

A COMPARATIVE STUDY ON THE INTERACTION OF TSE TAI-INDEX FUTURES AND TIMEX TAI-INDEX FUTURES

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Abstract

TSE and SIMEX individually introduced the futures based on TAI-Index and MSCI Taiwan Index since July 21, 1998 and January 9th, 1997 respectively. We first investigate the hedging effectiveness of TSE TAI-Index and SIMEX MSCI Taiwan Index futures on TWSI cash index first in Bayesian approach using Gibbs Sampler. Secondly, we perform the variance ratio test about the two futures price behavior and examine the lead-lag relation and price transmission. The data period covers from July 21, 1998 to July 31, 1999. Our results show that TSE TAI-Index futures is significantly superior to SIMEX MSCI Taiwan index futures and both the two Taiwan index futures do not follow the random walk process. Also, we find that positive autocorrelation in both TSE TAI-Index and SIMEX MSCI futures. On the aspect of lead-lag relation, we find that cash market leads futures market and the cointegration with cash exists in each of the two index futures.

Keywords: Index futures; Bayesian approach; Gibbs Sampling

1. Introduction

TSE TAI-Index and SIMEX MSCI Taiwan Index futures were newly introduced since July 21, 1998 and January 9, 1997 respectively. In order to understand how these stock index futures might affect the cash index (TWSI) in Taiwan, we explore several issues in this study including the comparison of hedging effectiveness, futures price behavior, and the lead-lag relation. On comparing hedging effectiveness, we construct the estimator in Bayesian Approach using Gibbs sampler. On price behavior, we apply the variance ratio test proposed by Lo and MacKinlay (1988, 1989) to determine if the futures price corresponds to random walk. As for the lead-lag relation and price transmission, both the augmented Dickey-Fuller test and the Granger-causality test using Error Correction Model are applied.

On the issue of hedging effectiveness, the minimum-variance model has been tested empirically for currency futures by Naidu and Shin (1981) and Hill and Schneeweis (1982a, 1982b). All these articles show that substantial risk reduction with hedged position in comparison with the unhedged spot position. Hill and Schneeweis (1982a) compare the hedging effectiveness of the minimum-variance hedge with the naive hedged position. They find that both techniques are highly effective. Hill and Schneeweis (1982b) find that the hedging effectiveness increases with the investment horizon. Hedge ratios and measures of hedging effectiveness are examined by Ederington (1979), Hill, Lire, and Schneeweis (1983); by Hill and Schneeweis (1984) for T-bond futures; by Overdail and Starleaf (1986) for CD futures; by Ederington (1979), Franckle (1980), and Howard and D'Antonio (1984) for T-bill futures; by Figlewski (1984, 1985), and Junkus and Lee (1985) for stock market

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index futures; by Hill and Schneeweis (1982), Grammatikos and Saunders (1983), Grammatikos (1986), and Hwang et al. (1996) for foreign currency futures. On the issue of the holding horizon, the relationship between futures hedging effectiveness and investment horizon has been studied [See Geppert (1995)]. The cointegration between futures and cash prices is considered to exist. It shows that there is one unique hedge ratio to fit the specified investment horizon and the hedging effectiveness converges to one as the horizon length increases.

On the issue of the variance ratio, Lo and MacKinlay (1988) exploit the fact that the variance of the increments in a random walk is linear in the sampling interval and thereby they propose the variance ratio test. Liu and He (1991) apply this test in measuring the foreign exchange rate process to determine whether it fits the random walk under homoscedasticity and heteroscedasticity.

The argument on the lead-lag relationship between the futures market and the cash market has been extensively investigated. Most work has shown that price movements in the futures markets consistently lead the stock index movements [see Finnerty & Park (1987), Ng (1987), Kawaller and Koch and Koch (1987), Harris (1989), Stoll and Whaley (1990), Chan (1992), Shyy and Vijaraghavan and Scott-Quinn (1996)]. Several issues about the reason why the lead-lag relationship exists are also explored. These include the effect of nonsynchronous trading between cash market and futures market, the difference of the transaction cost between these two markets and the somewhat short selling restriction in cash market. According to Shyy et al. (1996), difference in trading mechanism used in cash and futures market may also cause the lead lag relationships. However, in the same research, it also shows that after eliminating the effect of the nonsynchronous trading, the lead-lag pattern vanishes and the reverse causality from cash to futures becomes significant. On the other hand, Chan (1992) suggests that the lead-lag pattern is not well explained by nonsynchronous trading, and market wide information tends to be reflected better in futures markets.

2. Data

In order to investigate the hedging effectiveness of TSE TAI-Index and SIMEX MSCI Taiwan Index Futures on TWSI cash index, we collect the daily data of TSE TAI-Index and SIMEX MSCI Taiwan Index Futures prices from *China Night Times* and *Chung Hsin Securites Co.* The Taiwan Weighted Stock Index are generated from the *TEJ* and *Reuters System*, Taiwan. The sample period of both TSE TAI-Index and SIMEX MSCI futures covers from July 21, 1998, to July 31, 1999.¹

In order to compare the hedging effectiveness and calculating the variance ratios, we take the sample starting from July 21, 1998 to July 31, 1999. When proceeding with the test of lead-lag relationships, the whole data are divided into several sub-time periods including a) July 21, 1998 – January 31, 1999 and b) February 1, 1999 – July 31, 1999.

¹Data collected covers from July 21, 1998 since TSE TAI-Index futures was initiated at that date and terminates on July 31, 1999 because of the NSC sponsored research project ends at that date.

3. Methodology

3.1 Comparison of Hedging Effectiveness

3.1.1. Estimator Derivation

First of all, we want to introduce minimum-variance approach into the model. The portfolio hedging theory has been developed from the mean-variance framework [see Ederington (1979)]. Hedging spot with futures is regarded as constructing a portfolio, in which the futures and the underlying cash are the components. From minimizing the variance of the value of the hedging portfolio, that is to take difference of the variance by the hedge position, $h = -X_f / X_s$, the optimal hedge

ratio h^* is derived. That is $h^* = \frac{\tau_{sf}}{\tau_f^2}$.²

Under the minimum-variance model, a commonly adopted measure of hedging effectiveness for futures is the R^2 of the following regression:

$$s_t = r + S f_t + v_t \quad (1)$$

where s_t and f_t , respectively, represent the changes in spot and futures prices, and $S = \text{cov}(s_t, f_t) / \text{var}(f_t)$ is the “minimum-variance” or the so-called “optimal” hedge ratio. r is the term of intercept, and v_t denotes the error term. According to portfolio hedging theory, the coefficient of determination of the regression equation, which is also a measure of hedging effectiveness R^2 , is denoted by $\tau_{sf}^2 / \tau_s \tau_f$.³

Generally speaking, a futures contract that yields a higher R^2 is considered a better hedging candidate. Comparison of hedging performances based on R^2 measures is valid because it is well known that, in the context of canonical simple linear regression models, a higher R^2 corresponds to a smaller residual variance, which is the variance of the hedged portfolio. Lindahl (1989) points out, however, that the R^2 is an appropriate measure only in certain restricted cases. Specifically, she points out that only in the “same cash-different futures” case can R^2 statistics be used as a measure of hedging effectiveness.

For the R^2 to be a valid measure of hedging effectiveness and the S to be optimal, some conditions on the joint distribution of s_t and f_t have to be met. For example, s_t and f_t have to be distributed in a way such that f_t is independent of the error term, i.e. $E(f_t v_t) = 0 \forall s, t$.⁴

Nevertheless, a higher R^2 of a futures contract over that another, calculated

² The optimal hedge ratio h^* in this paragraph is represented by β from the next paragraph.

³ Some may doubt why adopt constant hedging instead of time varying. Some studies have proved that a constant hedge ratio appears to be a suitable assumption. More complicated procedures for estimating time varying optimal hedge ratio may not be necessary. In the contrary, it may increase much more transaction cost when hedging[See Mcnew, and Fackler (1994)].

⁴ Notice that the linear regression is essentially defined as the conditional expectation of s_t given f_t . The linear conditional expectation property (i.e., $E(s_t | f_t) = r + S f_t$) does not hold for arbitrary joint distribution of s_t and f_t . The class of distributions that retain the linear conditional expectation property and validate the mean-variance analysis known thus far is the class of elliptical distributions in which the normal distribution is a “member”.

based on some particular samples, does not imply that the variance of the resulting hedged portfolio for the former is necessarily smaller. It can merely be a result of randomness. More specifically, one always obtains different R^2 's with different futures contracts and/or with different data (over different sample periods), but the differences may not be significant. In particular, the differences may not be significant in statistical sense.

There seems, however, no appropriate test yet available that has been proposed to examine if the variances of the hedged portfolios using different futures contracts are statistically significant. Here we propose a Bayesian approach using the Gibbs sampler algorithm, a recently developed simulation algorithm in statistics, to analyze some hypotheses of interest regarding the relative hedging effectiveness between two futures contracts.

To demonstrate our approach, let s_t , f_{1t} , and f_{2t} denote, respectively, the changes in SIMEX TWSE cash index, SIMEX MSCI Taiwan index futures, and CME Dow Jones Taiwan index futures price at time t , $t=1, \dots, T$. For simplicity's sake, assume that these three variables follow an *iid* trivariate normal distribution with constant mean and covariance matrix:

$$\begin{pmatrix} s_t \\ f_{1t} \\ f_{2t} \end{pmatrix} \sim N \left[\begin{pmatrix} \sim_s \\ \sim_1 \\ \sim_2 \end{pmatrix}, \begin{pmatrix} f_s^2 & f_{s1} & f_{s2} \\ f_{s1} & f_1^2 & f_{12} \\ f_{s2} & f_{12} & f_2^2 \end{pmatrix} \right] \quad (2)$$

The above relationship can be written more concisely in matrix notation as: $R_t \sim N(\sim, \Sigma)$.

Thus, the minimum-variance hedge relation based on futures i is: $S_i = f_{si} / f_i^2$, while the sample counterpart is $S_i = f_{si} / f_i^2$. The variance of the corresponding hedged portfolio is: $Var(s - S_i f_i) \equiv f_s^2 - f_{si}^2 / f_i^2$. Note that $Var(s - S_i f_i)$ is the residual variance of a regression of s on f_i , and f_{si}^2 / f_i^2 represents the reduction in variance by introducing futures i . The R^2 measure is merely an estimate of $R_i^{*2} \equiv \frac{f_{si}^2 / f_i^2}{f_s^2}$.⁵

To compare the hedging effectiveness of different futures, we need to compare $Var(s - S_i f_i)$, or equivalently, R_i^{*2} . The hypothesis to be tested is $H_0: Var(s - S_1 f_1) = Var(s - S_2 f_2)$, or $H_0: R_1^{*2} = R_2^{*2}$.

To test the above hypothesis, we propose a Bayesian approach using the Gibbs sampler to examine the posterior distribution of the ratio of R_1^{*2} and R_2^{*2} . Define

$$r_{12} \equiv \frac{R_1^{*2}}{R_2^{*2}} = \frac{f_{s1}^2 / f_1^2}{f_{s2}^2 / f_2^2} \quad (3)$$

If r_{12} is significantly greater than 1, it implies that using individual share futures is superior to using index futures in reducing the spot variance. Note that the ratio of R^2 's may always differ from 1, but it may not be different from 1 significantly.

⁵ Note that R_i^{*2} is just the square of the correlation coefficient between s and f_i ... f_{si} .

3.1.2 A Bayesian Approach Using the Gibbs Sampler

After the test statistic – the estimate r_{12} is developed, the second part of the methodology is proceeded with [See Chou and Shyy (1996)]. In order to calculate the approximate true value from the sample data, the Bayesian approach is introduced.

Our purpose is to test if the ratio r_{12} is significantly different from 1. In classical theories, this can be done by using a likelihood-ratio or lagrange-multiplier type test. However, this is a difficult job because the restrictions complicated and nonlinear (i.e., $f_{s1}^2 / f_1^2 = f_{s2}^2 / f_2^2$). In particular, the properties of the test will rely on the availability of large samples.

Here, we propose a Bayesian approach using the Gibbs sampling algorithm that directly looks at the exact distribution of the ratio. For simplicity's sake, suppose we have a flat (Jeffrey's) prior for the parameters μ and Σ , i.e.,

$$P(\sim, \Sigma) \propto |\Sigma|^{-2}. \quad (4)$$

Thus, the joint posterior distribution for the parameters is as follows:

$$\mathcal{f}(\sim, \Sigma | R_1, \dots, R_T) \propto |\Sigma|^{-(T+4)/2} \exp\left(-\frac{1}{2} \sum_{t=1}^T (R_t - \sim)' \Sigma^{-1} (R_t - \sim)\right). \quad (5)$$

Note that the purpose here is to obtain the marginal posterior distributions of the ratio $\mathcal{f}(r_{12})$. This can be obtained by applying change-of-variable technique to the above equation, which is, however, analytically intractable. As an alternative, the posterior distribution can be calculated with the Gibbs sampling algorithm. The idea of the Gibbs sampler is to simulate a sample from the joint posterior distribution. That is,

$$\mathcal{f}(\sim, \Sigma | R_1, \dots, R_T). \quad (6)$$

The simulation of the joint posterior distribution is done by sequentially simulating from the *full conditional distributions*.⁶ The initial value will be the OLS estimates of \sim and Σ , which is according to the minimum-variance framework. Specifically, in this case we need to generate draws from the conditional distribution of μ given Σ , and from that of Σ^{-1} given μ . From the joint posterior distribution, it is easy to verify that the conditional distribution of μ is trivariate normal and the conditional distribution of Σ^{-1} is Wishart.

After the iteration is finished, series $\{\Sigma^{(q)}\} = \{\Sigma^{(0)}, \Sigma^{(1)}, \dots, \Sigma^{(q)}\}$ can be simulated. Thus, we can construct the series of the estimator

$\{r_{12}^{(q)}\} = \{r_{12}^{(1)}, r_{12}^{(2)}, \dots, r_{12}^{(q)}\}$, in which $r_{12}^{(q)} = \frac{f_{s1}^{2(q)} / f_1^{2(q)}}{f_{s2}^{2(q)} / f_2^{2(q)}}$. We now summarize the

Gibbs sampler iteration implement into the following steps:

1. Let $q=1$. Let $\sim^{(0)}$ and $\Sigma^{(0)}$ be the OLS estimates of \sim and Σ , respectively.

2. Draw $\sim^{(q)}$ from the multivariate distribution: $\mathcal{N}(\bar{R}, \frac{1}{T} \Sigma^{(q-1)})$, where

$$\bar{R} = \frac{1}{T} \sum_{t=1}^T R_t \text{ is the sample mean vector of } R_t \text{ 's.}$$

3. Draw $\Sigma^{-1(q)}$ from the Wishart distribution:

⁶ Review of the Gibbs sampling algorithm is available in several studies. See, for example, Gelfand and Smith(1990) for a summary of the properties of the algorithm.

$$W(T, (\sum_{t=1}^T (R_t - \tilde{r}^{(q)})(R_t - \tilde{r}^{(q)})')^{-1}).$$

$$4. \text{ Let } r_{12}^{(q)} = \frac{f_{s1}^{2(q)} / f_1^{2(q)}}{f_{s2}^{2(q)} / f_2^{2(q)}}.$$

5. Set $q=q+1$, then go to step 2.

Noted that a superscript (q) of a variable represents a draw generated from the corresponding conditional distribution in the q^{th} iteration cycle. Under weak regularity conditions [Gelfand and Smith (1990)], $r_{12}^{(q)}$ will be a draw from the marginal posterior distribution of r_{12} as q approaches infinity. A sequence of $r_{12}^{(q)}$'s thus constitute a sample from the posterior distribution of r_{12} .⁷ Statistical inferences can be made by examining the properties of this sample. For example, suppose a sample of size M , $\{r_{12}^{(1)}, \dots, r_{12}^{(M)}\}$ is generated from the distribution of r_{12} . One can estimate its marginal density by density estimation techniques such as kernel estimation method [See, e.g., Silverman (1986)]. One can also calculate the highest posterior density (HPD) region from this sample. In our case, the simple hypothesis $r_{12}=1$ can be tested by merely checking if the corresponding 95% HPD region contains 1. If so, one cannot reject the null hypothesis that the performances of the two futures contracts are the same at the 5% significance level.

3.2. Variance Ratio Test

The variance ratio test has been used to examine if the underlying time series follow a random walk process. The intuition behind the variance ratio test is that if the natural logarithm of a time series, denoted Y_t , is a pure random walk of the form

$$Y_t = \tilde{r} + Y_{t-1} + v_t, \quad E[v_t] = 0, \quad \text{for all } t, \quad (7)$$

or

$$\Delta X_t = \tilde{r} + v_t, \quad \Delta X_t = X_t - X_{t-1}, \quad (8)$$

then, the variance of its q -differences grows linearly with the difference q . If the series follows a random walk, it must be the case that variance of the q -differences is q times the variances of the first-difference:

$$\frac{\text{Var}(Y - Y_{t-q})}{q \cdot \text{Var}(Y_t - Y_{t-1})} = 1 \quad (9)$$

In an efficient market, changes in spot prices over time are random and reflect the arrival of new information.

With the value of the variance ratio, one can determine if the futures prices follow a random walk process. The value of the variance ratio can also be used to identify if the futures prices are positively or negatively serial correlated. If there exists a negative autocorrelation relationship, we can say that there exists trading noise in the short term over the market. In accordance with the trading noise, the overreaction behavior of investors should happen when trading. However, these kind of overreaction trading behavior will be modified in the long term because

⁷ There are two ways to obtain the sample: single path and multiple path methods. There is, however, no well-accepted result suggesting which is better. Here in this paper, we adopt the single-path method. We iterate the simulation procedure 10,000 times, and discard the first 1,000 draws.

investors are considered to make the rational decision with more sufficient information compared to short term investment horizon.

3.3. Lead-lag Relation and Price Transmission

The two series, $\{Y_t\}$ and $\{X_t\}$, refer to TWSI cash index and MSCI (or Dow Jones) index futures price series, respectively. If $\{Y_t\}$ and $\{X_t\}$ are cointegrated, then there exists a constant A , such that $v_t = Y_t - AX_t$ is stationary. We then estimate

$$Y_t = r + sX_t + v_t \quad (10)$$

where Y_t is price series in cash index and X_t is price series in futures index.

Let us perform the residual from regression of cash to futures. The structure of $\{v_t\}$ is autoregressive:

$$\Delta v_t = r + s v_{t-1} + \sum_{i=1}^n w_i \Delta v_{t-i} + \epsilon_t \quad (11)$$

where v_t is the error-term from the cointegration equation, and ϵ_t is a stationary random error term. The augmented Dickey-Fuller test is used to examine the significance of s in this regression. If s is not significantly different from zero, which means not to reject the null hypothesis that s is zero, then $\{v_t\}$ is nonstationary and we say the two time series, $\{Y_t\}$ and $\{X_t\}$ are not cointegrated.

4. Empirical Results

4.1. Hedge Effectiveness

4.1.1. Hedge Effectiveness Comparison

Table 1 reports the empirical results under the OLS regression in Bayesian approach using Gibbs Sampler. The time horizon is from July 21,1998 to July 31,1999. The data generated are tested by daily, weekly, and bi-weekly (fortnightly) to see each of their hedge ratios, R-square coefficients, and posterior means (r_{12}), including the 95% and 90% Highest Posterior Density (HPD) regions.

Some phenomena can be observed from this table. First, the posterior mean, which can also be expressed in r_{12} , is rejected. It indicates that the futures of TSE TAI-Index futures is highly superior to that of SIMEX MSCI futures price in both

Table 1

The hedge ratio, r-square, and HPD region of TSE TAI-Index and SIMEX MSCI Taiwan index futures in Bayesian Approach using Gibbs Sampler – Daily, Weekly, and Bi-weekly Data

Horizon	SIMEX (MSCI)		TSE (TAI-Index)		Posterior mean ¹	HPD region	
	Hedge ratio	R^2 (HE)	Hedge ratio	R^2 (HE)		(95%)	(90%)
Daily	0.5221	0.4344	0.3849	0.1366	3.1570**	(2.5799, 3.8704)	(2.6539, 3.7192)
Weekly	0.7552	0.7948	0.6814	0.4067	1.9444**	(1.3832, 2.9159)	(1.4435, 2.6817)
Bi-weekly	0.7581	0.8150	0.8826	0.5564	1.4366*	(0.9926, 2.3972)	(1.0478, 2.1295)

1. Posterior mean is also denoted r_{12} .

*: denotes significant at 5% level.

**: denotes significant at 1% level.

daily data and monthly data. And the posterior mean generated by daily data is much higher than that by the weekly and bi-weekly ones. Second, though the hedge ratio and hedging effectiveness calculated from daily data are both less than that from weekly data, we can not conclude that we can get better correlation in daily data than in weekly data. It may be explained by the lack of enough trading volume in TSE. Third, the hedge effectiveness is not high in both futures in daily data. The SIMEX MSCI futures daily hedge effectiveness is about 0.4344 while the TSE's is 0.1366, which is even worse. According to the output above, we thus have no reason to say that SIMEX MSCI Taiwan index futures and TSE TAI-Index futures are good instruments for hedging the TWSI spot index during the period from July 21, 1998 to July 31, 1999 due to the low trading volume.

4.1.2. Hedging Effectiveness in Different Time Horizons

Hedgers who have different hedging period and hedging effect may be varied under different investment horizons. Several studies have shown that hedging effectiveness tend to increase as the investment horizon increases [See Ederington (1979)]. **Table 2** shows the change of the hedge ratio, R-square in 28 different time horizons. We can see that the longer period holding the cash hedged with either of the two futures, the higher the correlation in cash and futures, and the higher R-square one gets. It corresponds to the previous study that there is positive relationship between investment horizons and hedging effectiveness. Besides, we can find that as the time horizon increases, there will be no difference using SIMEX MSCI Taiwan Index futures or TSE TAI-Index's in hedging because of the hedge ratio becomes similar (See **figure 2**) and the hedging effectiveness converges to 1 (See **figure 1**).

Table 2

The hedge ratio & coefficient of determination (R^2) of SIMEX MSCI and TSE
TAI-Index futures calculated from OLS – Horizon: 1 to 28 days

Horizon (days)	SIMEX (MSCI)		TSE (TAI-Index)		$R_{1/2}^*$
	R^2	Hedge Ratio	R^2	Hedge Ratio	
1	0.4344	0.5221	0.1366	0.3850	3.1801
2	0.6090	0.6399	0.2497	0.5493	2.4389
3	0.6946	0.6896	0.3823	0.6827	1.8169
4	0.6909	0.6975	0.4401	0.7421	1.5699
5	0.7236	0.6911	0.4993	0.7484	1.4492
6	0.7217	0.6960	0.4989	0.7234	1.4466
7	0.6978	0.6896	0.5309	0.7346	1.3144
8	0.7262	0.7224	0.5619	0.7687	1.2924
9	0.7241	0.7355	0.6008	0.7556	1.2052
10	0.7253	0.7421	0.6028	0.7515	1.2032
11	0.7424	0.7619	0.6287	0.7801	1.1808
12	0.7445	0.7738	0.6565	0.8031	1.1340
13	0.7523	0.7904	0.6506	0.8090	1.1563
14	0.7584	0.7988	0.6779	0.8274	1.1187
15	0.7525	0.7993	0.6956	0.8416	1.0818
16	0.7630	0.8143	0.7382	0.8647	1.0336
17	0.7851	0.8438	0.7517	0.8835	1.0444
18	0.7967	0.8587	0.7639	0.8978	1.0429
19	0.8224	0.8937	0.7819	0.9104	1.0518
20	0.8299	0.9104	0.7886	0.9353	1.0524
21	0.8394	0.9185	0.8173	0.9522	1.0270
22	0.8530	0.9253	0.8229	0.9645	1.0366
23	0.8624	0.9242	0.8355	0.9695	1.0322
24	0.8710	0.9200	0.8529	0.9797	1.0212
25	0.8697	0.9242	0.8495	0.9697	1.0238
26	0.8795	0.9332	0.8488	0.9756	1.0362
27	0.8873	0.9430	0.8525	0.9821	1.0408
28	0.8913	0.9491	0.8623	0.9817	1.0336