Channel Interpolation and MMSE Multi-Input Multi-Output Frequency-Domain DFE for Wireless Data Communications Using OFDM

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***Abstract* – The cyclic prefix (CP) is typically employed in conventional orthogonal frequency division multiplex (OFDM) transmission systems. The CP, however, consumes bandwidth and power, and is undesirable for applications such as wireless communications. In this paper, we propose an MMSE multi-input multi-output frequency domain decision-feedback equalizer (MIMO FDDFE). A novel channel interpolation algorithm is also proposed. Simulation results show with the proposed algorithm, an OFDM system without CP actually outperforms a conventional OFDM system with CP in both bit error rate (BER) and bandwidth efficiency.**

I. INTRODUCTION

Orthogonal frequency division multiplex (OFDM) has been shown to be a very efficient scheme for mitigating the adverse effects of inter-symbol interference (ISI) caused by multipath propagation encountered in high-speed wireless communications. OFDM finds its applications in several existing wireless local area network (WLAN) standards, e.g., IEEE 802.11a[1] and HIPERLAN/2[2], and is also being considered for future high-speed wireless data networks such as wireless local loop (WLL)[3]. In these systems, a cyclic extension known as the cyclic prefix (CP)[4] is transmitted along with each OFDM symbol. As long as the CP is longer than the memory of the channel, successive OFDM symbols do not interfere with each other, and the receiver can be made very simple because no special processing is required for mitigating the effect of ISI.

Although employing CP greatly simplifies receiver signal processing, it also consumes power and reduces the effective data rate at a given bandwidth. The length of the CP is typically 25% of the information-bearing part of the OFDM symbol[1,2], which is significant in a situation where bandwidth and power are at a premium – such as in a wireless communication system. Furthermore, the length of the CP must be greater than the memory of the channel in order to be effective, therefore for wireless communication systems the appropriate length of the CP could be difficult to determine because the channel could vary rapidly.

Several papers are available in the literature on possible ways to reduce, or even completely eliminate, the CP in an OFDM transmission system by using more sophisticated receiver signal processing[5,6]. In this paper, we propose a minimum mean-square error (MMSE) multi-input multi-output (MIMO) frequency-domain decision-feedback equalizer (FDDFE) for OFDM that can very effectively eliminate the need of the CP. Unlike conventional frequency-domain equalizers, the proposed FDDFE requires an estimate of the *time-domain* impulse response of the discrete-time equivalent channel. In principle, the time-domain channel estimate can be obtained by taking the inverse discrete Fourier transform (IDFT) of the frequency-domain channel estimate, which, in turn, can be obtained using preamble or pilot symbols. However, for systems such as IEEE 802.11a or HIPERLAN/2, some subcarriers are not loaded with data or pilot symbols and interpolation is required to obtain the channel estimates at these subcarrier frequencies. In this paper, we also propose a novel iterative interpolation algorithm for obtaining the frequency-domain channel estimates. Simulation results show that the proposed method is very effective. With the proposed algorithm, an OFDM system without CP actually outperforms a conventional OFDM system with CP in both bit error rate and bandwidth efficiency.

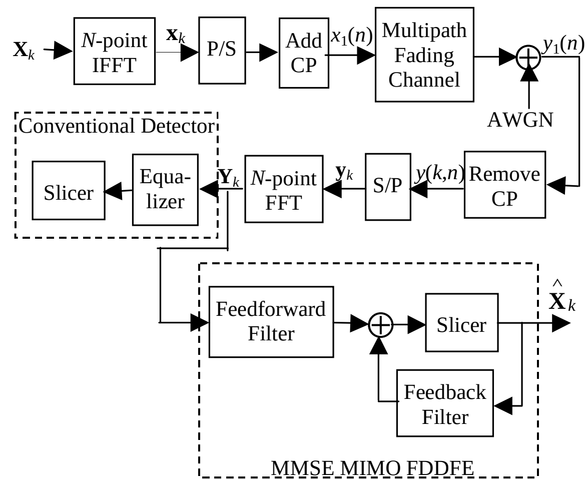

Fig. 1. Block diagram of an OFDM transmission system in which the DFT and IDFT are implemented using fast Fourier transform (FFT).

II. SYSTEM MODEL

A block diagram of an OFDM transmission system is shown in Fig. 1. Let $X(k,j)$, $j = 0,1,\ldots,(N-1)$, denote the modulation symbols on the j-th OFDM subcarrier of the k-th OFDM symbol, and let $x(k,n)$, $n = 0,1,\ldots, (N-1)$, be the time-domain samples of the k-th OFDM symbol (excluding

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CP), where N is the number of subcarriers (size of DFT). The samples $x(k,n)$ are passed through a parallel-to-serial (P/S) converter and a CP is prepended, resulting in a signal given by

$$x_1(n) = \sum_{k=0}^{K-1} x\big(k,(n-kN_T-N_g)_N\big)\Pi\big(n-kN_T\big) \quad (1)$$

where $(\bullet)_N$ is the modulo-N operator, N_g is the size of the CP, $N_T = N_g + N$ is the total number of samples in an OFDM symbol (including CP), K is the total number of OFDM symbols, and

$$\Pi(n) \equiv \begin{cases} 1, & 0 \le n < N_T \\ 0, & else \end{cases}. \quad (2)$$

As shown in Fig. 1, the OFDM symbols are transmitted through the wireless channel modeled as a multipath fading channel corrupted by additive white Gaussian noise (AWGN). The discrete-time baseband-equivalent received signal is given by

$$y_1(n) = \sum_{j=0}^{\nu} p_j x_1(n-j) + \eta(n) , \quad (3)$$

where ν is the memory of the channel, p_j, $j = 0,1,\ldots,\nu$ is the (time-domain) impulse response of the discrete-time equivalent channel, and $\eta(k)$ are samples of the additive Gaussian noise.

At the receiver, CP is removed from the received signal and the result is passed through a serial-to-parallel converter to obtain

$$y(k,n) = y_1(kN_T + N_g + n) \quad (4)$$

for $n = 0,1,\ldots,(N-1)$ and $k = 0, 1, \ldots, (K-1)$. IDFT of $y(k,\bullet)$ is computed and the result, $Y(k,\bullet)$, is processed by the proposed FDDFE or simply by a conventional OFDM detector.

III. MMSE MIMO Frequency-Domain DFE

Due to space limitations we only describe the case where $\nu \le (N_T+N_g)$, although the proposed scheme can be easily extended to the general case. Representing the received OFDM symbols by column vectors defined by

$$\mathbf{y}_k = \big[y(k,0) \quad \cdots \quad y(k,N-1)\big]^T , \quad (5)$$

where "T" denotes matrix transposition, it can be easily shown that,

$$\mathbf{y}_k = \mathbf{P}_0\mathbf{x}_k + \mathbf{P}_1\mathbf{x}_{k-1} + \mathbf{n}_k , \quad (6)$$

where

$$\mathbf{x}_k = \big[x(k,0) \quad \cdots \quad x(k,N-1)\big]^T , \quad (7)$$

$$\mathbf{n}_k = \big[\eta(kN_T+N_g) \quad \cdots \quad \eta((k+1)N_T-1)\big]^T , \quad (8)$$

and $\mathbf{P}_0$ and $\mathbf{P}_1$ are $N\times N$ matrices that are functions of p_j, $j = 0,1,\ldots,\nu$. Taking the DFT of both sides of (6), we have

$$\mathbf{Y}_k = \mathbf{Q}_0\mathbf{X}_k + \mathbf{Q}_1\mathbf{X}_{k-1} + \mathbf{N}_k , \quad (9)$$

where $\mathbf{Y}_k$, $\mathbf{X}_k$, and $\mathbf{N}_k$ are, respectively, the DFT of $\mathbf{y}_k$, $\mathbf{x}_k$, and $\mathbf{n}_k$, and

$$\mathbf{Q}_m = \frac{1}{N}\mathbf{W}\mathbf{P}_m\mathbf{W}^H \quad (10)$$

for $m = 0$ and 1, where "H" denotes Hermitian transposition and $\mathbf{W}$ is an $N \times N$ matrix whose (k,l)-th component is given by

$$W_{kl} = \exp\left(-\tfrac{j2\pi kl}{N}\right) \quad (11)$$

for $k,l = 0,1,\ldots,(N-1)$. Note that the first term in (9) is a linearly distorted version of the current transmitted OFDM symbol $\mathbf{X}_k$, while the second term represents inter-block interference (IBI) coming from the previous OFDM symbol. When $\nu \le N_g$, $\mathbf{Q}_0$ is a diagonal matrix while $\mathbf{Q}_1$ is a zero matrix, and there is no IBI. In this case, all that the receiver has to do is to undo the gains introduced by the wireless channel that are represented by the diagonal elements of $\mathbf{Q}_0$. When $\nu > N_g$, however, $\mathbf{Q}_0$ is no longer diagonal in general and $\mathbf{Q}_1$ is nonzero. In other words, subcarriers interfere with each other causing cross-talk (characterized by $\mathbf{Q}_0$), and successive OFDM symbols also interfere with each other causing IBI (characterized by $\mathbf{Q}_1$).

In this paper, we propose a DFE for minimizing cross-talk and IBI. As shown in Fig. 1, the proposed DFE consists of a feedforward filter and a feedback filter, both of which are MIMO filters with matrix coefficients. The input of the proposed DFE is the N-dimensional frequency-domain vectors $\mathbf{Y}_k$, and the output of the DFE is an N-dimensional column vector given by

$$\mathbf{Z}_k \equiv \sum_{i=0}^{N_f-1}\mathbf{w}_i^H\mathbf{Y}_{k+\Delta-i} + \sum_{i=1}^{N_b}\mathbf{b}_i^H\mathbf{X}_{k-i} , \quad (12)$$

where N_f and N_b are the number of coefficient matrices of the feedforward and feedback filters, respectively, and $\mathbf{w}_i$, $i = 0,1,\ldots,(N_f-1)$ and $\mathbf{b}_i$, $i=1,2,\ldots,N_b$, are, respectively, $N\times N$ matrix coefficients of the feedforward and feedback filters. Each component of the output $\mathbf{Z}_k$ is appropriately sliced to obtain the detected OFDM symbol $\hat{\mathbf{X}}_k$. Since the proposed DFE is a multi-input multi-output (MIMO) device whose input and output are frequency-domain quantities, it is referred to as the MIMO frequency-domain DFE (FDDFE). The matrix coefficients $\mathbf{w}_i$ and $\mathbf{b}_i$ are chosen such that the error covariance matrix

$$\mathbf{K} = E\Big[(\mathbf{Z}_k - \mathbf{X}_k)(\mathbf{Z}_k - \mathbf{X}_k)^H\Big] \quad (13)$$

has the property that $\mathbf{c}^H\mathbf{K}\mathbf{c}$ is minimum for any vector $\mathbf{c}$. The resulting FDDFE is referred to as the MMSE MIMO DFE.

For $\nu \le (N_T+N_g)$, choosing N_b=1 and $\Delta=N_f-1$ yields good performance. It can be shown that the coefficients of the MMSE MIMO DFE in this case is given by

$$\begin{cases} \begin{bmatrix} \mathbf{w}_0 \\ \vdots \\ \mathbf{w}_{N_f-1} \end{bmatrix} = \left(\mathbf{P}\mathbf{P}^H + \mathbf{R}_N\right)^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{Q}_1 \\ \mathbf{Q}_0 \end{bmatrix} \\ \mathbf{b}_1 = -\mathbf{Q}_1^H \mathbf{w}_{N_f-1} \end{cases} \tag{14}$$

where

$$\mathbf{P} = \begin{bmatrix} \mathbf{Q}_0 & \mathbf{Q}_1 & & \\ & \mathbf{Q}_0 & \ddots & \\ & & \ddots & \mathbf{Q}_1 \\ & & & \mathbf{Q}_0 \end{bmatrix} \tag{15}$$

is an $(NN_f)\times(NN_f)$ matrix and $\mathbf{R}_N$ is the autocorrelation matrix of the noise vector $\begin{bmatrix} \mathbf{N}_{k-N_f+1}^T & \cdots & \mathbf{N}_k^T \end{bmatrix}^T$.

IV. Iterative Channel Interpolation

It is clear from the previous section that computing the matrix coefficients for the MMSE MIMO FDDFE requires an estimate of the time-domain discrete-time equivalent channel. In principle, the time-domain channel estimate can be obtained by first computing the frequency-domain channel estimate using training symbols, and then taking the IDFT. In practice, however, many OFDM systems do not use all subcarriers. For example, in IEEE 802.11 and HIPERLAN/2 only 52 out of the 64 subcarriers are used for carrying data and/or pilot symbols as shown in Fig. 2[1,2]. The 12 unused band-edge subcarriers serve as guard band for implementation purposes. The channel at these unused subcarrier frequencies obviously cannot be estimated directly from the pilot or data symbols, and must be interpolated from the estimated channel at the active subcarrier frequencies. Note that perfect interpolation is not always possible because the unused subcarrier frequencies are contiguous.

In this paper, we propose the following simple yet effective iterative method for interpolating the channel estimates at unused subcarrier frequencies from channel estimates at active subcarrier frequencies. Initially, for k = 0,1, …, (N-1), let P_k be equal to the channel estimate of the k-th subcarrier if it is active, or zero if it is unused. The following procedure is next repeated a total of *MAX_ITER* times: For each unused subcarrier k, recompute P_k according to

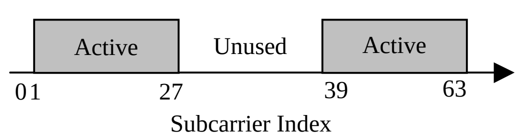

Fig. 2. Active and unused subcarriers in IEEE 802.11A and HIPERLAN/2. Subcarriers 0 and 28 to 38 are unused as shown in this figure.

$$P_k = \sum_{m=0}^{\lfloor N/L \rfloor - 1} P_{(mL+l)_N} G_{k-mL-l} \tag{16}$$

for l=0,1,…,(L-1), where

$$G_k = \left(\frac{L}{N}\right)\exp\left(-\frac{j\pi\nu k}{N}\right)\frac{\sin\left(\frac{\pi(\nu+1)k}{N}\right)}{\sin\left(\frac{\pi k}{N}\right)} \tag{17}$$

and L is an integer that is chosen empirically.

It should be noted that (16) and (17) are similar to the sinc interpolation formula[7]. However, since a contiguous block of subcarrier frequencies is unused, it is necessary to evaluate (16) repeatedly in order to gradually recover the unavailable channel estimates from the available estimates. Simulation results show that the proposed scheme is very effective and provides a significant performance gain when used in conjunction with the MMSE MIMO FDDFE.

V. Simulation Results

Performance of the proposed iterative channel interpolation and MMSE MIMO FDDFE are evaluated by computer simulation. Parameters in the transmitter basically follow those of IEEE 802.11A and HIPERLAN/2. The total number of carriers is N = 64, among which 52 are active and 12 are unused as shown in Fig. 2. Preamble OFDM symbols specified in IEEE 802.11a[1] are transmitted at the beginning of each burst of 100 OFDM symbols. The length N_g of the CP is equal to 0, 16, or 32. Uncoded quadrature phase-shift keying (QPSK) is used on active subcarriers.

The discrete-time baseband-equivalent wireless channel is simulated for ν=15 (spanning 25% of an OFDM symbol) and 31 (spanning 50% of an OFDM symbol). The channel taps are i.i.d. complex Gaussian distributed with zero-mean and complex variance $1/(\nu+1)$. The wireless channel is time-invariant within one burst and uncorrelated from burst to burst.

At the receiver, channel estimates at the active subcarrier frequencies are obtained from the preamble symbols available using the least-squares estimation algorithm. Gains at the unused subcarrier frequencies are then computed using the proposed iterative algorithm. After the entire frequency-domain channel estimate is obtained, IDFT is next performed to obtain the time-domain channel estimates. The MMSE MIMO FDDFE coefficient matrices are next computed according to (14). The output of the FDDFE is computed according to (12), and QPSK slicing is used on each component of the output to obtain the detected OFDM symbol. No frequency offset is assumed in this paper.

The performance of the iterative channel interpolation algorithm is shown in Fig. 3. The normalized mean-square error (NMSE), defined as the mean-square

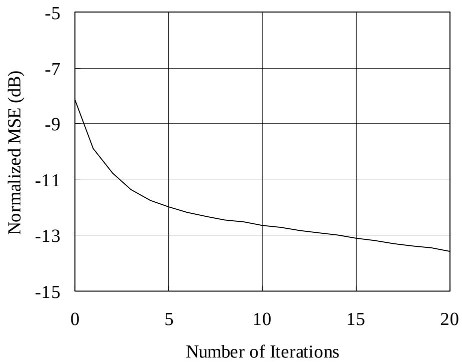

Fig. 3. Normalized MSE of the iterative channel interpolation algorithm as a function of the number of iterations.

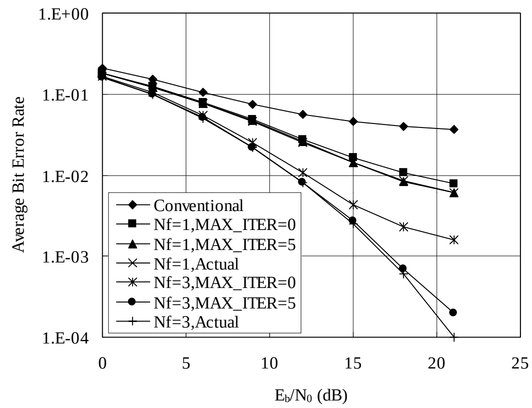

Fig. 5. Performance of MMSE MIMO FDDFE for ν=31, N_g=0.

error of the time-domain channel estimate normalized by the sum of magnitude-squares of the time-domain channel taps, is shown as a function of the number of iterations. No AWGN is assumed in this figure. It can be seen that without the proposed algorithm (i.e., *MAX_ITER*=0 and the channel estimates at unused subcarrier frequencies are simply set to 0) the NMSE is roughly –8 dB (15% error). For *MAX_ITER* = 5 the NMSE is reduced by approximately 4 dB. An additional 1 dB improvement is achievable by increasing *MAX_ITER* to 10. The benefit of increasing *MAX_ITER* beyond 10 is only marginal. We can thus conclude that the proposed iterative channel interpolation algorithm is effective as far as reducing NMSE is concerned.

The average bit error rate (BER) of the proposed MMSE MIMO FDDFE is shown in Fig. 4 as functions of E_b/N_0, where E_b is the transmitted energy per channel bit and $N_0/2$ is the two-sided power spectral density of the AWGN. No CP is used (N_g = 0), and ν=15 is assumed. The number of feedback coefficients is fixed at N_b = 1, while N_f = 1 and N_f = 3 are both simulated. For each value of N_f, three cases are investigated: 1) no channel interpolation (*MAX_ITER* = 0); 2) the proposed iterative method with *MAX_ITER* = 5; and 3) "perfect" channel interpolation in which the actual gains at

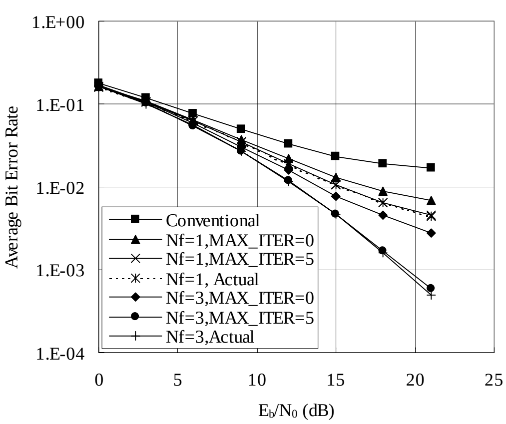

Fig. 4. Performance of MMSE MIMO FDDFE for ν=15, N_g=0.

the unused subcarrier frequencies are assumed to be known. In addition, the "conventional" OFDM receiver without the FDDFE (i.e., N_f = N_b = 0) is also simulated.

It can be seen from Fig. 4 that the average BER of a conventional OFDM receiver floors at approximately 2×10^{-2} when N_g = 0 (no CP) and ν=15. This is, of course, due to the effect of severe IBI. When an MMSE MIMO FDDFE with N_f = 1 and N_b = 1 is used, the BER floor is reduced by a factor of 3 to approximately 6×10^{-3} if *MAX_ITER*=0 (no channel interpolation). Furthermore, if the proposed iterative channel interpolation is used with *MAX_ITER* = 5, the BER floor is further reduced to 4×10^{-3}. It should also be noted that for N_f = 1 and N_b = 1, no significant performance difference is observed between *MAX_ITER* = 5 and perfect channel interpolation.

It can also be seen that for N_f = 3, N_b = 1, and *MAX_ITER* = 0, the BER floor is reduced to 2×10^{-3} – an order of magnitude lower than that of the conventional receiver. If iterative channel interpolation with *MAX_ITER* = 5 is used, however, the BER floor is completely removed. For this case the performance gain of the FDDFE over the conventional OFDM receiver is in excess of 10 dB at 2×10^{-2} average BER. This shows that the iterative channel interpolation in conjunction with the MMSE MIMO FDDFE is a very effective means of combating the effect of IBI for an OFDM transmission system. Furthermore, the performance difference between *MAX_ITER* = 5 and perfect channel interpolation is very small as previously observed.

Similar experiments are also performed for ν = 31 and N_g = 0 (no CP), and the results are plotted in Fig. 5. Observations similar to Fig. 4 can be made in this figure. In particular, the MMSE MIMO FDDFE with N_f = 3 and N_b = 1 in conjunction with iterative channel interpolation with *MAX_ITER* = 5 completely removes the BER floor, and the performance difference between *MAX_ITER*=5 and perfect channel interpolation is very small.

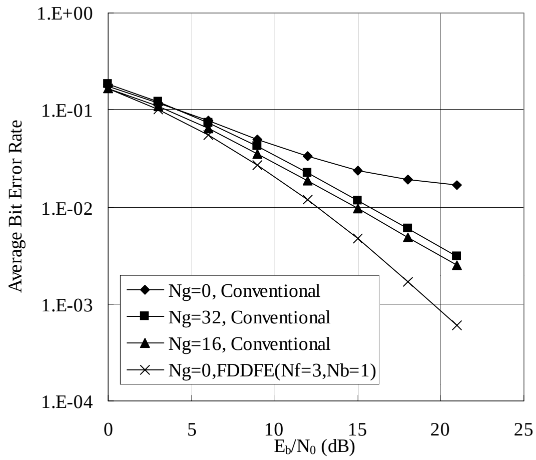

Fig. 6. Performance of MMSE MIMO FDDFE with N_g=0 and conventional receiver with N_g=16 and 32 for ν=15.

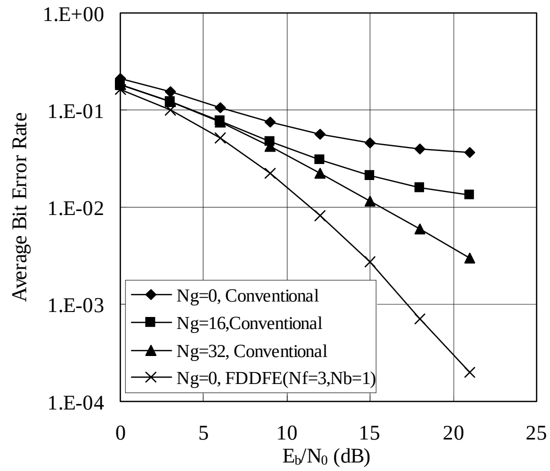

Fig. 7. Performance of MMSE MIMO FDDFE with N_g=0 and conventional receiver with N_g=16 and 32 for ν=31.

It is also interesting to note in Fig. 5 that while the BER of the conventional OFDM receiver is higher for ν = 31 than for ν = 15 (Fig. 4) due to the increased amount of IBI, the BER of the MMSE MIMO FDDFE with N_f = 3 and N_b = 1 is actually *lower* for ν = 31 than for ν = 15. This is because a channel with memory ν = 31 provides more path diversity (or implicit diversity) than a channel with ν = 15. The MMSE MIMO FDDFE effectively makes use of this increased amount of path diversity to overcome the effect of increased IBI, thus lowering the average BER. In order to probe this point further, the performance of an OFDM transmission system using the conventional receiver is also simulated with N_g = 16 and 32, and the results are plotted in Figs. 6 and 7 for ν = 15 and 31, respectively. Performance curves for N_g = 0 with and without the FDDFE are also duplicated. It can be seen from Fig. 6 that for ν=15, the performance of conventional OFDM receivers for N_g = 16 and N_g = 32 are approximately the same, with N_g = 32 being slightly inferior because more energy is used for transmitting the CP. Note that the average BER for these cases decrease one order of magnitude when E_b/N_0 is increased by 10dB. On the other hand, it can be seen that an MMSE MIMO FDDFE with N_f = 3 and N_b = 1 outperforms the conventional receivers even when N_g=0. In other words, an OFDM system with the MMSE MIMO DFE outperforms the conventional system in both average BER and bandwidth efficiency. Furthermore, the average BER of the FDDFE decreases faster than the conventional receivers. This observation suggests that the FDDFE is effective in making use of the path diversity of the channel. This is precisely the reason why an OFDM system with the MMSE MIMO DFE outperforms the conventional system in both average BER and bandwidth efficiency.

Similar observations can be made from Fig. 7 for ν =31. Here N_g = 32 significantly outperforms N_g = 16 for the conventional receiver because N_g = 16 is too small for ν=31. The average BER for N_g = 32 also decreases at a rate of an order of magnitude per 10-dB increase in E_b/N_0 just as in Fig. 6. However, the average BER for an MMSE MIMO FDDFE with N_f = 3 and N_b = 1 decreases faster than the conventional receiver and significantly outperforms the conventional receivers even when N_g = 0 just as in Fig. 6.

VI. CONCLUSION

An MMSE MIMO frequency-domain DFE is proposed for OFDM. A novel iterative interpolation algorithm is used for obtaining the frequency-domain channel estimates at unused subcarrier frequencies. Simulation results show that the proposed DFE in conjunction with iterative channel interpolation is very effective in making use of the available path diversity in a wireless channel to improve performance. In particular, with the proposed algorithm, an OFDM system without CP actually outperforms a conventional OFDM system with CP in both bit error rate (BER) and bandwidth efficiency for channels with memory ν = 15 and 31.

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