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運動律與組成律之正則架構(1/2)

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運動律與組成律之正則架構 (1/2)
Canonical framework for motion and constitutive laws

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摘要

本計畫旨在研究運動律與組成律的正則架構, 包括單一質點與多質點系統的摩擦運動, 以及其場論化 (連續化、反離散化) , 即連體的耗散變形與運動。本計畫的重心在於 (牛頓與愛因斯坦) 運動律的哈氏量 (Hamiltonian) 推廣爲含 (牛頓與愛因斯坦) 運動律與 (保守與耗散) 組成律的哈氏量。掌握正則架構後, 可以利用正則變換, 建構解析正解與高精度保群算法。

關鍵詞: 運動律、組成律、哈氏量、正則變換、保群算法、塑性、摩擦、耗散。

Abstract

The project was proposed to study the canonical framework of the laws of motion and constitution for a particle moving or trapped in a frictional environment, and its finitely many degree-of-freedom and continuum versions. The main focus is on the challenging extension of the Hamiltonians of the motion laws of Newton and Eienstein to Hamiltonians which account for both (conservative and dissipative) constitutive laws and (Newtonian and Eienstein) motion laws. Under the canonical framework, we take advantage of canonical transformations to find exact and analytical solutions and to devise highly accurate group-preserving aschemes.

Keywords: motion law, constitutive law, Hamiltonian, canonical framework, group-preserving schemes, plasticity, friction, dissipation.

一、背景

無論是靜態與擬靜態的靜不定結構分析或應力波分析或振動分析, 同時牽涉到 (靜或動的) 平衡律 (運動律) 與組成律, 兩者同時呈現在力學材料構件中, 一起規範其力學行爲。所以分析計算時, 兩者也要一起考慮。以近年來有長足發展的非線性動力學而言, 其中所謂遲滯現象, 在連體或結構之靜、動態反應中, 也屢見不鮮。但是此現象在連體、結構應力分析 (連體耗散變形運動或多質點系統摩擦運動) 問題中, 究竟應該溯源歸類爲變率效應 (黏滯性、黏彈性、黏塑性)? 還是混合有慣性效應? 譬如承受軸向變形的線性黏彈性桿子, 若左端固定, 右端作用一力量 Q, 則右端之位移 q 與力量 Q 有如下之關係:

$$Q(t) = \int_{-\infty}^{t} K(t,\tau) dq(\tau) \tag{1}$$

上式如果是組成律, 則 $K(t,\tau)$ 稱爲軸向鬆弛剛度函數。但是物質的反應是綜合呈現的, 含有組成律的成分, 也含有運動律的部分, 甚至可能有兩者之互制。($K(t,\tau)$ 若進一步混含慣性效應, 可稱之爲軸向動態鬆弛剛度函數。) 一般在進行材料試驗, 常常要求將速率、應變率放慢, 以便將運動律之慣性效應及組成律之變率效應拿掉, 專注於率無關之耗散, 即塑性或乾摩擦。理論上, 如果覺得變率效應必須考慮, 則再把變率效應放回去, 成爲率相關之耗散, 即黏塑性或黏摩擦。但是實務上, 則往往簡單地另加一項黏滯項, 聊表考慮了變率效應, 如質點之摩擦運動常見下述列式:

$$f_I + f_V + f_E + f_F = f \tag{2}$$

其中 $f_I = m\ddot{q}$, $f_V = c\dot{q}$, $f_E = kq$, $|f_F| \le const.$, f =外力。問題是組成力 f_C 是否可以分離爲 $f_C = f_V + f_F + f_E$, 分離是一種簡單地近似, 不分離可能很複雜, 但是是否可以找到內部的對稱性, 因而簡單而美而眞。進一步的問題是物質力 f_M 是否一定要分解成 $f_M = f_I + f_C$, 是否可能合起來而找到內部對稱性, 因而簡單而美而眞? 塑性組成律 (或乾摩擦定律) 雖已發展百年以上. 由於對象龐然、現象複雜, 其理論架構及實驗方法離完善尚遠。最近我們研究房屋支承在平面之摩擦運動發現質點之摩擦運動可能具有哈氏量 (Hamiltonian) 之正則架構 (canonical framework)。

二、目的

探討運動律與組成律合而爲一, 成爲渾然一體之物質律之可能性, 增進對於耗散物質非線性變形運動之了解, 改善材料試驗與連體、結構應力分析之方法。

三、重要性

結構物, 連體進入耗散狀態的分析、實驗相當棘手, 若能掌握運動律與組成律之正則架構, 可以利用保群變換, 建構解析正解及高精度數值算法, 並據以改進實驗方法與實驗分析方法 (數據處理、參數估測、初值識別、束制識別)。

四、研究方法、進行步驟及執行進度

1. 質點運動律

牛頓理論與愛因斯坦狹義相對論都說質點 (粒子) 之動量 p 之變率等於質點所受之力量 Q:

$$\dot{p} = Q \tag{3}$$

在符號上方加點表示時間變率。設質點靜止質量爲常數值 m_0, 若質點以速率 v 直線運動, 且 $0 \le v < c$, 其中 c 爲光速, 設爲常數值, 則根據愛因斯坦狹義相對論知質量 m 隨著 v 而增大:

$$m = \frac{m_0}{\sqrt{1-(\frac{v}{c})^2}} \tag{4}$$

因而質點之動量爲

$$p = mv = \frac{m_0 v}{\sqrt{1-(\frac{v}{c})^2}} \tag{5}$$

由

$$v = \frac{\partial H_1}{\partial p} \tag{6}$$

及 (5) 可得運動律之哈氏量 H_1 爲

$$H_1(p) = m_0 c^2 \sqrt{1 + (\frac{p}{m_0 c})^2} \tag{7}$$

當動量 p 很小時,

$$H_1(p) \cong m_0 c^2 + \frac{1}{2m_0} p^2 \tag{8}$$

其中第一項 $m_0 c^2$ 爲常數, 第二項 $\frac{p^2}{2m_0}$ 即爲牛頓理論之動能。

2. 單自由度組成律

庫倫乾摩擦組成律謂摩擦力 Q 與摩擦介面相對速率 v (質點因爲以速率 v 運動因而對周遭施加摩擦力 Q) 有如下之關係:

$$Q = \begin{cases} Q_y & \forall v > 0 \\ -Q_y & \forall v < 0 \end{cases} \tag{9}$$

其中摩擦力上限 Q_y 爲常數。(9) 未說明 $v=0$ 時 Q 值的大小, 一般以下式補正,

$$Q\begin{cases}=Q_y & \forall v>0\\ \in[-Q_y,Q_y] & \forall v=0\\ =-Q_y & \forall v<0\end{cases} \tag{10}$$

上述寫法雖然沒錯, 卻不完備, 沒有清楚表達 $v(t)$ 與 $Q(t)$ 的全部關係。文獻 [30] 借助有物理意義耗散率之拉氏乘子 $\dot{\Lambda}$ 將 (10) 完備化爲

$$\begin{aligned}&v=\frac{\dot{\Lambda}}{Q_y^2}Q,\\ &|Q|\le Q_y,\\ &\dot{\Lambda}\ge 0,\\ &|Q|\dot{\Lambda}=Q_y\dot{\Lambda}.\end{aligned} \tag{11}$$

容易證明 [30] 由 (11) 可導得 (10), 而 (10) 卻不能得到 (11)。

式 (10) 與模式 (11) 都表示在黏著時摩擦力 Q 介於 $\pm Q_y$ 之間, 反之亦然。這種摩擦力不唯一的情形, 僅是一種簡單的近似, 有人考慮在滑動前, 介於微觀凹凸表面間可能有可逆之彈性變形, 而將模式 (11) 的第一式加入彈性變形項, 修改成爲

$$\begin{aligned}&v=\frac{1}{k_0}\dot{Q}+\frac{\dot{\Lambda}}{Q_y^2}Q,\\ &|Q|\le Q_y,\\ &\dot{\Lambda}\ge 0,\\ &|Q|\dot{\Lambda}=Q_y\dot{\Lambda}.\end{aligned} \tag{12}$$

有些摩擦文獻 [38] 將模式 (11)[或 (10)] 黏著時摩擦力爲「集合值」, $Q\in[-Q_y,Q_y]$, 改正爲模式 (12), 使之成爲一對一關係 $Q=k_0q$ 的動作, 稱爲正規化 (regularization)。模式 (12) 可以等價地表示爲

$$\begin{aligned}&\frac{1}{k_0}\dot{Q}+\frac{Qv}{Q_y^2}Q=v && \text{if } |Q|=Q_y \text{ and } Qv>0\\ &\frac{1}{k_0}\dot{Q}=v && \text{if } |Q|<Q_y \text{ or } Qv\le 0 \text{ or both}\end{aligned} \tag{13}$$

事實上, 上述摩擦組成律與塑性組成律在形式上是完全一樣的, 模式 (11) 即爲完全剛塑性組成律, 模式 (12) 即爲完全彈塑性組成律。剛塑性組成律提出甚早, 而彈塑性組成律則遲至二十世紀二、三十年代才由 Prandtl 與 Reuss 提出。但是無論就材料彈塑行爲或材料介面摩擦行爲來看, 上述模式不見得是最接近眞實之簡單近似。Prager [64] 即提出不同的塑性組成律。由具體的應力應變曲線來觀察, 很多材料在加載時並沒有明顯的線性彈性段, 因此非線性彈性或塑性成爲另一種選擇。循著這條思路, 模式 (13) [即 (12)] 可以修改爲

$$\begin{aligned}&\frac{1}{k_0}\dot{Q}+\frac{Qv}{Q_y^2}Q=v && \text{if } Qv>0\\ &\frac{1}{k_0}\dot{Q}=v && \text{if } Qv\le 0\end{aligned} \tag{14}$$

或者更進一步修改爲

$$\begin{aligned}&\frac{1}{k_0\sqrt{1-\frac{QQ}{Q_y^2}}}\dot{Q}+\frac{Qv}{Q_y^2}Q>v && \text{if } Qv=0\\ &\frac{1}{k_0}\dot{Q}=v && \text{if } Qv\le 0\end{aligned} \tag{15}$$

模式 (14) 與模式 (15) 都表明: 當功率 $Qv>0$ 時, 摩擦力 Q 與相對位移 q 之關係爲非線性、是耗散的, 但當功率爲 0 或負時, 則爲線性彈性、是保守的; 兩模式的差異在於: 當功率爲正時, 亦即在耗散狀態時, 模式 (15) 的彈性部分爲非線性, 而模式 (14) 的彈性部分爲線性。模式 (15) 可以表示爲

$$\dot{q}=v \tag{16}$$

$$Q=-\frac{\partial H_2}{\partial q} \tag{17}$$

其中

$$H_2(q)=\begin{cases}-\frac{Q_y^2}{k_0}\sqrt{1+\left(\frac{k_0q}{Q_y}\right)^2}+c_2^{\mathrm{on}} & \text{if } Qv>0\\ -\frac{Q_y^2}{k_0}-\frac{1}{2}k_0q^2+c_2^{\mathrm{off}} & \text{if } Qv\le 0\end{cases} \tag{18}$$

爲組成律之哈氏量, 其中 c_2^{on}、c_2^{off}爲片段常數函數, 使得在切換時, 耗散狀態與保守狀態的哈氏量能夠銜接。當位移 q 很小時, 耗散狀態的 H_2 近似等於保守狀態的 H_2, 後者之第一項 Q_y^2/k_0 爲常數, 第二項 $\frac{1}{2}k_0q^2$ 即爲線性彈簧的位能。

附註: (18) 之第一式是一個非線性關係式, 規範摩擦力 Q 與摩擦位移 q 間的非線性關係, 似乎是全量形式, 這與耗散應是變率形式的認知不同, 但結合 (16) 與 (17), 以及切換機制 (注意 c^{on} 與 c^{off}) 後, 顯然是變率形式。因此全量形式的疑慮不存在。

3. 單自由度之正則形式

如果將質點與其周遭視爲一體, 爲一閉系統 (closed system), 則定義哈氏量 $H(p,q)$ 爲上述 H_1 與 H_2 之和:

$$H(p,q) = H_1(p) + H_2(q) = \begin{cases} m_0c^2\sqrt{1+(\frac{p}{m_0c})^2} - \frac{Q_y^2}{k_0}\sqrt{1+(\frac{k_0q}{Q_y})^2} + c^{\text{on}} & \text{if } Q\dot{q} > 0 \\ m_0c^2 + \frac{1}{2m_0}p^2 - \frac{Q_y^2}{k_0} - \frac{1}{2}k_0q^2 + c^{\text{off}} & \text{if } Q\dot{q} \le 0 \end{cases} \tag{19}$$

前者爲功率正時之哈氏量, 主控單自由度耗散變形滑動運動; 後者爲功率負或爲零時之哈氏量, 主控單自由度保守可逆變形。式中 c^{on}、c^{off}爲片段常數函數, 使得在切換時, 耗散狀態與保守狀態的哈氏量能夠銜接。有了 (19), 則由 (16) 與 (6), 以及 (3) 與 (17) 可分別導得

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p} \\ \dot{p} &= -\frac{\partial H}{\partial q} \end{aligned} \tag{20}$$

式 (20) 即爲單自由度耗散保守二相閉系統之正則運動方程式, (19) 爲其哈氏量。

4. 變率效應

變率效應相當重要, 在黏塑性或黏摩擦組成律都是重要課題。本計畫利用「橫貫臨界分叉」加以處理。所謂橫貫臨界分叉, 乃非線性動力學中的一種分叉現象, 餘維數爲1, 因爲具有穩定性交換 (exchange of stability) 的特性, 可借用改造之, 使之具有耗散與保守兩相切換的功能。但是不論那一相, 會因此引入黏性, 即變率效應。按不動點 (fixed point) 的橫貫臨界分叉有正規形 (normal form)

$$\dot{x} = \alpha_1 ux - \alpha_2 x^2 \tag{21}$$

其中 x 爲狀態變數,u 爲控制參數。上述 x, u 都應理解爲無因次, 所以例如彈塑性 Prandtl-Ruess 模式 (以應力單位爲單位) 的降伏函數 f 可利用剪切降伏應力 τ_y 來無因次化, 而 (以應變率單位爲單位) 塑性當量變率 $\dot{\lambda}$ 可利用剪切模數 G 與黏滯係數 η 的比值來無因次化。注意要滿足橫貫臨界分叉的先天限制, 以免變成同樣是餘維數爲1的鞍點節點分叉 (saddle-node bifurcation)。

研究橫貫臨界分叉之分叉圖 (bifurcation diagram), 發現其不動點即由 (21) 式描述, 但是其穩定不動點卻由下述互補三元描述

$$\alpha_1 u \le \alpha_2 x,\ x \ge 0,\ \alpha_1 ux = \alpha_2 x^2. \tag{22}$$

若取下述對應

$$\begin{array}{ccc} x & \leftrightarrow & \dot{\lambda} \\ u & \leftrightarrow & \dot{f} \\ \alpha_1 & \leftrightarrow & \langle f \rangle \\ \alpha_2 & \leftrightarrow & (1-f) \end{array}$$

其中$\langle f \rangle = (f + |f|)/2$, 則 (21) 式成爲

$$\ddot{\lambda} = \langle f \rangle \dot{f}\dot{\lambda} - (1-f)\dot{\lambda}^2, \tag{23}$$

而則 (22) 成爲

$$\langle f \rangle \dot{f} \le (1-f)\dot{\lambda}, \quad \dot{\lambda} \ge 0, \quad \langle f \rangle \dot{f}\dot{\lambda} = (1-f)\dot{\lambda}^2. \tag{24}$$

互補三元 (21) 描述降伏厚面開關機制, 是平衡開關機制。而 (23) 式含有變率效應, 是非平衡開關機制。

5.非平衡耗散動態系統

有了上節之 (23) 式, 可以參考動態系統

$$\dot{\mathbf{x}} = \mathcal{V}(t, \mathbf{x}, \mathbf{u}), \tag{25}$$

$$\mathbf{y} = \mathcal{Y}(t, \mathbf{x}, \mathbf{u}), \tag{26}$$

其中$\mathbf{x}$, $\mathbf{u}$, $\mathbf{y}$ 分別爲狀態向量, 輸入向量, 輸出向量, 寫出非平衡耗散系統的方程式如下:

$$\frac{d\mathbf{x}}{d\lambda} = \mathcal{V}(\lambda, \mathbf{x}, \mathbf{u}), \tag{27}$$

$$\mathbf{y} = \mathcal{Y}(\lambda, \mathbf{x}, \mathbf{u}), \tag{28}$$

$$\ddot{\lambda} = \langle f \rangle \dot{f} \dot{\lambda} - (1 - f)\dot{\lambda}^2 \tag{29}$$

6.平衡耗散動態系統

相對上節之非平衡耗散動態系統, 其對應之平衡耗散動態系統可以如下之方式描述:

$$\frac{d\mathbf{x}}{d\lambda} = \mathcal{V}(\lambda, \mathbf{x}, \mathbf{u}), \tag{30}$$

$$\mathbf{y} = \mathcal{Y}(\lambda, \mathbf{x}, \mathbf{u}), \tag{31}$$

$$\langle f \rangle \dot{f} \leq (1 - f)\dot{\lambda}, \quad \dot{\lambda} \geq 0, \quad \langle f \rangle \dot{f} \dot{\lambda} = (1 - f)\dot{\lambda}^2, \tag{32}$$

顯然, 這個動態系統具有開關機制, 而且是降伏厚面式的, 相當光滑, 不含有過方或振盪之不合理現象。

7.變分原理

以剛塑性組成律爲例, 組成律

$$Q_y \dot{\boldsymbol{q}} = \dot{\lambda} \boldsymbol{Q}, \tag{33}$$

$$\|\boldsymbol{Q}\| \leq Q_y,\ \dot{\lambda} \geq 0,\ \|\boldsymbol{Q}\|\dot{\lambda} = Q_y \dot{\lambda},$$

可以改寫爲

$$\begin{array}{l} \max \boldsymbol{Q}^T \dot{\boldsymbol{q}} \quad \text{w.r.t. } \boldsymbol{Q} \\ \text{subject to } \|\boldsymbol{Q}\| \leq Q_y \end{array}$$

若是完全彈塑性組成律

$$\frac{1}{k}\dot{\boldsymbol{Q}} + \dot{\lambda}\frac{1}{Q_y}\boldsymbol{Q} = \dot{\boldsymbol{q}}, \tag{34}$$

$$\|\boldsymbol{Q}\| \leq Q_y,\ \dot{\lambda} \geq 0,\ \|\boldsymbol{Q}\|\dot{\lambda} = Q_y \dot{\lambda},$$

則可以改寫爲

$$\begin{array}{l} \max \boldsymbol{Q}^T (\dot{\boldsymbol{q}} - \frac{1}{k}\dot{\boldsymbol{Q}}) \quad \text{w.r.t } \boldsymbol{Q}, \\ \text{subject to } \|\boldsymbol{Q}\| \leq Q_y. \end{array}$$

研究的下一個目標是尋求上二節之動態系統, 可否以變分原理描述之, 並由之彰顯正則架構。

8. 連體耗散運動

單自由度耗散運動正則形式, 容易推廣爲一長串一維之多自由度系統, 然後將之場論化爲承受軸向變形的桿子。然而往三維連體推廣卻不再單純 。本計畫參考現代組成律理論, 張量函數理論 (包括不變量理論) , 對偶理論 [11, 12, 13, 14] 來進行研究。在場論化的研究中, 先看線性彈性力學支配方程式, 將其化爲三維向量形式, 除了材料性質外, 使各變數只有三維向量, 沒有張量, 如應力張量或應變張量, 可以有各種微分或積分運算子。如等向性、擬靜態, 無外力下, 支配方程式爲:

$$\nabla^2 \mathbf{u} + \frac{1}{1 - \nu}\nabla(\nabla \cdot \mathbf{u}) = 0 \tag{35}$$

其中 ν 爲波松比, ∇ 爲梯度運算子, $\nabla\cdot$ 爲散度運算子, $\nabla^2 = \nabla\cdot\nabla$ 爲拉氏運算子, 此式有 Boussinesq-Galerkin 通解, Papkovich-Neuber 通解, Tep-Naghdi-Hsu 通解, Kelvin 特解。擬研究 (35) 加入外力及慣性項後之正則形式, 及其相應通解、特解之形式。由此出發嘗試得出對應之耗散運動正則形式。一般認爲 (例如參見: [16] 的 p.98, [9]) 哈氏量、哈氏運動方程式對研究統計力學 (降伏厚面、分佈元素模式)、微擾理論、量子力學 (奈米力學) 較爲方便, 對於研究場論 (連體力學), 則以拉氏量、拉氏運動方程式較佳。注意對應式 (19)、(20) 之拉氏量、拉氏運動方程式不難寫出來。重點是避開各種應力 、應力變率、應變、應變率之張量定義, 而以運算子之非線性爲之, 並保持對偶關係, 正則關係。

9. 初值識別與束制識別

一般材料組成律之實驗識別都假設其初始時刻爲零值時間, 爲達到此條件, 必須再對試體處理 (退火、重模), 使其處於零值態, 此動作便已破壞材料當下眞實之力學狀態, 而且處理過程會花費額外的金錢與時間。在工程實務上, 材料構件往往經過冷加工、搬運、組裝、施工等過程, 因此在工程中檢測工址工材之實際狀態, 進而預測遭受外力 (位移) 作用時材料之反應歷時, 對於材料之力學行爲掌握有重大的幫助。

根據黏彈性理論可知, 線性不老化材料之積分組成律可表示爲

$$\epsilon(t) = \int_{t_0}^{t} C(t-\tau)d\sigma(\tau) \tag{36}$$

與

$$\sigma(t) = \int_{t_0}^{t} E(t-\tau)d\epsilon(\tau) \tag{37}$$

其中 $C(t)$ 爲潛變順數, $E(t)$ 爲鬆弛模數, t 爲現在時刻, t_0 爲零值時間。通常, 材料都是處於非零值態, 故式 (36) 可改寫爲

$$\epsilon(t) = \int_{t_0}^{t_i} C(t-\tau)d\sigma(\tau) + \int_{t_i}^{t} C(t-\tau)d\sigma(\tau) \tag{38}$$

等號右邊第一項代表材料的初值狀態,

$$\epsilon_{t_i}(t) := \int_{t_0}^{t_i} C(t-\tau)d\sigma(\tau) \tag{39}$$

t_i 爲初始時間。相同地, 式 (37) 可以改寫爲此形式。注意: 式 (38) 中等號右邊兩項皆爲 t 之遞減函數。固然有可能因爲錯估初值狀態函數 $\epsilon_{t_i}(t)$ 之遞減性, 而將此不老化材料誤判成老化材料。由此可瞭解初值狀態的確定, 對於黏彈性材料之力學行爲掌握的確估有重大份量。而對於相似性極高卻又更複雜之彈塑性等保守耗散兩相力學系統, 其影響更爲顯著。本計畫利用線性系統可觀測之觀念來處理初值識別之問題。

線性時變系統之狀態空間表示式如下

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t) \tag{40}$$

$$\mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)\mathbf{u}(t) \tag{41}$$

其中 $\mathbf{x}: \mathbb{R} \to \mathbb{R}^x$ 爲狀態, $\mathbf{y}: \mathbb{R} \to \mathbb{R}^y$ 爲輸出, $\mathbf{u}: \mathbb{R} \to \mathbb{R}^u$ 爲輸入, 而 $\mathbf{A}: \mathbb{R} \to \mathbb{R}^{x\times x}$, $\mathbf{B}: \mathbb{R} \to \mathbb{R}^{x\times u}$, $\mathbf{C}: \mathbb{R} \to \mathbb{R}^{y\times x}$, $\mathbf{D}: \mathbb{R} \to \mathbb{R}^{y\times u}$ 則爲系統特性。若此系統在 $t \in (t_i, t_f]$ 爲可觀測, 則存在一非奇異 $\mathcal{O}$

$$\mathcal{O}(t_i, t_f) = \int_{t_i}^{t_f} \Phi^T(t, t_i)\mathbf{C}^T(t)\mathbf{C}(t)\Phi(t, t_i)dt, \quad t_i < t_f < \infty \tag{42}$$

其中 $\Phi(t, t_i)$ 爲狀態轉換矩陣。因此我們可以在任一時間 $t \in (t_0, t_f]$, 推求得初始時刻之狀態 $\mathbf{x}(t_0)$。

若一材料之力學行爲可表示成式 (40)、(41), 並存在一 $\mathcal{O}(t_i, t_f)$ 滿足式 (42), 我們便可以在任意時刻 t, $t \in (t_0, t_f]$ 推求的初始狀態 $\mathbf{x}(t_i)$。實則不然, 因彈塑性等保守耗散兩相力學系統實爲一非線性 、非平滑動態系統, 根據先前之研究我們可將如此複雜的行爲, 簡化爲位在歐氏空間與閔氏時空之混合時空的兩組線性系統相互切換來描述。此時是否可以在任意時刻 (不論切換與否、切換次數) 推求得初始狀態, 並且是否可利用彈

塑性組成模式之對稱群或前述正則形式辛群觀念來建立可觀測 (初值識別) 之準則, 有待本計畫第二年來釐清。若能再將降伏厚面開關機制加入, 則此結果更令人期待。

對於模式識別與參數估測, 如何將以上觀念加入並且整合物理特性 (如因果律、能量之存儲與耗散、記憶消退等) 所提供的束制, 進一步建立在非光滑、混合時空的識別理論, 本計畫第二年也會做深入的探討。

10. 保群算法之識別技術

本計劃第一年以保群算法識別熱傳導係數或擴散係數。將熱傳偏微分方程之空間部分離散, 利用微分方程

$$\dot{\boldsymbol{u}} = \boldsymbol{f}(\boldsymbol{u}, t)$$

的增廣式

$$\frac{d}{dt}\left\{\begin{array}{c}\boldsymbol{u}\\ \|\boldsymbol{u}\|\end{array}\right\} = \left[\begin{array}{cc}\mathbf{0} & \frac{\boldsymbol{f}}{\|\boldsymbol{u}\|}\\ \frac{\boldsymbol{f}^T}{\|\boldsymbol{u}\|} & 0\end{array}\right]\left\{\begin{array}{c}\boldsymbol{u}\\ \|\boldsymbol{u}\|\end{array}\right\}$$

之保群特性, 來建構識別算法, 其在效率與精度都有驚人的成果。本計劃第二年將會對此保群算法做更詳盡的研究。

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