values of the quark condensate a, while in Fig. 5(c) the sensitivity of the sum rule to the susceptibility χ is illustrated.

Figures 5(a)–5(c) indicate that the QCD sum rule for the strong πNN coupling, Eqs. (13) and (14), is numerically well behaved and may be used with some confidence. It yields a numerical prediction of $g_{\pi NN}=-(13.5\pm 1.5)$, with the error estimated by assuming $\Delta W=0.05\ \mathrm{GeV}^2$, $\Delta a=0.05\ \mathrm{GeV}^3$, and $\Delta\chi=1.0\ \mathrm{GeV}^{-1}$ and adding the errors quadratically. In analyzing the sum rule (13) or (14), it would also be useful to keep in mind that $g_{\pi NN}$ on the right-hand side could be replaced by $g_{\pi NN}+BM^2$ in order to take into account the strength of the transition from, e.g., the continuum to the ground state, a contribution which may not be

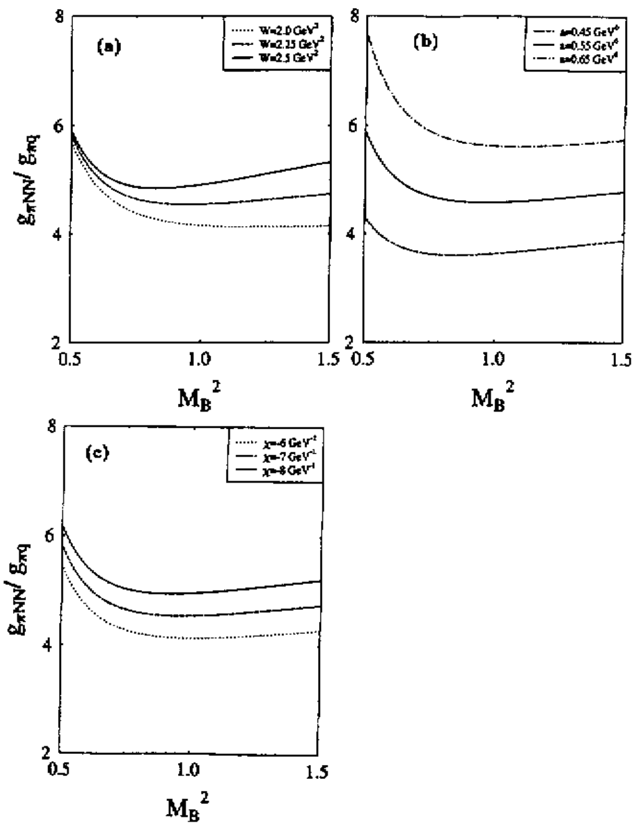

FIG. 5. $g_\pi NN/g_{\pi q}$ shown as a function of the Borel mass squared M_B^2 (in units of GeV^2): (a) for three different values of the continuum threshold; (b) for three different values of the quark condensate; and (c) for three different values of the induced condensate as characterized by the susceptibility χ.

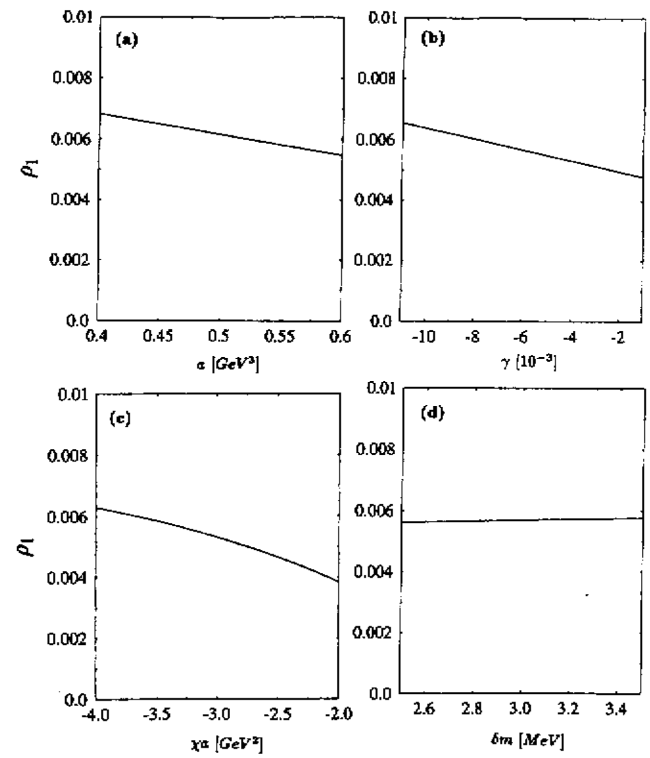

FIG. 6. The parameter ρ_1, which is essentially the left-hand side of Eq. (16), shown as a function of the given parameter while keeping all the other parameters fixed as standard values listed in the text: (a) by varying the quark condensate a, (b) by varying γ (which characterizes the difference between the up and down quark condensates), (c) by varying the susceptibility, and (d) by varying $\delta m(\equiv m_d - m_u)$.

sufficiently suppressed when the γ_5 πNN coupling is adopted. As already shown in [10], however, the resultant πNN does not differ much from the value quoted above and the cited error is more than adequate to also cover such uncertainty.

Adopting the standard set of parameter values (as listed earlier), we then find

$$\frac{g_{\pi^0 nn}/g_{\pi^0 uu} - g_{\pi^0 pp}/g_{\pi^0 dd}}{g_{\pi^0 NN}/g_{\pi^0 QQ}|_{\text{average}}} = 0.58\%, \tag{16}$$

$$\frac{(g_{\pi^+ np})/(g_{\pi^+ du}) - g_{\pi^0 pp}/g_{\pi^0 dd}}{g_{\pi^0 NN}/g_{\pi^0 QQ}|_{\text{average}}} = 0.35\%. \tag{17}$$

Combining with the difference in pion-quark couplings (from vertex renormalizations), we therefore conclude that $g_{\pi^0 nn}$ is numerically bigger than $g_{\pi^0 pp}$ by 0.80%, while $g_{\pi^+ np}$ is numerically greater than $g_{\pi^0 pp}$ by 0.15%.

Numerically, we may arbitrarily set $m_d = m_u$, $\langle \bar{d}d \rangle = \langle \bar{u}u \rangle$, and $\lambda_n = \lambda_p$, except keeping physical values for m_n and m_p. In this way, we find that $|g_{\pi^0 nn}|$ is bigger than $|g_{\pi^0 pp}|$ by 0.19%. Analogously, we find that $\lambda_n \neq \lambda_p$ (as found by Yang *et al.* [15]) has a contribution of 0.20%. $m_d \neq m_u$ and $\langle \bar{d}d \rangle \neq \langle \bar{u}u \rangle$ combine to give another contribution of 0.20%. Therefore, the overall contribution of 80% in fact comes from several sources of comparable magnitudes (and of the same sign in the present case).

It is of some interest to note that our prediction on $|g_{\pi^0 nn}| - |g_{\pi^0 pp}|$ (a positive value) differs in sign from the quark-model result [6], and also from a three-point correlation function approach [8]. It is also of interest to note that the difference in pion-quark couplings represents a source of numerical importance on the charge symmetry breaking effect.

It is of some importance to assess the error in our prediction on the charge-symmetry breaking effect as characterized by $|g_{\pi^0 nn}| - |g_{\pi^0 pp}|$. To analyze this question, it is helpful to keep in mind that, although the nucleon mass sum rule may yield a prediction on the nucleon mass with an error as large as (10–15)%, we are still able to make prediction on the nucleon-proton mass difference (which is only 0.1% of the average nucleon mass) and have a prediction which is also of (10–20)% in error [15]. The reason is simply that when we combine the sum rules for the proton and the neutron we may obtain another sum rule characterizing the difference—and such a sum rule is to be analyzed in its own light. The same situation occurs for the strong πNN couplings, where we have combined the sum rules for $g_{\pi^0 nn}$ and $g_{\pi^0 pp}$ to obtain a sum rule for the difference and this is what is behind the predictions cited for Eqs. (16) and (17).

Numerically, we may adopt the standard set of parameter values but choose to vary one particular parameter in order to estimate the error. In Figs. 6(a)–6(d), we display the sample results. In Fig. 6(a), the sensitivity of ρ_1, which determines $g_{\pi^0 nn}/g_{\pi^0 uu} - g_{\pi^0 pp}/g_{\pi^0 dd}$, to the quark condensate a is shown (while keeping γ, χ, δm, and other parameters fixed at the standard values). A variation of $\pm 0.05\ \mathrm{GeV}^3$ for a yields an error on ρ_1 of less than 0.05%. Analogously, we show in Figs. 6(b), 6(c), and 6(d), respectively, the sensitivity of ρ_1 to a reasonable variation in the parameters γ (in units of 10^{-3}), χa (in units of GeV^2), and δm ($\equiv m_d - m_u$, in units of MeV). Contrary to what has been found [6] in the naive quark model, the variation in δm does not produce visible change in ρ_1—a nontrivial aspect which manifests itself through the QCD sum rule. The variation of ρ_1 with respect to γ is much milder than that of the neutron-proton mass difference to γ. In fact, the change in γ is fairly restricted (i.e., of the order of 0.5×10^{-3} to ensure that the prediction on $m_n - m_p$ has an error of about 20%) if we wish to retain reasonable predictions on the neutron-proton mass difference. Even if we take the liberty to give a sizable uncertainty to γ, say as large as 4×10^{-3}, Fig. 6(b) suggests that the error on the prediction (16) would remain to be about 0.05%. Finally, we caution that the sensitivity of ρ_1 to a and χ as shown by Figs. 6(a) and 6(c) is caused by the nonzero value of γ (which breaks charge symmetry), rather than by a or χ itself—we have produced these figures only for the sake of estimating the errors.

To sum up, Figs. 6(a)–6(d) suggest that we may place an overall error of about 0.10% on our prediction in Eq. (16). Considering the different sources for the overall 0.58% as given by Eq. (16), such error estimate appears to be rather conservative since most of the sources actually do not involve any significant error and the 0.10% error estimated above arises primarily from possible error to a source of about 0.20%.

As a parenthetical remark, we note that the situation is very different in the three-point correlation function approach [8], in which the pion-nucleon couplings are determined predominantly by the quark condensate and as a result the uncertainty in γ translates itself directly into the error in $g_{\pi^0 pp} - g_{\pi^0 nn}$. In our case, the contribution from γ is only some part of the whole story and a large variation of γ does not cause as seriously a problem as in the three-point method. Nevertheless, it is amusing that different sum rules for the pion-nucleon coupling can be obtained based on different approaches and it is of some importance to pin down in future studies possible connections among these different sum rules.

III. LOW-ENERGY NUCLEON-NUCLEON SCATTERING

The one-pion-exchange potential (OPEP) in nucleon-nucleon interactions can be obtained in the standard manner [6]. For example, we have, in the $p-p$ case,

$$V_{pp}^{\pi}(r) = \frac{g_{\pi^0 pp}^2}{4\pi}\,\frac{\vec{\sigma}^1\cdot\vec{\nabla}_r\vec{\sigma}^2\cdot\vec{\nabla}_r}{4m_p^2}\,\frac{1}{r}\,(e^{-m_{\pi^0} r} - e^{-\Lambda r}), \tag{18}$$

where Λ is the cutoff at the short range which contributes negligibly to the isospin symmetry breakings [6]. The OPEP in the $n-n$ case may be obtained by simple substitution $p \to n$. On the other hand, the $n-p$ OPE potential is given by

$$\begin{aligned} V_{np}^{\pi}(r) = {} & \frac{g_{\pi^0 nn} g_{\pi^0 pp}}{4\pi}\,\tau_3^1\tau_3^2\,\frac{\vec{\sigma}^1\cdot\vec{\nabla}_r\vec{\sigma}^2\cdot\vec{\nabla}_r}{4m_n m_p}\,\frac{1}{r}\,(e^{-m_{\pi^0} r} - e^{-\Lambda r}) \\ & + \frac{g_{\pi^+ np}^2}{4\pi}\left\{ (\vec{\tau}^1\cdot\vec{\tau}^2 - \tau_3^1\tau_3^2)\,\frac{\vec{\sigma}^1\cdot\vec{\nabla}_r\vec{\sigma}^2\cdot\vec{\nabla}_r}{4M_N^2} \right. \\ & \left. + \frac{2\delta}{4M_N^2}\,(\vec{\tau}^1\times\vec{\tau}^2)_3\,\vec{\sigma}^1\times\vec{\sigma}^2\cdot\vec{L}\,\frac{1}{r}\,\frac{d}{dr}\right\}\frac{1}{r} \\ & \times (e^{-m_{\pi^+} r} - e^{-\Lambda r}). \end{aligned} \tag{19}$$

Here we have used $\delta \equiv (m_n - m_p)/(m_n + m_p)$ and $M_N = \frac{1}{2}(m_n + m_p)$. The ${}^1S_0$ phase shift δ_0, as used to describe low-energy nucleon-nucleon scatterings, can be parametrized in the standard manner:

$$k\cot\delta_0 = -\frac{1}{a} + \frac{1}{2}k^2 r_0 + \cdots, \tag{20}$$

with a the scattering length and r_0 the effective range. Experimentally, we have [17]

$$\begin{aligned} \delta a_{\mathrm{CSB}} &\equiv |a_{pp}| - |a_{nn}| = -(1.5\pm 0.5)\ \mathrm{fm}, \\ \delta a_{\mathrm{CD}} &\equiv |a_{np}| - |a_{nn}| = (5.0\pm 0.3)\ \mathrm{fm}. \end{aligned} \tag{21}$$

As already noted in [6], the differences in the OPEP's, such as $V_{pp}(r) - V_{nn}(r)$, may be treated as a perturbation, making it easy to compute δa_{CSB} or other low-energy isospin symmetry breaking observables. In this way, our prediction on the difference between $g_{\pi^0 nn}$ and $g_{\pi^0 pp}$ gives rise to about -1.0 fm on δa_{CSB}, consistent with the observation both in sign and roughly in magnitude.

As has been known [1,6] for some time, the bulk of the charge-dependent effect as revealed by a (relatively) large value of δa_{CD} can be understood quantitatively as caused by the pion mass difference $m_{\pi^-} \neq m_{\pi^0}$ while the differences as found in pion-nucleon couplings give rise to relatively small contribution to δa_{CD}. As a result, we may safely conclude that the observed δa_{CSB} and δa_{CD} as summarized recently in [17] can be understood, to a large extent, as manifestations of isospin asymmetries associated with the one-pion exchange potential.

IV. DISCUSSION AND SUMMARY

The observed differences [17] in the ${}^1S_0$ NN scatterings are by far the best known isospin symmetry breakings in which pion-nucleon couplings play a central role. There might be some isospin symmetry breakings in the ${}^1S_0$ effective ranges or in the ${}^3P_J$ scattering lengths, but until now such effects have not been extracted in any reliable manner.

The last term in $V_{np}^{\pi}(r)$ [in Eq. (19)], as induced by $m_p \neq m_n$, gives rise to isospin mixings such as ${}^3P_1 - {}^1P_1$ or ${}^3D_2 - {}^1D_2$ mixing, but only in a rather small way. On the other hand, charge-symmetry breakings observed in medium-energy elastic $n-p$ scattering (at TRIUMF and IUCF) may be dictated primarily [7] by the charge symmetry breaking force as caused by the $\rho-\omega$ mixing or by $g_{\omega pp} \neq g_{\omega nn}$. However, the short-range nature in this case may require a careful investigation of the quark-interchange mechanisms as addressed in [18], as it is difficult to reconcile the picture of exchanging, between two nucleons, a meson (ρ or ω) with a range of about 0.25 fermi, considerably smaller than the extent of a nucleon (with a radius of at least 0.5 fermi, even assuming a little bag picture for the nucleon).

In any event, the isospin symmetry breakings associated with the one-pion exchange potential (which is clearly on a very sound conceptual ground) can best be accessed through careful measurements of low-energy scattering parameters such as $\delta a_{\rm CSB}$, $\delta a_{\rm CD}$, etc. Such efforts should be further encouraged by the currently popular wisdom that the low-energy sector of the πN are NN systems can be described by the long -wave length limit of QCD via Goldstone theorem, yielding the so-called chiral perturbation theory, so that the low-energy scattering parameters seems to be more "fundamental" than those needed in, e.g., the description of medium-energy nucleon-nucleon scatterings.

In summary, we have adopted the method of QCD sum rules in the presence of an external pion field to investigate isospin symmetry breakings in pion- nucleon couplings. The resultant QCD sum rule for the strong pion-nucleon coupling is found to be well behaved and the prediction on $g_{\pi NN}$ is $-(13.5\pm1.5)$ in good agreement with nuclear force studies. The QCD sum rule for the difference between $g_{\pi^0 nn}$ and $g_{\pi^0 pp}$ can be analyzed in its own light and the most uncertain part (which is caused directly or indirectly by γ) is by itself fairly small, thereby resulting in a small overall error. Numerically, we find that $|g_{\pi^0 nn}|$ is bigger than $|g_{\pi^0 pp}|$ by $(0.80\pm0.10)\%$ (and bigger than $|g_{\pi^+ np}|$ by 0.65%), with its major portion determined reliably without significant error. This result accounts for the observed charge-symmetry breaking effect $\delta a_{\rm CSB}$ both in sign and (roughly) in magnitude. Contrary to many previous calculations in the literature, our result offers, for the first time, the possibility to understand both $\delta a_{\rm CSB}$ and $\delta a_{\rm CD}$ in a quantitative manner (both in sign and in magnitude).

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Ω and $\Sigma^0\Lambda$ transition magnetic moment in QCD sum rules

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The method of QCD sum rules in the presence of the external electromagnetic $F_{\mu\nu}$ field is used to calculate the Ω magnetic moment μ_Ω and the $\Sigma^0\Lambda$ transition magnetic moment $\mu_{\Sigma^0\Lambda}$, with the susceptibilities obtained previously from the study of octet baryon magnetic moments. The results $\mu_\Omega=-1.92\mu_N$ and $\mu_{\Sigma^0\Lambda}=1.5\mu_N$ are in good agreement with the recent experimental data. [S0556-2821(98)03003-3]

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According to the quark model the Ω^- is composed of three strange quarks with parallel spins. This state is particularly interesting because of the large symmetry breaking that cannot be accommodated consistently in the naive quark model. In Ref. [1], it has been shown that nonstatic baryon-dependent magnetic effects are large. In this paper we investigate these symmetry breaking effects and evaluate the Ω moment in the QCD sum rule [3]. The Ω moment has been studied in literaures [2] and various theoretical results range from $-1.3\mu_N$ to $-2.7\mu_N$. We find our result is in good agreement with the recent data [4,5].

In a typical hadronic scale the quantum chromodynamics (QCD) is highly nonperturbative which makes a direct analytical first-principles calculation impossible. In this work we adopt the method in the presence of an external electromagnetic field [6,7] to calculate the Ω and the $\Sigma^0\Lambda$ transition magnetic moment $\mu_{\Sigma^0\Lambda}$. The important aspect of this investigation is that the various susceptibilities in this calculation have already been determined from previous studies of octet baryon magnetic moments [6–9], and as a result our calculation is parameter free.

In the method of QCD sum rules [6,7], the two-point correlation function $\Pi(p)$ in the presence of an external electromagnetic field $F_{\alpha\beta}$ is written as

$$\Pi_\Omega(p)=i\int d^4x\langle 0|T\{\eta_\mu(x),\bar{\eta}^\mu(0)\}|0\rangle_{F_{\alpha\beta}}e^{ip\cdot x}$$
$$=\Pi_0(p)+\Pi_1(p)(\sigma\cdot F\hat{p}+\hat{p}\sigma\cdot F)+\cdots, \tag{1}$$

$$\Pi_{\Sigma\Lambda}(p)=i\int d^4x\langle 0|T\{\eta_{\Sigma^0}(x),\bar{\eta}_\Lambda(0)\}|0\rangle_{F_{\alpha\beta}}e^{ip\cdot x}$$
$$=\Pi_2(p)(\sigma\cdot F\hat{p}+\hat{p}\sigma\cdot F)+\cdots, \tag{2}$$

where $\Pi_0(p)$ is the polarization operator without the external field $F_{\alpha\beta}$. The η_μ, η_{Σ^0}, and η_Λ are the currents with Ω, Σ^0, and Λ quantum numbers

$$\eta_\mu(x)=\epsilon^{abc}[s^{aT}(x)C\gamma_\mu s^b(x)]s^c(x), \tag{3}$$

$$\eta_{\Sigma^0}(x)=\epsilon^{abc}\frac{1}{\sqrt{2}}\{[u^{aT}(x)C\gamma_\mu d^b(x)]\gamma_5\gamma^\mu s^c(x)$$
$$+[d^{aT}(x)C\gamma_\mu u^b(x)]\gamma_5\gamma^\mu s^c(x)\}, \tag{4}$$

$$\eta_\Lambda(x)=\epsilon^{abc}\sqrt{\frac{2}{3}}\{[u^{aT}(x)C\gamma_\mu s^b(x)]\gamma_5\gamma^\mu d^c(x)$$
$$-[d^{aT}(x)C\gamma_\mu s^b(x)]\gamma_5\gamma^\mu u^c(x)\}, \tag{5}$$

where $u^a(x)$, T, and C are the quark field, the transpose operators, and the charge conjugate operators. a,b,c are the color indices. The interpolating currents couples to the baryon states with the overlap amplititude λ:

$$\langle 0|\eta_\mu(0)|\Omega\rangle=\lambda_\Omega\nu_\mu(p), \tag{6}$$

$$\langle 0|\eta_{\Sigma^0}(0)|\Sigma\rangle=\lambda_\Sigma\nu_\Sigma(p), \tag{7}$$

$$\langle 0|\eta_\Lambda(0)|\Lambda\rangle=\lambda_\Lambda\nu_\Lambda(p), \tag{8}$$

where ν_μ is a vectorial spinor and satisfies $(\hat{p}-m_\Omega)\nu_\mu=0$, $\bar{\nu}_\mu\nu_\mu=-2m_\Omega$, and $\gamma_\mu\nu^\mu=p_\mu\nu^\mu=0$ in the Rarita-Schwinger formalism. $\nu(p)$ is a Dirac spinor.

On the hadronic level the correlators $\Pi_1(p)$ and $\Pi_2(p)$ are expressed in terms of the chirality-odd tensor structure $(\sigma\cdot F\hat{p}+\hat{p}\sigma\cdot F)$:

$$\Pi_1(p)=-\frac{1}{4}\mu_\Omega\frac{\lambda_\Omega^2}{(p^2-m_\Omega^2)^2}\left\{\frac{10}{9}+\frac{4}{9m_\Omega^2}(p^2-m_\Omega^2)^2\right\}$$
$$\times(\sigma\cdot F\hat{p}+\hat{p}\sigma\cdot F)+\cdots, \tag{9}$$

$$\Pi_2(p)=-\frac{1}{4}\mu_{\Sigma^0\Lambda}\frac{\lambda_\Sigma\lambda_\Lambda}{(p^2-\bar{m}^2)^2}(\sigma\cdot F\hat{p}+\hat{p}\sigma\cdot F)+\cdots, \tag{10}$$

where $\{\cdots\}$ is 1 for the nucleon magnetic moment. The deviation from unity in Eq. (9) is due to the Rarita-

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Schwinger formalism for the spin $\frac{3}{2}$ field, of which the details may be found in Ref. [10]. $\bar{m}=(m_{\Sigma^0}+m_\Lambda)/2$. We treat the Σ and Λ mass as degenerate due to their small mass difference since we never come across the poles and always work in the virtuality $p^2<0$. We denote the continuum and nondiagonal transition contributions simply by ellipses.

The external field $F_{\mu\nu}$ may induce changes in the physical vacuum and modify the propagation of quarks. Up to dimension six ($d\leqslant 6$), we introduce three induced condensates,

$$\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_{F_{\mu\nu}}=e_q\chi F_{\mu\nu}\langle 0|\bar{q}q|0\rangle,$$

$$g_s\langle 0|\bar{q}\frac{\lambda^n}{2}G^n_{\mu\nu}q|0\rangle_{F_{\mu\nu}}=e_q\kappa F_{\mu\nu}\langle 0|\bar{q}q|0\rangle,$$

$$g_s\epsilon^{\mu\nu\lambda\sigma}\langle 0|\bar{q}\gamma_5\frac{\lambda^n}{2}G^n_{\lambda\sigma}q|0\rangle_{F_{\mu\nu}}=ie_q\xi F^{\mu\nu}\langle 0|\bar{q}q|0\rangle, \tag{11}$$

where q refers to u, d, and s quark, and e_q is the charge. The χ, κ, and ξ in Eq. (5) are the quark condensate susceptibilities and their values have been the subject of various studies. Ioffe and Smilga [6] found $\chi\approx-8\ \ \mathrm{GeV}^{-2}$ with $\kappa=0$, $\xi=0$ in order to have $\mu_p=3.0\mu_N$ and $\mu_n=-2.0\mu_N$ ($\pm 10\%$). Balitsky and Yung [7] estimated

$$\chi=-3.3\ \ \mathrm{GeV}^{-2},\quad \kappa=0.22,\quad \xi=-0.44 \tag{12}$$

using the one-pole approximation. Belyaev and Kogan [8] extended the calculation and obtained an improved estimate $\chi=-5.7\ \ \mathrm{GeV}^{-2}$ using the two-pole approximation. Chiu *et al.* [9] also estimated the susceptibilities with the two-pole model and obtained

$$\chi=-4.4\ \ \mathrm{GeV}^{-2},\quad \kappa=0.4,\quad \xi=-0.8. \tag{13}$$

The values of these susceptibilities are consistent with one another except that the earliest result $\chi=-8\ \ \mathrm{GeV}^{-2}$ in [6], is considerably larger (in magnitude) due to their neglect of κ,ξ in the fitting procedure. In what follows, we shall adopt the condensate parameters $\chi=-4.5\ \ \mathrm{GeV}^{-2}$, $\kappa=0.4$, $\xi=-0.8$ which represent the average in the last three analyses.

The correlation functions $\Pi_1(p)$ and $\Pi_2(p)$ at the quark level are

$$\begin{aligned}\langle 0|T\eta_\mu(x)\bar{\eta}^\mu(0)|0\rangle_F &= -2i\epsilon^{abc}\epsilon^{a'b'c'}(\mathrm{Tr}\{S^{bb'}(x)\gamma_\mu C[S^{aa'}(x)]^TC\gamma_\mu\}\\ &\quad\times S^{cc'}(x)+2S^{bb'}(x)\gamma_\mu C[S^{aa'}(x)]^TC\gamma_\mu S^{cc'}(x)),\end{aligned} \tag{14}$$

$$\begin{aligned}\langle 0|T\eta_{\Sigma^0}(x)\bar{\eta}^\Lambda(0)|0\rangle_F &= -\frac{2}{\sqrt{3}}i\epsilon^{abc}\epsilon^{a'b'c'}\{\gamma_5\gamma^\mu S_s^{aa'}(x)\gamma_\nu C[S_u^{bb'}(x)]^T\\ &\quad\times C\gamma_\mu S_d^{cc'}(x)\gamma^\nu\gamma_5\\ &\quad-\gamma_5\gamma^\mu S_s^{aa'}(x)\gamma_\nu C[S_d^{bb'}(x)]^TC\gamma_\mu S_u^{cc'}(x)\gamma^\nu\gamma_5\},\end{aligned} \tag{15}$$

where $iS^{ab}(x)$ is the quark propagator in the presence of the external eletromagnetic field $F_{\mu\nu}$ [6]. We find

$$\begin{aligned}iS_q^{ab}(p)&=\delta^{ab}\frac{i}{\hat{p}-m_q}+\frac{i}{4}\frac{\lambda^n_{ab}}{2}g_sG^n_{\mu\nu}\frac{1}{(p^2-m_q^2)^2}[\sigma^{\mu\nu}(\hat{p}+m_q)+(\hat{p}+m_q)\sigma^{\mu\nu}]+\frac{i}{4}e_q\delta^{ab}F_{\mu\nu}\frac{1}{(p^2-m_q^2)^2}\\ &\quad\times[\sigma^{\mu\nu}(\hat{p}+m_q)+(\hat{p}+m_q)\sigma^{\mu\nu}]-\delta^{ab}\frac{\langle\bar{q}q\rangle}{12}(2\pi)^4\delta^4(p)-\delta^{ab}\frac{\langle g_s\bar{q}\sigma\cdot Gq\rangle}{192}(2\pi)^4g^{\mu\nu}\partial_\mu\partial_\nu\delta^4(p)-\delta^{ab}e_q\frac{\langle\bar{q}q\rangle}{192}\\ &\quad\times\left[\sigma\cdot Fg^{\mu\nu}-\frac{1}{3}\gamma^\mu\sigma\cdot F\gamma^\nu\right](2\pi)^4\partial_\mu\partial_\nu\delta^4(p)-\delta^{ab}e_q\chi\frac{\langle\bar{q}q\rangle}{24}\sigma\cdot F(2\pi)^4\delta^4(p)-\delta^{ab}e_q\kappa\frac{\langle\bar{q}q\rangle}{192}\\ &\quad\times\left[\sigma\cdot Fg^{\mu\nu}-\frac{1}{3}\gamma^\mu\sigma\cdot F\gamma^\nu\right](2\pi)^4\partial_\mu\partial_\nu\delta^4(p)+i\delta^{ab}e_q\xi\frac{\langle\bar{q}q\rangle}{768}\\ &\quad\times\left[\sigma^{\delta\rho}g^{\mu\nu}-\frac{1}{3}\gamma^\mu\sigma^{\delta\rho}\gamma^\nu\right]\gamma_5\epsilon_{\alpha\beta\delta\rho}F^{\alpha\beta}(2\pi)^4\partial_\mu\partial_\nu\delta^4(p)+\cdots\end{aligned} \tag{16}$$

in momentum space with $\hat{p}\equiv p_\mu\gamma^\mu$. Here we follow [6,7] and do not introduce induced condensates of higher dimensions, while we find that the strange quark mass correction is important so that we have explicitly kept it in our calculation.

Only the condensates with even dimensions contribute to the structure $(\sigma\cdot F\hat{p}+\hat{p}\sigma\cdot F)$. They are 1, $\chi\langle 0|\bar{q}q|0\rangle m_s$, $m_s\langle 0|\bar{q}q|0\rangle$, $\langle 0|g_s^2G^n_{\alpha\beta}G^n_{\alpha\beta}|0\rangle$, $\chi\langle 0|\bar{q}q|0\rangle^2$, $\chi\langle 0|\bar{q}q|0\rangle\langle 0|g_s\bar{q}\sigma\cdot Gq|0\rangle$, $\kappa\langle 0|\bar{q}q|0\rangle^2$, and $\xi\langle 0|\bar{q}q|0\rangle^2$ up to dimension eight. The up and down quark is treated as massless. The gluon condensate $\langle 0|g_s^2G^n_{\alpha\beta}G^n_{\alpha\beta}|0\rangle$ always appears with a small numerical factor $1/(2\pi)^4$ through the two-loop integration [6]. Its contribution was found to be negligible through the direct calculation [9]. Following Refs. [3,6,7] the four quark condensate $\langle 0|\bar{q}\Gamma_1q\bar{q}\Gamma_2q|0\rangle$ is treated by the factorization approximation as the susceptibilities are estimated under the the vacuum dominance hypothesis. The calculation is straightforward by substituting the quark propagator into Eqs. (9) and (14). Here we present the final result after Borel transformation:

$$\frac{9}{4}e_s\Bigg\{M_B^6E_2(y_1)L^{4/27}-\frac{6}{5}\chi m_sa_sM_B^4E_1(y_1)L^{-4/9}+\frac{6}{5}m_sa_sM_B^2E_0(y_1)L^{4/27}-\frac{4}{5}\chi a_s^2M_B^2E_0(y_1)L^{4/9}+a_s^2L^{28/27}\left[\frac{14}{15}+\frac{\chi m_0^2}{30}L^{-10/9}-\frac{22}{15}\kappa-\frac{4}{15}\xi\right]\Bigg\}=(2\pi)^4\lambda_\Omega^2e^{-m_\Omega^2/M_B^2}\mu_\Omega(1+A_1M_B^2), \tag{17}$$

$$\frac{1}{\sqrt{3}}(e_u-e_d)\Bigg\{M_B^6E_2(y_2)L^{-4/9}-\chi m_saM_B^4E_1(y_2)L^{-28/27}+\frac{4}{3}m_saM_B^2E_0(y_2)L^{-4/9}-m_sa_sM_B^2E_0(y_2)L^{-4/9}-\frac{2}{3}\chi aa_sM_B^2E_0(y_2)L^{-4/27}+\frac{1}{9}aa_sL^{4/9}\times\left[4+\kappa-2\xi+\frac{3}{4}\chi m_0^2L^{-10/9}\right]\Bigg\}=(2\pi)^4\lambda_\Sigma\lambda_\Lambda e^{-\bar{m}^2/M_B^2}\mu_{\Sigma^0\Lambda}(1+A_2M_B^2), \tag{18}$$

where $m_\Omega=1.672$ GeV, $\bar{m}=1.15$ GeV, $m_s=150$ MeV, $y_1=W_1^2/M_B^2$ and $y_2=W_2^2/M_B^2$. $E_n(y)=1-e^{-y}\Sigma_{k=0}^{n}1/k!y^k$ are the factors used to subtract the continuum contribution [6]. $W_1^2=5.0$ GeV^2 and $W_2^2=3.4$ GeV^2 are the continuum thresholds which are determined together with the overlap amplititudes $(2\pi)^4\lambda_\Omega^2=5.56$ GeV^6, $(2\pi)^4\lambda_\Sigma^2=1.88$ GeV^6, and $(2\pi)^4\lambda_\Lambda^2=1.64$ GeV^6 from the Ω, Σ, and Λ mass sum rules [11,12]. We adopt the "standard" values for the various condensates $a=-(2\pi)^2\langle 0|\bar{u}u|0\rangle=0.55$ GeV^3, $a_s=-(2\pi)^2\langle 0|\bar{s}s|0\rangle=0.55\times0.8$ GeV^3, $am_0^2=(2\pi)^2g_s\langle 0|\bar{u}\sigma\cdot Gu|0\rangle$, $m_0^2=0.8$ GeV^2. $L=\ln(M_B/\Lambda_{QCD})/\ln(\mu/\Lambda_{QCD})$, Λ_{QCD} is the QCD parameter, $\Lambda_{QCD}=100$ MeV, and $\mu=0.5$ GeV is the normalization point to which the used values of condensates are referred.

We further improve the numerical analysis by taking into account of the renormalization group evolutions of the sum rules (17) and (18) through the anomalous dimensions of the various condensates and currents. A_1 and A_2 are constants to be determined from the sum rule, which arise from the nondiagonal transitions $\Omega\gamma\to\Omega^*$, $\Sigma^*\to\Lambda\gamma$, or $\Sigma\to\Lambda^*\gamma$ [6]. The working intervals of the Borel mass M_B^2 for the sum rules (17) and (18) are 2.0 GeV$^2\leq M_B^2\leq4.0$ GeV^2 and 1.3 GeV$^2\leq M_B^2\leq3.0$ GeV^2, respectively, where both the continuum contribution and power corrections are controllable. Moving the factor $(2\pi)^4\lambda_\Omega^2e^{-m_\Omega^2/M_B^2}$ and $(2\pi)^4\lambda_\Sigma\lambda_\Lambda e^{-\bar{m}^2/M_B^2}$ on the right-hand side to the left and fitting the new sum rules with a straight line approximation we may extract the μ_Ω and $\mu_{\Sigma\Lambda}$. We show the new sum rules as a function of the Borel mass in Fig. 1. The new sum rules are almost stable and independent of M_B^2, which implies that the nondiagonal transition contributions are small.

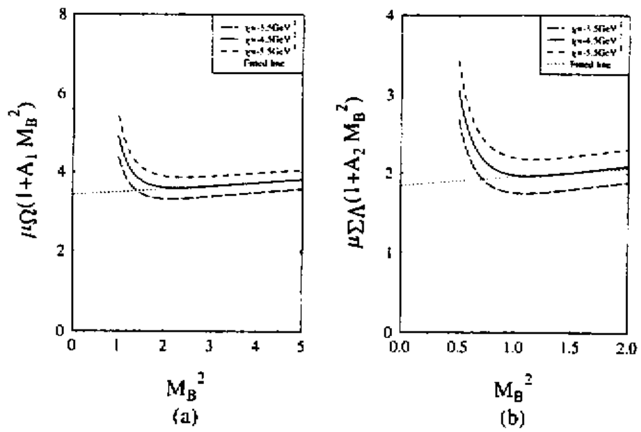

FIG. 1. (a) The Borel mass dependence of the Ω magnetic moment. The long-dashed, solid, and short-dashed curve are the QCD sum rule predictions for $\chi=-3.5$, -4.5, and -5.5 GeV^{-2}, respectively, from Eq. (17) after the numerical factor $(2\pi)^4\lambda_\Omega^2e^{-m_\Omega^2/M_B^2}$ is moved to the left-hand side. The dotted line is a straight-line approximation. The intersect with the Y axis is the Ω magnetic moment in units of $e/2m_\Omega$. The Borel mass M_B^2 is in units of GeV^2. (b) The Borel mass dependence of the $\Sigma\Lambda$ transition magnetic moment. The notations are the same as those in (a).

The sum rules are insensitive to the susceptibilities κ and ξ because of their small values. Their contributions are less than 5%. The dependence on χ is shown in Fig. 1. When χ varies from -4.5 to -3.5 GeV^{-2} or to -5.5 GeV^{-2}, the sum rules change within 10%. The correction from the strange quark mass is important and contributes about 20% to both of the sum rules. The SU(3)$_f$ flavor symmetry breaking parameter $\gamma=\langle\bar{s}s\rangle/\langle\bar{u}u\rangle$ needs to take the standard value of 0.8 in order to yield a good agreement with the experimental data in Eq. (17). The Ω magnetic moment would increase 30% if $\gamma=1$, in contradiction with the experimental data. Our final results are $\mu_\Omega=-3.41$ in units of $e/2m_\Omega$ and $\mu_{\Sigma^0\Lambda}=1.85$ in units of $e/2\bar{m}$, where $e/2m_B$ is a natural unit in QCD sum rule analyses. In unit of nuclear magneton $\mu_\Omega=-1.92\mu_N$ and $\mu_{\Sigma^0\Lambda}=1.5\mu_N$, in good agreement with the recent experimental data.

Baryon magnetic moments are important physical observables as masses. The method of QCD sum rules in the presence of an external electromagnetic field was successfully employed to calculate the octet baryon magnetic moments. The results are in reasonable agreement with the experimental data. In this work we have extended the same method to calculate the magnetic moment of the long-lived decuplet member, the Ω and $\Sigma^0\Lambda$ transition magnetic moments simultaneously, which may serve both as a consistency check of the various susceptibilities and a check of the method of the external field itself. Our results are in good agreement with the recent experimental data.

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Possible Σ^0-Λ mixing in the QCD sum rule

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We calculate the on-shell Σ^0-Λ mixing parameter θ with the method of the QCD sum rule. Our result is $\theta(m^2_{\Sigma^0})=(-)(0.5\pm0.1)$ MeV. The electromagnetic interaction is not included. [S0556-2821(98)03103-8]

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Isospin independence and charge symmetry are only approximate in the strong interaction. It is believed that the up and down quark mass difference and the electromagnetic interaction cause all the isospin violations [1,2]. Experimentally a strong signature for ρ^0-ω mixing has been observed in the cross-section measurement of the reaction $e^+e^-\to\pi^+\pi^-$ [3]. The effect from the electromagnetic interaction is of the opposite sign and much smaller than the experimentally determined $\langle\rho^0|H_{\rm CSB}|\omega\rangle$, while the current quark mass difference plays a dominant role [4]. Strong evidence for π-η-η' has also been obtained from studies of η' [5], ψ' [6], and ψ [7] decays. In this work we study possible Σ^0-Λ mixing with the method of QCD sum rules [8]. ρ^0-ω mixing has been analyzed within the same framework [8,9]. The isospin symmetry breaking sources in such an approach come from the current quark mass difference $\delta m=m_u-m_d\neq0$ and the quark condensate difference $\gamma=\langle0|\bar dd|0\rangle/\langle0|\bar uu|0\rangle-1\neq0$. We do not take into account the electromagnetic interaction in the present work.

We may study Σ^0-Λ mixing through the mixed propagator in the QCD vacuum

$$i\int d^4x e^{ipx}\langle0|\Sigma^0(x)\bar\Lambda(0)|0\rangle = (-)\frac{(\hat p+m_{\Sigma^0})\theta(\hat p+m_\Lambda)}{(p^2-m^2_{\Sigma^0}+i\epsilon)(p^2-m^2_\Lambda+i\epsilon)}, \tag{1}$$

where the mixing parameter $\theta(p^2)$ is defined through the following effective Lagragian:

$$L^{\rm mix}=\theta(\bar\Psi_{\Sigma^0}\Psi_\Lambda+\bar\Psi_\Lambda\Psi_{\Sigma^0}) \tag{2}$$

and may be measured using decays such as $\psi\to\Lambda\bar\Sigma^0+\bar\Lambda\Sigma^0$ in future experiments.

In order to calculate the mixing parameter we study the two-point correlator at the quark level as

$$\Pi(p)=i\int d^4x\langle0|T\{\eta_{\Sigma^0}(x),\bar\eta_\Lambda(0)\}|0\rangle e^{ip\cdot x}. \tag{3}$$

η_{Σ^0} and η_Λ are the currents with Σ^0 and Λ quantum numbers:

$$\eta_{\Sigma^0}(x)=\epsilon^{abc}\frac{1}{\sqrt2}\{[u^{aT}(x)C\gamma_\mu d^b(x)]\gamma_5\gamma^\mu s^c(x) + [d^{aT}(x)C\gamma_\mu u^b(x)]\gamma_5\gamma^\mu s^c(x)\}, \tag{4}$$

$$\eta_\Lambda(x)=\epsilon^{abc}\sqrt{\frac{2}{3}}\{[u^{aT}(x)C\gamma_\mu s^b(x)]\gamma_5\gamma^\mu d^c(x) - [d^{aT}(x)C\gamma_\mu s^b(x)]\gamma_5\gamma^\mu u^c(x)\}, \tag{5}$$

where $u^a(x)$, T, and C are the quark field, the transpose, and the charge conjugate operators. a, b, c are the color indices. The interpolating currents couple to the baryon states with the overlap amplititude λ:

$$\langle0|\eta_{\Sigma^0}(0)|\Sigma\rangle=\lambda_\Sigma\nu_\Sigma(p), \tag{6}$$

$$\langle0|\eta_\Lambda(0)|\Lambda\rangle=\lambda_\Lambda\nu_\Lambda(p), \tag{7}$$

where $\nu(p)$ is a Dirac spinor.

The correlation function $\Pi(p)$ may be expressed as

$$\begin{aligned}\langle0|T\eta_{\Sigma^0}(x)\,\bar\eta_\Lambda(0)|0\rangle &= -\frac{2}{\sqrt3}i\epsilon^{abc}\epsilon^{a'b'c'}\{\gamma_5\gamma^\mu S_s^{aa'}(x)\gamma_\nu C[S_u^{bb'}(x)]^T \\ &\quad\times C\gamma_\mu S_d^{cc'}(x)\gamma^\nu\gamma_5 \\ &\quad-\gamma_5\gamma^\mu S_s^{aa'}(x)\gamma_\nu C[S_d^{bb'}(x)]^T C\gamma_\mu S_u^{cc'}(x)\gamma^\nu\gamma_5\},\end{aligned} \tag{8}$$

where $iS^{ab}(x)$ is the quark propagator [10]:

$$iS_q^{ab}(x)=\langle0|T[q^a(x)\bar q^b(0)]|0\rangle = \frac{i\delta^{ab}}{2\pi^2x^4}\hat x+\frac{i}{32\pi^2}\frac{\lambda^n_{ab}}{2}g_sG^n_{\mu\nu}\frac{1}{x^2}(\sigma^{\mu\nu}\hat x+\hat x\sigma^{\mu\nu})$$

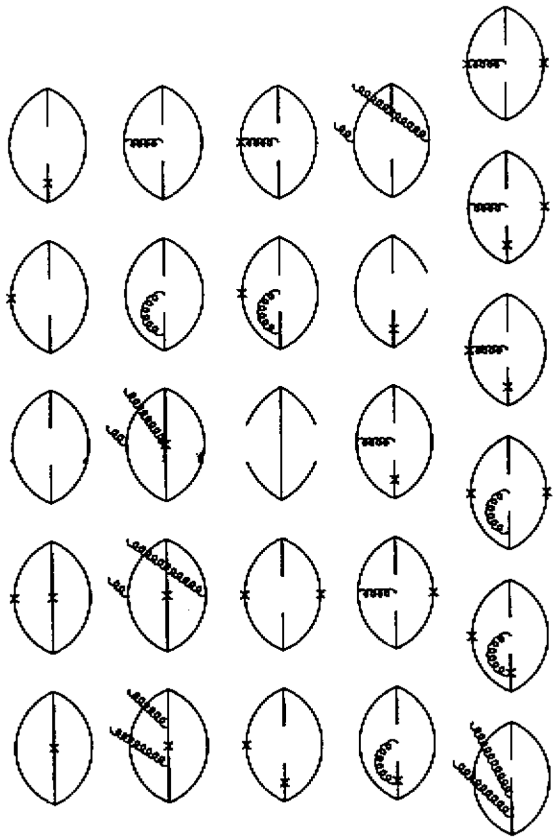

FIG. 1. Relevant diagrams in the QCD sum rules analysis of Σ^0-Λ ~~mixing up to dimension seven~~. The current quark mass correction is denoted by a cross. If two crosses appear in the same diagram, one of them comes from the strange quark.

$$
-\frac{\delta^{ab}}{12}\langle\bar{q}q\rangle+\frac{\delta^{ab}x^2}{192}\langle g_s\bar{q}\sigma\cdot Gq\rangle
-\frac{\langle\bar{q}q\rangle\langle g_s^2G^2\rangle x^4}{2^9\times 3^3}\delta^{ab}-\frac{m_q\delta^{ab}}{4\pi^2x^2}
+\frac{m_q}{32\pi^2}g_s\frac{\lambda^n_{ab}}{2}G^n_{\mu\nu}\sigma_{\mu\nu}\ln(-x^2)
-\frac{\delta^{ab}\langle g_c^2G^2\rangle}{2^9\times 3\pi^2}m_qx^2\ln(-x^2)+\frac{i\delta^{ab}m_q\langle\bar{q}q\rangle}{48}\hat{x}
-\frac{im_q\langle g_s\bar{q}\sigma\cdot Gq\rangle\delta^{ab}x^2\hat{x}}{2^7\times 3^2}+\cdots. \tag{9}
$$

At the hadronic level,

$$
\Pi(p)=(-)\lambda_{\Sigma^0}\lambda_\Lambda\frac{(\hat{p}+m_{\Sigma^0})\theta(p^2)(\hat{p}+m_\Lambda)}{(p^2-m^2_{\Sigma^0}+i\epsilon)(p^2-m^2_{\Lambda^0}+i\epsilon)}. \tag{10}
$$

The diagrams with nonzero contribution are presented in Fig. 1. In the limit of exact isospin symmetry $\Pi(p)$ vanishes. There are two isospin symmetry breaking parameters δm and γ. Each diagram is either propotional to δm or to γ. We explicitly keep the current quark mass term in the quark propagator up to order $O(m_q)$, which is denoted by a cross. The current quark mass enters a diagram either through expanding the free quark propagator up to $O(m_q)$ or through the equation of motion. Since the strange quark mass is not small, we keep terms such as $m_s(m_d-m_u)$. The calculation is standard as in the QCD sum rule analysis of the baryon mass. Equating the correlator $\Pi(p)$ at the quark level and $\Pi(p)$ at the hadronic level we arrive at two sum rules corresponding to two different structures 1 and $\hat{p}$. Here we present the final result after Borel transformation.

For structure 1,

$$
\begin{aligned}
&\frac{1}{\sqrt{3}}\Big\{\delta mM_B^8E_3L^{-8/9}+\gamma aM_B^6E_2-\frac{b}{8}\delta mM_B^4E_1L^{-8/9}\\
&\quad-\frac{4}{3}\delta ma_sm_sM_B^4E_1L^{-8/9}-\frac{1}{3}\gamma m_saa_sM_B^2E_0\\
&\quad+\frac{1}{3}\delta ma^2M_B^2E_0-\frac{1}{72}\gamma baM_B^2E_0\\
&\quad+\frac{7}{48}\delta ma_sm_sm_0^2M_B^2E_0L^{-38/27}\\
&\quad-\frac{5}{8}\delta mam_0^2m_sM_B^2\ln\frac{M_B^2}{\mu^2}E_0L^{-38/27}\Big\}\\
&=(2\pi)^4\lambda_\Sigma\lambda_\Lambda e^{-m^2/M_B^2}2m^2\theta(m^2_{\Sigma^0})(1+A_1M_B^2).
\end{aligned} \tag{11}
$$

For structure $\hat{p}$,

$$
\begin{aligned}
&-\frac{1}{\sqrt{3}}\Big\{\delta mm_sM_B^6E_2L^{-4/3}+4\gamma m_saM_B^4E_1L^{-4/9}\\
&\quad+4\delta ma_sM_B^4E_1L^{-4/9}+\frac{2}{3}\gamma aa_sM_B^2E_0L^{4/9}\\
&\quad-\frac{1}{4}\gamma m_sam_0^2M_B^2E_0L^{-26/27}-\frac{1}{4}\delta ma_sm_0^2M_B^2E_0L^{-26/27}\\
&\quad+\frac{1}{8}\delta mam_0^2M_B^2E_0L^{-26/27}\Big\}\\
&=(2\pi)^4\lambda_\Sigma\lambda_\Lambda 2me^{-m^2/M_B^2}\theta(m^2_{\Sigma^0})(1+A_2M_B^2),
\end{aligned} \tag{12}
$$

where $\delta m=m_d-m_u$ and $\gamma=\langle\bar{d}d\rangle/\langle\bar{u}u\rangle-1$, $m=m_{\Sigma^0}+m_\Lambda/2=1.15$ GeV is the average mass. $m_s=150$ MeV is the strange quark mass. $y=W^2/M_B^2$ and the factors $E_n(y)=1-e^{-y}\Sigma^n_{k=0}(1/k!)y^k$, are used to subtract the continuum contribution [11]. $W^2=3.4\ \mathrm{GeV}^2$ is the continuum threshold which is determined together with the overlap amplititude $(2\pi)^4\lambda_\Sigma^2=1.88\ \mathrm{GeV}^6$, $(2\pi)^4\lambda_\Lambda^2=1.64\ \mathrm{GeV}^6$ from the mass sum rules [12,13]. We adopt the "standard" values for the various condensates $b=\langle 0|g_s^2G^2|0\rangle=0.474\ \mathrm{GeV}^4$, $a=-(2\pi)^2\langle 0|\bar{u}u|0\rangle=0.55\ \mathrm{GeV}^3$, $a_s=-(2\pi)^2\langle 0|\bar{s}s|0\rangle=0.55\times 0.8\ \mathrm{GeV}^3$, $am_0^2=(2\pi)^2g_s\langle 0|\bar{u}\sigma\cdot Gu|0\rangle$, $m_0^2=0.8\ \mathrm{GeV}^2$. $L=\ln(M_B/\Lambda_{\mathrm{QCD}})/\ln(\mu/\Lambda_{\mathrm{QCD}})$, Λ_{QCD} is the

QCD parameter, $\Lambda_{QCD}=100$ MeV, $\mu=0.5$ GeV is the normalization point to which the used values of condensates are referred.

We further improve the numerical analysis by taking into account the renormalization group evolution of the sum rules (11) and (12) through the anomalous dimensions of the various condensates and currents. A_1 and A_2 are constants to be determined from the sum rule. They arise from the mixing with the excited states Σ^{0*}-Λ or Σ^0-Λ^* which were first introduced in the QCD sum rules analysis of the nucleon magnetic moments [11]. The working interval for the Borel mass M_B^2 is 1.3 $\text{GeV}^2 \leq M_B^2 \leq 2.5$ GeV^2 where both the continuum contribution and power corrections are controllable. Moving the factor $(2\pi)^4\lambda_\Sigma\lambda_\Lambda e^{-m^2/M_B^2}$ on the right-hand side to the left and fitting the new sum rule with a straight line approximation we may extract the mixing parameter θ.

Various theoretical approaches [2,10,14–18] yield consistent results for the quark mass difference $\delta m=3.2\pm0.4$ MeV. The difference of the up and down quark condensate has been analyzed with the chiral perturbation theory [14], the QCD sum rules for scalar and pseudoscalar mesons [17–19], effective models of QCD incorporating the dynamical breaking of chiral symmetry [20,21], and the QCD sum rules for baryons [10]. The numerical results from the above approaches are $\gamma=-(6-10)\times10^{-3}$ [14], $-(10\pm3)\times10^{-3}$ [17], $-(7-9)\times10^{-3}$ [20,21], and -6.57×10^{-3} [10].

In both of the sum rules (11) and (12) the contribution from δm and γ has opposite sign. The mixing parameters from Eqs. (11) and (12) would contradict each other if γ were too large or too small. In Fig. 2 the Borel mass dependence of the mixing parameter and the fitting straight line is shown. Through the intersection with the Y axis, we can obtain the value of θ directly. With $\gamma=-(7\pm1)\times10^{-3}$ and $\delta m=3.0\pm0.4$ MeV, our final result is $\theta(m_{\Sigma^0}^2)=(-)(0.5\pm0.1)$ MeV. It can be seen from Fig. 2 that Eqs. (11) and (12) yield almost the same value for θ. In summary we have calculated the on-shell Σ^0-Λ mixing parameter, which may be measured in future experiments.

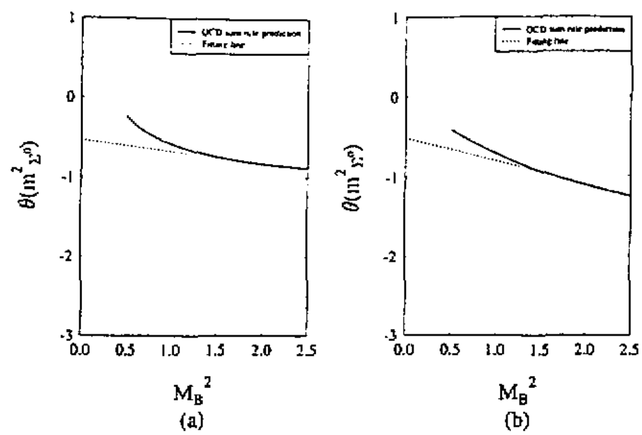

FIG. 2. (a) The Borel mass dependence of the mixing parameter. The solid curve is the QCD sum rule prediction for $\gamma=-7\times10^{-3}$ from Eq. (11]) for structure 1 with $\delta m=3.0$ MeV after the numerical factor $(2\pi)^4\lambda_{\Sigma^0}^2\lambda_\Lambda^2 e^{-m^2/M_B^2}$ is moved to the left-hand side. The dotted curve is the fitting straight line. The Borel mass M_B^2 is in unit of GeV^2. The mixing parameter θ is in unit of MeV. (b) The Borel mass dependence of the mixing matrix for the structure $\hat{p}$. The notation is the same as in (a).

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Electromagnetic decay of vector mesons as derived from QCD sum rules

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(i.e., in the rest frame of the vector meson).

In the original work of Shifman, Vainshtein, and Zakharov [3], the nonperturbative QCD effects are incorporated through the various non-zero condensates in the nontrivial QCD vacuum. Ioffe and Smilga

$$+\Pi_\mu^{\rho^* \to \pi\gamma}(p) + \cdots . \quad (7)$$

The $\rho\pi\gamma$ coupling is defined through the relation:

$\langle \pi'(p')| j_\lambda^{em}| \rho_\mu^m(p)\rangle$

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PHYSICS LETTERS B

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Electromagnetic decay of vector mesons as derived from QCD sum rules

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Abstract

The method of QCD sum rules in the presence of the external electromagnetic field is used to treat the problem of the electromagnetic decay of the various vector mesons, including $\rho \to \pi + \gamma$, $K^* \to K + \gamma$, and $\eta' \to \rho + \gamma$. The induced condensates obtained previously from the study of baryon magnetic moments are adopted, thereby ensuring the parameter-free nature of the present predictions. Further consistency is reinforced by invoking the various QCD sum rules for the meson masses. The numerical results on the various radiative decays agree very well with the experimental data. © 1998 Published by Elsevier Science B.V.

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Quantum chromodynamics (QCD) is believed to be the underlying theory of strong interactions. However, little is known concerning the structure of a meson (for which the naive quark-antiquark picture is at most only the leading approximation), since QCD is nonperturbative at the typical hadronic scale. In the absence of the first principle calculation, the electromagnetic decay of a vector meson, such as $\rho \to \pi\gamma$, may be treated via an effective Lagrangian [1,2], such as in the vector meson dominance model (VDM) [1] or in chiral perturbation theory [2]. Another approach, which is the focus of the present paper, is made possible via the method of QCD sum rules [3] in which the nonperturbative QCD physics is incorporated systematically as power corrections in the short-distance operator product expansion (OPE). Results from QCD sum rules will certainly serve as another useful reference point concerning radiative decays of vector mesons.

To be specific, we consider the electromagnetic decay of a vector meson in the flavor $SU(3)$ sector, i.e., in the sector of (u, d, s). Using $\rho \to \pi + \gamma$ as the illustrative example, we may describe the decay process by the effective Lagrangian,

$$\mathscr{L}_{\rho\pi\gamma} = e\frac{g_{\rho\pi\gamma}}{m_\rho}\epsilon_{\mu\nu\lambda\delta}\partial^\nu A^\mu \rho^\lambda \partial^\delta \pi , \quad (1)$$

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where A_μ is the electromagnetic field. The corresponding partial width is

$$\Gamma(\rho \to \pi\gamma) = \alpha \frac{g_{\rho\pi\gamma}^2}{3} \frac{P_{cm}^3}{m_\rho^2}, \tag{2}$$

with P_{cm} is the pion momentum in the CM frame (i.e., in the rest frame of the vector meson).

In the original work of Shifman, Vainshtein, and Zakharov [3], the nonperturbative QCD effects are incorporated through the various non-zero condensates in the nontrivial QCD vacuum. Ioffe and Smilga [4] and, independently, Balitsky and Yung [5] studied baryon magnetic moments by extending the method of QCD sum rules in the presence of the external electromagnetic field. It turns out that the QCD sum rule method as obtained in [4] or [5] can readily be applied to the problem of the electromagnetic decay of the various vector mesons, such as $\rho \to \pi + \gamma$, $K^* \to K + \gamma$, and $\eta' \to \rho + \gamma$. As a result, we wish to pursue along this direction a little further, hoping to obtain results which may help to clarify certain issues in relation to the effective Lagrangian approach [1,2].

We recall [4,5] that, in the method of QCD sum rules, the two-point correlation function $\Pi_\mu(p)$ in the presence of the constant electromagnetic field $F_{\mu\nu}$ may be introduced as follows,

$$\Pi_\mu(p) = i\int d^4x e^{ipx} \langle 0|T\left[j_\mu^\rho(x), j^\pi(0)\right]|0\rangle_F, \tag{3}$$

where we introduce the interpolating fields,

$$j_\mu^\rho(x) = \bar{u}^a(x)\gamma_\mu d^a(x), \quad j^\pi(x) = \bar{u}^a(x) i\gamma_5 d^a(x), \tag{4}$$

with a the color index. As in the QCD sum rule analyses for nucleon properties, we may introduce the overlap amplitude (coupling) of the meson current to the (physical) meson:

$$\langle 0|j_{\rho_\mu}|\rho\rangle = \lambda_\rho \epsilon_\mu, \tag{5}$$

$$\langle 0|j_\pi|\pi\rangle = \lambda_\pi, \tag{6}$$

where ϵ_μ is the polarization 4-vector of the ρ meson.

Embedding the system in the constant $F_{\mu\nu}$ field and introducing intermediate states we express the correlation function at the hadronic level as follows,

$$\Pi_\mu(p) = \lambda_\rho \lambda_\pi \frac{g_{\rho\pi\gamma}}{2m_\rho} \frac{1}{p^2 - m_\rho^2} \frac{1}{p^2 - m_\pi^2} \epsilon_{\alpha\beta\mu\sigma} F^{\alpha\beta} p^\sigma + \Pi_\mu^{\rho^* \to \pi\gamma}(p) + \cdots. \tag{7}$$

The $\rho\pi\gamma$ coupling is defined through the relation:

$$\langle \pi^l(p')|j_\lambda^{em}|\rho_\mu^m(p)\rangle \equiv -i\delta^{lm} e \frac{g_{\rho\pi\gamma}}{m_\rho} \epsilon_{\lambda\nu\sigma\mu} p^\nu q^\sigma K(q^2), \tag{8}$$

where $K(q^2)$ is the form factor normalized such that $K(0) = 1$. The superscripts l and m are isospin indices. Note that, in Eq. (7), we write down explicitly the leading $\rho \to \pi\gamma$ contribution, label the next transition term by $\Pi_\mu^{\rho^* \to \pi\gamma}(p)$, and denote the continuum contribution simply by the ellipsis.

The external field $F_{\mu\nu}$ may induce changes in the physical vacuum and modify the propagation of quarks. Up to dimension six ($d \le 6$), we may introduce three induced condensates as follows.

$$\langle 0|\bar{q}\sigma_{\mu\nu} q|0\rangle_{F_{\mu\nu}} = e_q e \chi F_{\mu\nu} \langle 0|\bar{q}q|0\rangle, \tag{9}$$

$$g_c \langle 0|\bar{q} \frac{\lambda^n}{2} G_{\mu\nu}^n q|0\rangle_{F_{\mu\nu}} = e_q e \kappa F_{\mu\nu} \langle 0|\bar{q}q|0\rangle, \tag{10}$$

$$g_c \epsilon_{\mu\nu\lambda\sigma} \langle 0|\bar{q}\gamma_5 \frac{\lambda^n}{2} G^{n\lambda\sigma} q|0\rangle_{F_{\mu\nu}} = ie_q e \xi F_{\mu\nu} \langle 0|\bar{q}q|0\rangle, \tag{11}$$

where q refers mainly to u or d (with suitable modifications for the s quark), e is the unit charge, $e_u = \frac{2}{3}$, and $e_d = -\frac{1}{3}$. The observed baryon magnetic moments have been used to fix these susceptibilities, leading to a value of χ in the range of $-(3.0\text{–}4.5)\,\mathrm{GeV}^{-2}$ and $2\kappa + \xi \approx 0$ [4–7]. Accordingly, we adopt in this paper $\chi = -3.5\,\mathrm{GeV}^{-2}$ and $2\kappa + \xi \approx 0$ with κ, ξ quite small, which represent the average of the optimal values in the literature.

On the other hand, the correlation function $\Pi_\mu(p)$ may also be evaluated at the quark level,

$$\Pi_\mu(p) = -i\int d^4x e^{ipx} \mathrm{Tr}\left[iS_u^{ab}(x)\gamma_\mu iS_d^{ba}(-x) i\gamma_5\right], \tag{12}$$

where $iS^{ab}(x)$ is the coordinate-space propagator in the presence of the $F_{\mu\nu}$ field [4,5].

Equating the correlation function obtained both at the quark level (l.h.s.) and at the hadron level (r.h.s.), performing the Borel transform on both sides (to accentuate the contribution from the leading $\rho \to \pi\gamma$ contribution), we finally obtain the QCD sum rule for $g_{\rho\pi\gamma}$:

$$
\begin{aligned}
&(e_u+e_d)a_q\frac{1}{24\pi^2}\left[-3\chi+(2+2\kappa+\xi)\frac{1}{M_B^2}\right] \\
&\quad+\cdots \\
&= \frac{g_{\rho\pi\gamma}}{2m_\rho}\frac{\lambda_\rho\lambda_\pi}{m_\rho^2-m_\pi^2}\left(e^{-\frac{m_\pi^2}{M_B^2}}-e^{-\frac{m_\rho^2}{M_B^2}}\right) \\
&\quad+\Pi_{\rho^*\to\pi\gamma}(M),
\end{aligned}
\tag{13}
$$

where $a_q \equiv -(2\pi)^2\langle 0|\bar{q}q|0\rangle$ and M_B is the Borel mass (with M_B^2 playing essentially the role of p^2). Note that in this sum rule (13) the dominant contribution comes from the induced quark condensate characterized by the susceptibility χ. The contribution from $\rho^* \to \pi\gamma$ is suppressed by the factor $(\frac{m_\rho}{m_{\rho^*}})^3$ and so is negligible to within 10% (which is the typical accuracy of the QCD sum rule approach). The sum rule (13) is stable within the working interval $M_B^2 \sim m_\rho^2$.

We proceed to note that in the framework of QCD sum rules the overlap amplitude λ_ρ can be determined in a self-consistent manner by making use of the mass sum rules for the ρ and π mesons [3,8],

$$
\begin{aligned}
\lambda_\rho^2 = \frac{1}{\pi^2}e^{\frac{m_\rho^2}{M_B^2}}&\left\{\frac{1}{4}\left(1+\frac{\alpha_s}{\pi}\right)M_B^4E_1-\frac{b}{48}\right. \\
&\left.+\frac{\alpha_s}{\pi}\frac{14}{81}a_q^2\frac{1}{M_B^2}\right\},
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
\lambda_\pi^2 = \frac{1}{\pi^2}e^{\frac{m_\pi^2}{M_B^2}}&\left\{\frac{3}{8}\left(1+\frac{11}{3}\frac{\alpha_s}{\pi}\right)M_B^4E_1+\frac{b}{32}\right. \\
&\left.+\frac{\alpha_s}{\pi}\frac{7}{27}a_q^2\frac{1}{M_B^2}\right\},
\end{aligned}
\tag{15}
$$

where $E_1 \equiv 1-(1+\frac{S_0^2}{M_B^2})e^{-\frac{S_0^2}{M_B^2}}$ (with $S_0^2 = 1.5\,\mathrm{GeV}^2$) is the factor arising from the continuum approximation, and $b \equiv (2\pi)^2\langle 0|\frac{\alpha_s}{\pi}G_{\alpha\beta}^n G_{\alpha\beta}^n|0\rangle$ is the gluon condensate. It was pointed out in [10] that the instanton contributions may invalidate the usual sum rule techniques for the pseudoscalar current. Nevertheless, it was suggested in [3,8] that Eq. (18) may still provide a rough estimate of λ_π with the experimental pion mass as input (despite the fact that it may fail to predict the pion mass). Moreover there are many careful analyses of this correlator which yield similar results for λ_π [11–13]. We may therefore adopt the value from the QCD sum rule for λ_π with the physical mass m_π as the input. We obtain the estimates $\lambda_\pi \approx 0.17 \pm 0.03\,\mathrm{GeV}^2$ and $\lambda_\rho \approx 0.17\,\mathrm{GeV}^2$. An independent phenomenological analysis in instanton models [14] yields $\lambda_\rho = f_\rho m_\rho = 0.17\,\mathrm{GeV}^2$, $\lambda_\pi = f_\pi \frac{m_\pi^2}{m_u+m_d} = 0.20 \pm 0.04\,\mathrm{GeV}^2$, values consistent with those from the present QCD sum rule approach.

Using the above values for λ_ρ and λ_π, the observed masses, and the condensate parameters discussed earlier, we finally obtain from the QCD sum rule (13),

$$g_{\rho\pi\gamma} = 0.59, \tag{16}$$

which is very close to the value $g_{\rho\pi\gamma} = 0.58$ as may be extracted from the partial width for the process $\rho \to \pi\gamma$.

We have used the familiar localized version of the 't Hooft instanton interaction [14] in connection with the correlator (3) to demonstrate that the sum rule (13) is modified only to the order linear in the current quark mass. Such modification is clearly of negligible numerical importance. This aspect, together with the above result on λ_π and λ_ρ, indicate that although the instanton effects may be quite significant in the pure pseudoscalar channel the present prediction in the $\rho \to \pi$ channel should be quite reliable.

Upon having established some confidence in the sum rule (13), we extend the method to describe other electromagnetic decays of flavor $SU(3)$ vector mesons, among which we have considered additional processes as listed in Table 1. The working interval for the Borel mass for the corresponding sum rules is $M_B^2 \sim m_{\mathrm{par}}^2$, where m_{par} is the mass of the parent particle in the decay process. In the case for the meson containing a strange quark or antiquark, we have taken into account the contributions due to the nonzero quark mass $m_s \approx 150\,\mathrm{MeV}$, which in the cases such as $K^* \to K\gamma$ may give rise to corrections of order of about 20%. We also use $\langle 0|\bar{s}s|0\rangle \approx 0.8 \times \langle 0|\bar{u}u|0\rangle$ (as suggested by chiral perturbation theory) which is essential for the $K^* \to K\gamma$ channel.

In Table 1, we present the experimental values for the various electromagnetic decays of vector mesons, the predictions from VDM [1], those from the effective Lagrangian approach [2], and what we have obtained from the QCD sum rules (this work). The results shown in this table indicate that the QCD sum rules yield predictions which are in better overall agreement with the experimental values.

We note that all parameters have been determined consistently in the framework of QCD sum rules as illustrated before. In particular, the various overlap amplitudes (couplings) are determined from the various mass sum rules: $\lambda_\pi = 0.17 \pm 0.03\,\mathrm{GeV}^{-2}$, $\lambda_K = 0.23 \pm 0.03\,\mathrm{GeV}^{-2}$, $\lambda_\rho = 0.17 \pm 0.01\,\mathrm{GeV}^{-2}$, $\lambda_{K^*} = 0.21 \pm 0.02\,\mathrm{GeV}^{-2}$. The above predictions and the other overlap amplitudes given in the following after taking into account the mixing effects are in good agreement with those in [14]. Note that the value for $\lambda_{\eta'}$ is of some specific interest in view of the $U_A(1)$ problem.

Table 1
Comparison between experimental data [9] and predictions from the Vector Meson Dominance Model (VDM) [1], effective chiral Lagrangian approach [2], and QCD sum rules. The unit is keV

Decay process	Experimental value	Vector Meson Dominance	Effective chiral Lagrangian	QCD sum rules
$\omega \to \pi\gamma$	716	888.0	800	720
$\omega \to \eta\gamma$	7	5.4	5	7.1
$\rho \to \pi\gamma$	68	63.0	80	65
$\rho \to \eta\gamma$	58	55.0	38	48
$\phi \to \eta\gamma$	57	71.0	68	80
$\phi \to \eta'\gamma$	< 1.8	0.24	1	0.67
$K^{0*} \to K^0\gamma$	116	149.0	117	102
$K^{-*} \to K^+\gamma$	50	68.0	29	60
$\eta' \to \rho\gamma$	60	119.0	77	60 [a]
$\eta' \to \omega\gamma$	6	12.5	7	6.1
$\phi \to \pi\gamma$	5.8	6.1	5	5.8 [b]

[a] Used as the input to determine η-η' mixing angle θ.
[b] Used as the input to determine ϕ-ω mixing parameter ϵ.

In the case of the ϕ-ω mixing, we introduce

$$|\omega\rangle = \cos\epsilon|\omega_0\rangle - \sin\epsilon|\phi_0\rangle,$$
$$|\phi\rangle = \sin\epsilon|\omega_0\rangle + \cos\epsilon|\phi_0\rangle, \tag{17}$$

and

$$j_\omega = \cos\epsilon\, j_{\omega_0} - \sin\epsilon\, j_{\phi_0},$$
$$j_\phi = \sin\epsilon\, j_{\omega_0} + \cos\epsilon\, j_{\phi_0}. \tag{18}$$

where

$$|\omega_0\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle), \quad |\phi_0\rangle = |s\bar{s}\rangle. \tag{19}$$

Using the interpolating field in (17), we derive the QCD sum rules for the relevant radiative decays and the mass sum rules for ω and ϕ mesons. We then use the observed value for the width of the decay $\phi \to \pi\gamma$ to determine the mixing angle. We obtain the mixing parameter $\epsilon = 0.063 \pm 0.005$ and the overlap amplitudes $\lambda_\omega = 0.16 \pm 0.01\,\mathrm{GeV}^{-2}$ and $\lambda_\phi = 0.24 \pm 0.02\,\mathrm{GeV}^{-2}$. As the Borel mass is varied in the interval $0.6\,\mathrm{GeV}^2 \le M_B^2 \le 1.1\,\mathrm{GeV}^2$, the mixing parameter ϵ changes from 0.068 to 0.058. Note that, as a consistency check, our result is consistent with the value $\epsilon = 0.079$ as determined from $\phi \to \rho\pi$ [2].

Furthermore, we use the experimental partial width for the decay process $\eta' \to \rho\gamma$ as the input in the derived QCD sum rules to extract the $\eta_0 - \eta_8$ mixing angle, which is parametrized as in [15,9]:

$$|\eta\rangle = \cos\theta|\eta_8\rangle - \sin\theta|\eta_0\rangle,$$
$$|\eta'\rangle = \sin\theta|\eta_8\rangle + \cos\theta|\eta_0\rangle, \tag{20}$$

and correspondingly for the interpolating fields,

$$j_\eta = \cos\theta j_{\eta_8} - \sin\theta j_{\eta_0},$$
$$j_{\eta'} = \sin\theta j_{\eta_8} + \cos\theta j_{\eta_0}. \tag{21}$$

At the same time we obtain the η, η' mass sum rules with the interpolating fields in (21) and use the physical meson masses to calculate the overlap amplitudes. We then use the extracted mixing angle to predict other processes. Our results are $\lambda_\eta = 0.23 \pm 0.03\,\mathrm{GeV}^{-2}$, $\lambda_{\eta'} = 0.33 \pm 0.05\,\mathrm{GeV}^{-2}$, $\theta = -19^\circ \pm 2^\circ$, a value consistent with that extracted from the decays $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$ [15] and in [9]. We find that such value on the $\eta_0 - \eta_8$ mixing is essential for the overall agreement with the experimental data. Nevertheless, we feel that the exact role of the so-called $U(1)$ problem is yet to be clarified.

We wish to emphasize the parameter-free nature of the present calculation, in that all parameters are determined consistently in the framework of QCD sum rules, with a single set of the values for the induced condensates obtained previously from studies of baryon magnetic moments. The sum rules so obtained are not modified in any serious manner by instantons [14] (and in fact are consistent with the phenomenological instanton picture of the QCD vacuum). We note that the uncertainty of the overlap amplitudes (couplings) derived in this way might be as large as 10% in some cases and the method of QCD sum rules, being an operator product expansion used for moderate Q^2, may have some inherent error of order of 10%. With this in mind, we may conclude that our results as shown in Table 1 agree very well with the experimental data.

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Isospin Symmetry Breakings as from QCD Sum Rules

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NUCLEAR PHYSICS A

Isospin Symmetry Breakings as from QCD Sum Rules

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We investigate isospin symmetry breakings, especially those connected with pion-nucleon couplings and the neutron-proton mass difference, by making use of the method of QCD sum rules in the presence of an external pion field. Manifestations of such isospin symmetry breakings are examined briefly in relation to low-energy nucleon-nucleon $^1S_0$ scatterings. We discuss the prospect of understanding the various isospin symmetry breaking phenomena observed in low-energy nuclear physics by extending chiral perturbation theory (χPT) to include isospin symmetry breakings (ISB). We emphasize that QCD sum rules help to establish χPT/ISB out of QCD by eliminating an excessive number of ISB condensate parameters, making χPT/ISB a worthy extension of χPT.

1. Introduction

Isospin symmetry and its possible violations constitute an integral part of nuclear physics [1], not only in the sense that good symmetry offers an effective classification of nuclear or hadronic structure but also in the proximity of the isospin symmetry concept to other fundamental ideas such as chiral symmetry. In general, isospin symmetry breaking effects observed in nuclear physics may arise from different sources such as, in addition to the well known electromagnetic interactions, strong interactions but augmented with hadron mass differences (such as the $n-p$ or $\pi^{\pm}-\pi^0$ mass difference) or with different meson-nucleon couplings (such as non-universal πNN couplings) or with isospin mixings (such as $\rho-\omega$ or $\pi-\eta$ mixing). Determination of isospin symmetry breaking effects requires information on quantities which may not be directly accessible from experiments, such as the meson-nucleon couplings or the isospin mixing parameters. Accordingly, reliable theoretical predictions become indispensible for advancing our understanding of the field.

A quantitative treatment of isospin symmetry breakings often involves the capability of treating strong interactions at a quantitative level or, in other words, the ability to handle the complications due to quantum chromodynamics (QCD) in a reliable manner. To some extent, the method of QCD sum rules, as proposed originally by Shifman, Vain-

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shtein, and Zakharov [2] and adopted, or extended, by many others [3–5], incorporates the nonperturbative QCD effects through various condensates (associated with the nontrivial QCD vacuum), thereby offering some hope of being able to treat strong interactions in a quantitative manner.

We wish in this work to focus on the determination of isospin symmetry breakings in pion-nucleon couplings, with some emphasis on manifestations of such symmetry breaking in low-energy nucleon-nucleon $^1S_0$ scattering. We discuss briefly the prospect of understanding the various isospin symmetry breaking phenomena observed in low-energy nuclear physics by extending chiral perturbation theory (χPT) to include isospin symmetry breakings (ISB). We emphasize that QCD sum rules help to establish χPT/ISB out of QCD by eliminating an excessive number of ISB condensate parameters, making χPT/ISB a worthy extension of χPT.

2. Isospin Symmetry Breakings in Pion-Nucleon Couplings

In the method of QCD sum rules in the presence of an external field, we attempt to evaluate, both at the quark and hadron levels, the correlation function specified by

$$\Pi(p) \equiv i \int d^4x e^{ip\cdot x} \langle 0|T(\eta_p(x)\bar{\eta}_p(0))|0\rangle \,|_\pi, \tag{1}$$

where we have

$$\begin{aligned} \eta_p(x) &= \epsilon^{abc}\{u^{aT}(x)C\gamma_\mu u^b(x)\}\gamma^5\gamma^\mu d^c(x) \\ \bar{\eta}_p(y) &= \epsilon^{abc}\{\bar{u}^b(y)\gamma_\nu C\bar{u}^{aT}(y)\}\bar{d}^c(y)\gamma^\nu\gamma^5. \end{aligned} \tag{2}$$

By evaluating the appropriate correlation function in the presence of the external pion field, we may determine the pion-nucleon couplings $g_{\pi^0 pp}$, $g_{\pi^0 nn}$, and $g_{\pi^+ pn}$. An alternative method is to evaluate correlation funtions with T product of currents sandwiched between the vacuum and the one-pion state, as first suggested by [6] and recently adopted by some authors [7]. It appears that the induced condensates involved in the external field method are now well understood and it becomes relatively straightforward to perform calculations to higher dimensions (as required for reliable predictions). This is what we have chosen to obtain isospin symmetry breakings in πNN couplings.

The above correlation function allows us to determine the $\pi^0 pp$ coupling. At the quark level, we use

$$\langle 0|T\eta_p(x)\bar{\eta}_p(0)|0\rangle_{\pi^0} = 2i\epsilon^{abc}\epsilon^{a'b'c'}\mathrm{Tr}\{S_u^{bb'}(x)\gamma_\nu C[S_u^{aa'}(x)]^T C\gamma_\mu\}\,\gamma_5\gamma_\mu S_d^{cc'}(x)\gamma_\nu\gamma_5. \tag{3}$$

Here the quark propagator is given by

$$S(x) = S^{(0)}(x) + S^{(\pi)}(x), \tag{4}$$

with

$$\begin{aligned} \mathrm{i}S^{(0)ab}(x) = \; & \tfrac{\mathrm{i}\delta^{ab}}{2\pi^2x^4}\hat{x} + \tfrac{\mathrm{i}}{32\pi^2x^2}\tfrac{\lambda^n_{ab}}{2}g_cG^n_{\mu\nu}(\sigma^{\mu\nu}\hat{x} + \hat{x}\sigma^{\mu\nu}) - \tfrac{\delta^{ab}}{12}\langle\bar{q}q\rangle \\ & + \tfrac{\delta^{ab}x^2}{192}\langle g_c\bar{q}\sigma\cdot Gq\rangle - \tfrac{m_q\delta^{ab}}{4\pi^2x^2} + \tfrac{m_q}{32\pi^2}\tfrac{\lambda^n_{ab}}{2}G^n_{\mu\nu}\sigma^{\mu\nu}\ln(-x^2) \\ & - \tfrac{\delta^{ab}\langle g_c^2G^2\rangle}{2^9\times 3\pi^2}m_q x^2\ln(-x^2) + \tfrac{i\delta^{ab}m_q}{48}\langle\bar{q}q\rangle\hat{x} - \tfrac{1}{2^7\times 3^2}im_q\langle g_c\bar{q}\sigma\cdot Gq\rangle\delta^{ab}x^2\,, \end{aligned} \tag{5}$$

and

$$
\begin{aligned}
\mathrm{i}S^{(\pi)ab}(x) = \; & -\tfrac{\mathrm{i}\delta^{ab}}{4\pi^2x^2}g_q\vec{\tau}\cdot\vec{\pi}\gamma_5 + \tfrac{\mathrm{i}\delta^{ab}}{24}g_q\vec{\tau}\cdot\vec{\pi}\gamma_5\chi\langle\bar{q}q\rangle \\
& -\tfrac{\mathrm{i}\delta^{ab}}{384}m_0^{\pi}g_q\vec{\tau}\cdot\vec{\pi}\gamma_5\langle\bar{q}q\rangle x^2 + \tfrac{\mathrm{i}}{2^5\pi^2}g_c g_q\vec{\tau}\cdot\vec{\pi}\gamma_5\tfrac{\lambda^n_{ab}}{2}G^n_{\mu\nu}\sigma^{\mu\nu}\ln(-x^2) \\
& -\tfrac{\mathrm{i}\delta^{ab}}{2^9\times 3\pi^2}g_q\vec{\tau}\cdot\vec{\pi}\gamma_5\langle g_c^2G^2\rangle x^2\ln(-x^2) - \tfrac{\delta^{ab}}{48}g_q\vec{\tau}\cdot\vec{\pi}\gamma_5\langle\bar{q}q\rangle\hat{x}\,,
\end{aligned}
\tag{6}
$$

where $\hat{x} \equiv x_\mu\gamma^\mu$, and we have also introduced $< \bar{\psi}(0)i\gamma^5\tau^j\psi(0) >\equiv g_q\chi\pi^j < \bar{q}q >$, and $< \bar{\psi}(0)i\gamma^5\tau^j g_c\sigma\cdot G\psi(0) >\equiv g_q m_0^{\pi}\tau^j < \bar{q}q >$ with ψ the isospin doublet consisting of u and d quarks.

We also note that, in the case of the π^-pn or π^+np coupling, we use

$$
\langle 0|T\eta_p(x)\bar{\eta}_n(0)|0\rangle_{\pi^-} = -4\mathrm{i}\epsilon^{abc}\epsilon^{a'b'c'}\gamma_\mu\gamma_5 S_d^{aa'}(x)\gamma_\nu C[S^{(\pi)bb'}(x)]^T C\gamma_\mu S_u^{cc'}(x)\gamma_5\gamma_\nu. \tag{7}
$$

We note the absence of $\mathrm{i}S^{(0)}(x)$ in the present case, due to a change in the electric charge. The structure (which does not involve the trace of some product) seems quite different from the π^0pp case, but it can be shown [8] that isospin symmetry is exact should we assume $m_u = m_d$ and $< \bar{u}u >=< \bar{d}d >$.

On the other hand, we may parametrize the correlation functions at the hadronic level in the standard manner.

$$
\int d^4x\langle 0|T\eta_p(x)\bar{\eta}_n(0)|0\rangle\,|_{\pi^-} = \lambda_n\lambda_p\frac{\mathrm{i}}{\hat{\mathrm{p}}-\mathrm{m_p}}(-\gamma_5\sqrt{2}g_{\pi^-pn})\frac{\mathrm{i}}{\hat{\mathrm{p}}-\mathrm{m_n}} + \cdots, \tag{8}
$$

$$
\int d^4x\langle 0|T\eta_p(x)\bar{\eta}_p(0)|0\rangle\,|_{\pi^0} = \lambda_p^2\frac{\mathrm{i}}{\hat{\mathrm{p}}-\mathrm{m_p}} + \lambda_p^2\frac{\mathrm{i}}{\hat{\mathrm{p}}-\mathrm{m_p}}(-\gamma_5 g_{\pi^0pp})\frac{\mathrm{i}}{\hat{\mathrm{p}}-\mathrm{m_p}} + \cdots. \tag{9}
$$

In this way, we obtain [8] the following QCD sum rule for the π^0pp coupling:

$$
\begin{aligned}
& M^6L^{-4/9}E_2 - \tfrac{\chi}{2}a_dM^4L^{2/9}E_1 - \tfrac{11}{24}bM^2E_0 + \tfrac{4}{3}a_u^2L^{4/9} \\
& +\tfrac{m_0^2}{3}a_u^2\tfrac{1}{M^2}L^{-2/27} + m_0^2\ln\tfrac{M^2}{\mu^2}\,L^{-26/27}[\tfrac{7}{8}(a_um_d + a_dm_u) - \tfrac{1}{4}m_ua_u] \\
& +m_0^2L^{-26/27}[\tfrac{5}{12}m_ua_u - \tfrac{1}{8}m_da_d + \tfrac{67}{48}m_da_u + \tfrac{115}{48}m_ua_d] \\
= \; & \tfrac{g_{\pi^0pp}}{g_{\pi^0dd}}\tilde{\lambda}_p^2e^{-\frac{m_p^2}{M^2}},
\end{aligned}
\tag{10}
$$

with $a_q \equiv -(2\pi)^2 < \bar{q}q >$, $< \bar{q}\sigma\cdot G >\equiv -m_0^2 < \bar{q}q >$ and $b \equiv< g_c^2G^2 >$. M is the Borel mass and $\tilde{\lambda}_p^2 = (2\pi)^4\lambda_p^2$. Analogously, we obtain the QCD sum rule for the π^+np coupling:

$$
\begin{aligned}
& M^6L^{-4/9}E_2 - \tfrac{\chi}{2}\bar{a}M^4L^{2/9} - \tfrac{11}{24}bM^2E_0 + \{\tfrac{4}{3}a_ua_d + \tfrac{1}{3}(a_d - a_u)a_d\}L^{4/9} \\
& -(m_u - m_d)a_uM^2L^{-4/9}E_0 + m_0^2L^{-26/27}\{-\tfrac{1}{2}a_um_u - \tfrac{5}{24}a_dm_d + \tfrac{29}{12}a_um_d + \tfrac{19}{8}a_dm_u\} \\
& +m_0^2\ln\tfrac{M^2}{\mu^2}\,L^{-26/27}\{a_um_d + a_dm_u - \tfrac{1}{4}m_u(a_d + a_u)\} + \tfrac{m_0^2}{3}a_ua_dL^{-2/27}\tfrac{1}{M^2} \\
= \; & \tfrac{g_{\pi^+np}}{g_{\pi^+du}}(2\pi)^4\lambda_n\lambda_pe^{-\frac{m^2}{M^2}}.
\end{aligned}
\tag{11}
$$

Here $E_0 = 1 - e^{-x}$, $E_1 = 1 - (1 + x)e^{-x}$, and $E_2 = 1 - (1 + x + \frac{x^2}{2})e^{-x}$ with $x = W/M^2$, and $L = 0.621ln(10M)$.

There are several sources for isospin symmetry breakings, viz.:

(1) At the hadronic level (r.h.s.), we have $m_n \neq m_p$, $\lambda_n \neq \lambda_p$, and $W_n \neq W_p$ (with W the threshold parameter in the continuum approximation for treating the excited states and the continuum). Apart from the well known neutron-proton mass difference, we use the parameters previously adopted by Yang et al. [9].

(2) At the quark level (l.h.s.), the isospin symmetry breakings caused by $<\bar{d}d> \neq <\bar{u}u>$ and $m_d \neq m_u$ have been made explicit in the formulae given above. In addition, there is an effect due to the non-universal pion-quark couplings. This gives rise to an important isospin symmetry breaking effect which has been taken into account in [8]. We finally obtain [8]

$$\frac{\frac{g_{\pi^0 nn}}{g_{\pi^0 uu}} - \frac{g_{\pi^0 pp}}{g_{\pi^0 dd}}}{\frac{g_{\pi^0 NN}}{g_{\pi^0 QQ}} |_{average}} = 0.58\% \tag{12}$$

$$\frac{\frac{g_{\pi^+ np}}{g_{\pi^+ du}} - \frac{g_{\pi^0 pp}}{g_{\pi^0 dd}}}{\frac{g_{\pi^0 NN}}{g_{\pi^0 QQ}} |_{average}} = 0.35\% \tag{13}$$

Combining with the difference in pion-quark couplings (from vertex renormalizations), we conclude that $g_{\pi^0 nn}$ is numerically bigger than $g_{\pi^0 pp}$ by 0.80%, while $g_{\pi^+ np}$ is numerically greater than $g_{\pi^0 pp}$ by 0.15%.

We proceed to note that the ${}^1S_0$ phase shift δ_0, as used to describe low-energy nucleon-nucleon scatterings, can be parametrized in the standard manner:

$$k\cot\delta_0 = -\frac{1}{a} + \frac{1}{2}k^2 r_0 + \cdots, \tag{14}$$

with a the scattering length and r_0 the effective range. Experimentally, we have [10]

$$\begin{aligned} \delta a_{\rm CSB} &\equiv \mid a_{pp} \mid - \mid a_{nn} \mid = -(1.5 \pm 0.5) fm; \\ \delta a_{\rm CD} &\equiv \mid a_{np} \mid - \mid a_{nn} \mid = (5.0 \pm 0.3) fm. \end{aligned} \tag{15}$$

As already noted in [11], the differences in OPEP's, such as $V_{pp}(r) - V_{nn}(r)$, may be treated as a perturbation, making it easy to compute $\delta a_{\rm CSB}$ or other low-energy isospin symmetry breaking observables. In this way, our prediction on the difference between $g_{\pi^0 nn}$ and $g_{\pi^0 pp}$ gives rise to about $-1.0\, fm$ on $\delta a_{\rm CSB}$, consistent with the obervation both in sign and roughly in magnitude.

As already known [1,11] for some time, the bulk of the charge-dependent effect as revealed by a (relatively) large value of $\delta a_{\rm CD}$ can be understood quantitatively as caused by the pion mass difference $m_{\pi^+} \neq m_{\pi^0}$ while the differences as found in pion-nucleon couplings give rise to relatively small contribution to $\delta a_{\rm CD}$. As a result, we may safely conclude that the observed $\delta a_{\rm CSB}$ and $\delta a_{\rm CD}$ as summarized recently in [10] can be understood, to a large extent, as manifestations of isospin asymmetries associated with the one-pion exchange potential.

3. Discussion and Summary

It may useful to mention that charge-symmetry breakings observed in medium-energy elastic $n - p$ scattering (at TRIUMF and IUCF) may be dictated primarily [12] by the charge symmetry breaking force as caused by the $\rho-\omega$ mixing or by $g_{\omega pp} \neq g_{\omega nn}$. However, the short-range nature in this case may require a careful investigation of the quark-interchange mechanisms as addressed in [13], as it is difficult to reconcile the picture of exchanging, between two nucleons, a meson (ρ or ω) with a range of about 0.25 fermi, considerably smaller than the extent of a nucleon (with a radius of at least 0.5 fermi, even assuming a little bag picture for the nucleon).

Nevertheless, the isospin symmetry breakings associated with the one-pion exchange potential (which is clearly on a very sound conceptual ground) can best be accessed through careful measurements of low-energy scattering parameters such as $\delta a_{\rm CSB}$, $\delta a_{\rm CD}$, etc. Such efforts should be further encouraged by the currently popular wisdom that the low-energy sector of the πN are NN systems can be described by the long-wave length limit of QCD via Goldstone theorem, yielding the so-called chiral perturbation theory (χPT), so that the low-energy scattering parameters seems to be more "fundamental" than those needed in, e.g., the description of medium-energy nucleon-nucleon scatterings. In this connection, it is of importance to emphasize that QCD sum rules help to establish χPT/ISB out of QCD by eliminating an excessive number of ISB condensate parameters, making χPT/ISB a worthy extension of χPT.

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Green functions in large N_c QCD

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In the presence of the nontrivial QCD ground state or vacuum, nonlocal condensates are used to characterize the quark or gluon propagator, or other Green functions of higher order. We wish to show in this paper that, provided that we may use the large N_c approximation to treat condensates of much higher order, there is a natural way of setting up closed sets of differential equations which govern the inter-related Green functions to a given order. As a specific example, the leading-order equations for the nonlocal condensates appearing in the quark propagator are derived and explicit solutions are obtained. Some applications of our analytical results are briefly mentioned.

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The ground state, or the vacuum, of QCD is known to be nontrivial, in the sense that there are non-zero condensates, including gluon condensates, quark condensates, and perhaps infinitely many higher-order condensates. In such a theory, propagators, i.e. causal Green's functions, such as the quark propagator

$$iS_{ij}^{ab}(x) \equiv < 0 \mid T(q_i^a(x)\bar{q}_j^b(0)) \mid 0 >, \tag{1}$$

carry all the difficulties inherent in the theory. Higher-order condensates, such as four-quark condensates, represent an infinite series of unknown parameters unless some useful ways for reduction can be obtained. As the vacuum, $\mid 0 >$, is highly nontrivial, there is little reason to expect that Wick's theorem (of factorization), as obtained for free quantum field theories, is still valid in general. Thus, we must look for alternative methods in order to obtain useful results.

It is known that the equations for Green functions up to a certain order usually involve Green functions of even higher order, thereby making such hierarchy of equations often useless in practice. In this paper, however, we wish to show that, provided that we may use the large N_c approximation to treat condensates of much higher order, there is in fact a natural way of setting up closed sets of differential equations which govern the inter-related Green functions to a given order. We consider this as an important accomplishment, both because we can always go over to the next level of sophistication in order to improve the approximation and because the large N_c expansion has been shown to yield desirable results for describing hadron physics.

In light of the nontrivial QCD vacuum, we begin by considering the feasibility of working directly with the various matrix elements such as the quark propagator of Eq. (1). Useful

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relations may be derived if we regard the equations for interacting fields [1],

$$\{i\gamma^\mu(\partial_\mu + ig\frac{\lambda^a}{2}A^a_\mu) - m\}\psi = 0; \tag{2}$$

$$\partial^\nu G^a_{\mu\nu} - 2gf^{abc}G^b_{\mu\nu}A^c_\nu + g\bar\psi\frac{\lambda^a}{2}\gamma_\mu\psi = 0, \tag{3}$$

as the equations of motion for *quantized* interacting fields, subject to the standard quantization rule that the equal-time commutators or anticommutators among these quantized interating fields are identical to those among non-interacting ones. As our basic example, we allow the operator $\{i\gamma^\mu\partial_\mu - m\}$ to act on the matrix element defined by Eq. (1) and obtain

$$\{i\gamma^\mu\partial_\mu - m\}_{ik} iS^{ab}_{kj}(x) = i\delta^4(x)\delta^{ab}\delta_{ij} + <0 \mid T(\{g\frac{\lambda^n}{2}A^n_\mu\gamma^\mu q(x)\}^a_i \bar q^b_j(0)) \mid 0> . \tag{4}$$

We should always keep in mind that the QCD vacuum $\mid 0>$ is a nontrivial ground state which is in general not annihilated by operating on it an annihilation operator.

Eq. (4) can be solved by splitting the propagator into a singular, perturbative part and a nonperturbative part:

$$iS^{ab}_{ij}(x) = iS^{(0)ab}_{ij}(x) + i\tilde S^{ab}_{ij}(x), \tag{5}$$

where

$$iS^{(0)ab}_{ij}(x) \equiv \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} iS^{(0)ab}_{ij}(p), \tag{6}$$

$$iS^{(0)ab}_{ij}(p) = \delta^{ab}\frac{i(\hat p + m)_{ij}}{p^2 - m^2 + i\epsilon}, \tag{7}$$

with $\hat a \equiv \gamma^\mu a_\mu$ for a four-vector a_μ. The nonperturbative part then satisfies the equation:

$$\{i\gamma^\mu\partial_\mu - m\}_{ik} i\tilde S^{ab}_{kj}(x) = <0 \mid T(\{g\frac{\lambda^n}{2}A^n_\mu\gamma^\mu q(x)\}^a_i \bar q^b_j(0)) \mid 0> . \tag{8}$$

This would be pretty much the end of the story unless we could find some way to proceed. We may note that Eq. (4) may also be derived by making use of, e.g., the path-integral formulation, and the issue of how to define renormalized composite operators, i.e. products of field operators, is by no means trivial (and fortunately we need not worry about such problem for the sake of this paper).

The approach which we suggest here [2] is based upon two key elements, namely, the set of interacting field equations *plus* the rule of canonical quantization (for interacting fields). The equations which we obtain, such as Eq. (8), are much the same as the set of Schwinger-Dyson equations (for the matrix elements). An important aspect in our derivation is that nontriviality of the vacuum $\mid 0>$ is observed at every step – a central issue in relation to QCD.

In what follows, we wish to focus on the quark propagator, as specified by Eqs. (1) and (4)-(7). We write

$$i\tilde S^{ab}_{ij}(x) = \delta^{ab}\{\delta_{ij}f(x^2) + i\hat x_{ij}g(x^2)\}, \tag{9}$$

with $x^2 \equiv x_0^2 - \vec x^{\,2}$. $f(x^2)$ and $g(x^2)$ are what we refer to as "nonlocal condensates" in connection with the quark propagator. For example, we have

$$<: \bar q(x)\, q(0) :> = -12 f(x^2),$$

which is the nonlocal quark condensate in the standard sense. To proceed further, we choose to work only with the leading term in the fixed-point gauge and introduce

$$<:\{gG_{\mu\nu}q(x)\}_i^a\bar{q}_j^b(0):>=\delta^{ab}\{(\gamma_\mu x_\nu-\gamma_\nu x_\mu)A(x^2)+i\sigma_{\mu\nu}B(x^2) \\ +(\gamma_\mu x_\nu-\gamma_\nu x_\mu)\hat{x}C(x^2)+i\sigma_{\mu\nu}\hat{x}D(x^2)\}, \tag{10}$$

with $G_{\mu\nu}\equiv(\lambda^n/2)G^n_{\mu\nu}$, an antisymmetric operator. The invariant functions $A(x^2)$, $B(x^2)$, $C(x^2)$, and $D(x^2)$ are additional nonlocal condensates which we have to deal with explicitly in this paper.

Under the assumption that we keep only the leading term in the fixed-point gauge (a simplifying assumption which can be removed if necessary), we have

$$\begin{aligned}\{i\gamma^\alpha\partial_\alpha-m\}_{ik}&<:\{gG_{\mu\nu}q(x)\}_k^a\bar{q}_j^b(0):>=-\tfrac{1}{2}x^\beta<:\{g^2G_{\mu\nu}G_{\alpha\beta}\gamma^\alpha q(x)\}_i^a\bar{q}_j^b(0):> \\ =&\quad -\tfrac{1}{2}\cdot\tfrac{1}{96}\cdot\tfrac{4}{3}\cdot<g^2G^2><:\{(\gamma_\mu x_\nu-\gamma_\nu x_\mu)q(x)\}_i^a\bar{q}_j^b(0):> \\ =&\quad -\tfrac{1}{144}<g^2G^2>\delta^{ab}(\gamma_\mu x_\nu-\gamma_\nu x_\mu)\{f(x^2)+i\hat{x}g(x^2)\}.\end{aligned} \tag{11}$$

Here the first equality follows from the field equation and the second one is based on the large N_c approximation that the contribution in which $G_{\mu\nu}$ and $G_{\alpha\beta}$ do not couple to color-singlet is suppressed by a factor of $1/N_c^2$. Thus, the factorization in the present case is justified to order $1/N_c^2$.

Now, we may use Eqs. (8) and (11) and obtain a closed set of equations:

$$2f'(x^2)-mg(x^2)=-\tfrac{3}{2}i(B-x^2C), \tag{12}$$
$$2x^2g'(x^2)+4g(x^2)+mf(x^2)=\tfrac{3}{2}x^2(A-D), \tag{13}$$
$$4iB'-2iC-2ix^2C'-mA=-\tfrac{1}{144}<g^2G^2>f(x^2), \tag{14}$$
$$2iA+2ix^2D'-mB=0, \tag{15}$$
$$-2iA'+4iD'-mC=-\tfrac{i}{144}<g^2G^2>g(x^2), \tag{16}$$
$$2iB'+2iC-mD=0, \tag{17}$$

where the derivatives are with respect to the variable x^2.

To leading order in m, we obtain, by eliminating A, B, C, and D,

$$x^2f_0'''+3f_0''-\xi_0^2x^2f_0'-2\xi_0^2f_0=0, \tag{18}$$
$$(x^2)^3g_0'''+5(x^2)^2g_0''+\{2x^2-\xi_0^2(x^2)^3\}g_0'-\{2+2\xi_0^2(x^2)^2\}g_0=0, \tag{19}$$

with $\xi_0^2\equiv<g^2G^2>/384$. Eq. (19) can be simplified considerably by introducing

$$g_0(x^2)\equiv(x^2)^{-2}\tilde{g}_0(x^2),$$

which leads to the equation:

$$x^2\tilde{g}_0'''-\tilde{g}_0''-\xi_0^2x^2\tilde{g}_0'=0. \tag{20}$$

Eqs. (18) and (20) can be solved by iteration, leading to the result:

$$f_0(t)=a_0\{1+\tfrac{1}{1\cdot3}(\xi_0t)^2+\tfrac{1}{1\cdot3^2\cdot5}(\xi_0t)^4+\cdots\}+a_1t\{1+\tfrac{1}{2\cdot4}(\xi_0t)^2+\tfrac{1}{2\cdot4^2\cdot6}(\xi_0t)^4+\cdots\}, \tag{21}$$
$$\tilde{g}_0'(t)=c_2t^2\{1+\tfrac{1}{2\cdot4}(\xi_0t)^2+\tfrac{1}{2\cdot4^2\cdot6}(\xi_0t)^4+\cdots\}, \tag{22}$$

with $t \equiv x^2$ and

$$a_0 = -\frac{1}{12} < \bar{q}q >, \quad a_1 = \frac{1}{192} < \bar{q}g_c\sigma \cdot Gq >, \quad c_2 = -\frac{g_c^2 < \bar{q}q >^2}{2^5 \cdot 3^4}. \tag{23}$$

Eq. (23) is obtained by comparing to the well-known series expansion for the quark propagator (see, e.g., [3]). Note that there are two integration constants, a_0 and a_1, for $f(x^2)$ but there is only one permissible constant for $g(x^2)$ (and c_2 is in fact a four-quark condensate taken in the large N_c limit).

For a number of applications, it is useful to obtain analytic expressions for $f_0(t)$ and $g_0(t)$. This turns out to be possible by way of Laplace transforms.

$$\bar{f}_0(s) \equiv \int_0^\infty ds e^{-st} f_0(t), \quad \bar{\tilde{g}}_0'(s) \equiv \int_0^\infty ds e^{-st} \tilde{g}_0'(t). \tag{24}$$

We obtain

$$\bar{f}_0(s) = -\frac{2a_1}{\xi_0^2} - \frac{a_0}{\xi_0} \frac{s}{\sqrt{s^2-\xi_0^2}} sec^{-1} \frac{s}{\xi_0} + \frac{\gamma_0 s}{\sqrt{s^2-\xi_0^2}}, \quad \bar{\tilde{g}}_0'(s) = \frac{2c_2}{(s^2-\xi_0^2)^{3/2}}, \tag{25}$$

with $\gamma_0 = 2a_1/\xi_0^2$. Looking up the table for Laplace transforms, we find

$$\tilde{g}_0' = \frac{2c_2}{\xi_0^2} \cdot \xi_0 t \cdot I_1(\xi_0 t), \tag{26}$$

with $I_1(z)$ the modified Bessel function of the first kind, to order one. It is straightforward to show that Eq. (26) yields the series expansion in Eq. (22). Also, the function $I_1(\xi_0 t)$ enters the second series in $f_0(t)$ as in Eq. (21).

As the first application, we note that that our analytical expressions for nonlocal condensates help to determine the induced condensates previously treated as new parameters in the the external-field QCD sum rule method, thereby making it more powerful than what it used to be. We also wish to mention that, making use of soft-pion theorems, the quark parton distributions for the Goldstone pions may also be evaluated in the massless limit.

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Large N_c QCD Sum Rules

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Abstract. I wish to stress that, in large N_c QCD (with N_c the number of color), a closed set of coupled differential equations may be derived for nonlocal condensates which, in the presence of the nontrivial QCD ground state or vacuum, are used to characterize the quark or gluon propagator, or other Green functions of higher order. It is pointed out that, by solving the coupled equations so obtained, the method of QCD sum rules, in its generalized sense, is free of additional parameters whenever we modify or generalize the method such as by considering the QCD vacuum in the presence of external fields. In addition, I wish to demonstrate a soft-pion theorem for the parton distributions of Goldstone pions.

INTRODUCTION

As of today, quantum chromodynamics (QCD) has been taken universally as the underlying theory of strong interaction physics. Although the asymptotically free nature of QCD allows us to test the candidate theory at high energies, the nonperturbative feature dominates for hadrons or nuclei at low energies, So far, it remains almost impossible to solve problems related to hadrons or nuclei.

The ground state, or the vacuum, of QCD is known to be nontrivial, in the sense that there are non-zero condensates, including gluon condensates, quark condensates, and perhaps infinitely many higher-order condensates. In such a theory, propagators, i.e. causal Green's functions, such as the quark propagator

$$iS_{ij}^{ab}(x) \equiv < 0 \mid T(q_i^a(x)\bar{q}_j^b(0)) \mid 0 >, \tag{1}$$

carry all the difficulties inherent in the theory. Specifically, we may write the quark propagator as a sum of a perturbative part and a nonperturbative part:

$$iS_{ij}^{ab}(x) = iS_{ij}^{(0)ab}(x) + i\tilde{S}_{ij}^{ab}(x), \tag{2}$$

where the perturbative part assumes the form of a free propagator:

$$iS_{ij}^{(0)ab}(x) \equiv \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} iS_{ij}^{(0)ab}(p), \tag{3a}$$

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$$iS_{ij}^{(0)ab}(p) = \delta^{ab}\frac{i(\hat{p}+m)_{ij}}{p^2 - m^2 + i\epsilon}, \tag{3b}$$

with $\hat{a} \equiv \gamma^\mu a_\mu$ for a four-vector a_μ, and the nonperturbative part may be parametrized as follows:

$$i\tilde{S}_{ij}^{ab}(x) \equiv \delta^{ab}\{\delta_{ij}f(x^2) + i\hat{x}_{ij}g(x^2)\}, \tag{4}$$

with $x^2 \equiv x_0^2 - \vec{x}^{\,2}$. The functions $f(x^2)$ and $g(x^2)$ are referred to as "nonlocal condensates". The solution to these functions would characterize the quark propagation completely and would thus represent the solution to the problem of nonperturbative QCD.

LARGE N_C QCD

In the well-known many-body theory developed for atomic or nuclear physics, the equations for Green functions of a given order usually involve Green functions of higher order, giving rise to an open-ended hierarchy of equations (which is often useless in practice). In what follows, however, we wish to show that, in large N_c QCD, there is in fact a natural way of setting up closed sets of differential equations which govern the inter-related Green functions to a given order.

In light of the nontrivial QCD vacuum, we begin by considering the feasibility of working directly with the various matrix elements such as the quark propagator of Eq. (1). Useful relations may be derived if we regard the equations for interacting fields [1],

$$\{i\gamma^\mu(\partial_\mu + ig\frac{\lambda^a}{2}A_\mu^a) - m\}\psi = 0; \tag{5}$$

$$\partial^\nu G_{\mu\nu}^a - 2gf^{abc}G_{\mu\nu}^b A_\nu^c + g\bar{\psi}\frac{\lambda^a}{2}\gamma_\mu\psi = 0, \tag{6}$$

as the equations of motion for *quantized* interacting fields, subject to the standard rule for quantization that the equal-time (anti-)commutators among these quantized interating fields are identical to those among non-interacting quantized fields.

Allowing the operator $\{i\gamma^\mu\partial_\mu - m\}$ to act on the matrix element defined by Eq. (1), we obtain

$$\{i\gamma^\mu\partial_\mu - m\}_{ik} iS_{kj}^{ab}(x) = i\delta^4(x)\delta^{ab}\delta_{ij} + <0 \mid T(\{g\frac{\lambda^n}{2}A_\mu^n\gamma^\mu q(x)\}_i^a \bar{q}_j^b(0)) \mid 0> . \tag{7}$$

Making use of Eqs. (2)-(4), we find

$$\{i\gamma^\mu\partial_\mu - m\}_{ik} i\tilde{S}_{kj}^{ab}(x) = <0 \mid T(\{g\frac{\lambda^n}{2}A_\mu^n\gamma^\mu q(x)\}_i^a \bar{q}_j^b(0)) \mid 0>, \tag{8}$$

an equation which may be used to investigate the nonlocal condensates $f(x^2)$ and $g(x^2)$.

As a useful benchmark, we shall work with the fixed-point gauge,

$$A_\mu^n(x) = -\frac{1}{2} G_{\mu\nu}^n x^\nu + \cdots. \tag{9}$$

As a result, the nonperturbative part $i\tilde{S}_{ij}^{ab}(x)$ may be solved immediately as a power series in x^μ,

$$i\tilde{S}_{ij}^{ab}(x) = -\frac{1}{12}\delta^{ab}\delta_{ij} < \bar{q}q > + \frac{i}{48} m\hat{x}_{ij}\delta^{ab} < \bar{q}q >$$
$$+ \frac{1}{192} < \bar{q}g\sigma \cdot Gq > \delta^{ab} x^2 \delta_{ij} + \cdots, \tag{10}$$

The first term is the integration constant which defines the so-called "quark condensate", while the mixed quark-gluon condensate appearing in the third term arises because of Eqs. (8) and (9). It is obvious that the series (10) is a short-distance expansion, which converges for sufficiently small x_μ. We note that Eq. (10) is just the standard quark propagator cited in most papers in QCD sum rules [2]. However, the aim of this paper is to show that we may do a much better job regarding how to obtain the solution to $f(x^2)$ and $g(x^2)$.

The approach which we suggest here [3] is based upon two key elements, namely, the set of interacting field equations *plus* the rule of canonical quantization (for interacting fields). The equations which we obtain, such as Eq. (7) or (8), are much the same as the set of Schwinger-Dyson equations (for the matrix elements). An important aspect in our derivation is that the nontriviality of the vacuum $\mid 0 >$ is observed at every step – a central issue in relation to QCD.

To proceed further, we shall work only with the leading term in the fixed-point gauge and introduce

$$<: \{gG_{\mu\nu}q(x)\}_i^a \bar{q}_j^b(0) :>$$
$$= \delta^{ab}\{(\gamma_\mu x_\nu - \gamma_\nu x_\mu)A(x^2) + i\sigma_{\mu\nu}B(x^2) + (\gamma_\mu x_\nu - \gamma_\nu x_\mu)\hat{x}C(x^2) + i\sigma_{\mu\nu}\hat{x}D(x^2)\}, \tag{11}$$

with $G_{\mu\nu} \equiv (\lambda^n/2)G_{\mu\nu}^n$, an antisymmetric operator. The invariant functions $A(x^2)$, $B(x^2)$, $C(x^2)$, and $D(x^2)$ are additional nonlocal condensates which we must deal with explicitly in this paper.

Under the assumption that we keep only the leading term in the fixed-point gauge (a simplifying assumption which can be removed if necessary), we have

$$\{i\gamma^\alpha\partial_\alpha - m\}_{ik} <: \{gG_{\mu\nu}q(x)\}_k^a \bar{q}_j^b(0) :>$$

$$= -\frac{1}{2}x^\beta <: \{g^2 G_{\mu\nu}G_{\alpha\beta}\gamma^\alpha q(x)\}_i^a \bar{q}_j^b(0) :> \tag{12a}$$

$$= -\frac{1}{2}\cdot\frac{1}{96}\cdot\frac{4}{3}\cdot < g^2G^2 ><: \{(\gamma_\mu x_\nu - \gamma_\nu x_\mu)q(x)\}_i^a \bar{q}_j^b(0) :> \tag{12b}$$

$$= -\frac{1}{144} < g^2G^2 > \delta^{ab}(\gamma_\mu x_\nu - \gamma_\nu x_\mu)\{f(x^2) + i\hat{x}g(x^2)\}. \tag{12c}$$

Here the second line, (12a), follows from the field equation and the third line, (12b), is based on the factorization property of large N_c QCD that the contribution in which $G_{\mu\nu}$ and $G_{\alpha\beta}$ do not couple to color-singlet is suppressed by at least a factor of $1/N_c^2$.

Now, we may use Eqs. (8) and (12c) and obtain a closed set of equations, six ordinary differential equations for six functions:

$$2f'(x^2) - mg(x^2) = -\frac{3}{2}i(B - x^2C), \tag{13a}$$

$$2x^2g'(x^2) + 4g(x^2) + mf(x^2) = \frac{3}{2}x^2(A - D), \tag{13b}$$

$$4iB' - 2iC - 2ix^2C' - mA = -\frac{1}{144} < g^2G^2 > f(x^2), \tag{13c}$$

$$2iA + 2ix^2D' - mB = 0, \tag{13d}$$

$$-2iA' + 4iD' - mC = -\frac{i}{144} < g^2G^2 > g(x^2), \tag{13e}$$

$$2iB' + 2iC - mD = 0, \tag{13f}$$

where the derivatives are with respect to the variable x^2.

Treating m as an expansion parameter,

$$F(x^2) = \sum_{k=0}^{\infty} m^k F_k(x^2), \tag{14}$$

we may solve the coupled equations, (13a)-(13f), order by order in m. To leading order in m, we obtain

$$x^2f_0''' + 3f_0'' - \xi_0^2x^2f_0' - 2\xi_0^2f_0 = 0, \tag{15}$$

$$(x^2)^3g_0''' + 5(x^2)^2g_0'' + \{2x^2 - \xi_0^2(x^2)^3\}g_0' - \{2 + 2\xi_0^2(x^2)^2\}g_0 = 0, \tag{16}$$

with $\xi_0^2 \equiv < g^2G^2 > /384$. The equations for A_0, B_0, C_0, and D_0 can easily be solved once we obtain f_0 and g_0.

Eq. (16) can be simplified considerably by introducing

$$g_0(x^2) \equiv (x^2)^{-2}\tilde{g}_0(x^2), \tag{17a}$$

which leads to the equation:

$$x^2\tilde{g}_0''' - \tilde{g}_0'' - \xi_0^2x^2\tilde{g}_0' = 0. \tag{17b}$$

Of course, Eqs. (15) and (17b) can be solved by iteration, again reproducing Eq. (10). However, it is often of critical importance to have analytic expressions for $f_0(x^2)$ and $g_0(x^2)$. This turns out to be possible by way of Laplace transforms. To

... perturbative contribution, its finite part is small compared to the second and third terms. The last term, which can be treated numerically, involves products of two condensates and it has a dimension higher than the second or third term by at least three (and is likely of less numerical significance).

SOFT-PION THEOREM FOR THE PARTON DISTRIBUTIONS OF GOLDSTONE PIONS

Consider the amplitude given by

$$T_{\mu\nu}(q^2, p\cdot q) = i\int d^4x e^{-iq\cdot x} < \pi^+(p) \mid T(J_\mu(x)J_\nu(0)) \mid \pi^+(p) > \tag{22a}$$

indicate such possibility, we consider Eq. (15) in the timelike region specified by $t \equiv x^2 > 0$:

$$\bar{f}_0(s) \equiv \int_0^\infty ds e^{-st} f_0(t). \tag{18}$$

We obtain

$$\bar{f}_0(s) = -\frac{2f_0'(0)}{\xi_0^2} - \frac{f_0(0)}{\xi_0}\frac{s}{\sqrt{s^2-\xi_0^2}} sec^{-1}\frac{s}{\xi_0} + \frac{\gamma_0 s}{\sqrt{s^2-\xi_0^2}}. \tag{19}$$

The spacelike region with $t \equiv x^2 < 0$ may be treated similarly. Analogously, Eq. (17b) may also be solved via Laplace transforms.

QCD SUM RULES

Thus far, we have described how to obtain a closed set of coupled equations for the nonlocal condensates which are relevant in the description of the quark propagator. We have also shown how these equations can be solved explicitly. Furthermore, some of the assumptions underlying our equations can be relaxed and more elaborate equations may then be obtained. Of course, some of our results are gauge dependent as the quark propagator (1) has been analyzed in a specific gauge (9). Nevertheless, our primary motivation for studying the quark propagator stems from our interest in the method of QCD sum rules [1], which may be regarded as the various approaches in which one tries to understand the roles played by the quark and gluon condensates for problems involving hadrons.

There are many variations in applying the method of QCD sum rules to specific problems in hadron physics, but the primary objective is quite clear and is to unravel the role of the nontrivial QCD vacuum in the problem. As the first approach, we may consider the Belyaev-Ioffe nucleon mass sum rules [5,4], where the short-distance expansion for the quark propagator is needed up to a certain (high) dimension. In this context, our analytical results on nonlocal condensates may be used to justify if the resultant series converges rapidly. The second approach is to consider the response of the QCD vacuum to some external fields, such as the method of QCD sum rules in the presence of an external axial field $Z_\mu(x)$ [6]. In this context, certain induced condensates are introduced (previously as new parameters) but the method offers a simple extension of the first approach in calculating magnetic moments, axial coupling constants, and other quantities by avoiding a need to treat explicitly the three-point Green's functions - a need which would involve a good deal of uncertainties. What is of great interest is that our analytical expressions for nonlocal condensates help to determine the induced condensates previously treated as new parameters, thereby making the external-field QCD sum rule method more powerful than what it used to be. To illustrate the point, we consider the external axial field Z_μ with the interaction,

$$\delta\mathcal{L}(x) = gZ^\mu(x)\bar{q}(x)\gamma_\mu\gamma_5 q(x). \tag{20}$$

For a constant Z^μ field, there are two major induced condensates [6]:

$$< 0 \mid \bar{q}(0)\gamma_\mu\gamma_5 q(0) \mid 0 >_{Z^\alpha} \quad \text{and} \quad < 0 \mid \bar{q}(0) g_c \tilde{G}_{\mu\nu}\gamma^\nu q(0) \mid 0 >_{Z^\alpha} .$$

We now have

$$\begin{aligned} & < 0 \mid \bar{q}(0)\gamma_\mu\gamma_5 q(0) \mid 0 >_{Z^\alpha} \\ &= i\int d^4x g Z^\alpha(x) < 0 \mid T(\bar{q}(x)\gamma_\alpha\gamma_5 q(x)\bar{q}(0)\gamma_\mu\gamma_5 q(0)) \mid 0 > \\ &= i\int d^4x g Z^\alpha(x)\{Tr[iS^{(0)}(-x)\gamma_\alpha\gamma_5 iS^{(0)}(x)\gamma_\mu\gamma_5] \\ &\quad + Tr[i\tilde{S}(-x)\gamma_\alpha\gamma_5 iS^{(0)}(x)\gamma_\mu\gamma_5] \\ &\quad + Tr[iS^{(0)}(-x)\gamma_\alpha\gamma_5 i\tilde{S}(x)\gamma_\mu\gamma_5] \\ &\quad + Tr[i\tilde{S}(-x)\gamma_\alpha\gamma_5 i\tilde{S}(x)\gamma_\mu\gamma_5]\}. \end{aligned} \tag{21}$$

The first term is the one-loop result which can be regularized (e.g., in d dimensions) and, as expected for a perturbative contribution, its finite part is small compared to the second and third terms. The last term, which can be treated numerically, involves products of two condensates and it has a dimension higher than the second or third term by at least three (and is likely of less numerical significance).

SOFT-PION THEOREM FOR THE PARTON DISTRIBUTIONS OF GOLDSTONE PIONS

Consider the amplitude given by

$$T_{\mu\nu}(q^2, p\cdot q) = i\int d^4x e^{-iq\cdot x} < \pi^+(p) \mid T(J_\mu(x)J_\nu(0)) \mid \pi^+(p) > \tag{22a}$$

$$\begin{aligned} &\equiv (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})T_1(q^2, p\cdot q) \\ &+ \frac{1}{p^2}(p_\mu - \frac{p\cdot q}{q^2}q_\mu)(p_\nu - \frac{p\cdot q}{q^2}q_\nu)T_2(q^2, p\cdot q), \end{aligned} \tag{22b}$$

which characterizes the forward Compton scattering off π^+ (a Goldstone boson) and also the parton distributions of π^+. Applying the soft-pion theorem (together with current algebra), we find, in the limit of $p_\mu \to 0$,

$$\begin{aligned} & T_{\mu\nu}(q^2, 0) \\ &= \frac{i}{f_\pi^2}\int d^4x e^{-iq\cdot x} < 0 \mid T(\{A_\mu^1(x) - iA_\mu^2(x)\}\{A_\nu^1(0) + iA_\nu(0)\} - 2V_\mu^3(x)V_\nu^3(0)) \mid 0 > \\ &= \frac{i}{f_\pi^2}\int d^4x e^{-iq\cdot x} Tr\{iS_d^{ba}(-x)\gamma_\mu\gamma_5 iS_u^{ab}(x)\gamma_\nu\gamma_5 \end{aligned}$$

$$-\frac{1}{2}iS_u^{ba}(-x)\gamma_\mu iS_u^{ab}(x)\gamma_\nu - \frac{1}{2}iS_d^{ba}(-x)\gamma_\mu iS_d^{ab}(x)\gamma_\nu\}. \tag{23}$$

The structure functions $W_i(q^2, p.q)$ (in the description of deep inelastic scattering off the π^+ target) is the imaginary part of $T_i(q^2, p\cdot q)$ divided by the factor π. Eq. (23) suggest that our analytical expressions for the nonlocal condensates may be useful for analyzing properties of Goldstone pions. We find, as a soft-pion limit,

$$W_1(q^2, p\cdot q) = \frac{1}{\pi}\mathrm{Im}\,T_1(q^2, p\cdot q)$$

$$\longrightarrow \frac{m < \bar{q}q >}{2f_\pi^2\zeta_0}, \quad \text{as} \quad p_\mu \to 0 \text{ and } q_\mu \to 0. \tag{24}$$

This limit is derived making use of our analytical expressions on the nonlocal condensates, again with Wick's rotation on the time integration.

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