行政院國家科學委員會專題研究計畫　成果報告

離散時間型之盈餘過程—可信度方法

計畫類別：個別型計畫
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計畫主持人：蔡啟良

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行政院國家科學委員會補助專題研究計畫 **□ 成 果 報 告**
□期中進度報告

離散時間型之盈餘過程─可信度方法

計畫類別：□ 個別型計畫　　□ 整合型計畫
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中　華　民　國　93　年　08　月　24　日

中文摘要

本計畫主要是想探討傳統離散型盈餘過程的一些問題。在傳統的離散型盈餘過程中，保險公司每期所收的保費是固定不變的，我們想應用 Buhlmann 可信度理論去計算 Buhlmann 可信度保費做為每期應收的純保費。利用此動態保費法，透過蒙地卡羅模擬可以計算出破產機率、在時間 n 以前破產的機率等重要議題。並且比較和解釋這些數值和當使用固定保費法時所產生的數值之差異性，也可以探討諸如”動態保費法可以顯著地降低破產的機率嗎?”等問題

關鍵詞：盈餘過程；破產時間；破產機率；Buhlmann 可信度理論

Abstract

This project is to study some problems based on the classical discrete time surplus process, in which the premium received in each period is assumed to be a constant. We will apply the Buhlmann credibility theory to calculate the so called Buhlmann credibility premium as the renewal pure premium received in each period. With the dynamic premium scheme, we want to calculate the probability of ruin, the probability of ruin before and at time *n*, etc, by Monte-Carlo simulation. Then, we compare each of these quantities with corresponding one calculated based on constant premium acheme, interpret the difference, and investigate some problems like “can the dynamic premium scheme significantly reduce the probability of ruin?” etc.

Keywords : Surplus process; Time of ruin; Probability of ruin; Buhlmann credibility theory;

計畫緣由與目的

Consider the surplus process of the insurer at time n, n=0, 1,2, ... in the classical discrete time risk model,

(1) $$U_n = u + n \times c - S_n,$$

where u = U_0 is the initial surplus, c is the amount of premiums received each period and S_n is the sum of all the claims in the first n periods. We also assume that

(2) $$S_n = W_1 + W_2 + \cdots + W_n,$$

where W_i is the sum of the claims in period i and W_1, W_2, …KW_n are independent, identically distributed random variable. Moreover, c= (1+η) $E[W]$, where W is a random variable distributed as the W_i and η is the relative security loading. Let T=min{n:U_n<0} denote the time of ruin (T=∞ if $U_n \geqq 0$ for all n), and let $\Psi(u)$=Pr{T<∞Û}denote the probability of ruin in this context. Obviously, $\Psi(u)$ is decrease in u.

From equations (1) and (2), for n = 0, 1, 2, …, we have the following recursive formula:

(3) $$U_{n+1} = U_n + c - W_{n+1},$$

with the initial value U_0 = u. Note that the amount of premiums received each period is a constant c. If we let the amount of premiums received in period (n+1) be c_{n+1}, then equation (3) becomes

(4) $$U_{n+1} = U_n + c_{n+1} - W_{n+1}, \quad \text{n=0, 1, ...}$$

The determination of c_{n+1} is based on the Buhlmann’¦s model of credibility theory. The idea is that the renewal premiums charged in casualty insurance are usually based on past experience. We want to know by Monte-Carlo simulation whether this dynamic premium scheme can significantly reduce the probability of ruin when compared with constant premium scheme.

We want to estimate the expected value of the random variable W_{n+1} for period $n+1$, given the realizations W_1=w_1, W_2=w_2, …KW_n=w_n, for the n prior periods. The expected value is denoted by E[W_{n+1}| W_1=w_1, W_2=w_2, …KW_n=w_n]. We make the following assumptions:

I. For j=1, 2,…K n1, the distribution of each W_j depends on a parameter Θ.
II. Given Θ, the random variables W_1, W_2, …KW_{n+1} are conditionally independent and identically distributed.

Define $\mu(\Theta) = E[W_j|\Theta]$ and $v(\Theta) = Var[W_j|\Theta]$. Let $\mu = E[\mu(\Theta)]$ denote the overall hypothetical mean, $a = Var[\mu(\Theta)]$ denote the variance of the hypothetical means, and $v = E[v(\Theta)]$ denote the expected process variance. Then the overall hypothetical mean $\mu = E[\mu(\Theta)] = E[E[W_j|\Theta]] = E[W_j]$, the Buhlmann’¦s credibility factor is Z=n/(n+ v/a), and the point estimator of the conditional expectation of $\mu(\Theta)$ is (see Klugman, Panjer and Willmot (1998)).

(5) $$E[\mu(\Theta) \mid W_1 = w_1, W_x = w_2, \cdots, W_n = w_n] = Z \times \overline{W} + (1 - Z) \times \mu,$$

where $\overline{W} = \frac{1}{n}\sum_{i=1}^{n} W_i$ is the mean of the previous observations. That is, the point estimator of the conditional expectation of $\mu(\Theta)$ is a weighted average of the sample mean and the hypothetical mean.

Also,
$$\begin{aligned}
& E[W_{n+1} \mid W_1 = w_1, W_x = w_2, \cdots, W_n = w_n] \\
&= E_\Theta[E[W_{n+1} \mid W_1 = w_1, W_x = w_2, \cdots, W_n = w_n, \Theta]] \\
&= E_\Theta[E[W_{n+1} \mid \Theta] \mid W_1 = w_1, W_x = w_2, \cdots, W_n = w_n] \\
&= E[\mu(\Theta) \mid W_1 = w_1, W_x = w_2, \cdots, W_n = w_n] \\
&= Z \times \overline{W} + (1 - Z) \times \mu.
\end{aligned}$$

Thus, $c_{n+1} = (1+\eta)\ E[W_{n+1} \mid W_1 = w_1, W_x = w_2, \cdots, W_n = w_n]$.

Now, if we assume the claim size is Exponentially distributed, and the number of claim in a period follows the Poisson distribution with parameter λ (denoted by Poisson(λ)). We know that if X_1, X_2, …KX_m, are identical and independent distribution with mean β, then $\sum_{i=1}^{m} X_i$ is a Gamma distribution with parameter m and β (denoted by Gamma(m,β)). Therefore, we may assume that Λ follows Poisson(λ), and given ΛNthe random variables W_1, W_2, …KW_{n+1} are independent Gamma(ΛNβ). Then $\mu(\Lambda) = E[W_j \mid \Lambda] = \Lambda\beta$, $v(\Lambda) = Var[W_j \mid \Lambda] = \Lambda\beta^2$, $\mu = E[\mu(\Lambda)] = \lambda\beta$, $v = E[v(\Lambda)] = \lambda\beta^2$, and $a = Var[\mu(\Lambda)] = \lambda\beta^2$. Therefore, Z=n/(n+1) and

(6) $$E[W_{n+1} \mid W_1 = w_1, W_2 = w_2, \cdots, W_n = w_n] = \frac{n}{n+1} \times \overline{W} + \frac{1}{n+1} \times \lambda\beta.$$

Note that when n is large, $n/(n+1)$ is close to one, and equation (6) is close to the sample mean $\overline{W}$ and is stable over time. Some casualty insurers only consider the most recent k years of claim experiences to renew the premium for the next coming year. In this case, we may apply the same approach of Klugman, Panjer and Willmot (1998) to obtain

(7) $$E[W_{n+1} \mid W_h = w_h, \cdots, W_n = w_n] = \frac{m}{m+1} \times \overline{W}_m + \frac{1}{m+1} \times \lambda\beta$$

where $m=min(n, k)$, $h=max(n-k, 0)+1=n-m+1$ and $\overline{W}_m = \frac{1}{m}\sum_{i=h}^{n} W_i$; and

$c_{n+1} = (1+\eta)\ E[W_{n+1} \mid W_h = w_h, \cdots, W_n = w_n]$. Note that equations (6) and (7) are identical when $n \le k$.

To get the numerical results, we perform Monte-Carlo simulation for 1000 times with the assumptions λ=10, β=10, k=3, 10 and η=-0.05, 0, 0.1, 0.2 (see table 1 and figures 1-4). We also assume that the study period is 100, that is, if $U_n > 0$ for all n=1, 2, …100, then ruin does not occur for this simulation.

Table 1: ruin probabilities $\Psi(u)$ for η = -0.05, 0, 0.1 and 0.2

η =-0.05											
u	0	100	200	300	400	500	600	700	800	900	1000
classical	99.3%	95.4%	90.2%	82.7%	71.7%	61.2%	50.7%	40.6%	31.8%	24.3%	16.9%
cred	100.0%	100.0%	95.5%	86.2%	74.4%	62.4%	52.1%	41.2%	28.7%	19.2%	10.8%
cred10	100.0%	100.0%	100.0%	99.8%	95.6%	79.7%	43.5%	12.9%	2.5%	0.1%	0.0%
cred3	100.0%	100.0%	100.0%	98.9%	91.5%	70.1%	36.9%	14.2%	2.6%	0.5%	0.2%
η =0											
u	0	50	100	150	200	250	300	350	400	450	500
classical	93.2%	85.7%	77.2%	69.5%	60.7%	52.7%	46.5%	39.9%	33.9%	29.1%	24.2%
cred	100.0%	99.7%	90.5%	76.7%	64.7%	54.0%	45.6%	38.8%	31.7%	24.9%	19.5%
cred10	100.0%	99.5%	93.0%	72.7%	50.4%	28.7%	11.9%	3.7%	1.0%	0.2%	0.0%
cred3	99.8%	95.3%	79.9%	54.3%	32.9%	15.6%	7.3%	1.7%	0.5%	0.2%	0.0%
η =0.1											
u	0	20	40	60	80	100	120	140	160	180	200
classical	70.8%	61.5%	50.6%	43.8%	36.2%	30.3%	25.3%	20.9%	17.0%	14.6%	11.9%
cred	93.4%	82.7%	64.2%	47.5%	34.6%	25.8%	19.2%	14.2%	11.1%	8.4%	6.1%
cred10	89.1%	75.1%	54.4%	38.4%	25.7%	17.1%	11.5%	6.3%	4.1%	2.1%	0.8%
cred3	79.5%	62.8%	42.5%	27.1%	14.7%	8.5%	5.4%	2.6%	2.0%	1.0%	0.4%
η =0.2											
u	0	20	40	60	80	100	120	140	160	180	200
classical	51.7%	39.1%	28.1%	20.5%	15.7%	11.2%	8.6%	5.8%	3.9%	2.8%	2.1%
cred	61.2%	42.4%	25.9%	15.6%	9.2%	5.6%	2.8%	1.3%	0.8%	0.4%	0.1%
cred10	58.7%	40.7%	24.4%	14.6%	8.6%	5.1%	2.5%	1.0%	0.7%	0.4%	0.1%
cred3	53.1%	35.4%	21.2%	12.2%	7.4%	4.4%	2.0%	0.8%	0.6%	0.2%	0.0%

From table 1 and figures 1-4, we observe the following:

(1) the ruin probabilities based on credibility scheme with m=10 is larger than the one with m=3, that is $\Psi_{cred10}(u) \geq \Psi_{cred3}(u)$, except very few cases, in which $\Psi_{cred10}(u)$ is slightly smaller than $\Psi_{cred3}(u)$.

(2) credibility scheme can NOT reduce ruin probability unless when u is large. That is, these exists a value u_0 such that the ruin probability based on credibility scheme $\Psi_{cred}(u)<\Psi_{classical}(u)$ for all $u>u_0$ and $\Psi_{cred}(u)> \Psi_{classical}(u)$ for all $u<u_0$. This situation also applies to $\Psi_{cred10}(u)$ vs $\Psi_{classical}(u)$ and $\Psi_{cred3}(u)$ vs $\Psi_{classical}(u)$.

(3) $\Psi_{cred}(u) \geq \Psi_{cred10}(u) \geq \Psi_{cred3}(u)$ for security loading $\eta \geq 0$ and almost all of u. For $\eta<0$, observation (2) applies, that is, these exists a value u_0 such that $\Psi_{cred}(u)>\Psi_{cred10}(u)$ (or $\Psi_{cred}(u)>\Psi_{cred3}(u)$) for all $u>u_0$, and $\Psi_{cred}(u)<\Psi_{cred10}(u)$ (or $\Psi_{cred}(u)<\Psi_{cred3}(u)$) for all $u<u_0$.

Fig. 1: ruin probabilities $\Psi_Z(u)$ for η=-0.05

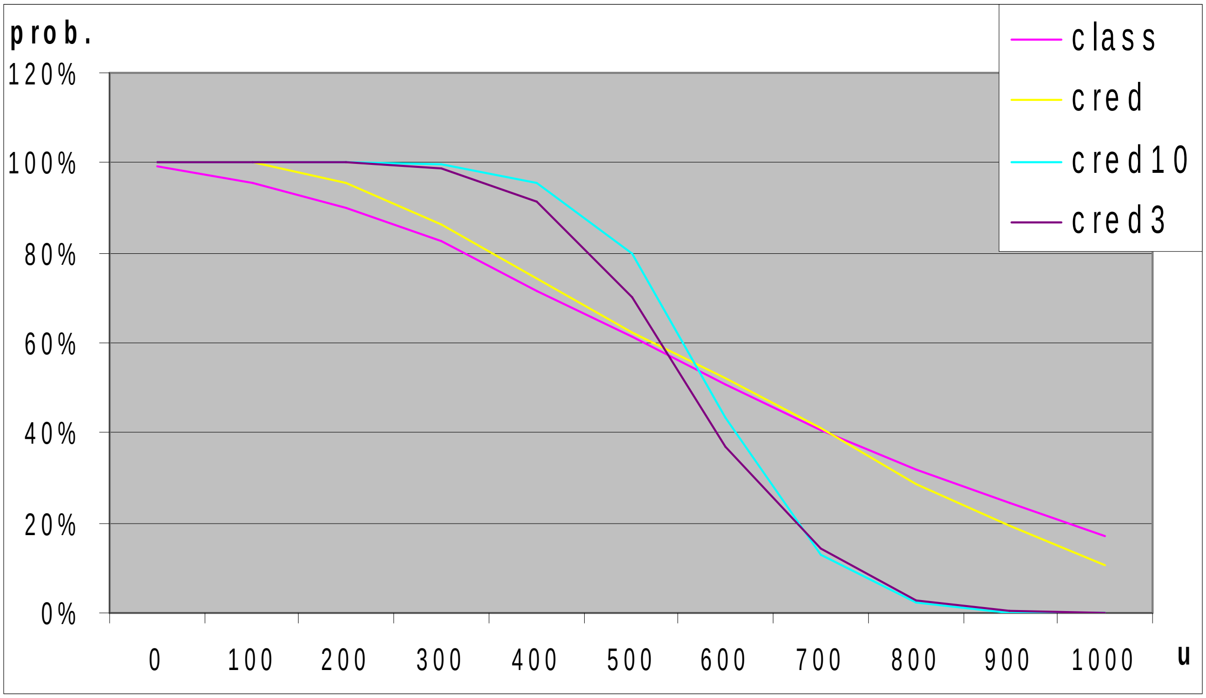


Fig. 2: ruin probabilities $\Psi_Z(u)$ for ηb 0

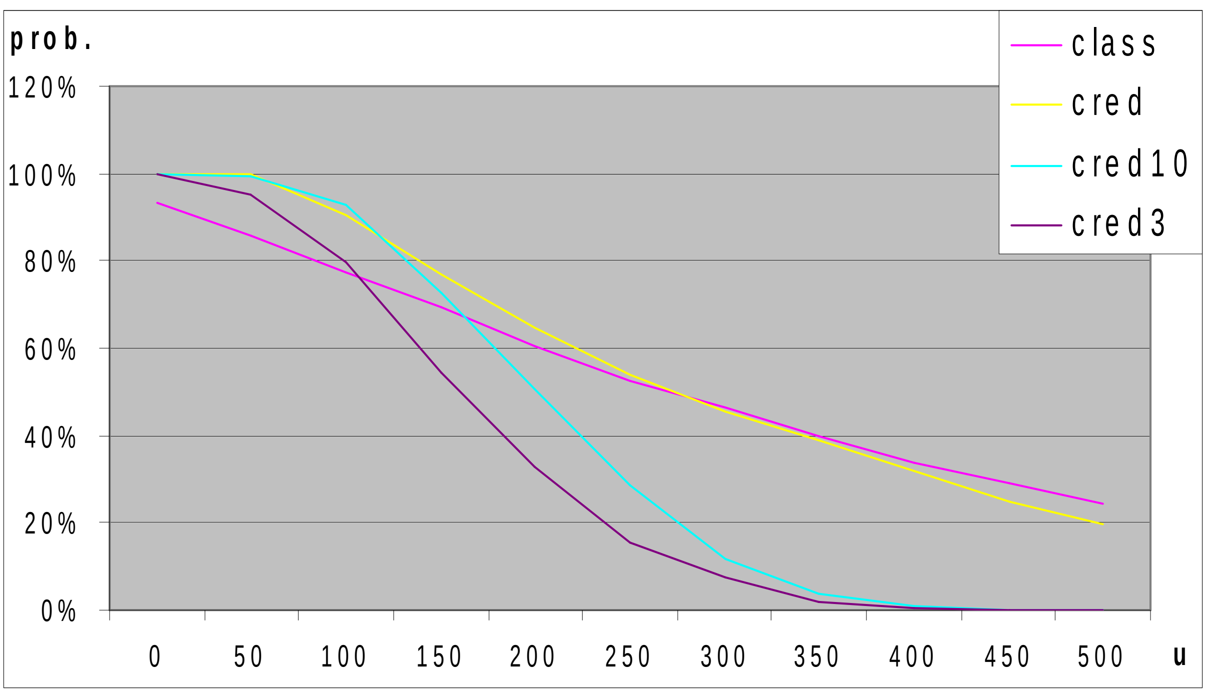


Fig. 3: ruin probabilities $\Psi_Z(u)$ for η_b 0.1

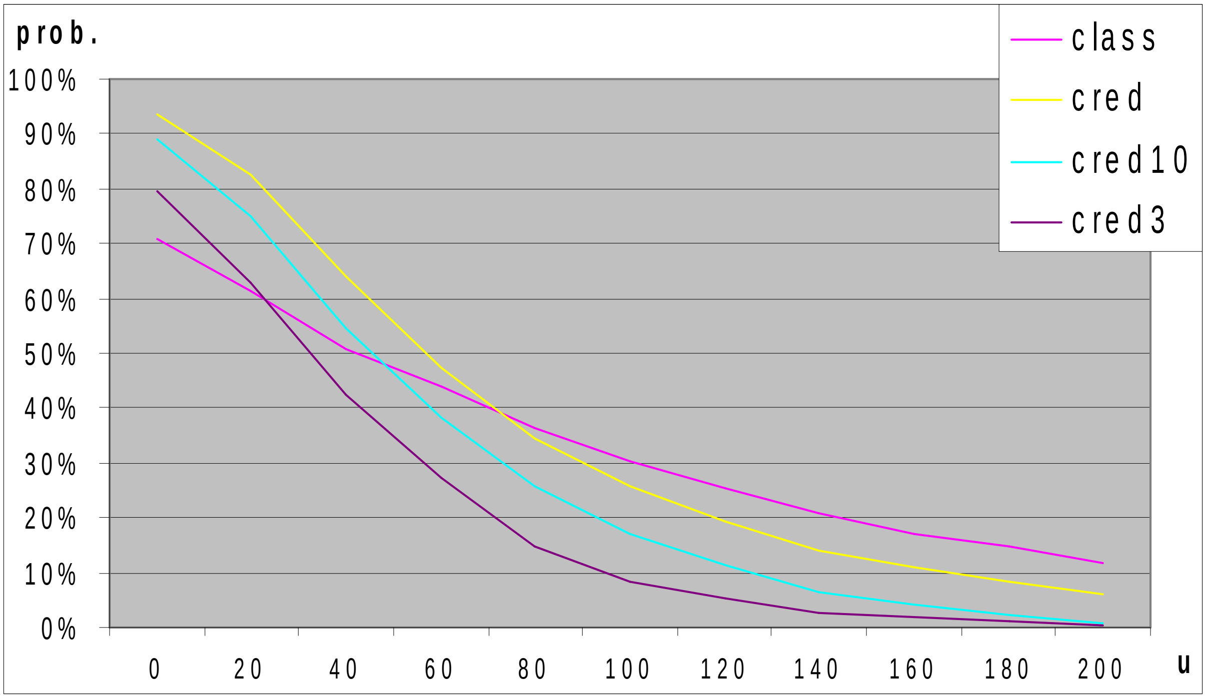
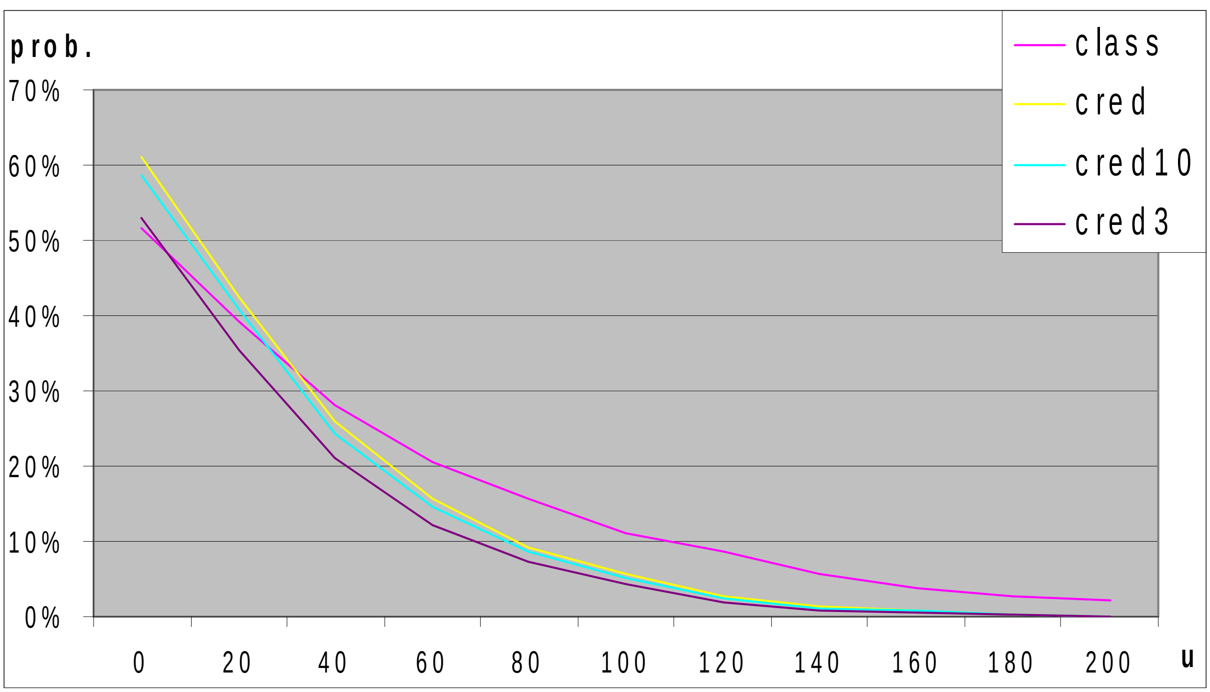


Fig. 4: ruin probabilities $\Psi_Z(u)$ for η_b 0.2



計畫成果自評

I have finished the following tasks as expected:

(a) apply Buhlmann credibility method to classical surplus process;

(b) perform Monte-Carlo simulation based on both dynamic premium scheme and constant premium scheme to calculate the probability of ruin. We conclude that credibility scheme (a dynamic premium scheme) can NOT reduce the probability of ruin, which disagree with out intuition.

參考文獻

Bowers, N., Gerber, H., Hickman, J., Jones, D., Nesbitt, C., 1997. Actuarial Mathematics, 2nd Edition, Society of Actuaries, Schaumburg.

Dickson, D. and Egidio dos Reis, A. 1996. On the distribution of the duration of negative surplus, Scandinavian Actuarial Journal, 148-164.

Dickson, D., Egidio dos Reis, A. and Waters, H. 1995. Some stable algorithms in ruin theory and their applications, ASTIN Bulletin, 25, 153-175.

Dickson, D. and Waters, H. 1991. Recursive calculation of survival probabilities, ASTIN Bulletin, 21, 199-221.

Dickson, D.C.M., Waters, H.R., 1992. The probability and severity of ruin in finite and in infinite time, ASTIN Bulletin 22, 177-190.

Dufresne, F., Gerber, H.U., 1988. The probability and severity of ruin for combinations of exponential claim amount distributions and their translations.
Insurance: Mathematics and Economics 7, 75-80.

Dufresne, F., Gerber, H.U., 1988. The surpluses immediately before and at ruin, and the amount of the claim causing ruin. Insurance: Mathematics and Economics 7, 193-199.

Feller, W., 1971. An Introduction to Probability Theory and Its Applications, Vol. 2, 2nd Edition. John Wiley, New York.

Herzog, T. N., 1996. Introduction to credibility theory. ACTEX publications, Winsted.

Klugman, S.A., Panjer, H.H., Willmot, G.E., 1998. Loss Models - From Data to Decisions, John Wiley, New York.

Water, H. R., 1993. Credibility Theory. Edinburgh, Heriot-Watt University.