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Electro-Elastic Modeling of Annular Piezoceramic Actuating Disk Transducers

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ABSTRACT: The static and dynamic characteristics of the annular piezoceramic actuating disk transducers are theoretically modeled and analyzed. The transducer consists of an annular piezoceramic disk laminated on an unequal radius isotropic elastic disk to form the asymmetric configuration. Piezoceramic actuator excites the transducer in extension and flexure coupled vibrations. An electroelastic theory is developed to analyze the mechanical, electrical and electromechanical behaviors. Resonant and antiresonant frequencies and the electromechanical coupling coefficients are computed and shown sensitive to geometric parameters of the elastic and piezoceramic materials. Numerical results are presented by the easy-to-use figures. Comparisons between experimental measurements of the transducer's resonant and antiresonant frequencies and modeling results exhibit a good agreement.

INTRODUCTION

THE piezoelectric materials with their strong electromechanical coupling effects make them possible to be used in the scope of sensors and actuators. The devices based on piezoelectricity have been expanded rapidly in the applications such as ultrasonic transducer (Mitra, 1996), vibratory gyrosensor (Sato et al., 1996), actuators (Matsko, Xu and Newnham, 1995; Liang and Rogers, 1997) in distributed form (Lee, 1990; Tzou and Zhong, 1993; Chang and Chou, 1999; Chang and Tung, 1999), a self-sensing actuator (Dosch, Inman and Garcia, 1992) to suppress the vibrations (Yang and Jeng, 1997), and to control the shape (Batra and Liang, 1997), and precision positioning mechanisms (Chang and Du, 1998). To measure the practicality of a piezoelectric element, the efficiency of electromechanical energy conversion between elastic and dielectric ones is an essential index (Chang, Rogacheva and Chou, 1995). The bimorphic disks made of one piezoelectric and one metal layers with the same radius and with different radii were studied under the simplification that both materials had the same Poisson's ratios (Rudnitskii, Sharapov and Shul'ga, 1990; Evseichik et al., 1991). This hypothesis is also found in Adelman and Stavsky (1980) and usually is suitable for limited applications.

This paper studies the static displacements and electromechanical coupling coefficient (EMCC) of the elastic-piezoceramic laminated disk transducer. The transducer consists of an annular piezoceramic disk laminated on an unequal radius isotropic elastic metal disk for actuator applications. The Poisson's ratios of piezoceramic and elastic material are not treated identical. The major motion of the

transducer is the extensional and flexural coupled vibration under a sinusoidal electrical excitation.

An electroelastic theory is developed first to analyze the static and dynamic behaviors of the composite piezoceramic plates, including static displacements, resonant frequencies, antiresonant frequencies and EMCC. This theory was formulated by modifying the ordinary plate theory to include the piezoelectric effects. The distributed form of electric potential and electric displacement in the electrical field direction for each piezoceramic disk was assumed quadratic and constant, respectively with respect to the thickness coordinate. These characteristics are numerically evaluated for different values of geometric variables, such as thickness and radius ratios of the piezoceramic and elastic materials.

THEORY

Basic Electroelastic Theory

A complete electroelastic theory consists of the equilibrium equations, the strain-displacement relationships, the constitutive relations and the electrostatic equations. For piezoceramic materials of class 6 mm and polarized in the thickness direction α_3 , the constitutive relations (Rogacheva, 1994) are:

$$S_{11} = s_{11}^E T_{11} + s_{12}^E T_{22} + s_{13}^E T_{33} + d_{31} E_3$$

$$S_{22} = s_{12}^E T_{11} + s_{11}^E T_{22} + s_{13}^E T_{33} + d_{31} E_3$$

$$S_{33} = s_{13}^E (T_{11} + T_{22}) + s_{33}^E T_{33} + d_{33} E_3$$

(continued)

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$$\begin{aligned}
S_{23} &= s_{44}^E T_{23} + d_{15} E_2 \\
S_{13} &= s_{44}^E T_{13} + d_{15} E_1 \\
S_{12} &= s_{66}^E T_{12} \\
D_1 &= d_{15} T_{13} + \epsilon_{11}^T E_1 \\
D_2 &= d_{15} T_{23} + \epsilon_{11}^T E_2 \\
D_3 &= d_{31} (T_{11} + T_{22}) + d_{33} T_{33} + \epsilon_{33}^T E_3
\end{aligned} \quad (1)$$

where S_{ij} , T_{ij} , D_i and E_i denote the components of strain, stress, electric displacements and electric fields, respectively, and s_{ij}^E , d_{ij} and ϵ_{ij}^T denote the elastic compliance constants at constant electric field, the piezoceramic constants, and the dielectric constant at constant stress field, respectively. The constitutive relations for isotropic elastic materials are:

$$\begin{aligned}
S_{11} &= \frac{1}{E} (T_{11} - \nu_m T_{22} - \nu_m T_{33}) \\
S_{22} &= \frac{1}{E} (T_{22} - \nu_m T_{11} - \nu_m T_{33}) \\
S_{33} &= \frac{1}{E} (T_{33} - \nu_m T_{11} - \nu_m T_{22}) \\
S_{23} &= \frac{2(1+\nu_m)}{E} T_{23} \\
S_{13} &= \frac{2(1+\nu_m)}{E} T_{13} \\
S_{12} &= \frac{2(1+\nu_m)}{E} T_{12}
\end{aligned} \quad (2)$$

where E and ν_m are the Young's modulus and Poisson's ratio of the elastic material. The general electrostatic equations are:

$$\begin{aligned}
\nabla \cdot \bar{D} &= 0 \\
\bar{E} &= -\nabla \psi
\end{aligned} \quad (3)$$

where ψ is the electrical potential. Having chosen the mutually perpendicular lines on the middle surface as coordinates α_1 and α_2 , and the normal to the middle surface as the third coordinate α_3 , for flat plates without surface traction, the equilibrium equations are:

$$\begin{aligned}
&\frac{1}{A_1} \frac{\partial T_{11}}{\partial \alpha_1} + \frac{1}{A_2} \frac{\partial T_{12}}{\partial \alpha_2} + \frac{1}{A_1 A_2} \frac{\partial A_2}{\partial \alpha_1} (T_{11} - T_{22}) \\
&\quad + \frac{2T_{12}}{A_1 A_2} \frac{\partial A_1}{\partial \alpha_2} + \frac{\partial T_{13}}{\partial \alpha_3} = \rho \frac{\partial^2 U_1}{\partial t^2} \\
&\frac{1}{A_2} \frac{\partial T_{22}}{\partial \alpha_2} + \frac{1}{A_1} \frac{\partial T_{21}}{\partial \alpha_1} + \frac{1}{A_1 A_2} \frac{\partial A_1}{\partial \alpha_2} (T_{22} - T_{11}) \\
&\quad + \frac{2T_{12}}{A_1 A_2} \frac{\partial A_2}{\partial \alpha_1} + \frac{\partial T_{23}}{\partial \alpha_3} = \rho \frac{\partial^2 U_2}{\partial t^2} \\
&\frac{1}{A_1} \frac{\partial T_{13}}{\partial \alpha_1} + \frac{1}{A_2} \frac{\partial T_{23}}{\partial \alpha_2} + \frac{1}{A_1 A_2} \frac{\partial A_2}{\partial \alpha_1} T_{13} \\
&\quad + \frac{1}{A_1 A_2} \frac{\partial A_1}{\partial \alpha_2} T_{23} + \frac{\partial T_{33}}{\partial \alpha_3} = \rho \frac{\partial^2 U_3}{\partial t^2}
\end{aligned} \quad (4)$$

where A_i , U_i and ρ are the Lamé parameters, the components of displacements, and material density, respectively. The strain-displacement relationships are:

$$\begin{aligned}
S_{11} &= \frac{1}{A_1} \left(\frac{\partial U_1}{\partial \alpha_1} + \frac{U_2}{A_2} \frac{\partial A_1}{\partial \alpha_2} \right) \\
S_{22} &= \frac{1}{A_2} \left(\frac{\partial U_2}{\partial \alpha_2} + \frac{U_1}{A_1} \frac{\partial A_2}{\partial \alpha_1} \right) \\
S_{33} &= \frac{\partial U_3}{\partial \alpha_3} \\
S_{12} &= \frac{A_1}{A_2} \frac{\partial}{\partial \alpha_2} \left(\frac{U_1}{A_1} \right) + \frac{A_2}{A_1} \frac{\partial}{\partial \alpha_1} \left(\frac{U_2}{A_2} \right) \\
S_{13} &= \frac{\partial U_1}{\partial \alpha_3} + \frac{1}{A_1} \frac{\partial U_3}{\partial \alpha_1} \\
S_{23} &= \frac{\partial U_2}{\partial \alpha_3} + \frac{1}{A_2} \frac{\partial U_3}{\partial \alpha_2}
\end{aligned} \quad (5)$$

HYPOTHESES

To solve this electroelastic problem, four assumptions are proposed. The first two are Kirchhoff-Love hypotheses used in classical plate theory for elastic plates and the others are used for piezoceramic materials particularly. The first hypothesis is that the normal stress acting on the surface elements parallel to the middle surface is small and can be neglected as compared to other stresses:

$$T_{33} = 0 \quad (6)$$

The second hypothesis is that a line normal to the middle surface before deformation remains perpendicular to the strained surface and is not extended after deformation:

$$S_{13} = S_{23} = 0 \quad (7)$$

The third hypothesis is that the electrical potential varies with thickness by the square law:

$$\psi = \psi^0 + \alpha_3 \psi^1 + (\alpha_3)^2 \psi^2 \quad (8)$$

where superscript 0, 1 and 2 denote that functions are coefficients of constant, α_3 , and $(\alpha_3)^2$, respectively, and ψ^0 , ψ^1 and ψ^2 are independent of α_3 . The fourth hypothesis is that the electric displacement is constant along the plate thickness:

$$D_3 = D_3^0 \quad (9)$$

where D_3^0 is a function independent of α_3 . Besides, the membrane forces N_{ij} and bending moments M_{ij} are considered instead of stress in this electroelastic plate theory and they are defined as:

$$\begin{aligned} N_{ij} &= \int_{\alpha_3} T_{ij} d\alpha_3 \\ M_{ij} &= \int_{\alpha_3} T_{ij} \alpha_3 d\alpha_3 \end{aligned} \quad (10)$$

THEORY FOR ELASTIC-PIEZOCERAMIC DISKS

An electroelastic theory can be derived from Equations (1) to (10), but only those relations for the particular elastic-piezoceramic disks are shown in this paper. Figure 1 shows the annular piezoceramic actuating disk transducer. This transducer consists of an isotropic solid elastic disk with thickness h_m and an annular piezoceramic disk with thickness h_p . The outside radius of piezoceramic disk r_o is not larger than that of the elastic disk R . The Poisson's ratios of piezoceramic and elastic materials are not necessarily identical. The major surfaces of the annular piezoceramic disk with the outside radius and inside radius r_i are fully covered by thin electrodes with neglected thickness. The piezoceramic is polarized in the thickness direction as shown by the arrowhead in Figure 1. The elastic disk is treated as an electrode to the piezoceramic disk. As shown in Figure 1, the cylindrical coordinates r , θ and z are used for derivation. Moreover, for convenience of analysis the transducer is divided into three sectors as shown in Figure 1. Sector one is the proportion whose radius is larger than r_o and consists of the elastic layer only. Sector two includes all piezoceramic layers and part of the elastic layer. Sector three is the proportion whose radius is less than r_i and consists of the elastic layer only. In Figure 1, the reference planes of sectors one, two and three are indicated by r_1 , r_2 , r_3 , respectively. If the distance between the reference plane of sector two and the bottom surface of the elastic layer is $h_n^{(2)}$, the electrical boundary conditions can be expressed as:

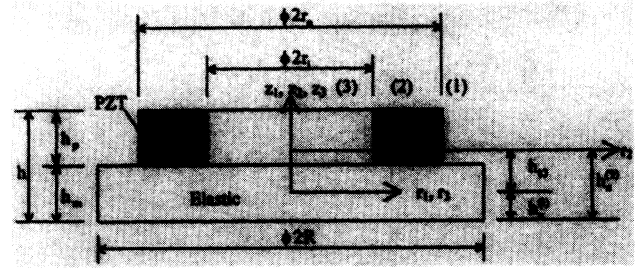


Figure 1. Side view of the annular piezoceramic actuating disk transducer. An annular piezoceramic disk is laminated on a solid elastic disk.

$$\begin{aligned} \psi \Big|_{z_3=h-h_n^{(2)}} &= V \\ \psi \Big|_{z_3=h_m-h_n^{(2)}} &= -V \end{aligned} \quad (11)$$

where V is the applied voltage. Under this asymmetric configuration and the applied sinusoidal electric excitation, the major motion of the disk caused by extension of the piezoceramic disk is the coupled extension and flexure vibration. In addition, the dynamic responses in the θ direction are omitted due to axis-symmetry:

$$\frac{\partial f}{\partial \theta} = 0 \quad (12)$$

where f is any physical quantity of the transducer. The physical quantities of the three sectors are related by the kinematic and static matching conditions at the boundary between them:

$$\begin{aligned} u_z^{(1)} \Big|_{r=r_o} &= u_z^{(2)} \Big|_{r=r_o} \\ u_r^{(1)} \Big|_{r=r_o} &= u_r^{(2)} \Big|_{r=r_o} - h_{12} \beta_r^{(2)} \\ \beta_r^{(1)} \Big|_{r=r_o} &= \beta_r^{(2)} \Big|_{r=r_o} \\ N_{rr}^{(1)} \Big|_{r=r_o} &= N_{rr}^{(2)} \Big|_{r=r_o} \\ Q_r^{(1)} \Big|_{r=r_o} &= Q_r^{(2)} \Big|_{r=r_o} \\ M_{rr}^{(1)} \Big|_{r=r_o} &= M_{rr}^{(2)} \Big|_{r=r_o} + h_{12} N_{rr}^{(2)} \\ u_z^{(2)} \Big|_{r=r_i} &= u_z^{(3)} \Big|_{r=r_i} \end{aligned}$$

(continued)

$$u_r^{(2)} \Big|_{r=r_i} = u_r^{(3)} \Big|_{r=r_i} + h_{12} \beta_r^{(3)} \quad S_{rr} = S_{rr}^0 + z k_{rr} \quad (17)$$

$$\beta_r^{(2)} \Big|_{r=r_i} = \beta_r^{(3)} \Big|_{r=r_i} \quad S_{\theta\theta} = S_{\theta\theta}^0 + z k_{\theta\theta}$$

$$N_{rr}^{(2)} \Big|_{r=r_i} = N_{rr}^{(3)} \Big|_{r=r_i} \quad (13)$$

$$Q_r^{(2)} \Big|_{r=r_i} = Q_r^{(3)} \Big|_{r=r_i}$$

$$M_{rr}^{(2)} \Big|_{r=r_i} = M_{rr}^{(3)} \Big|_{r=r_i} - h_{12} N_{rr}^{(3)}$$

where

$$\beta_r^{(i)} = -\frac{\partial u_z^{(i)}}{\partial r} \quad (14)$$

Q_r is the transverse shear force and h_{12} is the distance between reference planes of sectors one and two as shown in Figure 1. Superscripts (1), (2) and (3) denote that physical quantities belong to sectors one, two and three, respectively. Because it is symmetric for sectors one and three but is asymmetric for sector two with respect to their own neutral planes, the locations of the chosen reference planes are different. Since sectors one and three are simply the elastic material, their reference planes have equal distance to the bottom surface of the elastic layer, that is, $h_n^{(3)} = h_n^{(1)}$. To determine the dynamic responses, boundary conditions of the transducer should be denoted. If the transducer is free from constraints, the boundary conditions are:

$$\begin{aligned} N_{rr}^{(1)} \Big|_{r=R} &= 0 \\ Q_r^{(1)} \Big|_{r=R} &= 0 \\ M_{rr}^{(1)} \Big|_{r=R} &= 0 \end{aligned} \quad (15)$$

By using Equations (1) to (12), the equilibrium equations can be simplified. From Kirchhoff-Love hypotheses, the displacement in the α_i direction, U_i , in any place of plates can be expressed as:

$$\begin{aligned} U_1(\alpha_1, \alpha_2, \alpha_3) &= u_1(\alpha_1, \alpha_2) + \alpha_3 \beta_1(\alpha_1, \alpha_2) \\ U_2(\alpha_1, \alpha_2, \alpha_3) &= u_2(\alpha_1, \alpha_2) + \alpha_3 \beta_2(\alpha_1, \alpha_2) \\ U_3(\alpha_1, \alpha_2, \alpha_3) &= u_3(\alpha_1, \alpha_2) \end{aligned} \quad (16)$$

where u_i is the displacement in the α_i direction of the reference plane in the thickness direction. Using cylindrical coordinates and substituting Equations (12) and (16) into Equation (5), using the thickness coordinate, the strains for all three sectors are expressed as:

where the membrane strains S_{ij}^0 are:

$$\begin{aligned} S_{rr}^0 &= \frac{\partial u_r}{\partial r} \\ S_{\theta\theta}^0 &= \frac{u_r}{r} \end{aligned} \quad (18)$$

and the bending strains k_{ij} are:

$$\begin{aligned} k_{rr} &= \frac{\partial \beta_r}{\partial r} = -\frac{\partial^2 u_z}{\partial r^2} \\ k_{\theta\theta} &= \frac{\beta_r}{r} = -\frac{1}{r} \frac{\partial u_z}{\partial r} \end{aligned} \quad (19)$$

To simplify equilibrium equations, we first change coordinates of Equation (4) into cylindrical ones and combine Equations (6), (7), (12) and (16) into Equation (4), then integrate Equation (4) with respect to the thickness coordinate, and finally multiply Equation (4) with the thickness coordinate and integrate results with respect to the thickness coordinate. At the same time, Equation (10) was used to keep derived equations more clearly. Besides, the considered transducer is thin in thickness compared with its radius and the rotating inertia is omitted. After the above procedures, the equilibrium equations can be formulated as:

$$\begin{aligned} \frac{\partial N_{rr}^{(i)}}{\partial r} + \frac{1}{r} (N_{rr}^{(i)} - N_{\theta\theta}^{(i)}) &= R_0^{(i)} \frac{\partial^2 u_r^{(i)}}{\partial t^2} + R_1^{(i)} \frac{\partial^2 \beta_r^{(i)}}{\partial t^2} \\ \frac{1}{r} \frac{\partial}{\partial r} (r Q_r^{(i)}) &= R_0^{(i)} \frac{\partial^2 u_z^{(i)}}{\partial t^2}, \quad i = 1, 2, 3 \end{aligned} \quad (20)$$

where

$$\begin{aligned} Q_r^{(i)} &= \frac{\partial M_{rr}^{(i)}}{\partial r} + \frac{1}{r} (M_{rr}^{(i)} - M_{\theta\theta}^{(i)}) - R_1^{(i)} \frac{\partial^2 u_r^{(i)}}{\partial t^2} \\ R_0^{(1)} &= R_0^{(3)} = \rho_m h_m \\ R_0^{(2)} &= \rho_m h_m + \rho_p (h - h_m) \\ R_1^{(1)} &= R_1^{(3)} = \frac{1}{2} \rho_m h_m (h_m - 2h_n^{(1)}) \end{aligned} \quad (21)$$

$$R_1^{(2)} = \frac{1}{2} [\rho_m h_m (h_m - 2h_n^{(2)}) + \rho_p (h - h_m)(h + h_m - 2h_n^{(2)})]$$

and ρ_m and ρ_p are the density of elastic and piezoceramic material, respectively. The integration in Equation (10) can be carried out by combining Equations (1), (2), (3), (8), (9) and (17)–(19). During the derivation, the stresses were all expressed in terms of membrane and bending strains. The constant and linear coefficients such as those in Equation (8) were used to separate the parameters in different order. The derivation is rather tedious, we omit the detail and only write the results as:

$$\begin{bmatrix} N_{rr}^{(i)} \\ N_{\theta\theta}^{(i)} \\ M_{rr}^{(i)} \\ M_{\theta\theta}^{(i)} \end{bmatrix} = \begin{bmatrix} A_{11}^{(i)} & A_{12}^{(i)} & B_{11}^{(i)} & B_{12}^{(i)} \\ A_{12}^{(i)} & A_{11}^{(i)} & B_{12}^{(i)} & B_{11}^{(i)} \\ B_{11}^{(i)} & B_{12}^{(i)} & D_{11}^{(i)} & D_{12}^{(i)} \\ B_{12}^{(i)} & B_{11}^{(i)} & D_{12}^{(i)} & D_{11}^{(i)} \end{bmatrix} \cdot \begin{bmatrix} S_{rr}^{(i)} \\ S_{\theta\theta}^{(i)} \\ k_{rr}^{(i)} \\ k_{\theta\theta}^{(i)} \end{bmatrix} + \begin{bmatrix} E_{11}^{(i)} \\ E_{11}^{(i)} \\ F_{11}^{(i)} \\ F_{11}^{(i)} \end{bmatrix} \cdot V, \quad (22)$$

$i = 1, 2, 3$

where for sectors one ($i = 1$) and three ($i = 3$), the coefficients are:

$$\begin{aligned} A_{11}^{(i)} &= \frac{E_m h_m}{1 - \nu_m^2} \\ A_{12}^{(i)} &= \frac{E_m \nu_m h_m}{1 - \nu_m^2} \\ B_{11}^{(i)} &= \frac{E_m h_m (h_m - 2h_n^{(1)})}{2(1 - \nu_m^2)} \\ B_{12}^{(i)} &= \frac{E_m \nu_m h_m (h_m - 2h_n^{(1)})}{2(1 - \nu_m^2)} \\ D_{11}^{(i)} &= \frac{E_m h_m [h_m^2 - 3h_m h_n^{(1)} + 3(h_n^{(1)})^2]}{3(1 - \nu_m^2)} \\ D_{12}^{(i)} &= \frac{E_m \nu_m h_m [h_m^2 - 3h_m h_n^{(1)} + 3(h_n^{(1)})^2]}{3(1 - \nu_m^2)} \\ E_{11}^{(i)} &= 0 \\ F_{11}^{(i)} &= 0 \end{aligned} \quad (23)$$

and for sector two, the coefficients are:

$$\begin{aligned} A_{11}^{(2)} &= \frac{E_m h_m}{1 - \nu_m^2} + \frac{h_p}{s_{11}^E (1 - \nu_p^2)} \\ A_{12}^{(2)} &= \frac{E_m \nu_m h_m}{1 - \nu_m^2} + \frac{\nu_p h_p}{s_{11}^E (1 - \nu_p^2)} \end{aligned}$$

$$\begin{aligned} B_{11}^{(2)} &= \frac{E_m h_m (h_m - 2h_n^{(2)})}{2(1 - \nu_m^2)} + \frac{(h - h_m)(h + h_m - 2h_n^{(2)})}{2s_{11}^E (1 - \nu_p^2)} \\ B_{12}^{(2)} &= \frac{E_m \nu_m h_m (h_m - 2h_n^{(2)})}{2(1 - \nu_m^2)} + \frac{\nu_p (h - h_m)(h + h_m - 2h_n^{(2)})}{2s_{11}^E (1 - \nu_p^2)} \\ D_{11}^{(2)} &= \frac{E_m h_m [h_m^2 - 3h_m h_n^{(2)} + 3(h_n^{(2)})^2]}{3(1 - \nu_m^2)} \\ &+ \frac{h - h_m}{12s_{11}^E (1 - \nu_p^2)(1 - 2B)} \{-3B(1 + \nu_p)(h + h_m - 2h_n^{(2)})^2 \\ &+ 4[(h^2 + hh_m + h_m^2) - 3(h + h_m)h_n^{(2)} + 3(h_n^{(2)})^2] \\ &\times (1 - B + B\nu_p)\} \\ D_{12}^{(2)} &= \frac{E_m \nu_m h_m [h_m^2 - 3h_m h_n^{(2)} + 3(h_n^{(2)})^2]}{3(1 - \nu_m^2)} \\ &+ \frac{h - h_m}{12s_{11}^E (1 - \nu_p^2)(1 - 2B)} \{-3B(1 + \nu_p)(h + h_m - 2h_n^{(2)})^2 \\ &+ 4[(h^2 + hh_m + h_m^2) - 3(h + h_m)h_n^{(2)} + 3(h_n^{(2)})^2] \\ &\times (\nu_p + B - B\nu_p)\} \\ E_{11}^{(2)} &= \frac{2d_{31}}{s_{11}^E (1 - \nu_p)} \\ F_{11}^{(2)} &= \frac{d_{31}(h + h_m - 2h_n^{(2)})}{s_{11}^E (1 - \nu_p)} \end{aligned} \quad (24)$$

where

$$\begin{aligned} B &= \frac{d_{31}^2}{\epsilon_{33}^T s_{11}^E (1 - \nu_p)} = \frac{1}{2} k_p^2 \\ \nu_p &= -\frac{s_{12}^E}{s_{11}^E} \end{aligned} \quad (25)$$

Using $W_r^{(i)}$ and $W_z^{(i)}$ to denote the displacement magnitudes of the reference plane, the displacements can be expressed as:

$$u_r^{(i)}(r, t) = W_r^{(i)}(r) e^{j\omega t} \quad (26)$$

$$u_z^{(i)}(r, t) = W_z^{(i)}(r) e^{j\omega t}, \quad i = 1, 2, 3$$

where ω is the angular frequency of vibration of the transducer. Finally, substituting Equations (18), (19), (22) and (26) into Equation (20), the governing equations for each sector of the transducer can be expressed as:

$$\begin{bmatrix} L_{11}^{(i)} & L_{12}^{(i)} \\ L_{21}^{(i)} & L_{22}^{(i)} \end{bmatrix} \cdot \begin{bmatrix} W_r^{(i)} \\ W_z^{(i)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad i = 1, 2, 3 \quad (27)$$

where

$$\begin{aligned} L_{11}^{(i)} &= \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{r^2} \right) + (\lambda_r^{(i)})^2 \\ L_{12}^{(i)} &= -b_{rz}^{(i)} \left(\frac{\partial^3}{\partial r^3} + \frac{1}{r} \frac{\partial^2}{\partial r^2} - \frac{1}{r^2} \frac{\partial}{\partial r} \right) - \frac{R_1^{(i)}}{A_{11}^{(i)}} \omega^2 \frac{\partial}{\partial r} \\ L_{21}^{(i)} &= -b_{rz}^{(i)} \left(\frac{\partial^3}{\partial r^3} + \frac{2}{r} \frac{\partial^2}{\partial r^2} - \frac{1}{r^2} \frac{\partial}{\partial r} + \frac{1}{r^3} \right) - \frac{R_1^{(i)}}{D_{11}^{(i)}} \omega^2 \left(\frac{\partial}{\partial r} + \frac{1}{r} \right) \\ L_{22}^{(i)} &= \left(\frac{\partial^4}{\partial r^4} + \frac{2}{r} \frac{\partial^3}{\partial r^3} - \frac{1}{r^2} \frac{\partial^2}{\partial r^2} + \frac{1}{r^3} \frac{\partial}{\partial r} \right) - (\lambda_z^{(i)})^4 \end{aligned} \quad (28)$$

and

$$\begin{aligned} (\lambda_r^{(i)})^2 &= \frac{R_0^{(i)}}{A_{11}^{(i)}} \omega^2 \\ (\lambda_z^{(i)})^4 &= \frac{R_0^{(i)}}{D_{11}^{(i)}} \omega^2 \\ b_{rz}^{(i)} &= \frac{B_{11}^{(i)}}{A_{11}^{(i)}} \\ b_{rz}^{(i)} &= \frac{B_{11}^{(i)}}{D_{11}^{(i)}} \end{aligned} \quad (29)$$

To solve Equation (27), the governing equations can be further simplified without loss of validity by selecting the location of reference plane. The location of reference plane is determined by letting:

$$R_1^{(i)} = 0 \quad (30)$$

and the chosen results from Equation (21) are:

$$\begin{aligned} h_n^{(1)} &= h_n^{(3)} = \frac{1}{2} h_m \\ h_n^{(2)} &= \frac{1}{2} \frac{h_m^2 + (h^2 - h_m^2) \rho_p / \rho_m}{h_m + (h - h_m) \rho_p / \rho_m} \end{aligned} \quad (31)$$

By substituting Equation (31) into Equations (23), (24) and (28), the governing equations of the extensional and flexural motions of sectors one and three are decoupled, and their general solutions ($i = 1, 3$) are:

$$\begin{aligned} W_r^{(i)} &= C_1^{(i)} J_1(\lambda_r^{(i)} r) + C_2^{(i)} Y_1(\lambda_r^{(i)} r) \\ W_z^{(i)} &= C_3^{(i)} J_0(\lambda_z^{(i)} r) + C_4^{(i)} I_0(\lambda_z^{(i)} r) \\ &\quad + C_5^{(i)} Y_0(\lambda_z^{(i)} r) + C_6^{(i)} K_0(\lambda_z^{(i)} r) \end{aligned} \quad (32)$$

where J_n and Y_n are Bessel functions of the first and second kinds of order n , respectively, and I_n and K_n are modified Bessel functions of the first and second kinds of order n , respectively. But for sector two, $B_{11}^{(2)}$ is not zero for the chosen reference plane, and the extensional and flexural motions are still coupled. The detail procedure is similar to (Stavsky and Loewy, 1971) and omitted here, and the general solutions for the governing equations of sector two are:

$$\begin{aligned} W_r^{(2)} &= C_1^{(2)} P_1 J_1(\lambda_1 r) + C_2^{(2)} P_2 J_1(\lambda_2 r) - C_3^{(2)} P_3 I_1(m_3 r) \\ &\quad + C_4^{(2)} P_1 Y_1(\lambda_1 r) + C_5^{(2)} P_2 Y_1(\lambda_2 r) + C_6^{(2)} P_3 K_1(m_3 r) \\ W_z^{(2)} &= C_1^{(2)} J_0(\lambda_1 r) + C_2^{(2)} J_0(\lambda_2 r) + C_3^{(2)} I_0(m_3 r) \\ &\quad + C_4^{(2)} Y_0(\lambda_1 r) + C_5^{(2)} Y_0(\lambda_2 r) + C_6^{(2)} K_0(m_3 r) \end{aligned} \quad (33)$$

where

$$\begin{aligned} P_1 &= \frac{b_{rz}^{(2)} \lambda_1^3}{(\lambda_r^{(2)})^2 - \lambda_1^2} \\ P_2 &= \frac{b_{rz}^{(2)} \lambda_2^3}{(\lambda_r^{(2)})^2 - \lambda_2^2} \\ P_3 &= \frac{-b_{rz}^{(2)} m_3^3}{(\lambda_r^{(2)})^2 + m_3^2} \\ \lambda_1 &= \frac{1}{R} (\mu_1)^{1/2} \\ \lambda_2 &= \frac{1}{R} (\mu_2)^{1/2} \\ m_3 &= \frac{1}{R} |\mu_3|^{1/2} \\ \mu_1 &> \mu_2 > \mu_3 \end{aligned} \quad (34)$$

and μ_i are the roots of the following equation:

$$\begin{aligned} (1 - b_{rz}^{(2)} b_{rz}^{(2)}) \mu^3 - (R \lambda_r^{(2)})^2 \mu^2 \\ - (R \lambda_z^{(2)})^4 \mu + (R \lambda_r^{(2)})^2 (\lambda_z^{(2)*})^4 = 0 \end{aligned} \quad (35)$$

By substituting general solutions of the three sectors into Equations (13), (14) and (16), the matrix form of algebraic equations for coefficients $C_j^{(i)}$ can be written as:

$$[A] \cdot [C] = [F] \quad (36)$$

where

$$\{C\} = \{C_1^{(1)}, C_2^{(1)}, C_3^{(1)}, C_4^{(1)}, C_5^{(1)}, C_6^{(1)}, C_1^{(2)}, C_2^{(2)}, C_3^{(2)}, C_4^{(2)}, C_5^{(2)}, C_6^{(2)}, C_1^{(3)}, C_2^{(3)}, C_3^{(3)}, C_4^{(3)}\}^T \quad (37)$$

with $C_2^{(3)}$, $C_5^{(3)}$, and $C_6^{(3)}$ vanish to meet the finite condition at $r=0$, and $[A]$ is a 15th-order square matrix, $\{F\}$ is a 15×1 matrix. The elements of $[A]$ and $\{F\}$ are listed in the Appendix. From Equation (36), coefficients $C_j^{(i)}$ can be found and the resonant frequencies can be determined by solving

$$\det([A]) = 0 \quad (38)$$

The electric current I over a conduction area S_e can be calculated using:

$$I = \int_{S_e} D_3^0 dS_e \quad (39)$$

By expressing stresses by displacements, and using Equations (1), (2), (3), (8), (9), (11), and (17), Equation (39) can be expressed as:

$$I = -\frac{j\pi\omega d_{31}}{s_{11}^E(1-\nu_p)}(h+h_m-2h_n) \cdot \left[-\frac{2}{h+h_m-2h_n} (r_o W_r^{(2)}|_{r=r_o} - r_i W_r^{(2)}|_{r=r_i}) - (r_o \beta_r^{(2)}|_{r=r_o} - r_i \beta_r^{(2)}|_{r=r_i}) + \frac{2\varepsilon_{33}^T(1-2B)}{F_{11}^{(2)}(h-h_m)}(r_o^2 - r_i^2)V \right] \quad (40)$$

and the antiresonant frequencies can be determined by solving

$$I = 0 \quad (41)$$

In addition to dynamic characteristics, deformation of the transducer due to the static electrical voltage is another important information for the design of actuators. For static situation, by letting $\omega = 0$, Equation (28) becomes:

$$\begin{aligned} L_{11}^{(i)} &= \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{r^2} \right) \\ L_{12}^{(i)} &= -b_{rz}^{(i)} \left(\frac{\partial^3}{\partial r^3} + \frac{1}{r} \frac{\partial^2}{\partial r^2} - \frac{1}{r^2} \frac{\partial}{\partial r} \right) \\ L_{21}^{(i)} &= -b_{rz}^{(i)} \left(\frac{\partial^3}{\partial r^3} + \frac{2}{r} \frac{\partial^2}{\partial r^2} - \frac{1}{r^2} \frac{\partial}{\partial r} + \frac{1}{r^3} \right) \\ L_{22}^{(i)} &= \left(\frac{\partial^4}{\partial r^4} + \frac{2}{r} \frac{\partial^3}{\partial r^3} - \frac{1}{r^2} \frac{\partial^2}{\partial r^2} + \frac{1}{r^3} \frac{\partial}{\partial r} \right) \end{aligned} \quad (42)$$

Since the terms including $R_1^{(i)}$ disappeared, matching the

condition of Equation (30) is not necessary and the location of reference plane is determined by letting

$$B_{11}^{(i)} = 0 \quad (43)$$

The calculated results are:

$$\begin{aligned} h_n^{(1)} &= h_n^{(3)} = \frac{1}{2} h_m \\ h_n^{(2)} &= \frac{1}{2} \frac{h_m^2 + (h^2 - h_m^2)\eta_k}{h_m + (h - h_m)\eta_k} \end{aligned} \quad (44)$$

where

$$\eta_k = \frac{(1-\nu_m^2)}{Es_{11}^E(1-\nu_p^2)} \quad (45)$$

By this arrangement, the terms $b_{rz}^{(i)}$ and $b_{zr}^{(i)}$ in Equation (28) are vanished. The static governing equations of all the sectors have the same form that extension is decoupled from flexural motion. The general solutions under static situation are:

$$\begin{aligned} W_r^{(i)} &= B_1^{(i)} r + \frac{B_{-1}^{(i)}}{r} \\ W_z^{(i)} &= B_2^{(i)} r^2 + B_0^{(i)} + B_e^{(i)} \ln r + \frac{B_{-2}^{(i)}}{r^2} \end{aligned} \quad (46)$$

where coefficients $B_1^{(i)}$, $B_{-1}^{(i)}$, $B_2^{(i)}$, $B_0^{(i)}$, $B_e^{(i)}$, $B_{-2}^{(i)}$ can be found by meeting boundary conditions. Thus, static displacements of transducer can be solved.

EMCC

For piezoelectric elements, the EMCC is an important characteristic. There are several different definitions of EMCC. When vibration frequency is near the resonant region, the dynamic electromechanical coupling coefficient k_d is found by using the formula (Manson, 1950):

$$k_d^2 = \frac{f_a^2 - f_r^2}{f_a^2} \quad (47)$$

where f_r and f_a are the resonant and antiresonant frequencies, respectively.

NUMERICAL RESULTS

To illustrate the relationship between characteristics, material properties and geometric variables such as thickness and radius ratios, the dynamic characteristics of a particular elastic-piezoceramic disk transducer were numerically calculated as an example. We used the properties of

piezoceramic C-82 made by Fuji Ceramic Co., Japan. They are $d_{31} = -260 \times 10^{-12}$ (C/N), $s_{11}^E = 16.95 \times 10^{-12}$ (m²/N), $s_{12}^E = -5.76 \times 10^{-12}$ (m²/N), $\epsilon_{33}^T/\epsilon_0 = 3400$, $\rho_p = 7400$ (kg/m³), and $k_{31} = 0.3640$. The properties of metal Al are $E_m = 70 \times 10^9$ (N/m²), $\nu_m = 0.33$, and $\rho_m = 2700$ (kg/m³), and those of metal Fe are $E_m = 207 \times 10^9$ (N/m²), $\nu_m = 0.292$, and $\rho_m = 7800$ (kg/m³). The mathematical software MATLAB was used to handle the tedious computations. Since dependence of the thickness and radius ratios on the characteristics of the transducer has the similar tendency, we only show the results for thickness ratio $h_m/h = 0.5$. The calculated first resonant frequency constant, $f_r R^2/h$, the first antiresonant frequency constant, $f_a R^2/h$, and the dynamic EMCC, k_d , near the first resonant region are shown in Figures 2 to 4, respectively. The dynamic characteristics of transducers for the arbitrary geometry can be found from these figures. From the numerical results, it is shown that characteristics are sensitive to geometric variables r_i/R and r_o/R . The figures show similar tendency of resonant and antiresonant frequency constants and the EMCC. It's seen that for the inside radius ratio of below 0.5, the outside radius ratio of 0.7 gives the better dynamic performance for this particular transducer. It's also obvious from figures that the best configuration is at r_i

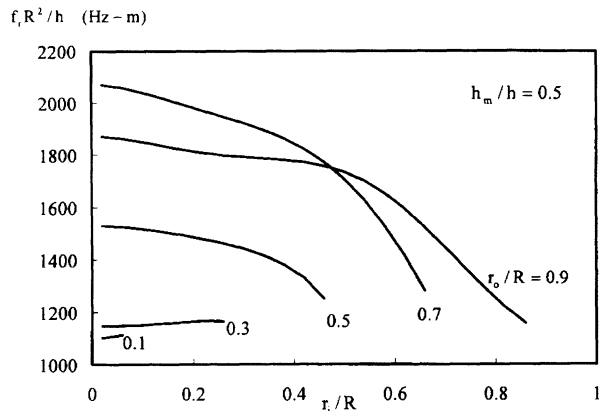


Figure 2. The first resonant frequency constant $f_r R^2/h$ versus radius ratios r_o/R and r_i/R for C-82 and Al materials.

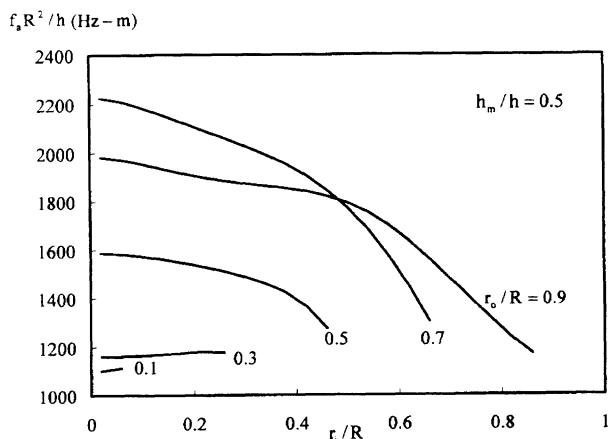


Figure 3. The first antiresonant frequency constant $f_a R^2/h$ versus radius ratios r_o/R and r_i/R for C-82 and Al materials.

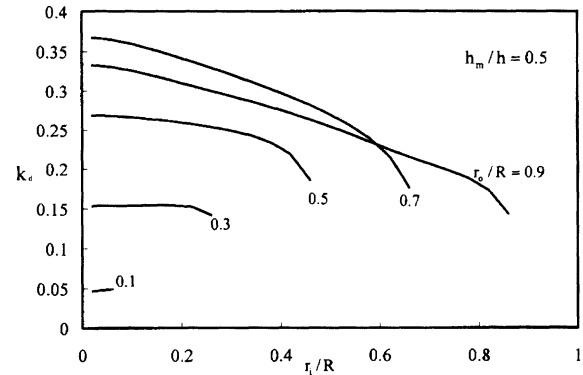


Figure 4. Electromechanical coupling coefficient k_d versus radius ratios r_o/R and r_i/R for C-82 and Al materials.

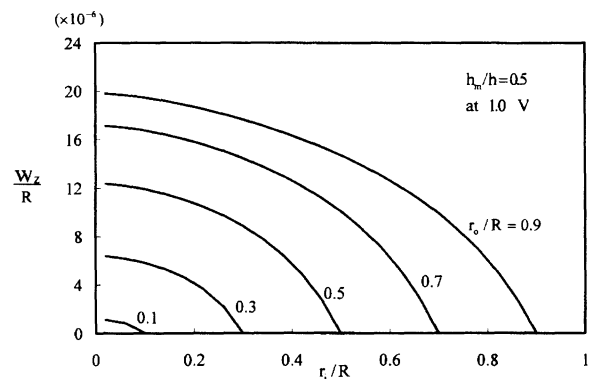


Figure 5. Nondimensional central displacement W_z/R versus radius ratios r_o/R and r_i/R for C-82 and Fe materials.

approaching zero. These figures are useful for the design of electroacoustic devices such as microphones, speakers, filters and high performance displacement actuators.

To study the relationship between static displacements and geometric variables, materials C-82 and Fe are used in numerical analysis and results are shown in Figure 5. The displacements increase monotonically with the increase of the piezoceramic proportion.

EXPERIMENTS

To verify the numerical results, some experiments were conducted. The specimens are composed of C-82 piezoceramic made by Fuji Ceramic Co., Japan and Al elastic materials. The piezoceramic and elastic layers are glued together under pressure and temperature of 40°C. The dimensions of four specimens are listed in Table 1. The experi-

Table 1. Dimensions of specimen.

Specimen	h_m (mm)	h (mm)	r_i (mm)	r_o (mm)	R (mm)
1	0.191	0.395	11.5	45	60
2	0.188	0.406	22.5	45	60
3	0.192	0.404	27.5	30	60
4	0.385	0.620	27.5	30	60

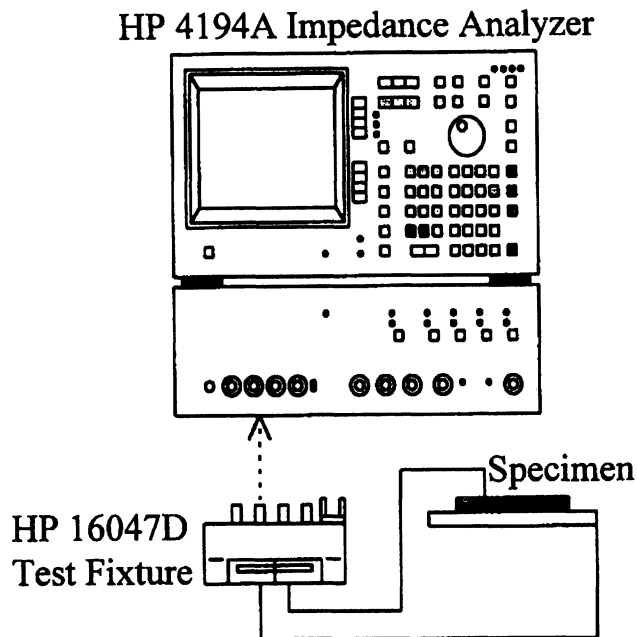


Figure 6. Experimental setup.

mental setup as shown in Figure 6, the HP 4194A impedance analyzer was used to measure the impedance spectrum. A typical recorded curve for specimen 1 is shown in Figure 7. In the impedance spectrum, the frequencies of the local minimum and maximum values of the impedance are resonant and antiresonant frequencies, respectively. The experimental data and the theoretical values for vibration modes 1 to 3 are all listed in Table 2. The maximum errors of 4.75% in resonant frequency and 4.41% in antiresonant frequency are found. This suggests the acceptance of the proposed theory. The piezoceramic properties provided by Fuji Ceramic Co. were checked by the experimental measurements on each individual before it's laminated.

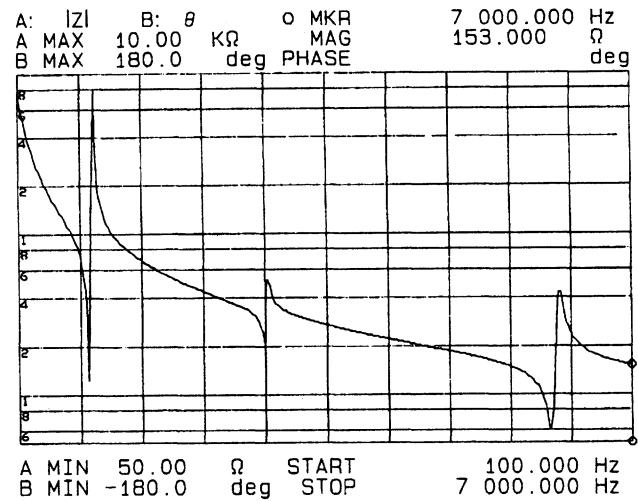


Figure 7. The measured impedance spectrum.

CONCLUSIONS

The electroelastic theory of an annular piezoceramic actuating disk transducers is presented. The problem under consideration is rather complex in the sense of membrane and flexural being coupled. Therefore two systems of cylindrical coordinates are defined for different sectors. The selection of reference plane in Equations (31) and (44) is of importance to make this problem solvable. The static and dynamic characteristics of the transducers are analyzed. The static analysis reveals that the use of larger proportion of piezoceramic material in the transducer will yield the larger displacement sensitivity. On the contrary, the dynamic analysis results do not illustrate the same tendency. For the transducer under consideration, the optimum geometry for dynamic characteristics exists at r_o/R near 0.7 and r_i/R approaching 0. Numerical results to include the thickness and radius ratios are presented

Table 2. Comparison of experimental and theoretical results

(a) resonant frequency									
Specimen	Theory (Hz)			Experiment (Hz)			Error (%)		
	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
1	1895	2957	16228	1881	2860	6069	-1.56	-3.28	-2.56
2	1876	2960	14645	1893	2820	4473	-1.94	-4.75	-3.70
3	1660	2475	15373	1642	2391	5156	-2.80	-3.40	-4.04
4	1159	4372	10202	1183	4333	9976	-2.03	-0.90	-2.21

(b) antiresonant frequency									
Specimen	Theory (Hz)			Experiment (Hz)			Error (%)		
	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
1	1949	2976	16304	1945	2895	16189	-0.40	-2.74	-1.82
2	1917	2988	14653	1935	2856	14522	-1.96	-4.41	-2.81
3	1681	2548	15374	1655	2480	15196	-3.78	-2.67	-3.31
4	1226	4456	10251	1261	4438	10055	-2.88	-0.42	-1.91

by easy-to-use figures. The results are quite significant comparing the use of solid piezoceramic disk laminated on the solid metal disk. These types of transducers are preferable due to sensitivity and the working frequency range can be controlled by geometrical parameters. They are particularly useful for the design of the laminated piezoelectric sensors and actuators, such as microphones, speakers, filters and high performance displacement actuators.

APPENDIX

Elements of matrix $[A]$ and vector $\{F\}$ in Equation (36). Elements are zero except:

$$A_{1,3} = J_0(\lambda_z^{(1)} r_o), \quad A_{1,4} = I_0(\lambda_z^{(1)} r_o), \quad A_{1,5} = Y_0(\lambda_z^{(1)} r_o)$$

$$A_{1,6} = K_0(\lambda_z^{(1)} r_o), \quad A_{1,7} = -J_0(\lambda_1 r_o), \quad A_{1,8} = -J_0(\lambda_2 r_o)$$

$$A_{1,9} = -I_0(m_3 r_o), \quad A_{1,10} = -Y_0(\lambda_1 r_o), \quad A_{1,11} = -Y_0(\lambda_2 r_o)$$

$$A_{1,12} = -K_0(m_3 r_o), \quad A_{2,1} = J_1(\lambda_r^{(1)} r_o), \quad A_{2,2} = Y_1(\lambda_r^{(1)} r_o)$$

$$A_{2,7} = (h_{12}\lambda_1 - P_1)J_1(\lambda_1 r_o), \quad A_{2,8} = (h_{12}\lambda_2 - P_2)J_1(\lambda_2 r_o)$$

$$A_{2,9} = (P_3 - h_{12}m_3)I_1(m_3 r_o), \quad A_{2,10} = (h_{12}\lambda_1 - P_1)Y_1(\lambda_1 r_o)$$

$$A_{2,11} = (h_{12}\lambda_2 - P_2)Y_1(\lambda_2 r_o), \quad A_{2,12} = (h_{12}m_3 - P_3)K_1(m_3 r_o)$$

$$A_{3,3} = \lambda_z^{(1)} J_1(\lambda_z^{(1)} r_o), \quad A_{3,4} = -\lambda_z^{(1)} I_1(\lambda_z^{(1)} r_o)$$

$$A_{3,5} = \lambda_z^{(1)} Y_1(\lambda_z^{(1)} r_o), \quad A_{3,6} = \lambda_z^{(1)} K_1(\lambda_z^{(1)} r_o)$$

$$A_{3,7} = -\lambda_1 J_1(\lambda_1 r_o), \quad A_{3,8} = -\lambda_2 J_1(\lambda_2 r_o)$$

$$A_{3,9} = m_3 I_1(m_3 r_o), \quad A_{3,10} = -\lambda_1 Y_1(\lambda_1 r_o)$$

$$A_{3,11} = -\lambda_2 Y_1(\lambda_2 r_o), \quad A_{3,12} = -m_3 K_1(m_3 r_o)$$

$$A_{4,1} = A_{11}^{(1)} \lambda_r^{(1)} J_0(\lambda_r^{(1)} r_o) - \frac{1}{r_o} (A_{11}^{(1)} - A_{12}^{(1)}) J_1(\lambda_r^{(1)} r_o)$$

$$A_{4,2} = A_{11}^{(1)} \lambda_r^{(1)} Y_0(\lambda_r^{(1)} r_o) - \frac{1}{r_o} (A_{11}^{(1)} - A_{12}^{(1)}) Y_1(\lambda_r^{(1)} r_o)$$

$$A_{4,7} = -(A_{11}^{(2)} P_1 + B_{11}^{(2)} \lambda_1) \lambda_1 J_0(\lambda_1 r_o) + [(A_{11}^{(2)} - A_{12}^{(2)}) P_1 + (B_{11}^{(2)} - B_{12}^{(2)}) \lambda_1] \frac{1}{r_o} J_1(\lambda_1 r_o)$$

$$A_{4,8} = -(A_{11}^{(2)} P_2 + B_{11}^{(2)} \lambda_2) \lambda_2 J_0(\lambda_2 r_o) + [(A_{11}^{(2)} - A_{12}^{(2)}) P_2 + (B_{11}^{(2)} - B_{12}^{(2)}) \lambda_2] \frac{1}{r_o} J_1(\lambda_2 r_o)$$

$$A_{4,9} = (A_{11}^{(2)} P_3 + B_{11}^{(2)} m_3) m_3 I_0(m_3 r_o) - [(A_{11}^{(2)} - A_{12}^{(2)}) P_3 + (B_{11}^{(2)} - B_{12}^{(2)}) m_3] \frac{1}{r_o} I_1(m_3 r_o)$$

$$A_{4,10} = -(A_{11}^{(2)} P_1 + B_{11}^{(2)} \lambda_1) \lambda_1 Y_0(\lambda_1 r_o) + [(A_{11}^{(2)} - A_{12}^{(2)}) P_1 + (B_{11}^{(2)} - B_{12}^{(2)}) \lambda_1] \frac{1}{r_o} Y_1(\lambda_1 r_o)$$

$$A_{4,11} = -(A_{11}^{(2)} P_2 + B_{11}^{(2)} \lambda_2) \lambda_2 Y_0(\lambda_2 r_o) + [(A_{11}^{(2)} - A_{12}^{(2)}) P_2 + (B_{11}^{(2)} - B_{12}^{(2)}) \lambda_2] \frac{1}{r_o} Y_1(\lambda_2 r_o)$$

$$A_{4,12} = (A_{11}^{(2)} P_3 + B_{11}^{(2)} m_3) m_3 K_0(m_3 r_o) + [(A_{11}^{(2)} - A_{12}^{(2)}) P_3 + (B_{11}^{(2)} - B_{12}^{(2)}) m_3] \frac{1}{r_o} K_1(m_3 r_o)$$

$$A_{5,3} = -D_{11}^{(1)} (\lambda_z^{(1)})^3 J_1(\lambda_z^{(1)} r_o), \quad A_{5,4} = -D_{11}^{(1)} (\lambda_z^{(1)})^3 I_1(\lambda_z^{(1)} r_o)$$

$$A_{5,5} = -D_{11}^{(1)} (\lambda_z^{(1)})^3 Y_1(\lambda_z^{(1)} r_o), \quad A_{5,6} = D_{11}^{(1)} (\lambda_z^{(1)})^3 K_1(\lambda_z^{(1)} r_o)$$

$$A_{5,7} = (B_{11}^{(2)} P_1 + D_{11}^{(2)} \lambda_1) \lambda_1^2 J_1(\lambda_1 r_o)$$

$$A_{5,8} = (B_{11}^{(2)} P_2 + D_{11}^{(2)} \lambda_2) \lambda_2^2 J_1(\lambda_2 r_o)$$

$$A_{5,9} = (B_{11}^{(2)} P_3 + D_{11}^{(2)} m_3) m_3^2 J_1(m_3 r_o)$$

$$A_{5,10} = (B_{11}^{(2)} P_1 + D_{11}^{(2)} \lambda_1) \lambda_1^2 Y_1(\lambda_1 r_o)$$

$$A_{5,11} = (B_{11}^{(2)} P_2 + D_{11}^{(2)} \lambda_2) \lambda_2^2 Y_1(\lambda_2 r_o)$$

$$A_{5,12} = -(B_{11}^{(2)} P_3 + D_{11}^{(2)} m_3) m_3^2 K_1(m_3 r_o)$$

$$A_{6,3} = \lambda_z^{(2)} \left[\frac{1}{r_o} (D_{12}^{(2)} - D_{11}^{(2)}) J_1(\lambda_z^{(2)} r_o) + D_{11}^{(2)} \lambda_z^{(2)} J_0(\lambda_z^{(2)} r_o) \right]$$

$$A_{6,4} = -\lambda_z^{(2)} \left[\frac{1}{r_o} (D_{12}^{(2)} - D_{11}^{(2)}) I_1(\lambda_z^{(2)} r_o) + D_{11}^{(2)} \lambda_z^{(2)} I_0(\lambda_z^{(2)} r_o) \right]$$

$$A_{6,5} = \lambda_z^{(2)} \left[\frac{1}{r_o} (D_{12}^{(2)} - D_{11}^{(2)}) Y_1(\lambda_z^{(2)} r_o) + D_{11}^{(2)} \lambda_z^{(2)} Y_0(\lambda_z^{(2)} r_o) \right]$$

$$A_{6,6} = \lambda_z^{(2)} \left[\frac{1}{r_o} (D_{12}^{(2)} - D_{11}^{(2)}) K_1(\lambda_z^{(2)} r_o) - D_{11}^{(2)} \lambda_z^{(2)} K_0(\lambda_z^{(2)} r_o) \right]$$

$$A_{6,7} = -[(h_{12} A_{11}^{(2)} + B_{11}^{(2)}) P_1 + (h_{12} B_{11}^{(2)} + D_{11}^{(2)}) \lambda_1] \lambda_1 J_0(\lambda_1 r_o) + [(A_{11}^{(2)} - A_{12}^{(2)}) h_{12} + B_{11}^{(2)} - B_{12}^{(2)}) P_1 + ((B_{11}^{(2)} - B_{12}^{(2)}) h_{12} + D_{11}^{(2)} - D_{12}^{(2)}) \lambda_1] \frac{1}{r_o} J_1(\lambda_1 r_o)$$

$$A_{6,8} = -[(h_{12}A_{11}^{(2)} + B_{11}^{(2)})P_2 + (h_{12}B_{11}^{(2)} + D_{11}^{(2)})\lambda_2]J_0(\lambda_2 r_o) \\ + [((A_{11}^{(2)} - A_{12}^{(2)})h_{12} + B_{11}^{(2)} - B_{12}^{(2)})P_2 + ((B_{11}^{(2)} - B_{12}^{(2)})h_{12} \\ + D_{11}^{(2)} - D_{12}^{(2)})\lambda_2]\frac{1}{r_o}J_1(\lambda_2 r_o)$$

$$A_{6,9} = [(h_{12}A_{11}^{(2)} + B_{11}^{(2)})P_3 + (h_{12}B_{11}^{(2)} + D_{11}^{(2)})m_3]m_3I_0(m_3 r_o) \\ - [((A_{11}^{(2)} - A_{12}^{(2)})h_{12} + B_{11}^{(2)} - B_{12}^{(2)})P_3 + ((B_{11}^{(2)} - B_{12}^{(2)})h_{12} \\ + D_{11}^{(2)} - D_{12}^{(2)})m_3]\frac{1}{r_o}I_1(m_3 r_o)$$

$$A_{6,10} = -[(h_{12}A_{11}^{(2)} + B_{11}^{(2)})P_1 + (h_{12}B_{11}^{(2)} + D_{11}^{(2)})\lambda_1]\lambda_1Y_0(\lambda_1 r_o) \\ + [((A_{11}^{(2)} - A_{12}^{(2)})h_{12} + B_{11}^{(2)} - B_{12}^{(2)})P_1 + ((B_{11}^{(2)} - B_{12}^{(2)})h_{12} \\ + D_{11}^{(2)} - D_{12}^{(2)})\lambda_1]\frac{1}{r_o}Y_1(\lambda_1 r_o)$$

$$A_{6,11} = -[(h_{12}A_{11}^{(2)} + B_{11}^{(2)})P_2 + (h_{12}B_{11}^{(2)} + D_{11}^{(2)})\lambda_2]\lambda_2Y_0(\lambda_2 r_o) \\ + [((A_{11}^{(2)} - A_{12}^{(2)})h_{12} + B_{11}^{(2)} - B_{12}^{(2)})P_2 + ((B_{11}^{(2)} - B_{12}^{(2)})h_{12} \\ + D_{11}^{(2)} - D_{12}^{(2)})\lambda_2]\frac{1}{r_o}Y_1(\lambda_2 r_o)$$

$$A_{6,12} = [(h_{12}A_{11}^{(2)} + B_{11}^{(2)})P_3 + (h_{12}B_{11}^{(2)} + D_{11}^{(2)})m_3]m_3K_0(m_3 r_o) \\ + [((A_{11}^{(2)} - A_{12}^{(2)})h_{12} + B_{11}^{(2)} - B_{12}^{(2)})P_3 + ((B_{11}^{(2)} - B_{12}^{(2)})h_{12} \\ + D_{11}^{(2)} - D_{12}^{(2)})m_3]\frac{1}{r_o}K_1(m_3 r_o)$$

$$A_{7,7} = J_0(\lambda_1 r_i), \quad A_{7,8} = J_0(\lambda_2 r_i), \quad A_{7,9} = I_0(m_3 r_i)$$

$$A_{7,10} = Y_0(\lambda_1 r_i), \quad A_{7,11} = Y_0(\lambda_2 r_i), \quad A_{7,12} = K_0(m_3 r_i)$$

$$A_{7,14} = -J_0(\lambda_z^{(3)} r_i), \quad A_{7,15} = -I_0(\lambda_z^{(3)} r_i)$$

$$A_{8,7} = P_1J_1(\lambda_1 r_i), \quad A_{8,8} = P_2J_1(\lambda_2 r_i), \quad A_{8,9} = -P_3I_1(m_3 r_i)$$

$$A_{8,10} = P_1Y_1(\lambda_1 r_i), \quad A_{8,11} = P_2Y_1(\lambda_2 r_i), \quad A_{8,12} = P_3K_1(m_3 r_i)$$

$$A_{8,13} = -J_1(\lambda_r^{(3)} r_i), \quad A_{8,14} = -h_{12}\lambda_z^{(3)}J_1(\lambda_z^{(3)} r_i)$$

$$A_{8,15} = h_{12}\lambda_z^{(3)}I_1(\lambda_z^{(3)} r_i), \quad A_{9,7} = \lambda_1J_1(\lambda_1 r_i)$$

$$A_{9,8} = \lambda_2J_1(\lambda_2 r_i), \quad A_{9,9} = -m_3I_1(m_3 r_i), \quad A_{9,10} = \lambda_1Y_1(\lambda_1 r_i)$$

$$A_{9,11} = \lambda_2Y_1(\lambda_2 r_i), \quad A_{9,12} = -m_3K_1(m_3 r_i)$$

$$A_{9,14} = -\lambda_z^{(3)}J_1(\lambda_z^{(3)} r_i), \quad A_{9,15} = -\lambda_z^{(3)}I_1(\lambda_z^{(3)} r_i)$$

$$A_{10,7} = (A_{11}^{(2)}P_1 + B_{11}^{(2)}\lambda_1)\lambda_1J_0(\lambda_1 r_i) \\ - [(A_{11}^{(2)} - A_{12}^{(2)})P_1 + (B_{11}^{(2)} - B_{12}^{(2)})\lambda_1]\frac{1}{r_i}J_1(\lambda_1 r_i)$$

$$A_{10,8} = (A_{11}^{(2)}P_2 + B_{11}^{(2)}\lambda_2)\lambda_2J_0(\lambda_2 r_i) \\ - [(A_{11}^{(2)} - A_{12}^{(2)})P_2 + (B_{11}^{(2)} - B_{12}^{(2)})\lambda_2]\frac{1}{r_i}J_1(\lambda_2 r_i)$$

$$A_{10,9} = -(A_{11}^{(2)}P_3 + B_{11}^{(2)}m_3)m_3I_0(m_3 r_i) \\ + [(A_{11}^{(2)} - A_{12}^{(2)})P_3 + (B_{11}^{(2)} - B_{12}^{(2)})m_3]\frac{1}{r_i}J_1(m_3 r_i)$$

$$A_{10,10} = (A_{11}^{(2)}P_1 + B_{11}^{(2)}\lambda_1)\lambda_1Y_0(\lambda_1 r_i) \\ - [(A_{11}^{(2)} - A_{12}^{(2)})P_1 + (B_{11}^{(2)} - B_{12}^{(2)})\lambda_1]\frac{1}{r_i}Y_1(\lambda_1 r_i)$$

$$A_{10,11} = (A_{11}^{(2)}P_2 + B_{11}^{(2)}\lambda_2)\lambda_2Y_0(\lambda_2 r_i) \\ - [(A_{11}^{(2)} - A_{12}^{(2)})P_2 + (B_{11}^{(2)} - B_{12}^{(2)})\lambda_2]\frac{1}{r_i}Y_1(\lambda_2 r_i)$$

$$A_{10,12} = -(A_{11}^{(2)}P_3 + B_{11}^{(2)}m_3)m_3K_0(m_3 r_i) \\ - [(A_{11}^{(2)} - A_{12}^{(2)})P_3 + (B_{11}^{(2)} - B_{12}^{(2)})m_3]\frac{1}{r_i}K_1(m_3 r_i)$$

$$A_{10,13} = -A_{11}^{(3)}\lambda_r^{(3)}J_0(\lambda_r^{(3)} r_i) + \frac{1}{r_i}(A_{11}^{(3)} - A_{12}^{(3)})J_1(\lambda_r^{(3)} r_i)$$

$$A_{11,7} = -(B_{11}^{(2)}P_1 + D_{11}^{(2)}\lambda_1)\lambda_1^2J_1(\lambda_1 r_i)$$

$$A_{11,8} = -(B_{11}^{(2)}P_2 + D_{11}^{(2)}\lambda_2)\lambda_2^2J_1(\lambda_2 r_i)$$

$$A_{11,9} = -(B_{11}^{(2)}P_3 + D_{11}^{(2)}m_3)m_3^2J_1(m_3 r_i)$$

$$A_{11,10} = (B_{11}^{(2)}P_1 + D_{11}^{(2)}\lambda_1)\lambda_1^2Y_1(\lambda_1 r_i)$$

$$A_{11,11} = -(B_{11}^{(2)}P_2 + D_{11}^{(2)}\lambda_2)\lambda_2^2Y_1(\lambda_2 r_i)$$

$$A_{11,12} = -(B_{11}^{(2)}P_3 + D_{11}^{(2)}m_3)m_3^2K_1(m_3 r_i)$$

$$A_{11,14} = D_{11}^{(3)}(\lambda_z^{(3)})^3J_1(\lambda_z^{(3)} r_i)$$

$$A_{11,15} = D_{11}^{(3)}(\lambda_z^{(3)})^3I_1(\lambda_z^{(3)} r_i)$$

$$A_{12,7} = (B_{11}^{(2)}P_1 + D_{11}^{(2)}\lambda_1)\lambda_1J_0(\lambda_1 r_i) \\ - [(B_{11}^{(2)} - B_{12}^{(2)})P_1 + (D_{11}^{(2)} - D_{12}^{(2)})\lambda_1]\frac{1}{r_i}J_1(\lambda_1 r_i)$$

$$A_{12,8} = (B_{11}^{(2)}P_2 + D_{11}^{(2)}\lambda_2)\lambda_2J_0(\lambda_2 r_i) \\ - [(B_{11}^{(2)} - B_{12}^{(2)})P_2 + (D_{11}^{(2)} - D_{12}^{(2)})\lambda_2]\frac{1}{r_i}J_1(\lambda_2 r_i)$$

$$A_{12,9} = -(B_{11}^{(2)}P_3 + D_{11}^{(2)}m_3)m_3I_0(m_3 r_i) \\ + [(B_{11}^{(2)} - B_{12}^{(2)})P_3 + (D_{11}^{(2)} - D_{12}^{(2)})m_3]\frac{1}{r_i}I_1(m_3 r_i)$$

$$A_{12,10} = (B_{11}^{(2)}P_1 + D_{11}^{(2)}\lambda_1)Y_0(\lambda_1 r_i) \\ - [(B_{11}^{(2)} - B_{12}^{(2)})P_1 + (D_{11}^{(2)} - D_{12}^{(2)})\lambda_1] \frac{1}{r_i} Y_1(\lambda_1 r_i)$$

$$A_{12,11} = (B_{11}^{(2)}P_2 + D_{11}^{(2)}\lambda_2)Y_0(\lambda_2 r_i) \\ - [(B_{11}^{(2)} - B_{12}^{(2)})P_2 + (D_{11}^{(2)} - D_{12}^{(2)})\lambda_2] \frac{1}{r_i} Y_1(\lambda_2 r_i)$$

$$A_{12,12} = -(B_{11}^{(2)}P_3 + D_{11}^{(2)}m_3)m_3K_0(m_3 r_i) \\ - [(B_{11}^{(2)} - B_{12}^{(2)})P_3 + (D_{11}^{(2)} - D_{12}^{(2)})m_3] \frac{1}{r_i} K_1(m_3 r_i)$$

$$A_{12,13} = h_{12} \left[A_{11}^{(3)}\lambda_r^{(3)}J_0(\lambda_r^{(3)}r_i) - \frac{1}{r_i}(A_{11}^{(3)} - A_{12}^{(3)})J_1(\lambda_r^{(3)}r_i) \right]$$

$$A_{12,14} = -\lambda_z^{(3)} \left[\frac{1}{r_i}(D_{12}^{(3)} - D_{11}^{(3)})J_1(\lambda_z^{(3)}r_i) + D_{11}^{(3)}\lambda_z^{(3)}J_0(\lambda_z^{(3)}r_i) \right]$$

$$A_{12,15} = \lambda_z^{(3)} \left[\frac{1}{r_i}(D_{12}^{(3)} - D_{11}^{(3)})I_1(\lambda_z^{(3)}r_i) + D_{11}^{(3)}\lambda_z^{(3)}I_0(\lambda_z^{(3)}r_i) \right]$$

$$A_{13,1} = A_{11}^{(1)}\lambda_r^{(1)}J_0(\lambda_r^{(1)}R) - \frac{1}{R}(A_{11}^{(1)} - A_{12}^{(1)})J_1(\lambda_r^{(1)}R)$$

$$A_{13,2} = A_{11}^{(1)}\lambda_r^{(1)}Y_0(\lambda_r^{(1)}R) - \frac{1}{R}(A_{11}^{(1)} - A_{12}^{(1)})Y_1(\lambda_r^{(1)}R)$$

$$A_{14,3} = -D_{11}^{(1)}(\lambda_z^{(1)})^3 J_1(\lambda_z^{(1)}R)$$

$$A_{14,4} = -D_{11}^{(1)}(\lambda_z^{(1)})^3 I_1(\lambda_z^{(1)}R)$$

$$A_{14,5} = -D_{11}^{(1)}(\lambda_z^{(1)})^3 Y_1(\lambda_z^{(1)}R)$$

$$A_{14,6} = D_{11}^{(1)}(\lambda_z^{(1)})^3 K_1(\lambda_z^{(1)}R)$$

$$A_{15,3} = \lambda_z^{(1)} \left[\frac{1}{R}(D_{12}^{(1)} - D_{11}^{(1)})J_1(\lambda_z^{(1)}R) + D_{11}^{(1)}\lambda_z^{(1)}J_0(\lambda_z^{(1)}R) \right]$$

$$A_{15,4} = -\lambda_z^{(1)} \left[\frac{1}{R}(D_{12}^{(1)} - D_{11}^{(1)})I_1(\lambda_z^{(1)}R) + D_{11}^{(1)}\lambda_z^{(1)}I_0(\lambda_z^{(1)}R) \right]$$

$$A_{15,5} = \lambda_z^{(1)} \left[\frac{1}{R}(D_{12}^{(1)} - D_{11}^{(1)})Y_1(\lambda_z^{(1)}R) + D_{11}^{(1)}\lambda_z^{(1)}Y_0(\lambda_z^{(1)}R) \right]$$

$$A_{15,6} = \lambda_z^{(1)} \left[\frac{1}{R}(D_{12}^{(1)} - D_{11}^{(1)})K_1(\lambda_z^{(1)}R) - D_{11}^{(1)}\lambda_z^{(1)}K_0(\lambda_z^{(1)}R) \right]$$

$$F_{4,1} = E_{11}^{(2)}V, \quad F_{6,1} = (F_{11}^{(2)} + h_{12}E_{11}^{(2)})V$$

$$F_{4,1} = -E_{11}^{(2)}V, \quad F_{6,1} = -(F_{11}^{(2)} + h_{12}E_{11}^{(2)})V$$

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