Asia-Pacific Journal of Financial Studies (2010) **39**, 70–89 doi:10.1111/j.2041-6156.2009.00004.x

Intraday Volatility Patterns in the Taiwan Stock Market and the Impact on Volatility Forecasting

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Received 28 January 2008; Accepted 20 October 2009

Abstract

Given the growing importance of the Taiwan stock market, the present study sets out to provide a comprehensive investigation of the intraday time series of the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX). We begin by exploring the intraday volatility patterns and then go on to examine their impact on intraday volatility forecasting. We find that the volatility of the TAIEX returns exhibits an L-shaped intraday periodic pattern, which is distinct across each day of the week. Our empirical results indicate that taking the intraday periodic pattern into account in a generalized autoregressive conditional heteroskedasticity model can substantially improve the precision of intraday volatility forecasting.

Keywords Forecasting; Generalized autoregressive conditional heteroskedasticity; Intraday; Taiwan; Volatility

JEL Classification: G10, C10, C30

1. Introduction

The availability of high-frequency prices in financial assets promotes continuing growth in our knowledge of price reactions to information and the evaluation of intraday market risk. This knowledge is of particular benefit to market participants who are involved in intraday trading, such as day traders, high-frequency portfolio managers, and program traders. Because volatility is a key input for market risk evaluation (particularly in the value-at-risk arena) and derivatives pricing, intraday

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volatility modeling and forecasting become increasingly important for those market participants. They become increasingly important also because the market risk of their positions should be evaluated much more frequently than a daily interval. Comparing the performance of intraday and interday models for various applications, Angelidis and Degiannakis (2008) find that the intraday model consistently produces the most accurate volatility forecasts. This finding for the superiority of intraday information also motivates us to investigate the potential problems for intraday volatility modeling and forecasting.

Following the work of Engle (1982), Bollerslev (1986) and Taylor (1986), a few generalized autoregressive conditional heteroskedasticity (GARCH)-type volatility models have been extensively studied and proven to be quite successful in daily volatility forecasting, although none of them in particular is found to be superior across datasets and applications. However, a periodic volatility pattern can seriously affect the volatility modeling of high-frequency time series (Areal and Taylor, 2002) if the models are utilized without taking the pattern into account, if the latter exists. It is already well documented that for some major stock markets, such as the USA and the UK, return volatility that varies systematically over the trading day are generally U-shaped (McInish and Wood, 1990; Abhyankar *et al.*, 1997; Andersen and Bollerslev, 1997; Cai *et al.*, 2004). As a consequence, standard time-series models of volatility have proven to be inadequate when applied to high-frequency returns of financial assets with such periodic patterns (Andersen and Bollerslev, 1997).

This study sets out to determine whether there are intraday volatility patterns present in the Taiwan stock market, and, if so, whether they have any impact on intraday volatility forecasting. Although Taiwan encompasses a very small geographical area, the importance and trading properties of its stock market provide strong motivation for our study. In terms of market value, the Taiwan stock market usually ranks somewhere between the 10th and 15th largest in the world. Furthermore, as indicated by Barber *et al.* (2005) and Chou *et al.* (2008), intraday trading is very prevalent in the Taiwan market.[1] Therefore, intraday volatility forecasting is important for risk management practice in this market.[2]

The motivation to study the Taiwan stock market is further enhanced by the fact that it shares several common characteristics with other Asian markets, such

[1]Barber *et al.* (2005) find that day traders account for over 20% of the total volume in the Taiwan stock market from 1995 through 1999. Chou *et al.* (2008) also show that the percentage is over 30% in the Taiwan futures market from 2001 through 2006. According to a research report of the Taiwan Futures Exchange, the percentage for the TAIEX futures is as high as 56% during 1998–2005.

[2]According to the 2007 survey of the World Federation of Exchanges, TAIEX options and futures were, respectively, ranked the 4th and 14th most frequently traded index options on a global scale. Therefore, the demand for futures and options risk management in this market is not negligible.

as a high turnover rate, price limits, and a high participation rate by individual investors.[3] The characteristics are markedly different from those of the major stock markets in North America or Europe. In addition, what we learn from Taiwan's stock market has important implications for other Asian markets.[4] As well as providing a comprehensive examination of the high-frequency stock prices of the Taiwan stock market, the present study offers important information on, and insights into, areas such as intraday volatility factors, which should be useful to those interested in modeling or forecasting the intraday volatility characteristics of such markets.

Based upon our examination of the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX), our empirical results reveal the existence of an L-shaped intraday periodic pattern in the volatility of TAIEX returns, which differs significantly from that found in many prior studies on the US and UK equity markets. We also find that the intraday volatility pattern is distinct across each day of the week, with Monday being the highest and Friday being the lowest, which should also prove to have an association with the protracted period of non-trading during weekends. Taking into account both the intraday and day-of-the-week effects of volatility modeling, we further investigate the impact on intraday volatility forecasting and discover that filtering the intraday periodic pattern can substantially improve forecasting performance, whereas the incremental contribution of the day-of-the-week effect is not always significant.

To the best of our knowledge, only a few studies have set out to investigate intraday volatility forecasting.[5] In one such study, Martens *et al.* (2002) undertake a comparison of the intraday volatility forecasts produced by "flexible Fourier functions" (FFF) (Andersen and Bollerslev, 1997), the "periodic GARCH" (P-GARCH) model (Bollerslev and Ghysels, 1996), and the common GARCH model (Bollerslev, 1986; Taylor, 1986).[6] They conclude that the FFF approach is computationally more efficient than, and only marginally inferior to, the P-GARCH model, in terms of forecasting performance.

[3]For example, the stock markets in South Korea and Thailand have high turnover rates as well as high participation rates of individual investors. Price limits are also adopted in the stock markets of China, Japan, South Korea, and Thailand.

[4]We thank the referee for this revision suggestion.

[5]Several studies also investigate intraday volatility forecasting, but do not focus on the impact of the periodic intraday pattern on volatility forecasting. For example, they deal with extreme value distribution (Bali and Weinbaum, 2007), the fractal model (Gordon, 2004), model comparison (Angelidis and Degiannakis, 2008), and the multivariate model for a lognormal-normal mixture distribution (Andersen *et al.*, 2003), by using high-frequency intraday data on volatility forecasting.

[6]Martens *et al.* (2002) compare volatility forecasts over a 30-min horizon for the spot exchange rates of the deutsche mark and the Japanese yen against the US dollar.

Martens *et al.* (2002) present that modeling the intraday periodic pattern explicitly improves the out-of-sample forecasting performance, but they fail to consider the distinct intraday patterns that are discernible across different days of the week: patterns that have been extensively documented for various equity markets by Ho and Cheung (1994) and Clare *et al.* (1997). As stressed by Andersen and Bollerslev (1997), taking the inter-day effect into account may be equally important and could in principle be incorporated along the same lines. This study therefore sets out with the aim of helping to fill this gap, by considering both the intraday and day-of-the-week patterns.

The remainder of the study is organized as follows. Section 2 provides background information on the Taiwan Stock Exchange (TSE) and summary statistics of the TAIEX returns. The analysis of intraday volatility pattern shows up in section 3. Section 4 offers the volatility modeling. An examination of the day-of-the-week volatility effect is undertaken in section 5, followed in section 6 by an analysis of the impact of the intraday and day-of-the-week patterns on intraday volatility forecasting. Finally, section 7 provides the conclusions drawn from this study.

2. Background Information on the Taiwan Stock Exchange and Data Description

Before proceeding further, it is useful to understand the institutional background of the TSE, which is an order-driven call market. Transactions are executed in strict price and time priority. During our sample period, the market operates from 09.00 hours to 13.30 hours, Monday through Friday (excluding public holidays). There is a 7% price limit in each direction from the previous day's closing price.[7] The most distinctive characteristic is the high participation of individual traders. According to a 2005 TSE report ("2005 Major Stock Markets Regulations"), individual domestic traders accounted for 69% of the total annual trading value, whereas they were 31% for the Hong Kong Exchange, 24% for the London Stock Exchange, and 35.1% for the New York Stock Exchange.

Our investigation of volatility behavior in the Taiwan stock market focuses on the TAIEX, comprising all common shares listed on the TSE. Both intraday and daily TAIEX data are obtained from the *Taiwan Economic Journal* database. The intraday data run from 2 January 2001 to 29 September 2006, while the daily time series cover the period from 4 January 1990 to 29 September 2006.

Our primary intraday dataset comprises a time series with 5-min frequency intervals and is a base for generating time series with various lower frequencies. Based upon the trading period on the TSE, which runs from 9.00 hours to

[7]In our sample, there are 3 days with an upper price limit of 7% and a lower price limit of 3.5%. The change of the lower price limit from 7% to 3.5% was in response to the re-opening of the US markets after 11 September 2001.

13.30 hours, our sample period includes 1423 trading days with 54 5-min returns per day. In total, we have 76 842 5-min, 38 421 10-min, 25 614 15-min, and 12 807 30-min observations.

Table 1 presents the summary statistics of the returns for various frequencies, from which we find that the properties of the returns are in general consistent with the stylized facts found in many prior studies (Andersen and Bollerslev, 1997; Mian and Adam, 2001; Taylor, 2005) on equity returns and that they do not rely on data frequency. All of the distributions are negatively skewed and leptokurtic, which is statistically supported by the Jarque–Bera normality tests.

Following a comparison of the summary statistics of returns across various frequencies, we find that with an increase in frequency, there is a monotonic increase in the magnitudes of both skewness and kurtosis. In other words, the higher the frequency is, the greater non-Gaussian the distribution will be. Finally, the Ljung-Box *Q*-statistics for absolute returns indicate the existence of an ARCH effect for all return series.

3. Intraday Volatility Patterns

Regardless of which frequency is considered, we consistently find that the intraday average absolute return is extremely high at the market opening, whereas it is at a lower and generally very stable level for the remainder of the day. In other words, the volatility of TAIEX returns exhibits an L-shaped intraday pattern, which is consistent with the finding of Huang and Yang (1999) in the Taiwan stock market and is similar to that of the Australian stock market (Mian and Adam, 2001) and the Shanghai Stock Exchange (Tian and Guo, 2007), but very different from the common finding of a U-shaped pattern for the USA and UK equity markets (McInish and Wood, 1990; Abhyankar *et al.*, 1997; Andersen and Bollerslev, 1997; Cai *et al.*, 2004). Figure 1 illustrates the intraday volatility periodicity for 5-min returns.[8] The correlogram of the 5-min returns represented by a solid line in Figure 2 also confirms the existence of intraday periodicity.

The high open volatility within the Taiwan stock market could come as a result of longer daily non-trading hours (a 19.5-h period). Fan and Lai (2006) find that such high open volatility is associated with a high level of information asymmetry. This is consistent with the Taiwan stock market as the market has a considerable number of individual investors. However, what is the reason leading to the stable post-open volatility? In particular, there is no increase in volatility just before the market closes. The absence of an increase in volatility just before the market closes may be due to some characteristics in TSE. Huang and Yang (1999) suggest that shorter trading hours and investor composition might be able to explain the

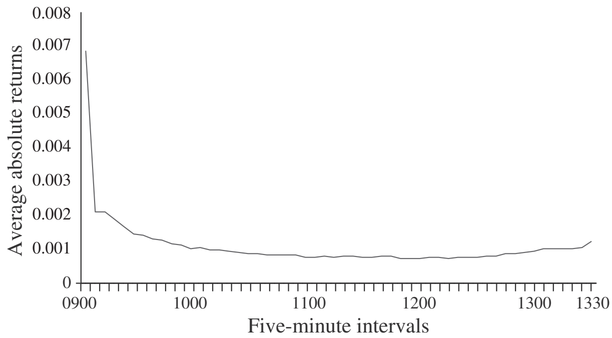
[8]Details of the intraday volatility periodicity for other frequencies are also available upon request.

Table 1 Summary statistics

This table presents the summary statistics of the raw returns of various frequencies. The intraday data sample period covers from 2 January 2001 to 29 September 2006, while the daily time series covers from 4 January 1990 to 29 September 2006. ***Indicates significance at the 1% level. The Ljung and Box (1978) test for up to tenth-order serial correlation is denoted by $Q(10)$ for the returns and by $Q^A(10)$ for the absolute returns.

Intervals	Total number of observations	Mean	Standard deviation	Skew	Kurtosis	Jarque-Bera	$Q(10)$	$Q^A(10)$
Panel A: Intraday intervals								
5-min	76842	0.00000486	0.0020	−0.179	76.713	17397645***	1030.2***	14816***
10-min	38421	0.00000971	0.0028	−0.153	35.224	1662454***	122.39***	5763.3***
15-min	25614	0.00001460	0.0035	−0.183	28.434	690560***	171.53***	2214.3***
30-min	12807	0.00002910	0.0047	−0.099	19.070	137825***	42.369***	2086.8***
Panel B: Inter-day intervals								
1-day	4517	−0.00003450	0.0079	−0.197	5.413	1125***	56.846***	3542.4***
2-day	2259	−0.00006750	0.0115	−0.144	5.931	817***	37.887***	149.75***
5-day	903	−0.00016800	0.0186	−0.268	5.335	216***	13.627	648.06***
8-day	564	−0.00027600	0.0243	−0.615	6.182	273***	22.913	410.53***

Figure 1 Averages of intraday absolute returns.



This figure exhibits the intraday pattern of 5-min absolute returns.

phenomenon, whereas Fan and Lai (2006) examine the effect of the extension of trading hours[9] on TSE's intraday patterns and find no evidence to suggest that shorter trading hours affect the volatility pattern.[10]

4. Volatility Modeling

Before taking into account the observed L-shaped periodicity for intraday volatility forecasting, we first employ the widely-used GARCH(1,1)[11] model to determine what it is that makes standard time-series volatility models unsuitable for modeling intraday time series. Indeed, there are prior studies that have established theoretical relationships between the GARCH parameters at various frequencies (Nelson, 1990; Drost and Nijman, 1993; Drost and Werker, 1996).

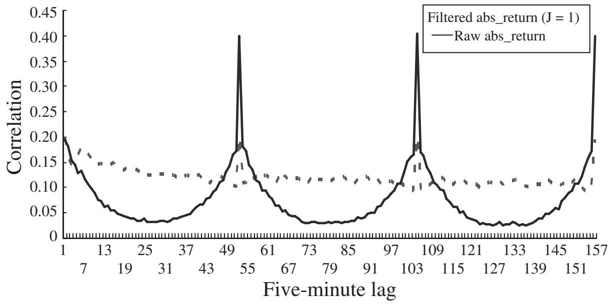
First, with an increase in data frequency, volatility persistence ($\alpha + \beta$) should tend towards 1. Second, with a move from daily to increasingly fine intraday frequencies, parameter α should tend towards 0, and parameter β should tend towards 1. Third, the estimates of half life, mean lag, and median lag should be stable for returns at different frequencies. Table 2 presents the estimation results of the GARCH model for the TAIEX returns for various frequencies. We determine from

[9]TSE cancelled Saturday trading and extended trading by 1.5 h per day from 1 January 2001.

[10]We find that the intraday volatility patterns of both the daily returns not hitting price limits and those days with lower price limits are still L-shaped, and that the period with a higher holding ratio of institutional traders has a steeper volatility pattern just before the market closes. Although these results are just some preliminary findings for exploring the potential reasons of the intraday volatility pattern, they provide some interesting and challenging ideas for future research.

[11]We have tried several other GARCH model specifications, such as ARMA-GARCH, GARCH(p,q), and GJR-GARCH(p,q). These results are similar to those of GARCH(1,1) and are available upon request from the authors.

Figure 2 Three-day correlogram of intraday absolute returns.



This figure exhibits the intraday pattern of autocorrelations of five-min absolute returns.

Table 2 that the results of the theoretical aggregation work satisfactorily for multiple-day frequencies, but break down at intraday frequencies. For example, the sum of α and β does not exhibit any monotonic increase when the frequency moves from 30 to 5 min.

There is no stability in the estimates of half life, mean lag, and median lag for the intraday series. In particular, the half life estimate of the 30-min series is roughly 8.39 times that of the 5-min series, while the estimates of half life, mean

Table 2 Generalized autoregressive conditional heteroskedasticity (GARCH)-(1,1) model estimations

This table presents the estimation results of the GARCH(1,1) model for the raw returns of various frequencies. The estimated GARCH models are of the following form: $R_t = z_t\sigma_t, \sigma_t^2 = w + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$, where $z_t \sim i.i.d.\ N(0,1)$. The reported estimates for α and β are obtained by quasi-maximum likelihood methods under the assumption that the innovations have conditional normal distributions; all P-values are <0.0001. The half life of a shock to the conditional variance is calculated as $-\log(2)/\log(\alpha + \beta)$; the mean lag of a shock to the conditional variance is calculated as $\alpha/(1 - \alpha - 2\beta + \alpha\beta + \beta)$; and the median lag of a shock to the conditional variance is calculated as $½ + [\log(1 - \beta) - \log\alpha - \log(2)]/\log(\alpha + \beta)$.

Intervals	Total number of observations	α	β	$\alpha + \beta$	Half life	Mean lag	Median lag
Panel A: Intraday intervals							
5-min	76842	0.150049	0.600126	0.750175	2.41	0.44	−0.50
10-min	38421	0.326202	0.543135	0.869337	4.95	1.06	3.04
15-min	25614	0.150005	0.600488	0.750493	2.41	0.44	−0.50
30-min	12807	0.037002	0.929308	0.966310	20.23	0.54	1.84
Panel B: Inter-day intervals							
1-day	4517	0.088905	0.897645	0.986550	51.19	0.95	41.28
2-day	2259	0.118414	0.856751	0.975165	27.56	0.94	20.49
5-day	903	0.097591	0.869802	0.967393	20.91	0.83	12.71
8-day	564	0.118286	0.836321	0.954607	14.92	0.82	8.43

lag, and median lag are much more stable across those return series with a frequency covering a complete day. In short, such inconsistency in the theoretical predictions of the estimates of the intraday return series suggests that the intraday periodic pattern in volatility cannot be ignored when modeling intraday time series (Andersen and Bollerslev, 1997).

To effectively overcome the above problem, we use the general framework developed by Andersen and Bollerslev (1997) to analyze high-frequency return series. Consider the following decomposition for the intraday returns:

$$R_{t,n} = E(R_{t,n}) + N^{-\frac{1}{2}}\sigma_t s_{t,n} Z_{t,n}, \tag{1}$$

where $R_{t,n} = \ln\left(\frac{P_{t,n}}{P_{t,n-1}}\right)$. N denotes the number of high-frequency returns per day, σ_t is the conditional volatility on day t, $s_{t,n}$ refers to the periodic intraday component of the nth interval on day t, and $Z_{t,n}$ is an i.i.d. zero-mean, unit-variance error term.

We define a new variable as:

$$x_{t,n} \equiv 2\log\left[|R_{t,n} - E(R_{t,n})|\right] - \log\sigma_t^2 + \log N, \tag{2}$$

which can be obtained by approximating $E(R_{t,n})$ with the average of all intraday returns, and σ_t from the estimate from a daily volatility model. We then have the following regression:

$$\hat{x}_{t,n} = \sum_{j=0}^{J} \sigma_t^j \left[\mu_{0j} + \mu_{1j}\frac{n}{N_1} + \mu_{2j}\frac{n}{N_2} + \sum_{i=1}^{D}\lambda_{ij} I_{n=d_i} + \sum_{i=1}^{P}\left(\gamma_{ij}\cos\frac{2\pi in}{N} + \delta_{ij}\sin\frac{2\pi in}{N}\right)\right], \tag{3}$$

where $N_1 = (N + 1)/2$ and $N_2 = (N + 1)\ (N + 2)/2$ are normalizing constants, from which each of the corresponding $J + 1$ FFF are parameterized by the quadratic component (μ coefficients) and a number of sinusoids (γ and δ coefficients).

It may be expedient to also include time-specific dummies for applications in which some intraday intervals do not fit well within the overall regular periodic pattern (λ coefficients). We present the intraday periodic components, or factors, as follows:

$$\hat{s}_{t,n} = \frac{T \times \exp\left(\frac{\hat{x}_{t,n}}{2}\right)}{\sum_{t=1}^{\frac{T}{N}} \sum_{n=1}^{N} \exp\left(\frac{\hat{x}_{t,n}}{2}\right)}. \tag{4}$$

Therefore, the filtered return series can be obtained as:

$$\widetilde{R}_{t,n} = \frac{R_{t,n}}{\hat{s}_{t,n}}. \tag{5}$$

Figure 2 provides a correlogram illustration of the raw and filtered absolute 5-min returns, from which we clearly see that, although not entirely eliminated, the

Table 3 Generalized autoregressive conditional heteroskedasticity (GARCH)-(1,1) estimates of filtered returns

This table presents the estimation results of the GARCH(1,1) model for the filtered returns of various frequencies. The estimated GARCH models are of the following form: $R_t = z_t\sigma_t, \sigma_t^2 = w + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$, where $z_t \sim i.i.d.\ N(0,1)$. The reported estimates for α and β are obtained by quasi-maximum likelihood methods under the assumption that the innovations are conditionally normally distributed.

Intervals	Total number of observations	α	β	$\alpha + \beta$	Half life	Mean lag	Median lag
5-min	76842	0.152041	0.626347	0.778388	2.77	0.48	−0.32
10-min	38421	0.15	0.6	0.75	2.41	0.44	−0.50
15-min	25614	0.15	0.599999	0.749999	2.41	0.44	−0.50
30-min	12807	0.149999	0.599995	0.749994	2.41	0.44	−0.50

periodic pattern has been significantly mitigated. This suggests that the intraday periodic pattern of volatility has been purged by the de-periodicity procedure.

Much more convincing evidence is provided by the estimates of the GARCH models reported in Table 3, which when compared to the results provided in Table 2 are more consistent with the various theoretical predictions. For example, we clearly see that the volatility parameters now display a more coherent pattern across various frequencies, with the sum of α and β increasing monotonically when the frequency moves from 30 to 5 min. In addition, the estimates of half life, mean lag, and median lag are relatively stable across the return frequencies.

In summary, taking account of the intraday periodic pattern should help make a substantial contribution to modeling high-frequency financial time series.

5. Day-of-the-week Volatility Effect

For those undertaking analyses of the long-run phenomena associated with intraday time series, taking account of the day-of-the-week periodic pattern is equally as important as the intraday periodic pattern (Andersen and Bollerslev, 1997). Therefore, we should further examine the TAIEX returns to determine whether they exhibit any obvious day-of-the-week volatility patterns.

To test for the day-of-the-week volatility effect, we use the modified Levene test (Brown and Forsythe, 1974), because this is one of the most powerful tests (even for small samples) and it is also a test that remains robust on the departure from normality of the underlying distribution (Ho and Cheung, 1994). The modified Levene test statistic is defined as:

$$F = \left[\sum_{j=1}^{J} n_j (D_j - D)^2 \bigg/ \sum_{j=1}^{J} \sum_{i=1}^{n_j} (D_{ij} - D_j)^2 \right] [(N-J)/(J-1)] \sim F(J-1,\ N-J), \tag{6}$$

where $D_{ij} = |R_{ij} - M_{ij}|$, R_{ij} is the return for week i and weekday j, for $j = 1, 2, 3, 4,$

5; M_j is the sample median return for weekday j computed over n_j weeks; $D_j = \sum_{i=1}^{n_j} D_{ij}/n_j$, which is the mean absolute deviation (from the median) for weekday j; $D = \sum_{j=1}^{J} \sum_{i=1}^{n_j} D_{ij}/N$, which is the grand mean; and $N = \sum_{j=1}^{J} n_j$.

Table 4 reports the descriptive statistics of the daily returns across weekdays, from which we see that Monday has the highest volatility, while Friday has the lowest. This is consistent with the argument that the markets tend to be more volatile at the beginning of the week and less volatile with the approach of the weekend (Ho and Cheung, 1994; Clare *et al.*, 1997).

Focusing on the relationship between volatility and mean returns shows that the day with the highest volatility is also the day with the lowest return, and vice versa, which is one of the remarkable characteristics of emerging markets (Ho and Cheung, 1994; Clare *et al.*, 1997). Most importantly, from the Levene test result, the hypothesis of homoskedasticity is strongly rejected at the 1% significance level. Therefore, such evidence suggests the existence of a day-of-the-week effect for TAIEX returns.

To provide more convincing evidence, we must further examine whether the intraday volatility pattern varies across different days of the week. Table 5 presents the mean intraday periodic components on each intraday interval from Monday to Friday, from which we determine that the null hypothesis of equal periodic components from Monday to Friday is strongly rejected for all intraday intervals.

In summary, we therefore find the existence of both an intraday periodic pattern and a day-of-the-week effect in the volatility of returns on the TAIEX.

6. Intraday Volatility Forecasting

The results of intraday return modeling show that taking intraday volatility periodicity into account leads to the GARCH model meeting the econometric requirements

Table 4 Descriptive statistics of daily returns across weekdays

This table presents the descriptive statistics of daily returns across weekdays. As in the case of the intraday data, the sample period runs from 2 January 2001 to 29 September 2006.

Time	Monday	Tuesday	Wednesday	Thursday	Friday
Mean	−0.001421	−0.000476	0.001140	0.000588	0.001465
Median	−0.000230	−0.000460	−0.000570	0.000199	0.001014
Standard deviation	0.017244	0.013654	0.015036	0.014294	0.013506
Skewness	−0.225	0.555	0.308	0.028	−0.354
Kurtosis	4.358	5.162	4.236	5.129	4.322
Jarque–Bera	24.034	70.242	22.508	53.841	26.738
	(0.000006)	(<0.0001)	(0.000013)	(<0.0001)	(<0.0001)
Modified Levene test statistic	F-value = 4.155 $\sim$F(4, 1418) The hypothesis of homoskedasticity has to be rejected at the 1% level				

Table 5 Intraday periodic component means across weekdays

This table presents the intraday periodic component means of volatility across weekdays. The *F*-values are the test statistics for the null hypothesis that the intraday volatility means for different weekdays are equal. All *P*-values <0.0001.

Time	Weekdays					*F*-value
	Monday	Tuesday	Wednesday	Thursday	Friday	
0905	1.993	2.091	2.181	2.148	2.087	26.76
0910	2.212	2.322	2.421	2.384	2.316	26.29
0915	2.196	2.306	2.404	2.368	2.300	25.61
0920	1.971	2.070	2.158	2.126	2.064	25.03
0925	1.661	1.745	1.819	1.792	1.740	24.92
0930	1.384	1.454	1.516	1.493	1.450	25.29
0935	1.193	1.253	1.306	1.286	1.249	25.87
0940	1.090	1.144	1.193	1.175	1.142	26.32
0945	1.054	1.106	1.154	1.136	1.104	26.56
0950	1.055	1.107	1.154	1.137	1.105	26.56
0955	1.059	1.111	1.159	1.141	1.109	26.41
1000	1.039	1.091	1.138	1.121	1.089	26.25
1005	0.990	1.040	1.084	1.068	1.037	26.16
1010	0.924	0.970	1.011	0.996	0.968	26.26
1015	0.862	0.905	0.943	0.929	0.903	26.54
1020	0.820	0.861	0.898	0.884	0.859	26.86
1025	0.804	0.844	0.880	0.867	0.842	27.10
1030	0.808	0.848	0.884	0.871	0.846	27.22
1035	0.819	0.858	0.896	0.882	0.857	27.23
1040	0.820	0.860	0.897	0.883	0.859	27.24
1045	0.802	0.841	0.877	0.864	0.840	27.24
1050	0.766	0.803	0.838	0.825	0.802	27.24
1055	0.724	0.759	0.792	0.780	0.758	27.03
1100	0.690	0.724	0.755	0.744	0.722	26.42
1105	0.673	0.707	0.737	0.726	0.705	25.24
1110	0.676	0.711	0.741	0.730	0.708	23.88
1115	0.693	0.729	0.759	0.748	0.726	22.89
1120	0.713	0.750	0.782	0.770	0.747	22.77
1125	0.725	0.762	0.794	0.782	0.760	23.73
1130	0.721	0.758	0.790	0.778	0.756	25.32
1135	0.705	0.740	0.771	0.760	0.738	26.63
1140	0.684	0.718	0.749	0.737	0.717	27.18
1145	0.670	0.703	0.733	0.722	0.701	27.24
1150	0.668	0.700	0.730	0.719	0.699	27.11
1155	0.676	0.710	0.740	0.729	0.708	26.18
1200	0.690	0.725	0.755	0.744	0.722	23.87
1205	0.698	0.734	0.765	0.754	0.731	21.18
1210	0.694	0.731	0.761	0.750	0.727	19.54

Table 5 (*Continued*)

Time	Weekdays					F-value
	Monday	Tuesday	Wednesday	Thursday	Friday	
1215	0.679	0.715	0.744	0.733	0.711	19.61
1220	0.661	0.695	0.724	0.713	0.693	21.31
1225	0.653	0.686	0.715	0.704	0.684	23.88
1230	0.663	0.697	0.726	0.715	0.695	25.94
1235	0.696	0.731	0.762	0.750	0.729	26.78
1240	0.745	0.781	0.815	0.803	0.780	26.82
1245	0.794	0.833	0.869	0.856	0.831	25.99
1250	0.825	0.867	0.903	0.890	0.864	24.02
1255	0.827	0.870	0.907	0.893	0.867	21.53
1300	0.808	0.851	0.886	0.873	0.847	19.79
1305	0.791	0.833	0.868	0.855	0.829	19.54
1310	0.805	0.847	0.883	0.870	0.844	20.80
1315	0.879	0.924	0.963	0.948	0.920	23.03
1320	1.037	1.089	1.135	1.118	1.086	25.22
1325	1.296	1.360	1.418	1.397	1.357	26.44
1330	1.642	1.722	1.796	1.769	1.719	26.84
Total	0.949	0.996	1.039	1.023	0.994	94.74

for various frequencies. In this section, as opposed to exploring the marginal impact of the day-of-the-week volatility effect on modeling, we examine the intraday volatility forecasting performance for those GARCH models in which we consider only the intraday periodic pattern, or both the intraday periodicity and the day-of-the-week effect, by evaluating both in-sample and out-of-sample performance.

We compare the volatility levels generated from the GARCH models with three intraday returns series: comprised of raw returns, returns filtered by the intraday components, and returns filtered by both the intraday components – and the day-of-the-week dummies with the realized volatility, which is computed as the sum of the squared intraday returns (Andersen and Bollerslev, 1997) for alternative frequencies.

There is no universal agreement about the best measure for evaluating the volatility forecasting performance. Hence, we compare the different forecasting performances based on five measures that have commonly been used in the literature: (i) root mean square error (RMSE); (ii) mean absolute error (MAE); (iii) adjusted R^2 from the regression of realized variance on the forecast variance; (iv) adjusted R^2 from the regression of log(realized variance) on log(forecast variance); and (v) adjusted R^2 from the regression of square root(realized variance) on square root(forecast variance). The log and square root transformations are used as variance stabilizing transformations in the regressions because the realized variance is very heteroskedastic (Deo *et al.*, 2006).

Root mean square error and MAE are defined as follows:

$$\text{RMSE} = \sqrt{\frac{1}{T}\sum_{i=1}^{T}\left(\hat{\sigma}_t^2 - R_t^2\right)^2} \tag{7}$$

$$\text{MAE} = \frac{1}{T}\sum_{i=1}^{T}|\hat{\sigma}_t^2 - R_t^2|, \tag{8}$$

where $\hat{\sigma}_t^2$ and R_t^2 denote the respective forecasted and realized variances, with a better forecast having a lower error measure.

The regression methods are defined as below:

$$R_t^2 = \alpha_1 + \beta_1\hat{\sigma}_t^2 + \varepsilon_t \tag{9}$$

$$\log\left(R_t^2\right) = \alpha_2 + \beta_2\log\left(\hat{\sigma}_t^2\right) + \varepsilon_t \tag{10}$$

$$\textit{square root}\left(R_t^2\right) = \alpha_3 + \beta_3\ \textit{square root}\left(\hat{\sigma}_t^2\right) + \varepsilon_t. \tag{11}$$

Our in-sample window for the parameter estimation covers the period from 2 January 2001 to 31 May 2006. Table 6 presents the full sample results.

In regards to out-of-sample forecasting, we take the last 4 months (i.e. from 1 June 2006 to 29 September 2006) as the out-of-sample period. By rolling over day by day, we obtain the one-period volatility forecasts of the out-of-sample period. Table 7 provides these results.

First, to transform the volatility forecasts for the filtered returns into the volatility forecasts that are supposed to be generated by the original returns, the variance forecasts generated by the filtered returns are multiplied by the square of the periodic component, $s_{t,n}$. We begin with a comparison of the results of the raw returns with those obtained for returns filtered by the intraday components.

We find that for both the in-sample and out-of-sample forecasts, the use of filtered returns creates much lower forecasting errors, and they are not dependent on performance measures or data frequencies. For example, the MAE measures for filtered returns improve by 24.22[12] to 41.07%[13] for in-sample forecasting and by 2.52[14] to 17.00%[15] for out-of-sample forecasting. The R^2 measures in panels C, D and E for filtered returns are 1.99[16] to 73.66[17] times the raw returns for in-sample forecasting and 1.60[18] to 79.28[19] times that for out-of-sample forecasting. In short,

[12]Table 6, panel B, 10-min: (8.536410E−06 – 1.126486E−05)/1.126486E−05 = −24.22%.
[13]Table 6, panel B, 5-min: (4.416060E−06 – 7.493876E−06)/7.493876E−06 = −41.07%.
[14]Table 7, panel B, 15-min: (8.512061E−06 – 8.732018E−06)/8.732018E−06 = −2.52%.
[15]Table 7, panel B, 10-min: (6.505367E−06 – 7.838249E−06)/7.838249E−06 = −17.00%.
[16]Table 6, panel D, 10-min: 0.27207/0.13688 = 1.99.
[17]Table 6, panel C, 15-min: 0.11564/0.00157 = 73.66.
[18]Table 7, panel D, 5-min: 0.12945/0.08102 = 1.60.
[19]Table 7, panel C, 5-min: 0.04757/0.0006 = 79.28.

Table 6 Full-sample forecasting

This table presents the full-sample results of volatility forecasting based on raw returns and filtered returns for various frequencies. Five measures that have commonly been used in the literature: (i) root mean square error (RMSE); (ii) mean absolute error (MAE); (iii) adjusted R^2 from the regression of realized variance on the forecast variance; (iv) adjusted R^2 from the regression of log(realized variance) on log(forecast variance); and (v) adjusted R^2 from the regression of square root(realized variance) on square root (forecast variance). The log and square root transformations were used as variance stabilizing transformations in the regressions because the realized variance is very heteroskedastic (Deo *et al.*, 2006). *Indicates better forecasting performance.

		Filtered return	
Intervals	Raw return	With weekday dummy	Without weekday dummy
Panel A: RMSE			
5-min	3.421529E−05	3.414798E−05	3.408764E−05*
10-min	5.003266E−05	4.713272E−05*	4.714150E−05
15-min	6.115605E−05	5.636153E−05*	5.637477E−05
30-min	8.628790E−05	7.732583E−05*	7.743387E−05
Panel B: MAE			
5-min	7.493876E−06	4.504511E−06	4.416060E−06*
10-min	1.126486E−05	8.534125E−06*	8.536410E−06
15-min	1.645382E−05	1.051467E−05	1.050454E−05*
30-min	2.425551E−05	1.582514E−05*	1.583720E−05
Panel C: Adjusted R^2 from the regression of realized variance on the forecast variance			
5-min	0.00081	0.00542*	0.00538
10-min	0.00175	0.04269*	0.04229
15-min	0.00157	0.11586*	0.11564
30-min	0.00732	0.20193*	0.19971
Panel D: Adjusted R^2 from the regression of log(realized variance) on log(forecast variance)			
5-min	0.07725	0.16415*	0.16318
10-min	0.13688	0.27228*	0.27207
15-min	0.11818	0.37140*	0.36753
30-min	0.12338	0.58896*	0.58876
Panel E: Adjusted R^2 from the regression of square root(realized variance) on square root(forecast variance)			
5-min	0.04157	0.12866	0.12949*
10-min	0.06124	0.26799*	0.26733
15-min	0.04509	0.38763*	0.38707
30-min	0.05328	0.55948*	0.55774

taking into consideration the intraday periodic pattern substantially improves not only the model specification, but also the volatility prediction.

Second, we compare the results for the returns filtered by both the intraday components and the day-of-the-week dummies with those for returns filtered only by the intraday components, in an attempt to determine the incremental contribution of the day-of-the-week effect. As Table 6 shows, 15 out of the 20 in-sample

Table 7 Out-of-sample forecasting

This table presents the out-of-sample results of volatility forecasting based on raw returns and filtered returns for various frequencies. Five measures that have commonly been used in the literature: (i) root mean square error (RMSE); (ii) mean absolute error (MAE); (iii) adjusted R^2 from the regression of realized variance on the forecast variance; (iv) adjusted R^2 from the regression of log(realized variance) on log(forecast variance); and (v) adjusted R^2 from the regression of square root(realized variance) on square root (forecast variance). The log and square root transformations were used as variance stabilizing transformations in the regressions because the realized variance is very heteroskedastic (Deo *et al.*, 2006). *Indicates better forecasting performance.

		Filtered return	
Intervals	Raw return	With weekday dummy	Without weekday dummy
Panel A: RMSE			
5-min	1.684959E−05	1.486018E−05	1.484525E−05*
10-min	2.334985E−05	2.065974E−05	2.063166E−05*
15-min	2.656079E−05	2.382986E−05	2.380092E−05*
30-min	3.898061E−05	3.396531E−05	3.394391E−05*
Panel B: MAE			
5-min	4.179418E−06	3.548642E−06	3.500925E−06*
10-min	7.838249E−06	6.505367E−06*	6.540230E−06
15-min	8.732018E−06	8.476391E−06*	8.512061E−06
30-min	1.702455E−05	1.447323E−05*	1.448875E−05
Panel C: Adjusted R^2 from the regression of realized variance on the forecast variance			
5-min	0.0006	0.04735	0.04757*
10-min	0.00041	0.02776	0.02797*
15-min	0.00961	0.22879	0.2334*
30-min	0.01017	0.34795*	0.34748
Panel D: Adjusted R^2 from the regression of log(realized variance) on log(forecast variance)			
5-min	0.08102	0.12935	0.12945*
10-min	0.08536	0.22423*	0.22387
15-min	0.07377	0.37557*	0.37398
30-min	0.06869	0.5014	0.50239*
Panel E: Adjusted R^2 from the regression of square root(realized variance) on square root(forecast variance)			
5-min	0.03419	0.15109*	0.15055
10-min	0.02809	0.14765*	0.14732
15-min	0.03989	0.40736*	0.40583
30-min	0.03387	0.53212	0.53438*

measures indicate that performance is better if the day-of-the-week pattern is taken into consideration.

If we undertake a much more careful comparison of the forecast errors, then we find that the increments are rather small. For example, the RMSE for those returns that are also filtered by the day-of-the-week dummies are 0.00000001 smaller than

Table 8 The results of Wilcoxon's sign-rank test

This table presents the results of median equality tests (Wilcoxon's sign-rank test) for both full-sample and out-of-sample volatility forecasting errors generated by the models based on raw returns and various filtered returns. *P*-values are given in parentheses. *Indicates better forecasting performance.

	5-min	10-min	15-min	30-min
Panel A: Full-sample forecasting				
Median of absolute error				
Raw return	5.691873E−06	6.450113E−06	1.089500E−05	1.303700E−05
With weekday dummy	1.239334E−06	3.277231E−06*	3.896705E−06*	4.402470E−06
Without weekday dummy	1.192252E−06*	3.277643E−06	3.900373E−06	4.393672E−06*
Statistics of Wilcoxon's signed-rank test				
Raw return versus With weekday dummy	1.3131E+09 (<0.0001)	2.1770E+08 (<0.0001)	1.0916E+08 (<0.0001)	2.2363E+07 (<0.0001)
Raw return versus Without weekday dummy	1.3490E+09 (<0.0001)	2.1698E+08 (<0.0001)	1.0883E+08 (<0.0001)	2.2417E+07 (<0.0001)
With weekday dummy versus Without weekday dummy	9.5152E+08 (<0.0001)	−5.6650E+07 (<0.0001)	18365389 (<0.0001)	1.0667E+06 (<0.0001)
Panel B: Out-of-sample forecasting				
Median of absolute error				
Raw return	1.808247E−06	4.243433E−06	4.870579E−06	1.065400E−05
With weekday dummy	1.248845E−06	2.490858E−06*	3.672139E−06	6.191703E−06*
Without weekday dummy	1.210952E−06*	2.533244E−06	3.623938E−06*	6.258513E−06
Statistics of Wilcoxon's signed-rank test				
Raw return versus With weekday dummy	1.8324E+06 (<0.0001)	5.5233E+05 (<0.0001)	1.1426E+05 (<0.0001)	5.6516E+04 (<0.0001)
Raw return versus Without weekday dummy	1.9539E+06 (<0.0001)	5.3883E+05 (<0.0001)	1.1881E+05 (<0.0001)	5.5885E+04 (<0.0001)
With weekday dummy versus Without weekday dummy	2.4082E+06 (<0.0001)	−3.8518E+05 (<0.0001)	49835.5 (0.0053)	−3.7469E+04 (<0.0001)

those for returns filtered only by the intraday components. As Table 7 shows, nine out of the 20 out-of-sample measures demonstrate better performance with consideration of the day-of-the-week pattern. Therefore, we conclude that a consideration of the day-of-the-week effect does not really provide significant help in forecasting intraday volatility.

To test the robustness of our empirical results, we further conduct Wilcoxon's signed-rank test.[20] Diebold and Mariano (1995) stress that Wilcoxon's signed-rank test is powerful and robust when comparing predictive accuracy. From Table 8, the results of Wilcoxon's signed-rank test are consistent with our findings; that is, the forecasting errors of the filtered return are all significantly lower than the raw return. Regarding whether to have a weekday dummy or not, half of the forecasting errors with a weekday dummy are significantly lower than those without a weekday dummy, and vice versa.

In summary, we find that filtering the intraday periodic pattern substantially improves the intraday volatility forecasting performance, whereas the incremental contribution stemming from a consideration of the day-of-the-week effect is not always significant.

7. Conclusions

Although it is has already been well documented that a U-shaped periodic pattern is exhibited by intraday volatility within the USA and UK stock markets, it is natural to suspect that alternative market characteristics could create very different periodicities in terms of intraday volatility. The present study has investigated the intraday volatility patterns of the TAIEX returns and their impact on intraday volatility forecasting. We believe this study should be useful as the Taiwan stock market cannot be ignored given its ranking in the world markets; and also because it shares several common characteristics with other Asian markets. These characteristics are markedly different from those of the major stock markets in North America or Europe and, therefore, what we learn from the Taiwan stock market has important implications for other Asian markets as well.

We have found that the volatility in TAIEX returns exhibits an L-shaped intraday pattern, very different from the U-shaped pattern typically found in the major stock markets in North America and Europe. This pattern also differs across the various days of the week. Furthermore, we also see that when modeling high-frequency data, consideration of intraday patterns substantially improves the

[20]The test statistics of Wilcoxon's signed-rank test are:

$$WSRT = \frac{\sum_{t=1}^{T} I_{+}(\mathrm{d}_t)\mathrm{rank}(|\mathrm{d}_t|) - \frac{T(T+1)}{4}}{\sqrt{\frac{T(T+1)(2T+1)}{24}}} \sim N(0,1),$$

where $I_{+}(\mathrm{d}_t) = 1 \quad \text{if } \mathrm{d}_t > 0$
$\qquad\quad = 0 \quad \text{otherwise.}$

$$\mathrm{d}_t \equiv \{e_{it}\}_{t=1}^{T} - \{e_{jt}\}_{t=1}^{T} \equiv \{\hat{\sigma}_{it}^2 - R_t^2\} - \{\hat{\sigma}_{jt}^2 - R_t^2\}.$$

precision of intraday volatility forecasting, whereas it seems that the day-of-the-week effect does not always have such an important role to play.

In addition to providing a comprehensive investigation of high-frequency stock prices in the Taiwan stock market, the present study offers important insights for those with an interest in modeling or forecasting intraday volatility within such markets. Future studies might use certain elements of the information provided by the estimations undertaken in the present study, such as intraday components, in order to directly adjust the raw returns when modeling the high-frequency data.

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