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Angular Momentum Distributions in the Atomic Nuclei

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The problem of how many protons (neutrons) of a given orbital angular momentum $l\hbar$ are to be found in a nucleus of a given proton number Z (neutron number N) is reinvestigated by means of the improved Thomas-Fermi method.

Three kinds of nuclear models are calculated: The types of potentials chosen arc (i) square well potential, (ii)-the Green's potential, and (iii) the Green's potcntial plus 45 times the Thomas-Frenkel spin orbit term. The parameters involved in the potentials are adopted from the paper of Hwang and Yang, which may exactly reproduce the nuclear radii, trends of binding energies, and location of 3s and 4s maxima in the neutron scattering cross section. The eigenvalues calculated by Green are used to simplify the procedure of calculation. The results are then compared with the empirical data compiled by Klinkenbeig. For the last type of potential, the calculation is almost perfectly exact. The first appearance of particles of the next higher angular momentum is almost exact for each model. The puzzle in the Yang's treatment of the first appearance problem is then solved. The crigin of the appearance of magic numbers in the Yang's calculation is explained in the light of the present theory.

ANGULAR MOMENTUM DISTRIBUTIONS

THE number of neutrons (or protons) n_l with a given angular momentum $l\hbar$ in the atomic nuclei is determined by means of the improved Thomas-Fermi method[1]. Three kinds of nuclear models with

(i) the square well potential

$$V(r) = -V_0 \qquad \text{for } r \leq a,$$
$$V(r) = 0 \qquad \text{for } r > a,$$

(ii) the Green's potential

$$V(r) = -V_0 \qquad \text{for } r \leq a,$$
$$V(r) = -V_0 \exp[-(r-a)/d] \qquad \text{for } r > a,$$

and (iii) the Green's potential plus 4.5 times the Thomas-Frenkel spin orbit term

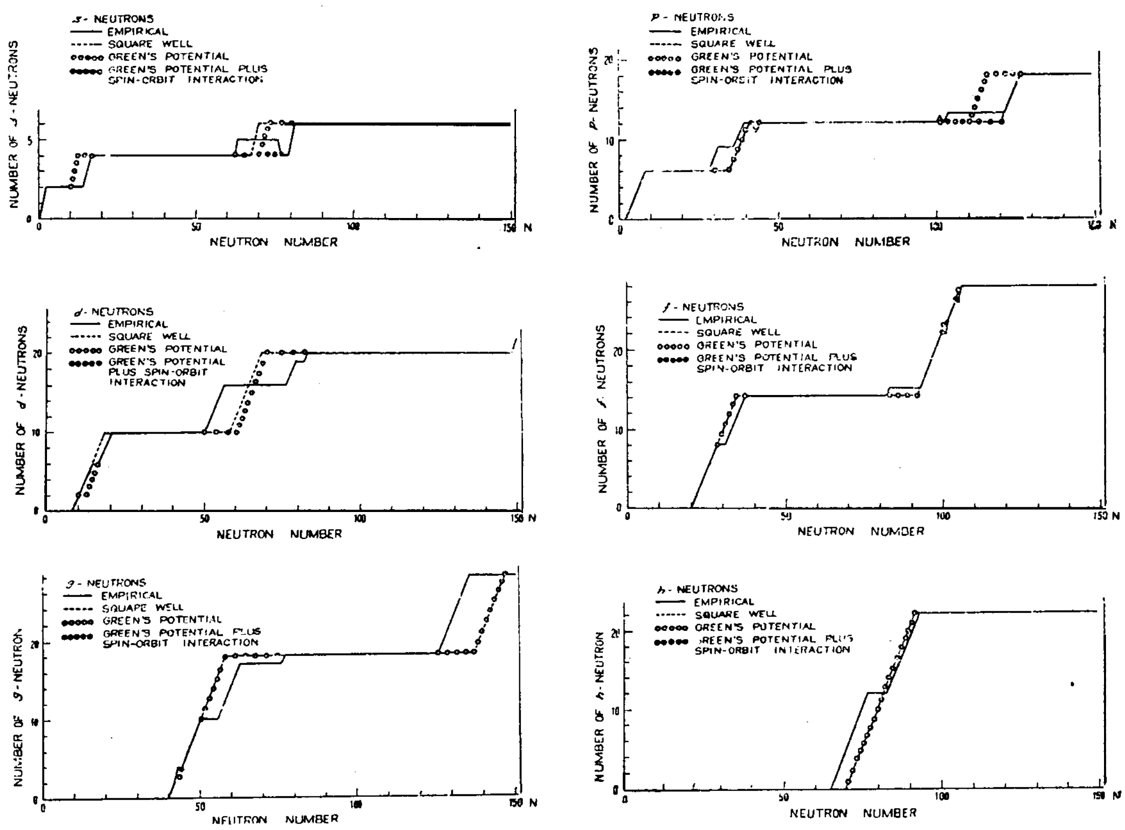
are investigated. The eigenvalues of the Schrödinger equation with these potentials have been calculated and illustrated in diagrams by Green[2]. These eigenvalues serve

1) J. L. Hwang, Chin. J. Phys. 1, 74 (1963) and 2, 28 (1964)
2) A. E. S. Green and K. Lee, Phys. Rev. 99, 772 (1955)
A. E. S. Green, Phys. Rev. 102, 1325 (1956)
A. E. S. Green, Phys. Rev. 104, 1617 (1956).

us the first guess for determining the maximum energy $(E_l)_{max}$ involved in the expression[1])

$$n_l = \frac{2(2l+1)}{h}\left\{2\int\left[2m\left((E_l)_{max} - V(r)\right) - \frac{(l+{}^1/_2)^2 h^2}{r^2}\right]^{\frac{1}{2}} dr + \frac{h}{2}\right\}.$$

The parameters V_0, a, and d involved in the potentials are adopted from the paper of Hwang and Yang[3], which may exactly reproduce the nuclear radii, trends of binding energies, and location of 3s and 4s maxima in the neutron scattering cross section. n_l's for nuclei of ten different neutron (proton) numbers are calculated and are checked with the calculated values of Green. Since no significant error is found, the diagrams of Green are used instead of evaluating Eq.(1) for other nuclei.



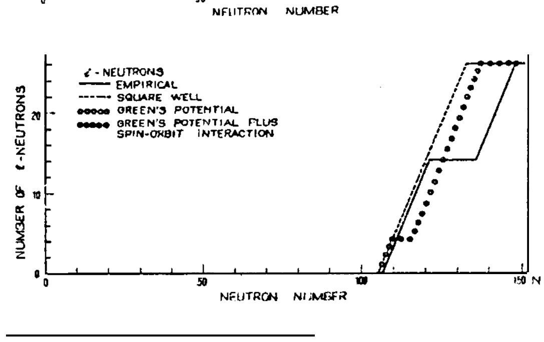


Fig. 1 Number of s-, *p-*, *d-...* neutrons in a nucleus of a given neutron number N. For the case of nuclear models, only the deviation from the empirical value is illustrated.

3) J. L. Hwang and I. S. Yang, Chin. J. Phys. 2, 3.2 (1964)

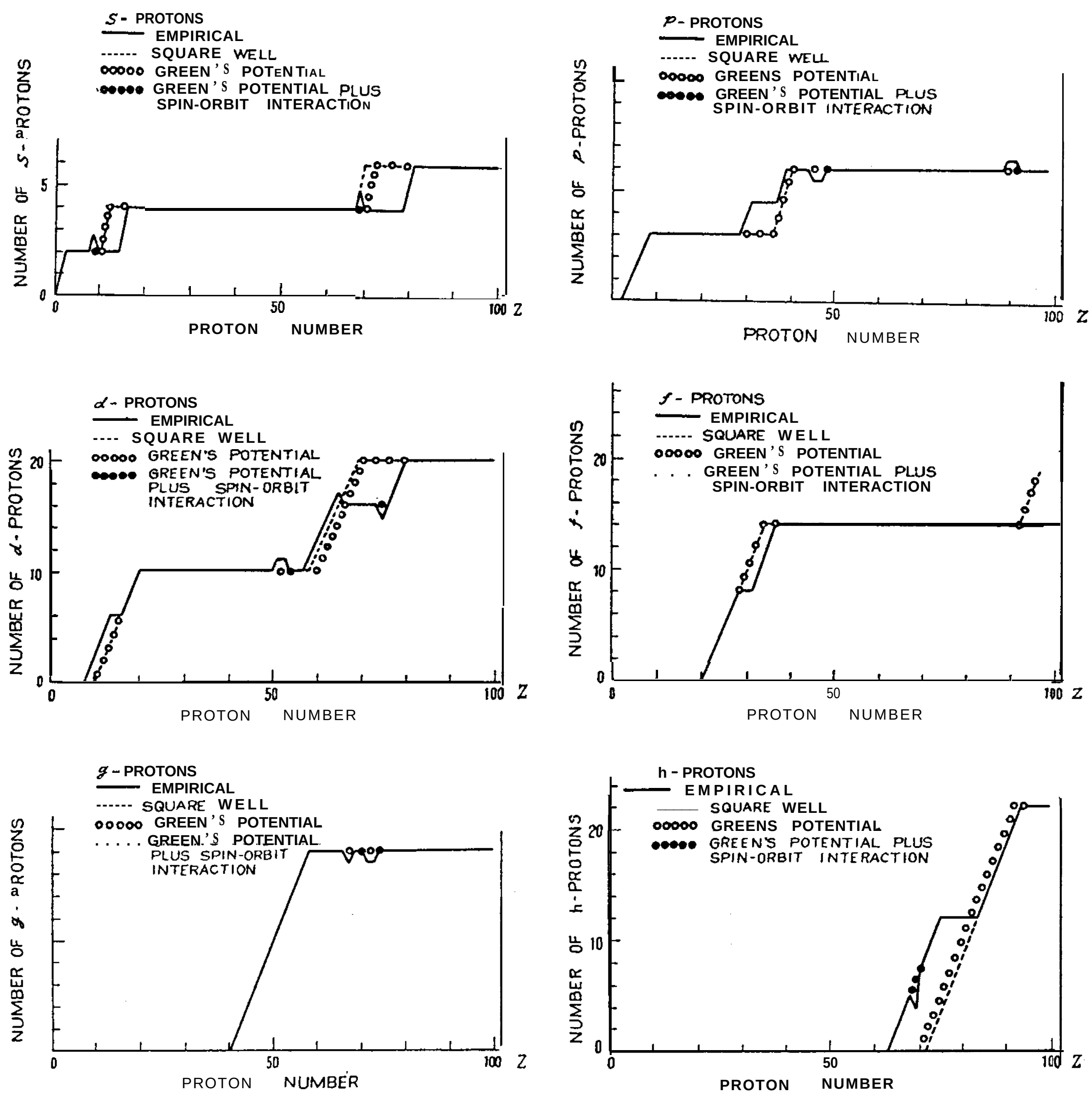


Fig. 2 Number of s-, p-, d-··· protons in a nucleus of a given proton number Z. For case of nuclear models, only the deviation from the empirical valus is illustrated.

The results are illustrated in Figs. 1-4. The empirical curves determined from the paper of Klinkenberg[4] are also given there. Since the conformity between the calculated and empirical values is very good, only deviations form the empirical values are illustrated. For the last type of potential, the conformity is almost complete. The first remarkable evidence revealed by Figs. 1-2 is that the difference between the square well potential and Green potential is small. The second is that the first appearance of the next higher angular momentum is not so different from the exact values. This is a result of our correct choice for the potential parameters.

THE FIRST APPEARANCE PROBLEM

In the early stage of the theory of nuclear shell structure, Yang[5] was guided by a principle that the successive appearance of particles of higher angular momentum is associated with the successive beginning of new shells, and found that the first appearance of neutrons (protons) with angular momentum states s, p, d, f···occurs respectively at the nuclei of neutron (proton) number 1, 3, 9, 21, 29, 51 83······(magic number plus one). From the empirical data or the former calculation Figs. l-2 it is, however, seen that the first appearance of these nucleons would occur at 1, 3, 9, 21, 41, 71, 113···, if the magnitude of spin-orbit coupling were suitably chosen. The truth of Yang's discovery is thus now denied. However, it seems that the origin of this interesting accident, the appearance of magic numbers, remains still unclarified.

The equation for the number of particles with the angular momentum $l\hbar$ employed by him was

$$n_l=\frac{8}{h}(l+\tfrac{1}{2})\int\left[P^2(r)-(l+\tfrac{1}{2})^2\hbar^2/r^2\right]^{\frac{1}{2}}dr,$$

which has just the same form as Eq.(1) used by us except for lacking the last term $(2l+1)$. This last term is the key of the problem. In our distibution curves (Figs. 1-2), if shifting the abscissa upward by $(2l+1)+1$ units, that is, if raising two units for s-state, four units for p-state, six units for d-state and so on, it can readily be seen that the s-, p-, d-,··· neutrons (protons) appear firstly at N (or Z) = 3, 7, 15, 29, 51, 83, 127···. Apart from the first three numbers, they all reproduce the Yang's discovery.

This may more precisely be stated with the level scheme[6]. In the place where a higher angular momentum l appears for the first time, the splitting of the Z-state level into two levels with $j=l+\frac{1}{2}$ and $j=l-\frac{1}{2}$ is particularly large, and the higher magic number will occur there. On the other hand, the level with $j=l+\frac{1}{2}$ *is lower* than the level with $j=l-\frac{1}{2}$, and therefore the particles, $2(l+\frac{1}{2})+1=2l+2$ in number, occupying the lower level precede to appear in the distribution curve. When the abscissa is raised, these $2l+2$ particles sink and the upper edge of the lower major shell becomes a sign of the first appearance of Z-state $(j=l-\frac{1}{2})$ particles. The occurrence of the magic number is thus explained. As to the first three magic numbers, Yang attributed them to a modification of nuclear shape, that is, he assumed that for light nuclei the nuclear density shrinks to a gaussian form.

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4) P. F. A. Klinkenerg Revs. Modern Phys. 24, 63 (1952)
5) L. M. Yang, Proc. Phys. Soc. (London) 64, 632 (1951); M. Born and L. Yang, Nature 166, 339 (1950)
6) See for example, M.G. Mayer and J.H.D. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley and Sons, Inc., New York, 1955) p. 58