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Design of real FIR filters with arbitrary complex frequency responses by two real Chebyshev approximations

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Abstract. Since the real coefficients of a FIR filter with arbitrary complex-valued desired frequency responses are neither symmetric nor antisymmetric, the Remez exchange algorithm cannot be applied directly. The problem can be solved by dividing the original complex approximation into two real ones such that the Remez exchange algorithm can be applied by slightly modifying the Parks–McClellan program. This method is much easier than the currently existing methods using linear programming or complex Chebyshev approximation, and the performance is satisfactory. More importantly, the magnitudes of the resultant complex errors are also equiripple as the direct complex Chebyshev approximation designs. Several numerical examples including a low-pass filter, a full-band differentiator, a wide-band Hilbert transformer, and a chirp-delay and sine-delay FIR all-pass phase equalizer are given to show the effectiveness of this approach.

Zusammenfassung. Da die reellen Koeffizienten eines FIR-Filters mit beliebigem komplexwertigem Wunschfrequenzgang weder symetrisch noch antimetrisch sind, kann der Remez-Algorithmus nicht angewendet werden. Das Problem kann dadurch gelöst werden, daß die ursprüngliche komplexe Approximation so in zwei reelle aufgeteilt wird, daß der Remez-Algorithmus nach einer geringfügigen Modifikation des Parks–McClellan-Programms angewendet werden kann. Diese Methode ist wesentlich einfacher als die bekannten Verfahren, die auf linearer Programmierung oder auf komplexer Tschebyscheff-Approximation beruhen, und die Ergebnisse sind zufriedenstellend. Wichtig ist, daß die Beträge der resultierenden komplexen Fehler Equiripple-Eigenschaften wie bei direkter komplexer Teschbyscheff-Approximation aufweisen. Zum Nachweis der Effizienz dieses Verfahrens werden mehrere Beispiele angeführt wie ein Tiefpaß-Filter, ein breitbandiger Differenzierer und Hilbert-Transformator sowie ein Chirp-delay und Sine-delay FIR Allpaß Phasenentzerrer.

Résumé. Du fait que les coefficients réels d'un filtre FIR à réponse fréquentielle complexe arbitraire ne sont ni symétriques ni antisymétriques, l'algorithme d'échange de Remez ne peut pas être appliqué directement. Le problème peut être résolu en divisant l'approximation complète originelle en deux approximations réelles de telle sorte que l'algorithme de Remez puisse être appliqué en modifiant légèrement la procédure de Parks–McClellan. Cette méthode est bien plus aisée que les méthodes existantes utilisant la programmation linéaire ou une approximation de Chebyshev complexe et les performances sont satisfaisantes. Un aspect plus important réside dans le fait que les modules des erreurs complexes résultantes sont à oscillation uniforme comme le sont les conceptions par approximation de Chebyshev complexe directe. Plusieurs exemples numériques incluant un filtre passe-bas, un différentiateur pleine bande, un filtre de Hilbert à bande large, ainsi que des égaliseurs de phase FIR passe-tout chirp-retard et sinus-retard sont donnés pour montrer l'efficience de cette approche.

Keywords. FIR filter, real Chebyshev approximation, differentiator, Hilbert transformer, all-pass phase equalizer.

1. Introduction

Conventionally, we often use the well-known McClellan–Parks program [4] to design the linear phase FIR digital filters. But these filters need large length and long time delay when designed with sharp cut-offs. Hermann and Schuessler proposed the method for designing minimum phase FIR filters [3] which cause less delay but introduce delay distortion because the delay is not a constant in the passband. In order to design filters which have less delay than linear phase filters and have approximately constant group delay in the filter passband, recently Chen and Parks have used a standard linear programming algorithm to solve this complex approximation problem [1], and then Preuss designed them in a more general approach by the complex Remez exchange algorithm [5], which also has recently been improved by Schulist [7].

In this paper, we divide the complex Chebyshev approximation problem into two real Chebyshev approximations, and each of them can be solved by using Remez exchange algorithm. This method is fast and easy, also the powerful McClellan–Parks program can be applied after slight modification. The overall performance is satisfactory and the magnitude of the total complex error is also equiripple in the Chebyshev sense. Moreover, the method can also be applied to design all-pass phase equalizers.

The problem formulation for designing general FIR filters with arbitrary complex frequency responses is given in Section 2, in which several examples including lowpass filter, full-band differentiator and wide-band Hilbert transformer designs are demonstrated here. Section 3 presents the design of FIR allpass filters with prescribed phase characteristics and approximately unit magnitude response; these FIR allpass filters are very useful for phase equalization and chirp processing. Two examples including chirp delay and sine delay filters are demonstrated and compared with the designs by the Steiglitz algorithm [8]. Finally, a summary is given in Section 4.

2. Problem formulation for FIR digital filter designs with constant group delay in passband

The frequency response of a FIR digital filter with real impulse response $h(n)$, $n=0, 1, \ldots, N-1$ is given by

$$H(w)=\sum_{n=0}^{N-1} h(n)\,\mathrm{e}^{-jnw}. \tag{1}$$

For simplicity, we consider only odd-length filter designs, let $N=2L+1$ and

$$h(n)=h_{\mathrm{e}}(n)+h_{\mathrm{o}}(n), \quad n=0, 1, \ldots, N-1, \tag{2}$$

where $h_{\mathrm{e}}(n)$ and $h_{\mathrm{o}}(n)$ are the even part and odd part of $h(n)$, respectively, and are given by

$$h_{\mathrm{e}}(L-n)=h_{\mathrm{e}}(L+n)=\tfrac{1}{2}[h(L-n)+h(L+n)], \quad n=0, 1, \ldots, L \tag{3a}$$

and

$$h_{\mathrm{o}}(L-n)=-h_{\mathrm{o}}(L+n)=\tfrac{1}{2}[h(L-n)-h(L+n)], \quad n=0, 1, \ldots, L. \tag{3b}$$

Obviously $h_{\mathrm{e}}(L)=h(L)$ and $h_{\mathrm{o}}(L)=0$. Thus

$$\begin{aligned} H(w) &= \sum_{n=0}^{2L} h_{\mathrm{e}}(n)\,\mathrm{e}^{-jnw}+\sum_{n=0}^{2L} h_{\mathrm{o}}(n)\,\mathrm{e}^{-jnw} \\ &= \left\{h_{\mathrm{e}}(L)+\sum_{n=1}^{L}[h_{\mathrm{e}}(L-n)\,\mathrm{e}^{jnw} + h_{\mathrm{e}}(L+n)\,\mathrm{e}^{-jnw}]\right\}\mathrm{e}^{-jLw} \\ &\quad + \left\{\sum_{n=1}^{L}[h_{\mathrm{o}}(L-n)\,\mathrm{e}^{jnw} + h_{\mathrm{o}}(L+n)\,\mathrm{e}^{-jnw}]\right\}\mathrm{e}^{-jLw} \\ &= \mathrm{e}^{-jLw}\left[\sum_{n=0}^{L}\hat{h}_{\mathrm{e}}(n)\cos nw + \mathrm{j}\sum_{n=1}^{L}\hat{h}_{\mathrm{o}}(n)\sin nw\right], \end{aligned} \tag{4}$$

where

$$\hat{h}_{\mathrm{e}}(n)=\begin{cases} h_{\mathrm{e}}(L), & n=0, \\ 2h_{\mathrm{e}}(L-n), & n=1, \ldots, L \end{cases} \tag{5a}$$

and

$$\hat{h}_{\mathrm{o}}(n)=2h_{\mathrm{o}}(L-n), \quad n=1, \ldots, L. \tag{5b}$$

Then we use (4) to approximate the desired complex-valued frequency response $D(w)$:

$$D(w)=\begin{cases} M(w)\,e^{jP(w)} \\ \quad =e^{-jLw}\{M(w)\cos[Lw+P(w)] \\ \qquad +jM(w)\sin[Lw+P(w)]\}, \\ \qquad\quad w\in\text{passbands.} \\ 0, \quad w\in\text{stopbands,} \end{cases} \tag{6}$$

where $M(w)$ and $P(w)$ are the amplitude response and phase response of $D(w)$, respectively. Then the design problem can be separated into two real approximated criteria, which are called by even and odd approximation, respectively, i.e., even approximation:

$$H_e(w)=\sum_{n=0}^{L}\hat{h}_e(n)\cos nw \simeq D_e(w), \tag{7a}$$

for

$$D_e(w)=\begin{cases} M(w)i\cos[Lw+P(w)], & w\in\text{passband,} \\ 0, & w\in\text{stopband,} \end{cases} \tag{7b}$$

and odd approximation:

$$H_o(w)=\sum_{n=1}^{L}\hat{h}_o(n)\sin nw \simeq D_o(w), \tag{7c}$$

for

$$D_o(w)=\begin{cases} M(w)\sin[Lw+P(w)], & w\in\text{passband,} \\ 0, & w\in\text{stopband.} \end{cases} \tag{7d}$$

The overall filter impulse response $h(n)$ can be obtained by combining the resultant $\hat{h}_e(n)$ and $\hat{h}_o(n)$. Equations (7a) and (7c) are formulated to find $\hat{h}_e(n)$ and $\hat{h}_o(n)$ such that to minimize the maximum absolute weighted errors defined by

$$\|E_e(w)\| = \max W_e(w)|D_e(w)-H_e(w)|, \quad w\in\text{passband/stopband} \tag{8a}$$

and

$$\|E_o(w)\| = \max W_o(w)|D_o(w)-H_o(w)|, \quad w\in\text{passband/stopband,} \tag{8b}$$

for even and odd approximation, respctively, where $W_e(w)$ and $W_o(w)$ are the weighting functions.

The main differences between this problem and the conventional filter approximation problem are in the desired responses for even and odd approximation, the original McClellan–Parks program can be easily modified to fit this problem. Also the weighted errors for two real approximations are each equiripple, if the weighting functions are chosen the same for both even and odd approximations ($W_e(w)=W_o(w)=W(w)>0$), then the magnitudes of the overall complex errors are also equiripple in the complex Chebyshev sense. This separate approximation approach has the simplicity advantages and easy implementation for practical applications.

Due to the fact that the degree of freedom for odd approximation is one less than that for even approximation, the peak error of the former is generally larger than that of the latter. Suppose the peak errors of even and odd approximation are δ_e and δ_o respectively, i.e.,

$$W(w)|D_e(w)-H_e(w)| \leqslant \delta_e \tag{9a}$$

and

$$W(w)|D_o(w)-H_o(w)| \leqslant \delta_o, \tag{9b}$$

then the peak magnitude of the overall complex error is

$$W(w)|D(w)-H(w)| \leqslant \sqrt{\delta_e^2+\delta_o^2}. \tag{10}$$

EXAMPLE 1. Design of low-pass filters.
A 31 point low-pass filter with $L=15$, group delay $\tau=12$ ($P(w)=-12w$), a passband [0, 0.06] and a stopband [0.12, 0.5] is considered in the design specifications. If the passband weighting is 1 and the stopband weighting is 10 for both even approximation and odd approximation, the frequency magnitude and group delay responses are shown in Figs. 1(a) and 1(b), respectively. Figure 1(c) shows the magnitude of the overall complex error, in which the peak value is 0.04404 in passband and 0.004401 in stopband. The traces of overall complex error in passband and stopband are shown in Figs. 1(e)

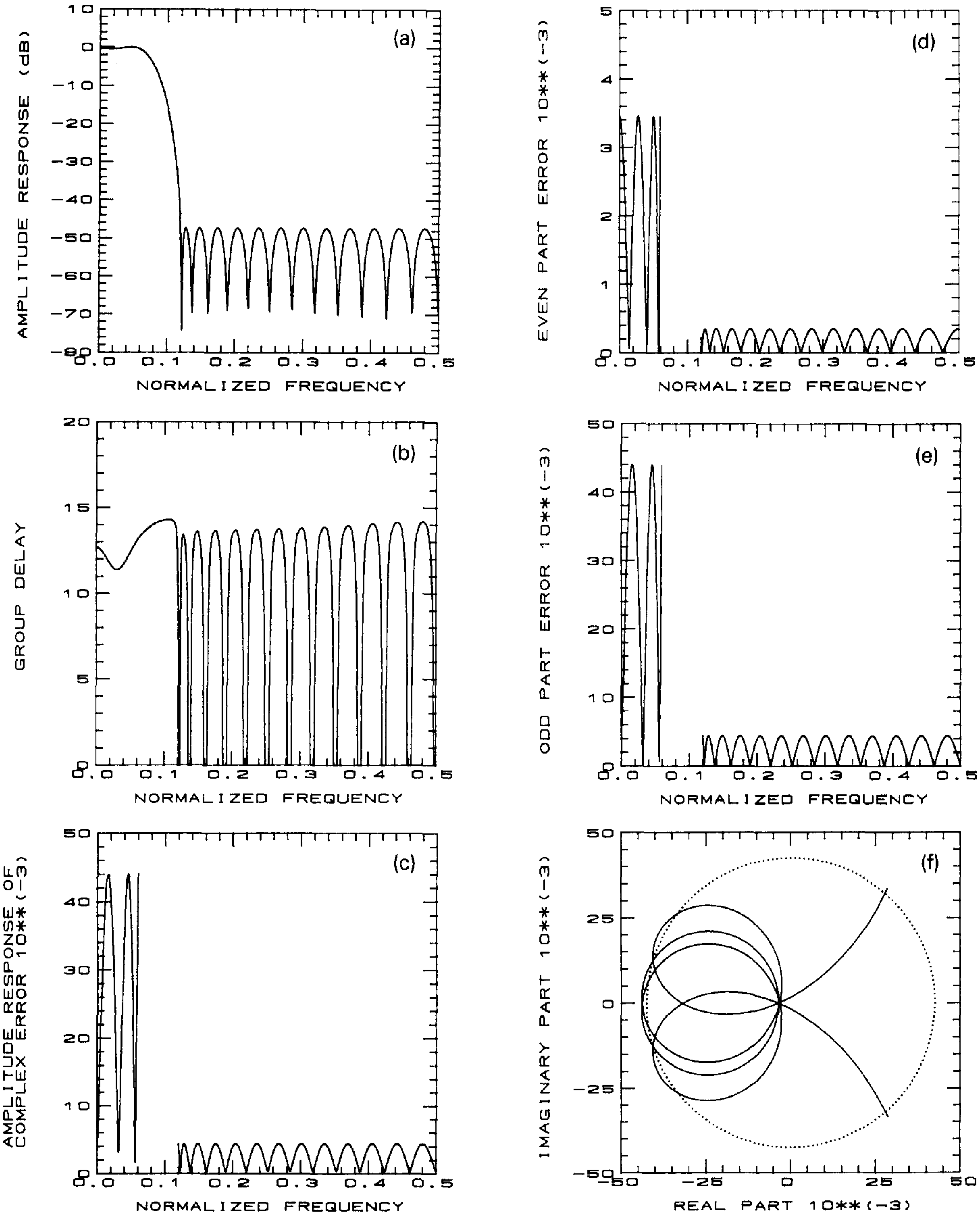


Fig. 1. Example 1. A 31 point low-pass filter with $f_p = 0.06$, $f_s = 0.12$ and $\tau = 12$. (a) Magnitude response, (b) group delay response, (c) equiripple magnitude response of complex error, (d) equiripple error for even approximation, (e) equiripple error for odd approximation, (f) trace of complex error in the passband [−0.06, 0.06] (dotted line: error radius of [7]), (g) trace of complex error in the stopband [0.12, 0.88] (dotted line: error radius of [7]).

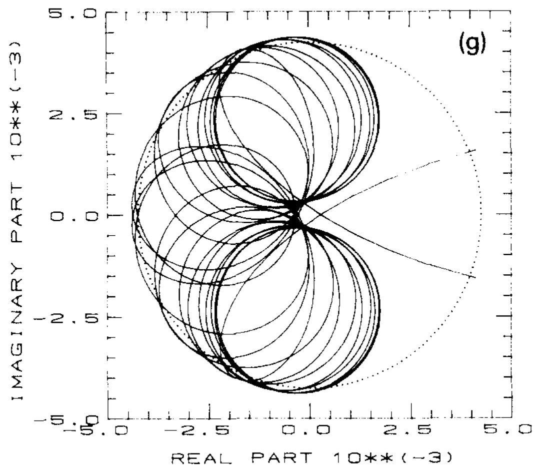


Fig. 1. Continued.

and 1(f), respectively, companying the radius of the complex error in [7] (dotted lines). For details, the performance between this new method and the existing methods [1, 5, 7] are listed in Table 1, and the peak magnitude error is slightly larger than their errors. However, the peak delay error is better and smaller than the Preuss algorithm [5], and the overall performance is satisfactory and comparable to the current methods. The low-pass filter coefficients are listed in Table 2 for reference companying those in Examples 2 and 3, and all the related results of the design examples are summarized in Table 3 for clear illustration. Also Table 4 shows comparison of these examples with linear phase FIR filters.

EXAMPLE 2. Design of differentiators.
The desired frequency response of a differentiator with a constant group delay τ in passband is given by

$$D(w)=\mathrm{j}w\,\mathrm{e}^{-\mathrm{j}\tau w}=\mathrm{j}w\,\mathrm{e}^{-\mathrm{j}Lw}\,\mathrm{e}^{\mathrm{j}(L-\tau)w},\quad |w|\leqslant w_p,$$

$$=\mathrm{e}^{-\mathrm{j}Lw}[-w\sin(L-\tau)w+\mathrm{j}w\cos(L-\tau)w],$$

$$|w|\leqslant w_p, \qquad (11)$$

where w_p is the cut-off frequency. Hence the method described in this section can be applied here by defining

$$D_e(w)=-w\sin(L-\tau)w,\quad 0\leqslant w\leqslant w_p \qquad (12a)$$

and

$$D_o(w)=w\cos(L-\tau)w,\quad 0\leqslant w\leqslant w_p. \qquad (12b)$$

When $N=31$, $L=15$, $\tau=12$, $w_p=0.9\pi$ and $W(w)=1$ are used in the design specifications, the resultant magnitude and group delay responses are shown in Figs. 2(a) and 2(b), respectively. While Fig. 2(c) shows the magnitude of the complex error, and the related results are tabulated in Table 3. In order to design an odd-length full-band differentiator, the addition of half-sample delay is needed such that the phase discontinuity at folding frequency can be eliminated [2, 6]. Figures 3(a) and 3(b) show the magnitude and group delay responses of a 31 point full-band differentiator with $\tau=11.5$ and $L=15$. Figure 3(c) presents the equiripple magnitude of the complex error. Due to an unavoidable phase

Table 1

Comparison of the new method and the existing algorithms [1, 5, 7] for low-pass filter design with $N=31$, $L=15$, $\tau=12$, $f_p=0.06$, $f_s=0.12$

Method	New method	Chen–Parks algorithm	Preuss algorithm	Schulist algorithm
Peak magnitude of complex error in passband	0.04404	0.0436	0.0426	0.0425
Peak magnitude of complex error in stopband	0.004401	0.00436	0.00426	0.00425
Group delay in passband (peak delay error)	11.376–13.063 (1.063)	11.37–12.97 (0.97)	11.389–13.111 (1.111)	Unknown
Design time (computer)	0.74 sec (Vax 8700)	18 min (Vax 11/750)	3 min 10 sec (Vax 11/750)	15 sec (Vax 11/750)

Table 2

Filter coefficients in Examples 1–3

n	Low-pass filter	Non-full band differentiator	Full-band differentiator	Hilbert transformer
0	−2.9260379E−03	−1.0091288E−02	−1.3531666E−02	−3.3941267E−03
1	−9.8825069E−03	1.1886250E−02	3.2449083E−03	−4.0436308E−03
2	−1.3881510E−02	−1.8827861E−02	−3.8333663E−03	−6.4458265E−03
3	−1.9693393E−02	2.8432570E−02	4.6755876E−03	−9.8042637E−03
4	−2.0491123E−02	−4.1576695E−02	−5.8965888E−03	−1.4459278E−02
5	−1.5141083E−02	5.9554216E−02	7.7542798E−03	−2.0930812E−02
6	−3.9204481E−04	−8.4429681E−02	−1.0730968E−02	−3.0084996E−02
7	2.4004953E−02	0.1198350	1.5912993E−02	−4.3525711E−02
8	5.6648027E−02	−0.1729465	−2.6164776E−02	−6.4658344E−02
9	9.3365692E−02	0.2604411	5.1097807E−02	−0.1023066
10	0.1283565	−0.4322625	−0.1416289	−0.1887903
11	0.1551370	0.9383774	1.273391	−0.6151260
12	0.1682457	5.4668047E−02	−1.273388	0.6559071
13	0.1646001	−1.046978	0.1416173	0.2290364
14	0.1444263	0.5387052	−5.1074747E−02	0.1414943
15	0.1112726	−0.3633616	2.6128136E−02	0.1023119
16	7.1212150E−02	0.2711155	−1.5860323E−02	7.9208553E−02
17	3.1348690E−02	−0.2121423	1.0661341E−02	6.3420743E−02
18	−1.8581077E−03	0.1699732	−7.6711178E−03	5.1622480E−02
19	−2.4049312E−02	−0.1376432	5.7974458E−03	4.2291015E−02
20	−3.3727024E−02	0.1117274	−4.5487881E−03	3.4635030E−02
21	−3.2274164E−02	−9.0407021E−02	3.6765132E−03	2.8225839E−02
22	−2.3139339E−02	7.2579376E−02	−3.0445959E−03	2.2792621E−02
23	−1.0682135E−02	−5.7601929E−02	2.5707730E−03	1.8178353E−02
24	1.0674495E−03	4.5020834E−02	−2.2060969E−03	1.4268538E−02
25	9.4494168E−03	−3.4513429E−02	1.9247511E−03	1.0979116E−02
26	1.3160582E−02	2.5828628E−02	−1.7039170E−03	8.2442425E−03
27	1.3230813E−02	−1.8759197E−02	1.5276431E−03	6.0060788E−03
28	9.6369786E−03	1.3115450E−02	−1.3995871E−03	4.2114430E−03
29	7.4154162E−03	−8.7163160E−03	1.2983035E−03	2.8070202E−03
30	2.1086752E−03	8.0967303E−03	−8.1132576E−03	2.6191624E−03

discontinuity at zero frequency, it causes a large pulse for group delay response, this discontinuity at the origin is of no consequence since the magnitude is zero at origin.

EXAMPLE 3. Design of Hilbert transformers. For a Hilbert transformer with group delay τ, the desired frequency response is

$$D(w)=\begin{cases} \mathrm{j}\,\mathrm{e}^{-\mathrm{j}\tau w}, & -\pi<w<0,\\ -\mathrm{j}\,\mathrm{e}^{-\mathrm{j}\tau w}, & 0<w<\pi. \end{cases} \tag{13a}$$

Due to the symmetric and anti-symmetric properties for the magnitude and phase responses, respectively, we only care for the positive frequency design, and

$$D(w)=-\mathrm{j}\,\mathrm{e}^{-\mathrm{j}Lw}\,\mathrm{e}^{\mathrm{j}(L-\tau)w}$$
$$=\mathrm{e}^{-\mathrm{j}Lw}[\sin(L-\tau)w-\mathrm{j}\cos(L-\tau)w],$$
$$0<w_{\mathrm{L}}\leqslant w\leqslant w_{\mathrm{H}}\leqslant\pi, \tag{13b}$$

where w_{L} and w_{H} are the lower and upper cut-off frequencies. Thus the design procedure is similar to that of the differentiators. Also a half-sample delay is added to solve the phase discontinuity at $w_{\mathrm{H}}=\pi$ for designing a wide-band Hilbert transformer. Figures 4(a) and 4(b) show the frequency magnitude and group delay responses of a 31 point

Table 3

FIR filter design examples with arbitrary complex frequency responses

Example	Type of filter (length)	Desired group delay in passband	Peak magnitude of complex error[a] in passband/stopband	Peak magnitude of absolute error[b] in passband/stopband	Group-delay (peak delay error)	Design time in seconds on VAX 8700	Fig.
1	Low-pass filter (31)	12	0.04404/0.004401	0.0333/0.0044	11.376–13.063 (1.063)	0.74	1
2	Non-full-band differentiator (31)	12	0.01027	0.009749	11.6417–950.9291[c] 11.647–12.432[d]	0.67	2
2	Full-band differentiator (31)	11.5	0.01952	0.01952	10.8225–62.7261[e] 11.05058–11.81223[f]	0.74	3
3	Wide-band Hilbert transformer (31)	11.5	0.003392	0.003385	11.412–11.816 (0.316)	0.68	4
4	Chirp all-pass filter (61)	$30+(16/\pi)\times(w-\frac{1}{2}\pi)$	0.001595	0.0007944	22.1146–37.8854 (0.1146)	1.4	5
5	Sine-delay all-pass filter (61)	$30-2\pi \sin w$	0.001528	0.0005249	23.8427–29.8742 (0.1258)	1.06	6

[a] Peak magnitude of complex error: $|D-H|$.
[b] Peak magnitude of absolute error: $||D|-|H||$.
[c] Delay range in [0.00, 0.45].
[d] Delay range in [0.01, 0.45].
[e] Delay range in [0.0, 0.5].
[f] Delay range in [0.05, 0.5].

Table 4

Comparison of Examples 1–3 with linear phase filters

Filter	Low-pass filter		Non-full-band differentiator		Full-band differentiator		Hilbert transformer	
	Nearly linear phase	Linear phase	Nearly linear phase	Linear phase	Nearly linear phase	Linear phase	Nearly linear phase	Linear phase
Length	31	33	31	31	31	32	31	30
δ_p	0.04404	0.04155	0.01027	0.009337	0.01952	0.01802	0.003392	0.003539
δ_s	0.004401	0.004155	none	none	none	none	none	none
Delay in passband	11.376–13.063	16	11.647–12.432	15	11.051–11.812	16	11.412–11.816	15

wide-band Hilbert transformer with $\tau=11.5$, $L=15$, $w_L=0.1\pi$, $w_H=\pi$ and $W(w)=1$.

3. Design of FIR all-pass phase equalizers

For a FIR all-pass filter, its magnitude response is approximately unity with some prescribed phase characteristics, i.e.,

$$D(w)=e^{-jLw}\, e^{j\Phi(w)} = e^{-jLw}[\cos \Phi(w)+j \sin \Phi(w)], \quad 0\leqslant w\leqslant\pi, \tag{14}$$

where $\Phi(w)$ is the phase response and a prescribed function of w. The design problem is similar to that

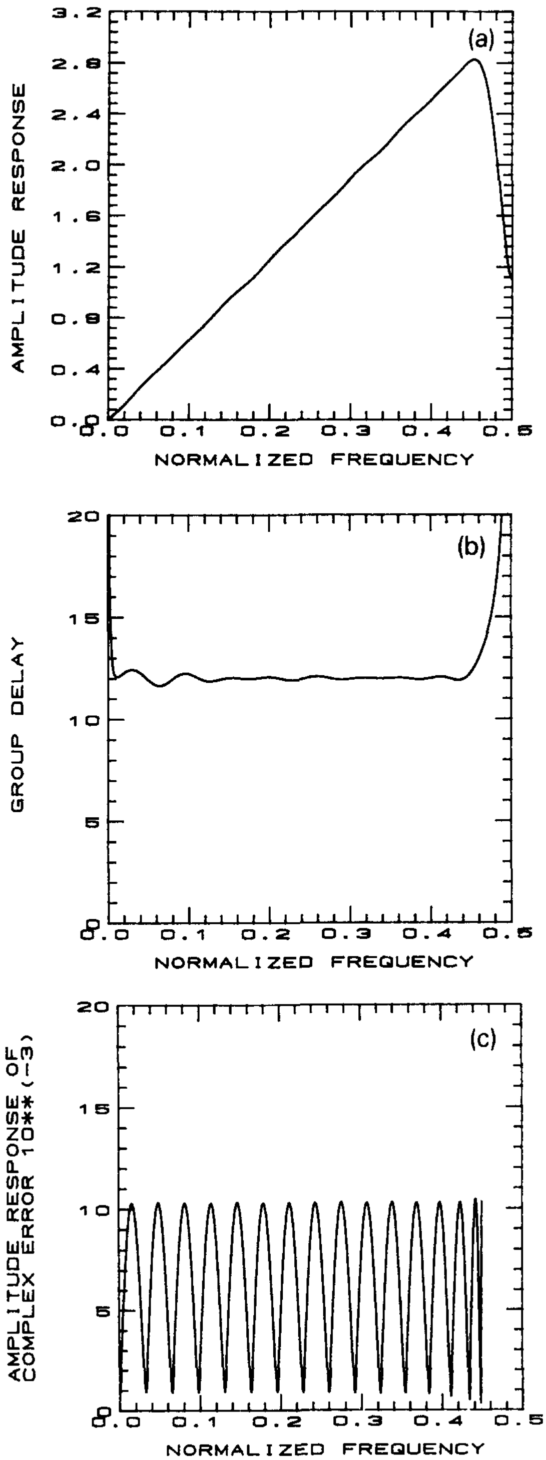


Fig. 2. Example 2. A 31 point differentiator with $f_p=0.45$ and $\tau=12$. (a) Magnitude response, (b) group delay response, (c) equiripple magnitude response of complex error.

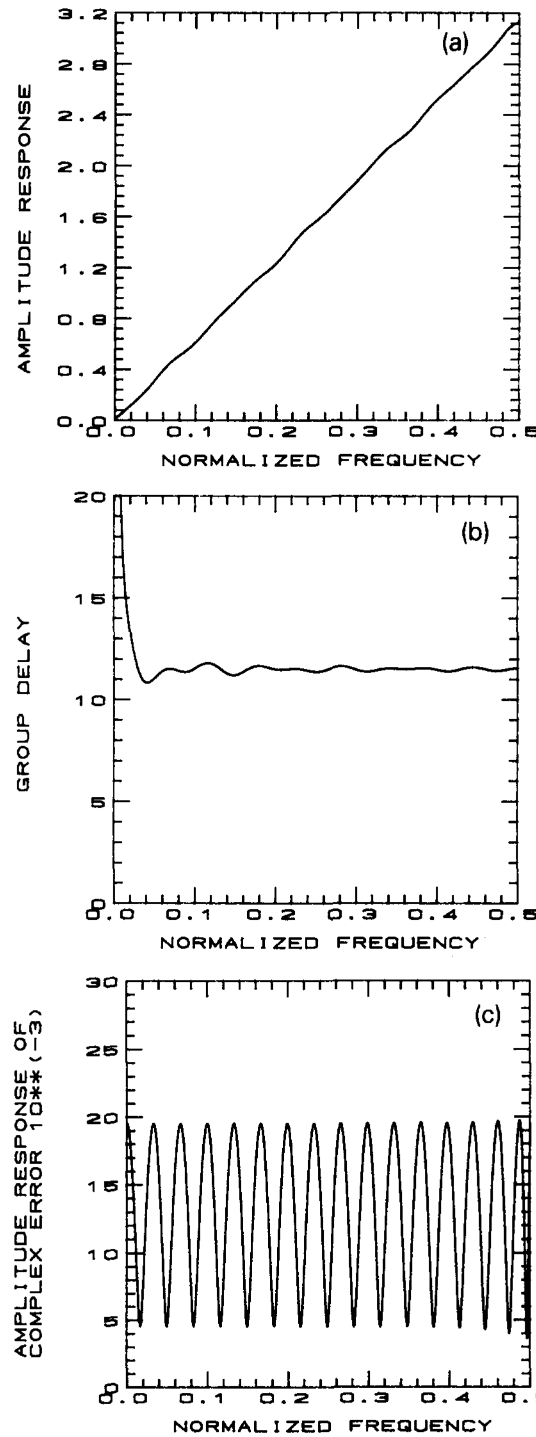


Fig. 3. Example 2. A 31 point full-band differentiator with $\tau=11.5$ by adding a half-sample delay. (a) Magnitude response, (b) group delay response, (c) equiripple magnitude response of complex error.

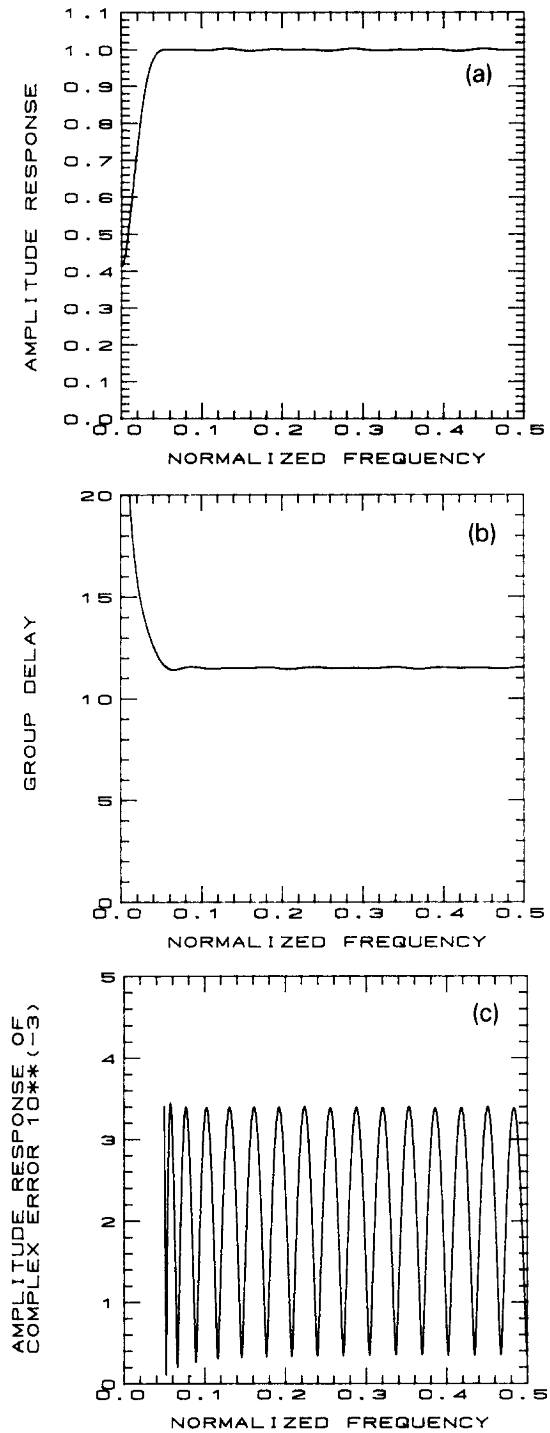


Fig. 4. Example 3. A 31 point wide-band Hilbert transformer with $f_L = 0.05$, $f_H = 0.5$ and $\tau = 11.5$. (a) Magnitude response, (b) group delay response, (c) equiripple magnitude response of complex error.

in Section 2, i.e., we wish to minimize the maximum absolute error ($W(w) = 1$) defined as

$$\|E_e(w)\| = \max\left|\cos \Phi(w) - \sum_{n=0}^{L} \hat{h}_e(n) \cos nw\right|,$$

$$0 \leqslant w \leqslant \pi \tag{15a}$$

for even approximation, and

$$\|E_o(w)\| = \max\left|\sin \Phi(w) - \sum_{n=0}^{L} \hat{h}_o(n) \sin nw\right|,$$

$$0 \leqslant w \leqslant \pi \tag{15b}$$

for odd approximation.

Due to the phase discontinuity at zero and folding frequencies, relaxation of the band edge specification for the odd approximation is permitted such that a better result will be obtained, that is to say, (15b) can be reformulated as below to minimize

$$\|E_o(w)\| = \max\left|\sin \Phi(w) - \sum_{n=0}^{L} \hat{h}_o(n) \sin nw\right|,$$

$$w_o \leqslant w \leqslant \pi - w_o \quad \text{and} \quad w_o \ll \pi. \tag{16}$$

If the phase $\Phi(w)$ is symmetric or antisymmetric about $w = \pi/2$, the FIR all-pass filter can be implemented with one-half the usual number of multiplications, in a manner analogous to the linear phase case [8]. These results are summarized below:

$$\left.\begin{array}{ll} h(L-k) = h(L+k), & k \text{ even} \\ h(L-k) = -h(L-k), & k \text{ odd} \end{array}\right\}$$

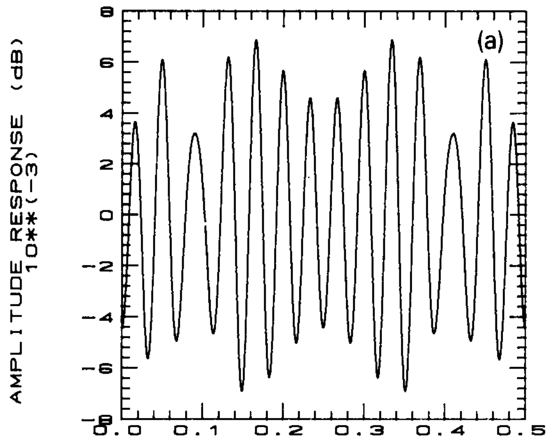
$$\text{for } \Phi(w) \text{ even about } \pi/2 \tag{17}$$

and

$$h(L-k) = h(L+k) = 0, \quad k \text{ odd},$$

$$\text{for } \Phi(w) \text{ odd about } \pi/2. \tag{18}$$

The output coefficients of (18) are not actually zero in practice, however they are generally very small, we simply set these coefficients to zero for keeping the antisymmetry of the phase $\Phi(w)$.



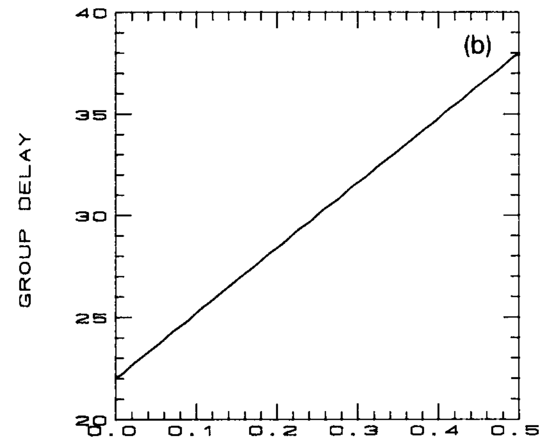
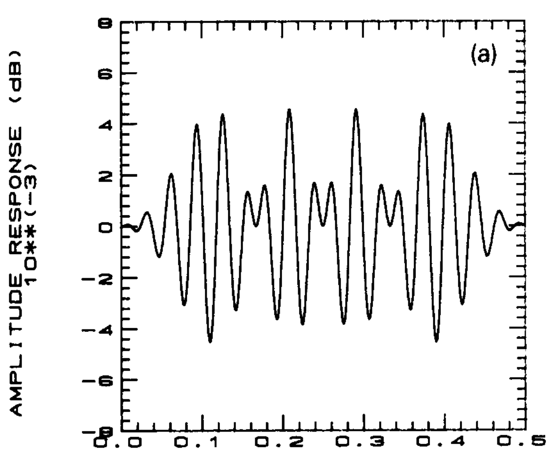


Fig. 5. example 4. a 61 point chirp all-pass phase equalizer for $0.015 \leqslant f \leqslant 0.485$. (a) Magnified magnitude response, (b) group delay response.



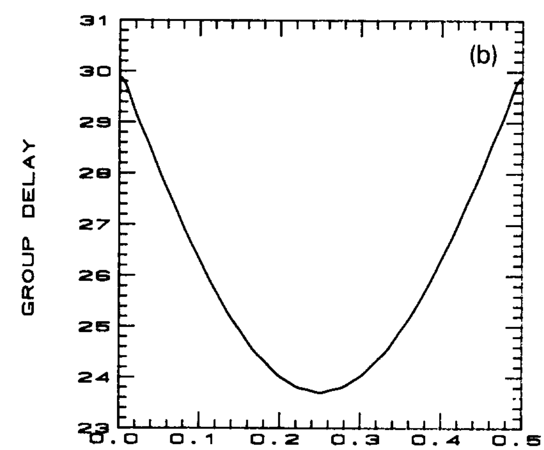


Fig. 6. Example 5. A 61 point sine-delay all-pass phase equalizer for $0.02 \leqslant f \leqslant 0.48$. (a) Magnified magnitude response, (b) group delay response.

EXAMPLE 4. Design of chirp all-pass phase equalizers.

This example deals with the important class of digital chirp filter with phase specification

$$D(w) = e^{-jLw} e^{j\Phi(w)},$$

$$\text{with } \Phi(w) = -\beta(w - \tfrac{1}{2}\pi)^2,\ L = \tfrac{1}{2}(N-1). \tag{19}$$

This phase response is symmetric about $w = \pi/2$.

$$\therefore \text{Arg } D(w) = -\tfrac{1}{2}(N-1)w - \beta(w - \tfrac{1}{2}\pi)^2. \tag{20}$$

Then the desired group delay is

$$\tau(w) = -\frac{d}{dw}\text{Arg } D(w) = \tfrac{1}{2}(N-1) + 2\beta(w - \tfrac{1}{2}\pi). \tag{21}$$

This delay is linearly increased with the frequencies. Consider a 61 point chirp all-pass filter design with $L=30$, $\beta=8/\pi$ and $w_o=0.03\pi$, the desired phase characteristics of (20) is

$$\operatorname{Arg} D(w) = -30w - \frac{8}{\pi}(w-\tfrac{1}{2}\pi)^2, \quad 0.03\pi \leqslant w \leqslant 0.97\pi, \tag{22}$$

and group delay

$$\tau(w) = 30 + \frac{16}{\pi}(w-\tfrac{1}{2}\pi). \tag{23}$$

The resultant magnitude response (magnified version) and group delay responses are shown in Figs. 5(a) and 5(b), respectively, the peak magnitude error is 0.0007944 which is smaller than 0.0008769 in [8].

EXAMPLE 5. Design of sine-delay all-pass phase equalizers.
The desired frequency response is a unit amplitude and a sinusoidial phase characteristic

$$\operatorname{Arg} D(w) = -\tfrac{1}{2}(N-1)w - \beta\cos w, \tag{24}$$

where $\Phi(w) = -\beta\cos w$ is antisymmetric about $w=\pi/2$, and the group delay is

$$\tau(w) = -\frac{\mathrm{d}}{\mathrm{d}w}\operatorname{Arg} D(w) = \tfrac{1}{2}(N-1) - \beta\sin w. \tag{25}$$

A 61 point sine-delay all-pass filter is designed with $L=30$, $\beta=2\pi$ and $w_o=0.04$; Figs. 6(a) and 6(b) show the magnitude response (magnified version) and group delay response, respectively, and the peak magnitude error is 0.0005249 which is much smaller than 0.000931 in [8].

4. Conclusion

By separately approximating the real and imaginary parts of FIR filter complex-valued frequency response, the Parks–McClellan program can be slightly modified to design general FIR filters and all-pass phase equalizers very effectively. This approach is much easier than the current existing linear programming techniques, and the performance is very satisfactory, and more importantly the overall complex errors are also equiripple in the complex Chebyshev sense. This approach has several practical advantages such as fast design time and easy implementation with comparable accuracy.

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