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SW—Soil and Water

Application of the Simulated Annealing Method to Agricultural Water Resource Management

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This work presents a model based on the on-farm irrigation scheduling and the simulated annealing (SA) optimization method for agricultural water resource management. The proposed model is applied to an irrigation project located in Delta, Utah of 394·6 ha area for optimizing economic profits, simulating the water demand and crop yields and estimating the related crop area percentages with specified water supply and planted area constraints.

The application of SA to irrigated project planning in this study can be divided into nine steps: (1) to receive the output from the on-farm irrigation scheduling module; (2) to enter three simulated annealing parameters; (3) to define the design 'chromosome' representing the problem; (4) to generate the random initial design 'chromosome'; (5) to decode the design 'chromosome' into a real number; (6) to apply constraints; (7) to apply an objective function and a fitness value; (8) to implement the annealing schedule by the Boltzmann probability; and (9) to set the 'cooling rate' and criterion for termination.

The irrigation water requirements from the on-farm irrigation scheduling module are: (1) 1067·9, 441·7, and 471·8 mm for alfalfa, barley and maize, respectively, in one unit command area; and (2) 1039·5, 531·4, 490·9, and 539·4 mm for alfalfa, barley, maize and wheat, respectively, in the other unit command area. The simulation results demonstrate that the most appropriate parameters of SA for this study are as follows: (1) initial simulation 'temperature' of 1000; (2) number of moves equal to 90; and (3) 'cooling rate' of 0·95.

1. Introduction

Agricultural water resource planning or irrigation planning can be described as a process for simulating complex climate–soil–plant relationships, applying mathematical optimization techniques to determine the most beneficial crop patterns and water allocations. Such a determination can be non-trivial when large irrigated areas with significant crop diversification are considered, especially with the typical temporal and volumetric restrictions on water supply. A computer-based model to simulate the climate–soil–plant systems with a new mathematical optimization technique could be an effective tool to help irrigation planners to make sound decisions prior to each crop season. Maidment and Hutchinson (1983) stated that irrigation water management models may be classified into two types: (1) demand simulation models, and (2) economic optimization models. Demand simulation models pertain to the climate–soil–plant system, and can be used to deduce the amount and timing of irrigation needed to ensure adequate crop growth. Economic optimization studies relate the cost of irrigation to the benefits derived from increased crop productivity, among other possible factors, to determine the economically optimal patterns of crops and irrigation water application.

Irrigation scheduling is a basic component of agricultural water resource planning. Many existing models

Notation

a	empirical coefficient
a_i, b_i	minimum and maximum values of decoded decimal
A	crop planted area, ha
$A_{j,\%}$	crop planted area within command area, %
$A_{j,ha}$	crop planed area within command area, ha
A_{uca}	area of each unit command area, ha
$A_{\min}$	minimum percentage area of crop within command area, %
$A_{\max}$	maximum percentage area of crop within command area, %
d_i	depth of irrigation water, mm
d_n	maximum net depletable depth, mm
D	soil moisture depletion, mm
D_a	soil allowable depletion, mm
$D_{\max}$	soil maximum allowable depletion, mm
E	energy during annealing scheduling, dimensionless
E_c, E_a	conveyance and application coefficient, %
E_{move}	project benefit at current move during annealing scheduling, $
E_T	evapotranspiration, mm
E_{To}	daily reference crop evapotranspiration, mm day^{-1}
E_{Tc}	potential crop evapotranspiration, mm day^{-1}
E_{Tca}	actual crop evapotranspiration, mm day^{-1}
	daily reference crop evapotranspiration at each stage, mm day^{-1}
$E_{T,c,stage}$	potential crop evapotranspiration at each stage, mm day^{-1}
$E_{T,ca,stage}$	actual crop evapotranspiration at each stage, mm day^{-1}
ΔE	change of project benefit from current and previous moves, $
f_{season}	cumulative seasonal infiltration, mm
$F_{i,j}$	fertilizer cost of the jth crop in the ith command area, $ ha^{-1}
i, j	command area and crop index
k	decision variable
K_a	soil moisture stress coefficient
K_{cb}	basal crop coefficient
K_{cb}^t	basal crop coefficient at day t
K_s	coefficient for evaporation rate from a wet soil surface
K_y	crop yield response factor
$K_{y,stage}$	crop yield response factor at current growth stage
$L_{i,j}$	labour cost of the jth crop in the ith command area, $ ha^{-1}
m_i	substring length
M	number of moves
M_A	available soil moisture, mm m^{-1}
N	number of command areas within irrigated project
N_c	number of crops within command area
$O_{i,j}$	operation cost of the jth crop in the ith command area, $ ha^{-1}
$P_{i,j}$	unit price of the jth crop in the ith command area, $ ha^{-1}
P_r	Boltzmann probability
Q_{dem}	cumulative crop water demand in command area, m^3
Q_{sup}	available water supply for command area, m^3
$Q_{i,j}$	cumulative water requirement of the jth crop in the ith command area, m^3
r	random number
R_a	extraterrestrial radiation, mm day^{-1}
R_z	root depth, mm
s	summation identifier for substring length
$S_{i,j}$	seed cost per hectare of the jth crop in the ith command area, $ ha^{-1}
t	day of year
t_1, t_n	Julian days at the beginning and end of the crop growth stage
t_d	time required for soil surface to dry after irrigation or rainfall, days
t_w	time in days since wetting due to irrigation or rainfall, days
T	daily air temperature, ^{0}C
$T_{\max}$, $T_{\min}$	maximum and minimum daily temperatures, °C
T_{sa}	simulation 'temperature' during cooling schedule, dimensionless
T_{new}, T_{old}	simulation 'temperatures' at the end and beginning, dimensionless
W	unit price of irrigation water, $ m^{-3}
x	decoded decimal
Y_{am}	crop yield reduction
$Y_{a,season}$	relative crop yield reduction due to infiltration over the entire season
$Y_{am,season}$	relative crop yield reduction due to water stress over the entire season
$Y_{am,stage}$	relative crop yield reduction due to water stress at each stage
$Y_{i,j}$	yields per hectare of the jth crop in the ith command area, ton ha^{-1}
α	'cooling' rate
θ_t	soil moisture at the tth day
θ_{fc}, θ_{wp}	soil moisture at field capacity and wilting point

determine on-field water demands based on climate–soil–plant systems. Hill *et al.* (1982) developed the crop yield and soil management simulation model (CRPSM) to estimate crop yield as a function of soil moisture content, crop phenology and climate during the growing periods. Keller (1987) developed the unit command area (UCA) model based partly on the concepts of the CRPSM model. The UCA model consists of two modules: the on-field module for water allocation and distribution; and the field and weather generation module. Prajamwong (1994) developed the command area decision support model (CADSM) with three main sub-models: (1) weather and field generation; (2) on-field crop–soil water balance simulation; and (3) water allocation and distribution. Smith (1991) developed the CROPWAT computer program to calculate crop water requirements and irrigation requirements from climatic and crop data.

Simulated annealing is a stochastic computational technique derived from statistical mechanics for finding near globally solutions to large optimization problems (Davis, 1991). The mathematical theory behind simulated annealing can be explained by the theory of Markov chains (Aarts *et al*., 1985; Otten and Ginneken, 1989) and influenced by the following three operators: (1) initial simulation 'temperature'; (2) the number of moves to allowable rearrangements of the atoms within each temperature; and (3) the 'cooling rate' to decrease the 'temperature'. In a mathematical context, these three operators are the required parameters in the simulated annealing method. Kirkpatrick *et al.* (1983) were the first to propose and demonstrate the application of simulation techniques from statistical physics of combinational optimizations. The mathematical theory to perform the idea of simulated annealing can be obtained using the theory of Markov chains (Laarhoven & Aarts, 1987). Bohachevsky *et al.* (1986) stated that the advantage of the simulated annealing method is the ability to migrate through a sequence of local extremes in search of the global solution and to recognize when the global extremum has been located.

It is interesting to review some papers in which simulated annealing has been applied to water resource management and irrigation scheduling (Dougherty & Marryott, 1991; Marryott *et al*., 1993). Dougherty & Marryott (1991) applied simulated annealing to three problems of optimal groundwater management: (1) a dewatering problem; (3) a contamination problem; and (4) contaminant removal with a slurry wall. Furthermore, they stated that five elements are needed to apply simulated annealing to a particular optimization problem: (1) a concise representation of the configuration of the system decision variables; (2) a scalar cost function; (3) a procedure for generating rearrangements of system; (4) an annealing schedule; and (5) a criterion for terminating the algorithm. It was concluded that simulated annealing had the potential for solving groundwater management problems and that because the application of simulated annealing to water resources problems was new and its development is immature further performance improvements could be expected. Walker (1992) applied the simulated annealing method to a peanut growth model for optimization of irrigation scheduling. The peanut growth model was first applied to determine the days to irrigate and the amount of irrigation during the season. Later, simulated annealing was implemented in the peanut model. The general procedures of this study can be summarized as follows: (1) an initial vector with a fixed 10-day irrigation schedule was chosen from planting to harvest to begin simulation for each year from 1974 to 1991; (2) the peanut model was run and a gross yield was obtained based on the initial vector selected; (3) a new vector of days was generated by the random number generator and a simulated crop yield was calculated; and (4) the Boltzmann distribution probability with generated random number was used to make the decision whether to update the irrigation days or not.

2. Model development

This study focuses mainly on developing an irrigation and planning model to simulate an on-farm irrigation system, and optimize the allocation of the irrigated area to alternative crops for maximum net benefit by the use of a customized simulated annealing method. Therefore, this work develops a model based on the on-farm irrigation scheduling and simulated annealing method to support the agricultural water resource planning and management. The model consists mainly of six basic modules: (1) a main module to direct the running of the model with pull-down menu ability; (2) a data module to enter the required data by a user-friendly interface; (3) a weather generation module to generate the daily weather data; (4) an on-farm irrigation scheduling module to simulate the daily water requirement and relative crop yield; (5) a simulated annealing module to optimize the project maximum benefit; and (6) a results module to present results by tables, graphs and printouts.

Six basic data types are required for the model: (1) project site and operation data; (2) command area data; (3) seasonal water supply data; (4) monthly weather data; (5) soil properties data; and (6) crop phenology and economic data. Herein, the weather generation module is adopted from CADSM (Prajamwong, 1994) to generate daily reference crop evapotranspiration and rainfall data based on the monthly mean and standard deviations data. The on-farm irrigation scheduling module receives

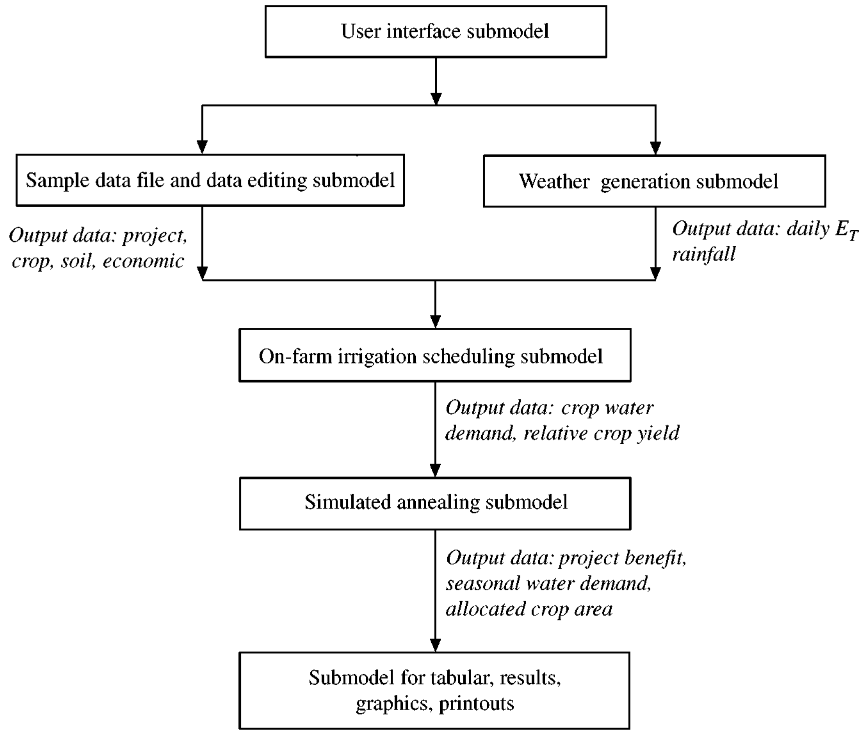

Fig. 1. The framework and logic employed in the agricultural water resource decision support model; E_T, evapotranspiration

the basic project data and generated daily weather data to simulate the on-farm water balance. The daily simulation procedure includes three programming loops: (1) number of command areas within the simulated irrigation project, (2) number of crops within each command area, and (3) number of days from planting to harvest for each crop type. The daily simulation begins from the first command area in the project, the first crop within the command area, and the first Julian day for each crop. The procedure continues until all crops in each command area and all command areas in the irrigation project are processed. The output from this module includes relative crop yield and crop irrigation water requirements. Both outputs are the required inputs for the following simulated annealing optimization module. *Figure 1* presents the framework and logic employed in the irrigation decision support model.

3. On-farm irrigation scheduling

These on-farm irrigation scheduling processes deal with the daily water balance to estimate relative crop yield and irrigation water requirements. The Julian day of planting for each crop type is calculated based on specified crop planting dates. Therefore, the Julian day at harvest is the sum of a crop planting day and cumulative days from each of the growth stages. The daily simulation begins from the first command area in the project, the first crop within the command area, and the initial Julian day for each crop. This procedure continues until all crops in each command area and all command areas in the irrigation project are processed. The relative crop yield and irrigation water requirements are the return values from this module. The results are subsequently sent to the simulated annealing optimization module.

The method developed by Keller (1987) and Prajamwong (1994) was used in this study to generate the daily weather data based on the mean monthly and standard deviation data. A normal distribution is assumed for generating the daily crop reference evapotranspiration and air temperature. A log-normal distribution is assumed for generating daily precipitation. Two important control factors are necessary for generating the daily weather data: (1) the arid probability; and (2) the random sowing date. The arid probability controls the aridity of the year and the random sowing date affects the sequence of the generated data. Based on the generated daily weather data, the Hargreaves equation (Hargreaves et al., 1985) was used to calculate the reference crop evapotranspiration:

$$E_{To} = 0{\cdot}0023 R_a (T + 17{\cdot}8)\sqrt{T_{\max} - T_{\min}} \quad (1)$$

where: E_{To} denotes the (grass) reference crop evapotranspiration in mm day^{-1}; R_a represents the extraterrestrial radiation in mm day^{-1}; T is the mean daily air temperature in °C; T_{max} denotes the maximum daily air temperature in °C; and T_{min} represents the minimum daily air temperature in °C.

The basal crop coefficient K_{cb} represents the effects of the crop canopy on evapotranspiration and varies with time of year. The rate of change of the basal crop coefficient with time can be approximated as a linear increase (or decrease), as expressed in the following equation (Prajamwong, 1994):

$$K_{cb}^{t} = K_{cb}^{stage-1} + (t - t_{stage-1}) \times \frac{K_{cb}^{stage} - K_{cb}^{stage-1}}{t_{stage} - t_{stage-1}} \tag{2a}$$

and,

$$t_{stage-1} \leqslant t \leqslant t_{stage} \tag{2b}$$

where: K_{cb}^{t} denotes the basal crop coefficient for day t; K_{cb}^{stage} represents the basal crop coefficient at the current stage; $t_{stage-1}$ is the first day of current crop stage; t_{stage} denotes the first day of the next crop growing stage; and t is the day of year.

The daily reference crop evapotranspiration E_{To} is used to calculate the potential E_{Tc} and actual crop evapotranspiration E_{Tca}, as given in Eqns (3) and (4), respectively,

$$E_{Tc} = (K_{cb} + K_s)E_{To} \tag{3}$$

$$E_{Tca} = (K_{cb}K_a + K_s)E_{To} \tag{4}$$

The soil moisture stress coefficient K_a and the coefficient for evaporation rate from a wet soil surface after irrigation or rainfall K_s are given by

$$K_a = \frac{\ln[100(\theta_t - \theta_{wp})/(\theta_{fc} - \theta_{wp}) + 1]}{\ln(101)} \tag{5}$$

$$K_s = (1 - K_c)\left\{1 - \left[\frac{t_w}{t_d}\right]^{1/2}\right\}F_w \tag{6}$$

where: θ_t denotes the soil moisture by volume at the tth day; θ_{fc} and θ_{wp} represent soil moisture by volume at field capacity and wilting point; t_w is the time in days since wetting due to irrigation and/or rainfall; and t_d denotes the time in days required for the soil surface to dry after an irrigation and/or rainfall event.

For on-demand irrigation scheduling, irrigation should be performed when the soil moisture depletion D initially exceeds the allowable depletion D_a. The required amount, or application depth, d_i in mm for a given irrigation, and allowable depletion D_a in mm, can be mathematically described by Eqns (7) and (8), respectively,

$$d_i = \frac{D}{E_c E_a} \tag{7}$$

$$D_a = (\theta_{fc} - \theta_{wp})R_z D_{max} \tag{8}$$

where: D represents the soil moisture depletion in mm; E_c is the conveyance coefficient; E_a denotes the water application efficiency; R_z represents the root depth of the crop in mm; and D_{max} is the maximum allowable soil water depletion in mm.

For each crop type, the cumulative water requirement in a growing season is the sum of the irrigation application depths at each time during the growing season. The cumulative water requirement for each command area is the sum of seasonal crop irrigation water requirements within the command area. Finally, the cumulative irrigation water requirement for the project is the sum of the water requirements of each command area within the project.

The amounts of infiltration and runoff are calculated based on the irrigation water or effective rainfall multiplied by the percentage of deep percolation and runoff due to irrigation and rainfall. The model user enters percentage values of infiltration and runoff. The cumulative amount of infiltration is used to calculate the crop yield reduction due to waterlogging.

Two factors influence the relative crop yield: (1) the water stress due to insufficient water for crop evapotranspiration; and (2) waterlogging due to infiltration, produced by over-irrigation and or precipitation. Although the percentage of relative crop yield starts at 100% at the beginning of a growing season, the value can be reduced to less than 100% if there is any water stress or waterlogging during the growing season.

The relative yield reduction due to water stress is calculated at the end of each growth stage based on the ratio of cumulative potential crop evapotranspiration $E_{T,o,stage}$, and actual crop evapotranspiration $E_{T,c,stage}$ in each stage. The relationships can be described by the following equations (Prajamwong, 1994):

$$Y_{am,stage} = 1 - K_{y,stage}\left[1 - \frac{E_{T,ca,stage}}{E_{T,c,stage}}\right] \tag{9}$$

$$E_{T,ca,stage} = \sum_{t=t_1}^{t_n} E_{Tca} \tag{10}$$

$$E_{T,c,stage} = \sum_{t=t_1}^{t_n} E_{Tc} \tag{11}$$

where: $Y_{am,stage}$ denotes the relative yield reduction due to water stress at each stage; $K_{y,stage}$ represents the crop yield response factor at the same stage; $E_{T,ca,stage}$ is the actual crop evapotranspiration at the end of the stage; $E_{T,c,stage}$ denotes the potential crop evapotranspiration at the end of the stage; t_1 and t_n represent the Julian days at the beginning and end of the stage; and E_{Tca} and E_{Tc} are daily crop potential and actual evapotranspiration in mm day^{-1}, respectively.

The minimum value of $Y_{am,stage}$ at each growth stage was chosen to be representative of the relative yield reduction due to water stress over the entire season $Y_{am,season}$ as given by

$$Y_{am,\ season} = \mathrm{Min}(Y_{am,1}\,;\ Y_{am,2}\,;\ldots;\ Y_{am,stage}) \quad (12)$$

The cumulative infiltration within the root zone will reduce soil aeration due to waterlogging and influence the crop yield. Based on the only consideration of total infiltration during the crop growth period, the relative yield reduction due to waterlogging is calculated at the end of the season based on the ratio of cumulative total infiltration f_{season} and the maximum net depletable depth d_n in the root zone. These relationships can be represented by the following equations (Prajamwong, 1994):

$$Y_{a,season} = 1 - a\left(\frac{f_{season}}{d_n}\right) \quad (13)$$

$$d_n = D_{\max} M_A R_z \quad (14)$$

where: $Y_{a,season}$ denotes the relative yield reduction due to infiltration over the entire season; a is the empirical coefficient; $D_{\max}$ represents the maximum allowable depletion (fraction); M_A is the available soil moisture in mm m^{-1}; and R_z denotes the maximum root depth in m.

The product of relative yield reduction due to water stress over the entire season $Y_{am,season}$ and relative yield reduction due to waterlogging over the entire season $Y_{a,season}$ is the final value of relative crop yield at the end of the growing season.

4. Implementation of simulated annealing

4.1. *Simulated annealing model*

Simulated annealing (SA) has recently been applied to functional optimization problems. Functional optimization problems can be described as 'real-world' problems with an objective of obtaining the minimum or maximum global values within specified constraints. For decision support in irrigation project planning, this 'real-world' problem attempts to obtain the optimal crop area-allocated values to maximize the benefit of an irrigation project, given various constraints (*e.g.* maximum and minimum planted areas by crop type and maximum volume of water supply). The SA module has been implemented with the on-farm irrigation scheduling module to maximize the project benefit. The computational procedure of the SA module can be divided into the following steps: (1) to receive the output from the irrigation scheduling module; (2) to enter simulated annealing parameters through an user interface; (3) to define the design 'chromosome' to represent the problem; (4) to generate the random initial design 'chromosome'; (5) to decode the design 'chromosome' into a real number; (6) to apply constraints; (7) to apply an objective function and a fitness value; (8) to implement the annealing schedule by the Boltzmann probability; and (9) to set the 'cooling rate' and criterion for termination. *Figure 2* shows the flowchart of simulated annealing module. The following sections provide descriptive details about each of the steps.

4.2. *Data requirements*

Three parameters must be specified for the simulated annealing module as follows.

(1) *Initial simulation 'temperature'*
Following the steel industry analogy, the initial simulation 'temperature' refers to the initial temperature for 'annealing' in the model. The simulation 'temperature' is gradually decreased depending on the simulation 'cooling rate'. Also, the initial simulation 'temperature' will influence the Boltzmann probability that dominates the annealing schedule.

(2) *Number of moves*
The number of moves is the allowable time for rearrangement of the atoms to a lower energy state within each temperature value. Certainly, a higher number of moves will have a higher opportunity to find a better fitness value, but it will take more computational time. Also, there should be an optimal number of moves to obtain the optimal results for different problems.

(3) *'Cooling rate'*
The 'cooling rate' is the coefficient to decide the rate of simulation 'temperature' decrease. A slow 'cooling rate' (*e.g.* 0·9) allows the molecules to align themselves into a completely ordered crystalline structure; this configuration is the state of minimum energy for the system. If the 'cooling' is too rapid (*e.g.* 0·1), the system does not reach the higher ordered state, but ends up in a high-energy state. The 'cooling' schedule can be mathematically described as follows:

$$T_{new} = \alpha T_{old} \quad (15)$$

where: T_{new} and T_{old} are the simulation 'temperatures' at the end and beginning of the 'cooling' schedule; and α is the 'cooling' rate, which can range from 0 to 1.

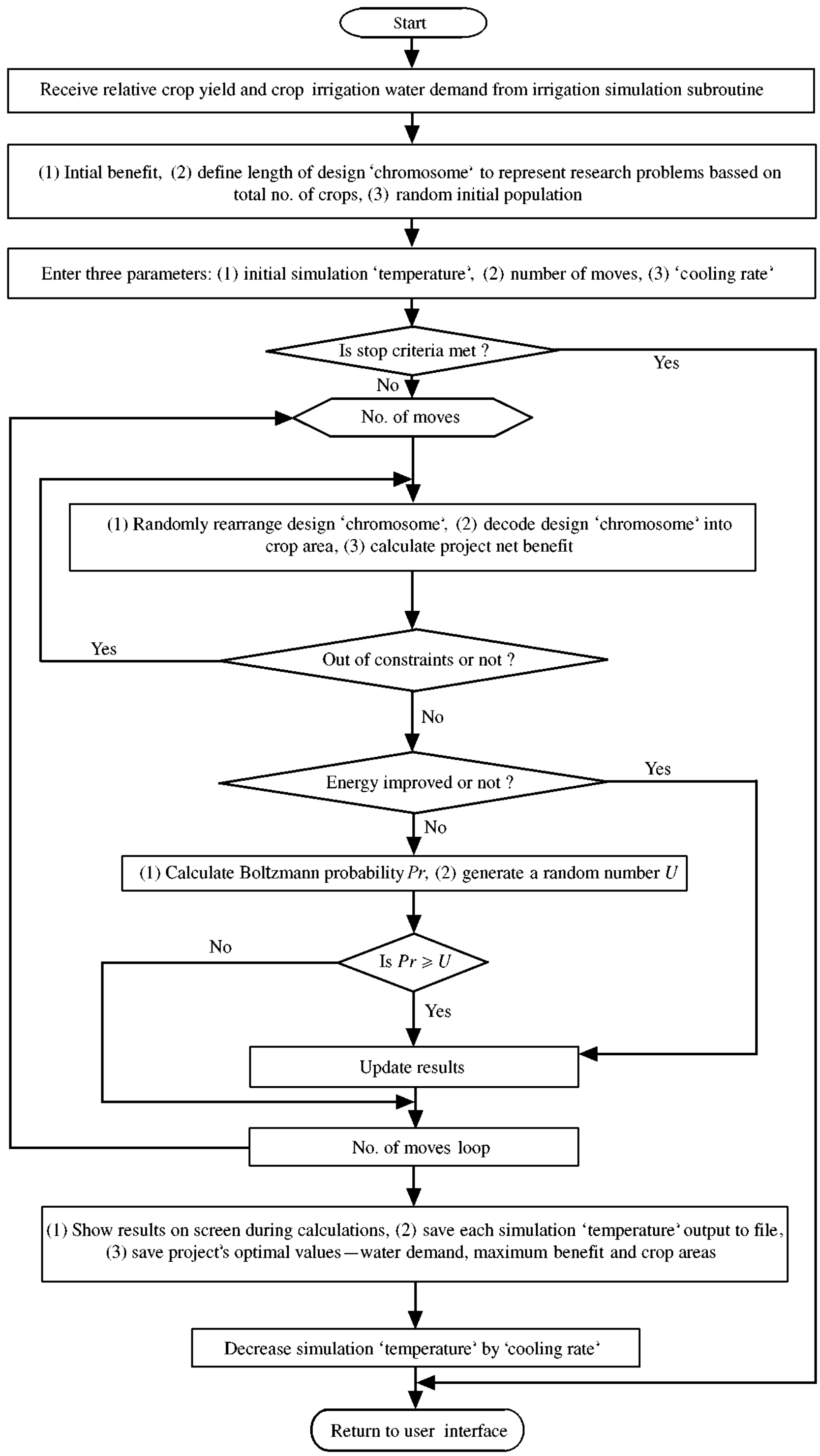

Fig. 2. Flowchart of simulated annealing

4.3. *Representative design 'chromosome'*

The length of a design 'chromosome' consists of a fixed number of binary digits. Also, the position and random number values influence the decoded value of the design 'chromosome'. To design a 'chromosome' length to represent an irrigation project, the cumulative numbers of crops within each command area are first calculated. Each crop is then assigned seven binary digits to represent its area, which can range from 1 to 100%, in all of the

Binary	0010010	1010111	0001111	1100000	0010100	1011001	1100110
Crop	1	2	3	4	5	6	7
Command Area	<--------- UCA 1 ----------->			<---------------- UCA 2 ----------------->			

Fig. 3. A sample chromosome coding scheme to represent seven crops in the Delta project

percentage points, of the cumulative area in each command area (seven binary digits give a value of $0\text{–}2^7 - 1$, or 0–127 in decimal). Finally, the length of a design 'chromosome' equals the cumulative number of crop types multiplied by seven.

For example, two command areas have been considered in the Delta, Utah irrigation project for testing the model. The first command area, UCA1, includes three crop types and the second command area, UCA2, includes four crop types. Therefore, seven crop types are within these two command areas of the Delta, Utah irrigation project, and the length of a design 'chromosome' should be 49. While considering a design 'chromosome' string of 49 binary digits, the seven crops in the two command areas can be depicted in coded form as shown in *Fig. 3*.

4.4. *Decoding a design 'chromosome' into a real number*

The design 'chromosome' can be decoded into a decimal number to represent the crop area within each command area. The conventional decoding method is used in this study. Consider a problem with k decision variables x_i, $i = 1, 2, \ldots, k$, defined on the intervals $x_i \in [a_i, b_i]$. Each decision variable can be decoded as a binary substring of length m_i. The decoded decimal x_i can be obtained from the following equation (McKinny & Lin, 1994):

$$x_i = a_i + \frac{b_i - a_i}{2^{m_i} - 1} \sum_{s=0}^{m_i} b_s \times 2^s \tag{16}$$

where: s is the summation identifier for substring length.

The following case study from the Delta, Utah project contains seven crop types in the two command areas. Therefore, this problem has seven decision variables x_i, and i can range from 1 to 7. Without considering inherent crop area constraints, the percentage area of each crop type can range from 1 to 100% of the total command area. Therefore, the interval for each decision variable can be represented as $x_i \in [1, 100]$, and a_i equals 1 and b_i equals 100. In conclusion, Eqn (16) can decode the binary digits into an actual number in the range from 1 to 100. The next step is to transfer this decimal number into crop area percentage $A_{j,\%}$, and area $A_{j,ha}$ within each command area. A simple averaging technique was used, as given by

$$A_{j,\%} = \frac{x_j}{\sum_{j=1}^{N_c} x_j} 100 \tag{17}$$

$$A_{j,ha} = \frac{A_{j,\%}}{100} A_{uca} \tag{18}$$

where: j is the crop index; N_c denotes the number of crops within each command area; and A_{uca} represents the area of each unit command area.

4.5. *Rearrangement of design 'chromosomes'*

Rearrangement of design 'chromosomes' is necessary to order the huge number of atoms within each simulation 'temperature' value for minimizing the energy of the system. In this study, the rearrangements can be treated as a change in the location of the binary digits within the design 'chromosome'; therefore, the newly allocated crop area can be obtained after decoding the rearranged design 'chromosome'.

There are three steps in the rearrangement of the design 'chromosome': (1) move the binary digits at regions I–III depending on the length from the second cut site to the end of the design 'chromosome'; (2) back the binary digits one position in region II; and (3) move the binary digits at regions III–I, depending on the length from the first digit to the first cut site. A design 'chromosome' with 15 binary digits and two random break points demonstrates the procedure. The old and new design 'chromosome', before and after rearrangement are shown in *Fig. 4*.

4.6. *Annealing scheduling by boltzmann probability*

Annealing scheduling is the heart of the simulated annealing method. This procedure is the major difference from the traditional optimization methods (*e.g.* iterative improvement or Monte Carlo methods) that allows perturbations to move uphill in a controlled fashion;

Region	I					II					III				
Old	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
New	11	12	13	14	15	10	6	7	8	9	1	2	3	4	5

Fig. 4. Rearrangement chromosome for the simulated annealing method

```
M is the number of moves;
T_sa is the current simulation 'temperature';
for (move = 1; move ≤ M; move++)
{
    Evaluate the change in energy, ΔE;
    if (ΔE < 0)
    {
        Accept this move, and update the benefit and crops' allocated area;
    }
    if (ΔE > 0)
    {
        Generating a uniform random number r;
        Calculate the Boltzmann Probability P_r;
        if (r < P_r)
        {
            Update the benefit and the crops' allocated area;
        }
    }
}
```

Fig. 5. Computer flow for annealing scheduling in the simulated annealing module

therefore, the simulated annealing method has the opportunity to escape from a local optimum toward a global optimum. *Figure 5* shows the computer flow for annealing scheduling in the SA module and can be described in the following steps.

(1) The simulated annealing method allows many moves within one simulation 'temperature' value; therefore, the first step is to compare the energy difference ΔE (*i.e.* difference of project benefit) from the previous move to the current move:

$$\Delta E = E_{move} - E_{move+1} \tag{19}$$

where: E_{move} and E_{move+1} represent the project benefit at current and previous moves.

(2) If the energy difference is negative ($\Delta E < 0$), the irrigation project maximum benefit and related crop areas are accepted at this move because the energy has been improved from the previous move to the current move.

(3) If the energy difference is positive ($\Delta E > 0$), this means the energy was not improved, but the irrigation project maximum benefit and related crop areas still have the opportunity to update if the Boltzmann probability P_r is greater than the generated uniform random number r. The Boltzmann probability can be defined as

$$P_r = e^{-\Delta E/T_{sa}} \tag{20}$$

From the above equation, it can be seen that P_r is influenced by simulation 'temperature' T_{sa}; that is, higher simulation 'temperatures' will have higher P_r values, and the system has a greater opportunity to update the configuration if $\Delta E > 0$. This also implies that the system at a higher simulation 'temperature' has a higher ability to rearrange the atoms (*i.e.* to jump away from local optima) for finding better, more optimal results. As the simulation 'temperature' continues to decrease, the system tends to equilibrium because the P_r value is small, and there is no more ability to update the configuration if $\Delta E > 0$. Finally, the global (or near global) optimum can be determined from this procedure.

4.7. *Objective function and fitness value*

In this study, the objective function includes the income from crop harvest, cost of irrigation water and crop production cost. The objective is to maximize the irrigation project benefit or the fitness value from the seven crops growing in the two command areas. Within the calculation loop of design 'chromosome' size, the objective function returns a fitness value to the model and then updates the fitness value and related crop-allocated area if this value is higher than previous ones. At the end of the design 'chromosome' loop, the subsequent fitness value is the highest benefit within the loop. Also, the maximum fitness value is selected from the generation number loop. Therefore, the fitness value and related crop area are the optimum results at the end of the calculations. The objective function is mathematically expressed as
Maximize:

$$\sum_{i=1}^{N}\sum_{j=1}^{N_c}(P_{i,j}Y_{i,j} - S_{i,j} - F_{i,j} - O_{i,j})A_{i,j} - W\sum_{i=1}^{N}\sum_{j=1}^{N_c}Q_{i,j} \tag{21}$$

where: i, j is the command area and crop index; N is the number of command areas within irrigated project; N_c is the number of crops within each command area; $P_{i,j}$ is unit price of the jth crop in the ith command area in $\$\,\text{ha}^{-1}$; $Y_{i,j}$ is yields per hectare of the jth crop in the ith command area in ton ha^{-1}; $S_{i,j}$ is seed cost per hectare of the jth crop in the ith command area in $\$\,\text{ha}^{-1}$; $F_{i,j}$ is fertilizer cost of the jth crop in the ith command area in $\$\,\text{ha}^{-1}$; $L_{i,j}$ is labour cost of the jth crop in the ith command area in $\$\,\text{ha}^{-1}$; $O_{i,j}$ is operation cost of the jth crop in the ith command area in $\$\,\text{ha}^{-1}$; $A_{i,j}$ is planted area of the jth crop in the ith command area in ha; W is unit price of irrigation water in $\$\,\text{m}^{-3}$; and $Q_{i,j}$ is cumulative water requirement of the jth crop in the ith command area in m^3.

The objective function is subject to the following constraints.

(1) To consider social factors and to prevent one high -value crop from dominating the search for maximum benefit, the maximum and minimum area percentages must be considered for the crops:

$$A_{\min} \leqslant A \leqslant A_{\max} \tag{22}$$

where $A_{\min}$ and $A_{\max}$ are the minimum and maximum percentage area values of crop j in command area i in %, respectively.

(2) The cumulative water demand of crop j in command area i should be less than the available water supply for each command area:

$$\sum_{j=1}^{N_c} Q_{dem} \leqslant Q_{sup} \tag{23}$$

where: Q_{dem} denotes the irrigation water requirement for crop j in command area i in m^3 ; and Q_{sup} represents the available water supply for command area i in m^3.

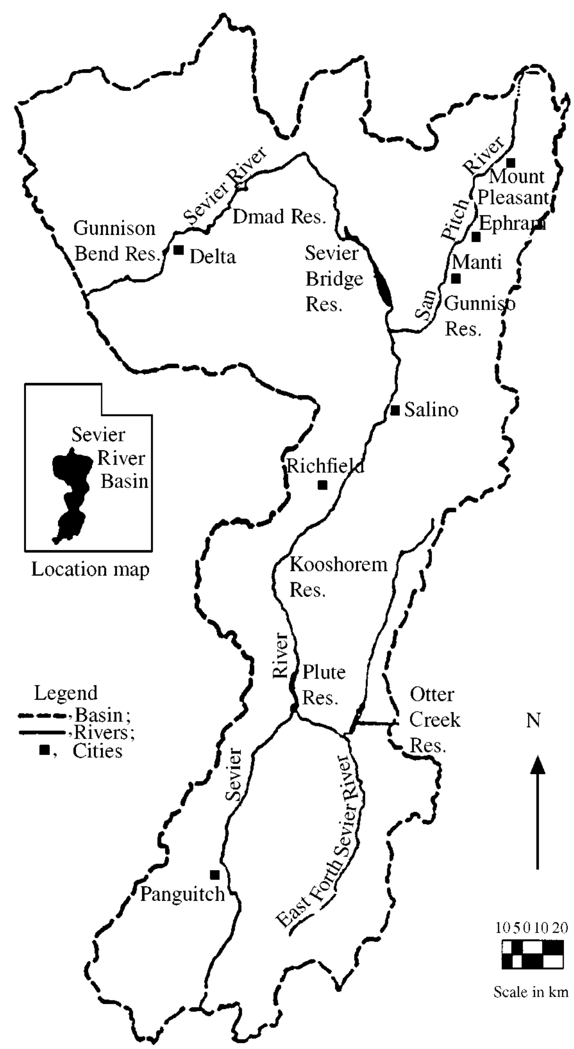

Fig. 6. Seiver River Basin, Utah (Tzou, 1989). ▬ ▬, Basin; ▬▬, Rivers; ■ , Cities

Table 1
Recorded monthly weather data for the Delta, Utah

Month	*Temperature,°C*		*No. of rainy days, day*		*Rainfall,mm*	
	Average	*SD*	*Average*	*SD*	*Average*	*SD*
Jan	− 3·5	3·4	6·0	3·1	14·0	10·4
Feb	0·0	3·1	5·0	2·7	13·8	13·1
Mar	4·1	2·0	6·8	3·7	21·4	15·4
Apr	9·0	1·7	6·5	3·2	20·8	14·1
May	14·6	1·6	5·7	3·2	23·7	19·9
Jun	19·4	1·5	3·5	2·9	12·1	12·2
Jul	24·4	1·0	3·5	2·0	10·3	10·7
Aug	23·1	1·0	4·1	2·4	13·3	14·6
Sep	17·5	1·7	4·1	3·1	16·6	18·9
Oct	10·7	1·6	4·9	3·4	21·3	21·8
Nov	3·0	1·9	4·8	2·6	15·1	12·2
Dec	− 2·2	2·3	5·4	3·0	17·0	18·1

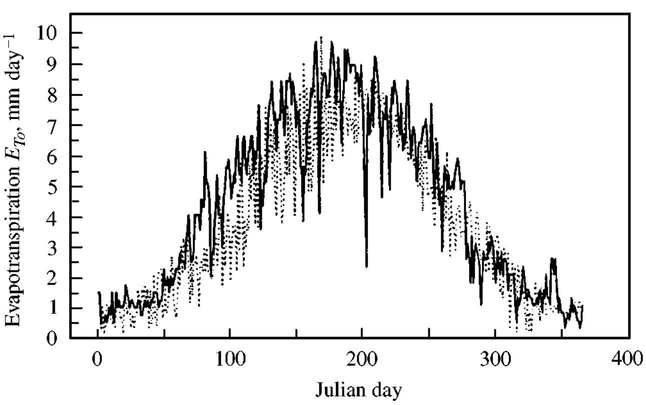

Fig. 7. The relationship between generated and recorded reference crop evapotranspiration for Delta irrigated project in 1993: ——, *recorded;* ...., *generated*

5. Application and results

5.1. *Site description*

The Wilson Canal System, close to the city of Delta in central Utah, was used in this study and is part of the many diversions in the Sevier River Basin operated by the Abraham Irrigation Company as an on-demand irrigation system with a good communications network (Tzou, 1989). The Wilson Canal is 11 480 m in length with water being supplied from the Gunnison Bend Reservoir. *Figure 6* depicts the location of the Sevier River Basin.

The climate in the Delta area is essentially a cold desert type, which is arid with cold winters and warm summers. The UCA1 and UCA2 command areas were selected within the Wilson Canal System for evaluating the model. The UCA1 command area has a 2896 m water course, 83·3 ha planted area, and three crop types are planted: alfalfa, barley and maize. On the other hand, the UCA2 command area has a 12 350 m water course and 311·3 ha planted area. In addition, four crop types are planted: alfalfa, barley, maize and wheat.

5.2. *Application of weather generation*

Based on the monthly recorded weather data, the weather generation module can generate daily weather data for use in the on-farm irrigation scheduling module. The long-term recorded monthly meteorological data in Delta, Utah, as shown in Table 1, were used to generate the daily reference crop evapotranspiration, precipitation and mean, maximum and minimum air temperatures for use in the on-farm irrigation scheduling submodel. According to the test, the most suitable arid probability was 78 for the weather generation module generating the dried year and the seed number was 50 in this study. *Figure 7* shows the relationships between the generated and 1993 recorded values of E_{TO}. As presented in *Fig. 7*, the generated E_{TO} was a little higher than the recorded E_{TO} and has a similar trend.

5.3. *Application to the on-farm irrigation scheduling module*

The on-farm irrigation scheduling module deals with the daily simulation of the water balance to estimate irrigation water requirement and relative crop yield. *Figure 8* shows the relationship between the daily soil moisture content, depth of irrigation and rainfall for alfalfa and barley crops in the UCA1 command area, respectively. *Figure 9* compares the potential and calculated evapotranspiration for maize and wheat in the

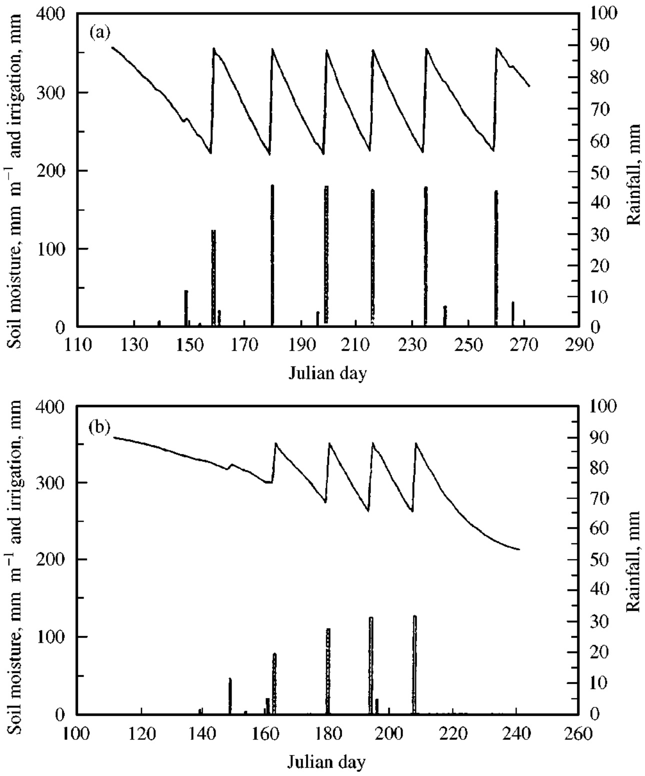

Fig. 8. The relationship between soil moisture, irrigation depth and rainfall for (a) alfalfa and (b) barley crops in the UCA2 command area: ——, soil moisture; ═══, irrigation depth rainfall

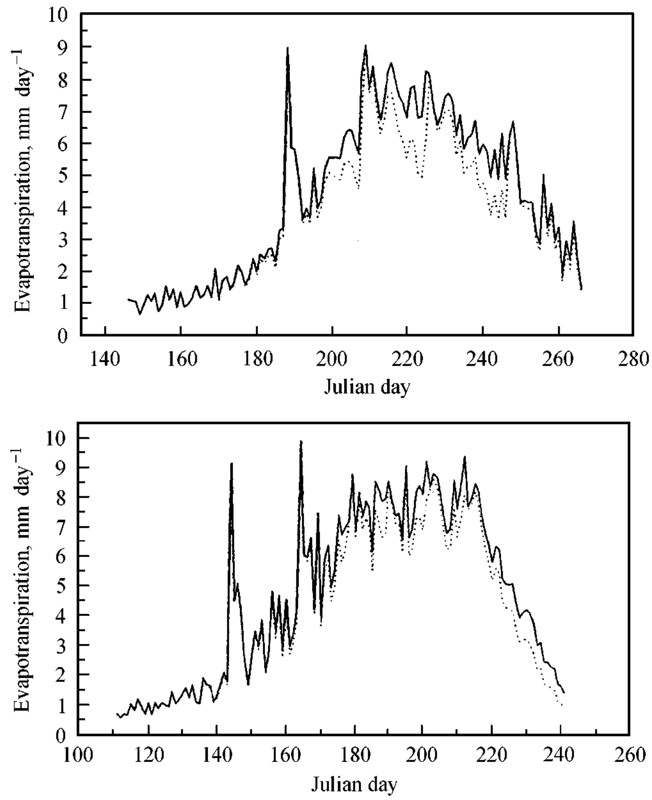

Fig. 9. The relationship between potential (——) and simulated (........) evapotranspiration for (a) maize and (b) wheat in the UCA4 command area

UCA2 command area. Tables 2 and 3 show the seasonal outputs from the on-farm irrigation scheduling module for the UCA1 and UCA2 command areas, respectively. The irrigation water requirements from the on-farm irrigation scheduling module show that: (1) crops within the UCA1 command area are 1067·9, 441·7 and 471·8 mm for alfalfa, barley and maize, respectively; and (2) crops within UCA2 command area are 1039·5, 531·4, 490·9 and 539·4 mm for alfalfa, barley, maize and wheat, respectively. Also, the relative crop yield due to water stress and waterlogging can be summarized as follows: (1) crops within the UCA1 command area are 86·27, 95·36 and 84·52% for alfalfa, barley and maize, respectively; and (2) crops within UCA2 command area are 85·59, 95, 84·68 and 93·11% for alfalfa, barley, maize and wheat, respectively.

Table 2
Seasonal outputs for the UCA1 command area from the on-farm irrigation scheduling module; E_T, evapotranspiration

	Alfalfa	*Barley*	*Maize*
Potential E_T, mm	1038·03	555·58	514·84
Actual E_T, mm	907	505·71	460·94
Evaporation from wet soil surface, mm	2·08	21·35	13·37
Number of irrigations	6	4	3
Total irrigation depth, mm	1067·92	441·71	471·82
Deep percolation, mm	70·14	29·39	37·19
Surface runoff, mm	28·49	11·94	15·11
Yield reduction due to water stress, %	11·43	3·59	14·54
Yield reduction due to waterlogging, %	2·6	1·09	1·09
Relative crop yield, %	86·27	95·36	84·52

Table 3
Seasonal outputs for the UCA2 command area from the on-farm irrigation scheduling module; E_T, evapotranspiration

	Alfalfa	*Barley*	*Maize*	*Wheat*
Potential E_T, mm	1039·33	572·08	523·35	611·18
Actual E_T, mm	906·15	528·68	469·61	558·05
Evaporation from wet soil surface, mm	3·38	37·85	21·88	34·4
Number of irrigations	7	6	4	6
Total irrigation depth, mm	1039·5	531·4	490·9	539·37
Deep percolation, mm	68·32	35·13	38·42	35·64
Surface runoff, mm	27·75	14·27	15·61	14·48
Yield reduction due to water stress, %	11·73	3·49	14·16	5·39
Yield reduction due to waterlogging, %	3·04	1·56	1·35	1·58
Relative crop yield, %	85·59	95·0	84·68	93·11

Table 4
Four data sets to test the simulated annealing submodel

Sets	*1*	*2*	*3*	*4*
Initial simulation 'temperature'	1000	1000	1000	1000
Number of moves	50	70	90	110
'Cooling rate'	0·95	0·95	0·95	0·95
Number of runs	10	10	10	10

Table 5
Summarized results from four data sets for the simulated annealing method

Set	*1*	*2*	*3*	*4*
Standard deviation, $	2103	1928	1581	2079
Average benefit, $	110 435	111 580	111 494	110 767
Maximum benefit, $	113 036	115 333	114 857	114 058
Minimum benefit, $	106 599	108 405	109 517	106 910
Number of moves	50	70	90	110
Initial simulation 'temperature'	1000	1000	1000	1000
'Cooling rate'	0·95	0·95	0·95	0·95

5.4. *Application of the simulated annealing*

Three parameters are necessary for the simulated annealing method: (1) the initial simulation 'temperature'; (2) the number of moves; and (3) the 'cooling rate'. Marryott *et al.* (1993) stated that the total number of simulations required by simulated annealing is controlled by the length of the Markov chain (*i.e.* the number of moves). The Markov chain length represents the number of simulations per annealing temperature step required to ensure equilibrium in the optimization process. Therefore, Arts and van Laarhoven (1985) suggested that the minimum value of the Markov chain length (*i.e.* number of moves) must be satisfied to ensure the location of an optimal (or near optimal) solution. In this study, four rules were followed to find suitable parameters.

(1) The length of the Markov chain can be in the range of 10–100 times the number of decision variables. Also, the chain length can be cut or increased by a factor of 10 and ten simulated annealing runs performed for the given problem.
(2) The 'cooling rate' can be chosen between 0·8 and 0·99 throughout the entire annealing run (Kirkpatrick *et al.*, 1983; Kirkpatrick, 1984).
(3) A series of several runs can be performed with the same parameters because the result for each run is essentially different and independent of the random starting point. If the standard deviation from all

Table 6

Simulated annealing results with initial simulation 'temperature' of 1000, number of moves 90, and 'cooling rate' of 0·95 for the whole project and for two unit command areas (UCA)

Run	*Project*		*UCA1*				*UCA2*				
	Net benefit, $	*Water demand, $\times 10^3$ m^3*	*Alfalfa %*	*Barley, %*	*Maize, %*	*Water demand, $\times 10^3$ m^3*	*Alfalfa %*	*Barley, %*	*Maize, %*	*Wheat %*	*Water demand, $\times 10^3$ m^3*
1	112,715	3017·371	67·19	17·97	14·84	722·143	42·65	27·21	27·21	2·94	2295·228
2	114 857	3048·221	72·90	21·50	5·61	749·610	42·67	28·89	24·89	3·56	2298·612
3	109 517	2990·306	70·69	5·17	24·14	742·742	39·81	26·85	29·63	3·70	2247·564
4	110 593	2999·315	64·66	27·82	7·52	707·130	42·54	19·74	29·39	8·33	2292·185
5	112 255	3014·310	72·55	13·73	13·73	749·830	39·27	43·98	9·95	6·81	2264·481
6	110 005	3024·033	71·00	12·00	17·00	742·571	42·62	10·13	38·82	8·44	2281·462
7	112 646	3017·384	68·87	19·81	11·32	730·025	41·05	37·89	14·21	6·84	2287·359
8	110 144	2957·283	58·39	35·77	5·84	674·015	41·92	26·26	27·78	4·04	2283·267
9	110 318	2980·368	68·57	7·62	23·81	731·610	38·15	48·55	8·09	5·20	2248·758
10	111 886	2974·774	60·51	29·94	9·55	685·982	41·48	38·07	17·61	2·84	2288·793
Avg	**111 494**	**3002·337**	**67·53**	**19·13**	**13·34**	**723·566**	**41·22**	**30·76**	**22·76**	**5·27**	**2278·771**
Max	**114 857**										
Min	**109 517**										
SD	**1581**										

the runs is high, the parameters may not be suitable to find the optimal or near-optimal solutions. On the other hand, the parameters with low standard deviation from all the runs will have more confidence to find the optimal or near-optimal results for the applied problems.

(4) Several sets of parameters are required to apply the model and the best set will have a higher average and a lower standard deviation of the project benefit from a series of runs.

Based on the above rules, four data sets are used in this study to test the simulated annealing submodel as shown in Table 4. After ten runs were performed for each data set, Table 5 summarizes the final results from the four data sets. Table 5 indicates the average benefit ranges from US$110 435 to 114 494 and the standard deviation ranges from US$2103 to 1581. For choosing the most suitable parameters in this study, set 3 is the best because it has the lowest standard deviation even if the average benefit is a little lower than that for set 2. Therefore, the most appropriate parameters for the SA method in this study are as follows: (1) the initial simulation 'temperature' equals 1000; (2) the number of moves equals 90; and (3) the 'cooling rate' equals 0.95. Table 6 summarizes the final results from ten runs for these parameters. As presented, the maximum benefit is up to US$ 114 857 on run 2, and standard deviation of benefit is as low as US$ 1581 from the ten runs. Therefore, the parameters are sure to obtain the near global optimal values for this irrigation project planning problem.

Figure 10 displays the sample graph from the SA method to represent the benefit from the Delta project during the searching process. Note that the graphs are read from right to left because the annealing proceeds from high to low simulation 'temperatures'. At higher

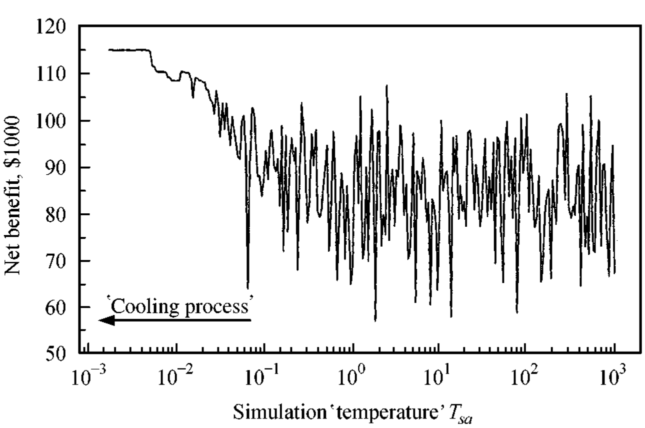

Fig. 10. Sample graphs from the simulated annealing method to obtain optimal net benefit with three parameters; initial temperature, $T_{sa} = 1000$; cooling rate $\alpha = 0{\cdot}95$; number of moves, $M = 90$

simulation 'temperatures', the annealing scheduling makes the graph climb either downward or upward, and the graphs tend toward equilibrium and reach the near global optimum results at the lower simulation 'temperatures'.

6. Conclusion

This work develops a model based on the on-farm irrigation scheduling and the simulated annealing optimization method to provide guidelines on agricultural water resource planning and management. The model consists mainly of six basic modules: (1) a main module to direct the running of the model, with pull-down menu facility; (2) a data module to enter the required data by a user-friendly interface; (3) a weather generation module to generate the daily weather data; (4) an on-farm irrigation scheduling module to simulate the daily water requirement and relative crop yield; (5) a simulated annealing module to optimize the project maximum benefits; and (6) a results module to present results by tables, graphs and printouts. The model is applied to Delta, Utah for optimizing the maximum crop production benefits and identifying the crop area-allocated percentages with the application of the simulated annealing method.

The simulated annealing method (SA) was implemented to optimize the agricultural water resource planning. As for the SA method, three parameters are needed in this method: (1) initial simulation 'temperature'; (2) number of moves; and, (3) 'cooling rate'. The SA module has been implemented with the on-farm irrigation scheduling module to maximize the project benefit. The computational procedure of the SA can be divided into the following steps: (1) to receive the output from the irrigation module; (2) to enter the simulated annealing parameters through a user interface; (3) to define the design 'chromosome' representing the problem; (4) to generate the random initial design 'chromosome'; (5) to decode the design 'chromosome' into a real number; (6) to apply constraints; (7) to apply an objective function and a fitness value; (8) to implement the annealing schedule by the Boltzmann probability; and (9) to set the 'cooling rate' and criterion for termination.

The ability of annealing scheduling allows the SA method to update its configurations even though the energy is not improved from that of the previous simulation 'temperature'. Therefore, the SA method can overcome the problem of traditional optimization methods that often get stuck in a local optimal because the traditional optimization method just allows the configurations to be updated at the time of energy improvement from the previous iteration. In this study, the most suitable data set has an average benefit of US$ 114 500 and the standard deviation of benefit equals US$ 1581. The relative controlling parameters are: (1) an initial simulation 'temperature' of 1000; (2) the number of moves equal to 90; and, (3) a 'cooling rate' of 0·95. The final results from the SA method for the Delta, Utah are: (1) a project benefit of US$ 114 500; (2) project water demand of $3{\cdot}002 \times 10^6$ m^3; (3) crop area percentages within the UCA1 command area of 67·5, 19·1, and 13·4% for alfalfa, barley and maize, respectively; and (4) crop area percentages within the UCA2 command area of 41·2, 30·8, 22·8, and 5·2% for alfalfa, barley, maize and wheat, respectively.

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