

Discussion

Letter to the Editor: Closure to discussion on the article “Rigid body concept for geometric nonlinear analysis of 3D frames, plates and shells based on the updated Lagrangian formulation” by Y.B. Yang, S.P. Lin, C.S. Chen [Comput. Methods Appl. Mech. Engrg. 196 (2007) 1178–1192]

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The discussor raises a very interesting question in the nonlinear analysis of three-dimensional framed structures, with regard to the appearance of the factor 1/2 for bisymmetric solid cross sections in Eq. (15), which relates to the rotational properties of internal moments. This is not an issue for the geometric nonlinear analysis of framed structures using the approach presented in the paper for the following reasons:

- (1) The factor 1/2 in Eq. (15) arises from the adoption of the equality in Eq. (5.5.48) of Yang and Kuo [4], which is mathematically correct, as was noted by the discussor.
- (2) In the nonlinear analysis of framed structures, distinction should be made between the “internal moments”, which are generated as the stress resultants over the cross section, and the “external moments”, which are generated by external mechanical devices. With regard to the rotational properties of the internal torsion, one needs to know the composition of shear stresses τ_{xy} and τ_{xz} over a cross section, which *cannot* be determined solely from the sectional shape, and are *not* known before the problem is solved. For I-sections, rather than for bisymmetrical solid sections, the parameter α cannot be directly related to I_y/I_z in the way used by the discussor. See Yang and McGuire [6] for a more general definition of α for I-sections.

- (3) In contrast, we *can* always specify the rotational properties of external moments because they are generated by external agencies.
- (4) For a nonlinear structure to be in equilibrium, we can “cut” the structure into a number of “elements” (members) and “joints”, and require each element and joint to be in equilibrium in the *deformed state*. The rotational components of internal moments on both sides of each “cut” will cancel out by themselves in the deformed state, as was stated in Section 5 of the paper or Sections 6.3 and 6.7 of Yang and Kuo [4], *regardless of whether they are defined as semitangential moments* [1], *generalized moments* [2], or the one including the α parameter mentioned by the discussor. In other words, in the nonlinear analysis of three-dimensional frames, if a rational approach is adopted, the definition of the internal moments is really up to the choice of the researchers.
- (5) The other difficulty encountered in defining the rotational properties of internal torsions is that they are *not* generally known before the solution is obtained. For instance, let us consider an I-beam subjected to an end torque. Depending on the extent of warping restraint at the two ends, which is unknown if the I-beam is part of a three-dimensional frame, the torque will be resisted by St. Venant torque and warping torque, which can be regarded as semi- and quasi-tangential torques, respectively. However, the ratio of St. Venant torque to warping torque is not known before the torsional equation is solved [5].

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- (6) Although some physical meanings were given for the terms containing the factor $1/2$ in Eq. (15), they were there only for the interest of researchers familiar with the rotational properties of internal moments.
- (7) With regard to “external torques”, i.e., those generated by mechanical devices, it is essential that the rotational increments generated by these devices upon three-dimensional rotations be duly taken into account in establishing the “joint equilibrium conditions” in the deformed state, as was stated in Section 5 of the paper, or in more details in Sections 6.3 and 6.7 of Yang and Kuo [4]. There is no argument that different “external moments” will result in different buckling loads. In fact, the same problem as the one presented by the discussor was analyzed by Yang and Kuo [3] via consideration of “element” equations and “joint” equations in the deformed state, in which distinction has been made between the “internal” and “external” moments with no consideration made for the factor $1/2$ or α .

The authors would like to thank the discussor for bringing this issue here. We wish that the explanations given above can help clarify the problem.

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