Corrections

Corrigendum to "Line Digraph Iterations and Connectivity Analysis of de Bruijn and Kautz Graphs"

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An error appears in the proof of Lemma 3.2 in our paper "Line Digraph Iterations and Connectivity Analysis of de Bruijn and Kautz Graphs" [Ding-Zhu Du, Yuh-Dauh Lyuu, and D. Frank Hsu, *IEEE Trans. Computers*, vol. 42, no. 5, pp. 612-616, May 1993]. To fix it, we need to modify Definition 3.1 and the Proof of Lemma 3.2 on p. 614. They should read as follows.

DEFINITION 3.1. *Given positive integers k and l, let* $\mathcal{L}\bullet(k, l)$ *be the class of graphs G so that for any* $m \le k$ *distinct nodes* $y_1, \cdots, y_m \in V(G)$, *a node* $x \in V(G)$, *any* $2q$ $(1 \le q \le k+1)$ *edges* $e_0, e_1, \cdots, e_{2q-1}$ *where* $e'_0 = e'_2 = \cdots = e_{2q-1} = x''$ *and*

$$\{e''_1, e''_3, \cdots, e''_{2q-1}\} \subseteq \{y'_1, y'_2, \cdots, y'_m\},$$

and m positive numbers $r_1, \cdots, r_m$ *such that* $\sum_i r_i = k - q + 1$, *there exist* $k - q + 1$ *node-disjoint paths each of positive length at most l from x to* $y_1, \cdots, y_m$, *where* r_i *of them go to* y_i *for* $1 \le i \le m$, *and none involves* e_j *for* $0 \le j \le 2q - 1$. *Note: x may be one of the* y_i. *(The introduction calls this notion spread*.)*

LEMMA 3.2. $L^h(G) \in \mathcal{L}\bullet(k, l + h)$ *if* $G \in \mathcal{L}\bullet(k, l)$, *for* $h \ge 0$.

PROOF. The proof is by induction on i, the depth of line graph iterations. This lemma surely holds for $i = 0$. Assume it is true for i up to $h - 1$ and consider $i = h > 0$.

Consider a node x and m distinct nodes $y_1, \cdots, y_m$, in $L^i(G)$. These nodes are edges in $L^{i-1}(G)$. Let $r_1, \cdots, r_m$ be positive integers such that $\sum_j r_j - k - 1 + 1$ $(1 \le q \le k + 1)$. Assume $x'' = y'_1 = \cdots = y'_s$ in $L^{i-1}(G)$ for some $0 \le s \le m$, with $s = 0$ meaning no (x, y_j) is an edge in $L^i(G)$. Hence, in $L^{i-1}(G)$, the edge x is incident to edges y_j for $1 \le j \le s$.

There are three cases to consider:

1) $s = 0$,
2) $s > 0$ and $\{(x, y_j) \mid 1 \le s\} \subseteq \{e_0, e_1, \cdots, e_{2q-1}\}$, and
3) $s > 0$ and there exists $1 \le j \le s$ such that $(x, y_j) \notin \{e_0, e_1, \cdots, e_{2q-1}\}$.

First, suppose 1) occurs, i.e., $s = 0$. By the induction hypothesis, in $L^{i-1}(G)$, there are $k - q + 1$ node-disjoint paths from x'' to $y'_1, \ldots, y'_m$ where r_j of them end at y'_j and each has positive length at most $l + i - 1$ (here we treat y'_a and y'_b with $a \ne b$ as distinct even if $y'_a = y'_b$) and none involves $2q$ edges $e''_0, e'_1, e''_2, \cdots, e'_{2q-1}$, in $L^{i-1}(G)$. Typical paths look like

$$\overbrace{x'' \to \cdots \to y'_j}^{\alpha}.$$

The paths extended to

$$x' \to \overbrace{x'' \to \cdots \to y'_j}^{\alpha} \to y''_j.$$

they induce, in $L^i(G)$, the desired node-disjoint paths from x to the y_j's.

Suppose 2) occurs. By the same argument as the above, we can prove the existence of the desired node-disjoint paths from x to y_js.

Now, suppose 3) occurs. Without loss of generality, assume $(x, y_1) \notin \{e_0, e_1, \cdots, e_{2q-1}\}$. If $q = k + 1$, then the desired $k - q + 1$ paths exist trivially. Thus, we may also assume $q < k + 1$. Define $y_{m+1} = y_1$ with $r_{m+1} = r_i - 1$ (if $r_{m+1} = 0$, just drop node y_{m+1} from the induction step). Consider the graph obtained from $L^{i-1}(G)$ by deleting edges x, y_1, and $2q$ edges $e''_0, e'_1, e''_2, \ldots, e'_{2q-1}$ and considering $y'_{s+1}, \cdots, y'_{m+1}$. By the induction hypothesis, there are $\sum_{j=2}^{m+1} = k - q$ node-disjoint paths in $L^{i-1}(G)$ from x'' to $y'_2, \cdots, y'_{m+1}$ where r_j of them end at y'_j (here we treat y'_a and y'_b with $a \ne b$ as distinct even if $y'_a = y'_b$), each has positive length at most $l + i - 1$, and none involves x, y_1, and $2q$ edges $e''_0, e'_1, e''_2, \cdots, e'_{2q-1}$. Typical paths look like

$$\overbrace{x'' \to \cdots \to y'_j}^{\alpha} \quad \text{for } 2 \le j \le m + 1.$$

The paths extend to

$$x' \to \overbrace{x'' \to \cdots \to y'_j}^{\alpha} \to y''_j, \quad \text{for } 2 \le j \le m + 1,$$

they induce, in $L^i(G)$, $k - q$ node-disjoint paths from x to y_js, each of positive length at most $l + i$ and none involving the edge (x, y_1) and $2q$ edges $e_0, e_1, \cdots, e_{2q-1}$. Add (x, y_1) to get the $(k - q + 1)$th path in $L^i(G)$ and the proof is completed. □

With the above modification, the remainder in the paper is correct except that in the introduction, the phrase "two consecutive edges" at line 28 of p. 613 should be replaced by "some edges."

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