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Transient Performance of Solar Systems with a Bang-Bang Controller

Bin-Juine Huang*

Keywords: transient analysis, solar system, bang-bang controller, thermal performance.

ABSTRACT

A transient analysis of a solar hot water system with Bang-Bang controller is reported. A variable-structure model of the collector is used in the analysis. In the pump ON mode, a first-order dynamic model based on Hottel-Whillier-Bliss model was used to evaluate the thermal performance of the collector. In the pump OFF mode, Huang and Lu's model was used. The occurrence of the pump limit cycle can be clearly shown by using a phase plane diagram and the transient performance of a solar hot water system was studied by a system simulation. It was found that pump oscillation becomes more frequently as the variation of solar irradiation is rough or the tank temperature reaches high. This oscillation can be reduced if the setting point ΔT_{ON} was raised. The effect of pump oscillation on the energy collecting efficiency was shown to be minimal.

INTRODUCTION

Differential-temperature (DT) controllers are used widely in forced-circulation solar hot water systems to control the pump in order to reduce energy loss during the periods of low or intermittent irradiation, especially in cloudy days. As shown in Fig.1 is a solar system used to supply hot water. In general, the DT pump controller is a ON/OFF relay (i.e. Bang-Bang controller) and thus is a nonlinear element. Most commercial designs allow the user to make a field adjustment for the ON point of the Bang-Bang controller, while the OFF point is fixed. The present study is based on this type of Bang-Bang controller.

Since the correct setting of a Bang-Bang controller relies heavily on an understanding of the dynamic behaviour of the collector, several methods have been developed. Schiller *et al.* (1980) applied a lumping technique to derive an approximate dynamic model consisting of a set of N ordinary differential equations for N segments of the collector. For a flat-plate collector at non-zero flow condition, the lumping was carried out simply by discretizing the Hottel-Whillier-Bliss transient equation for the N segments of the collector. Solving this N equations, the response of the collector at non-zero flow conditions can be determined. Using simulation in time domain and a lumped model for storage tank, Schiller *et al.* (1980) made a comparison of a solar hot water system with Bang-Bang and proportional control strategies. However, the results are still suspicious due to the use of the simplified collector dynamic model at zero-flow condition.

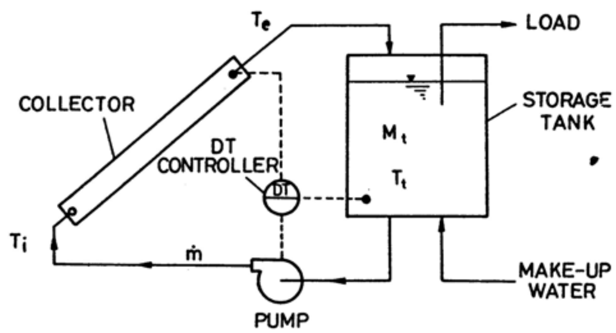

Fig.1. Schematic diagram of a solar system with DT controller.

Since the Bang-Bang controller is a nonlinear element in the feedback loop, limit cycle phenomena may occur and affect the thermal performance of the solar system. Winn (1983) and Kahwaji and Winn (1986) used the Hottel-Whillier-Bliss steady-flow model and a simple lumped model at zero flow (i.e. a quasi-steady collector model) to determine the effects of temperature settings on the cycling rates. The relation for the cycling period was then derived and used in the calculation of total number of pump cycles in a day for a given sinusoidal irradiation. However, it was assumed in this approach that the collec-

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* *Professor, Department of Mechanical Engineering National Taiwan University, Taipei, Taiwan ROC.*

tor response is infinitely fast at the instants of pump ON and OFF. This may cause an error due to transient effects of collector.

In this paper, the transient performance of solar systems with Bang-Bang controller is studied by using a variable-structure collector model. That is, two dynamic models, one for steady flow and the other for zero flow, were used in the analysis.

DYNAMIC MODEL OF COLLECTOR

Steady-flow Model

As the pump is turned on with a steady flow ($\dot{m}$= constant), the following first-order dynamic model of Klein *et al.* (1974) can be used to describe the dynamic behavior of collector:

$$\tau_c G C_p \frac{d\Delta T_f}{dt} + G C_p \Delta T_f$$

$$= K_a F_R(\tau\alpha)_n I_T - F_R U_L (T_i - T_a), \tag{1}$$

where τ_c is the time constant at steady flow; G is the mass flowrate per unit collector area (i.e. $\dot{m}/A_c$), $F_R(\tau\alpha)_n$ and $F_R U_L$ are the solar-noon collector absorptance-transmittance and heat loss coefficients, respectively, which can be determined by a performance test based on ASHRAE 93-77 or ANSI/ASHRAE 93-1986 Testing Standard. ΔT_f and K_a (the collector angle modifier) are defined as

$$\Delta T_f \equiv T_e - T_i, \tag{2}$$

$$K_a \equiv \frac{F_R(\tau\alpha)_e}{F_R(\tau\alpha)_n} = 1 - b_o \left(\frac{1}{\cos\theta} - 1\right). \tag{3}$$

Equation (1) can be written as

$$\tau_c G C_p \frac{d\Delta T_f}{dt} + G C_p \Delta T_f = F_R(\tau\alpha)_e (I_T - I_T^+), \tag{4}$$

where I_T^+ is the critical irradiation defined in Eq.(5) which is the minimum irradiation required for the positive energy collection of the collector in the pump ON mode,

$$I_T^+ \equiv \frac{F_R U_L}{F_R(\tau\alpha)_e}(T_i - T_a). \tag{5}$$

As $I_T \leq I_T^+$, the energy collection will cease and the collector irradiates the energy to the ambient. This can be avoided by turning off the pump through a DT controller.

Zero-flow Model

Schiller *et al.* (1980) and Klein *et al.* (1974) applied the Hottel-Whillier-Bliss model to describe the transient response of the collector during pump OFF periods as an asymptotic approximation; while Winn (1983) derived a simple lumped model to account for the transient effects. Both models remain to be verified experimentally and incur some difficulty in determining the collector thermal capacitance analytically.

In fact, as the pump is turned off, the circulating fluid becomes stagnant inside the collector and the heat transfer behaviour is entirely different from that at steady flow. A first-order semi-empirical model which was derived and verified experimentally by Huang and Lu (1982) can be applied. That is, for zero flow ($\dot{m} = 0$),

$$\tau_o \frac{dT_z}{dt} = E I_T - (T_z - T_a), \tag{6}$$

where T_z is the fluid mean temperature inside the collector; τ_o is the time constant of the collector at zero flow; E is the equilibrium constant of the collector. τ_o and E can be experimentally determined by the testing method described by Huang and Lu (1982). Equation (6) can be written as

$$\tau_o \frac{d\Delta T_z}{dt} + \Delta T_z = E(I_T - I_0^+), \tag{7}$$

where

$$\Delta T_z \equiv T_z - T_{init}. \tag{8}$$

T_{init} is the initial mean fluid temperature inside the collector; I_0^+ is the critical irradiation defined in Eq.(9) which is the minimum irradiation for positive energy collection of the collector in the pump OFF mode.

$$I_0^+ \equiv \frac{T_{init} - T_a}{E}. \tag{9}$$

LIMIT CYCLE PHENOMENON

The pump cycling caused by the limit cycle phenomenon of the DT controller can be understood clearly by the transient behavior of the collector.

Pump ON Mode

Assuming that the irradiation I_T is steady (i.e. I_T=constant). Then, for constant T_i, T_a, and coefficients $F_R(\tau\alpha)_e$ and $F_R U_L$, (i.e. constant I_T^+), the time solution of Eq.(4) is

$$\Delta T_f(t) = \Delta T_\infty + (\Delta T_{ON} - \Delta T_\infty) e^{-t/\tau_c}, \tag{10}$$

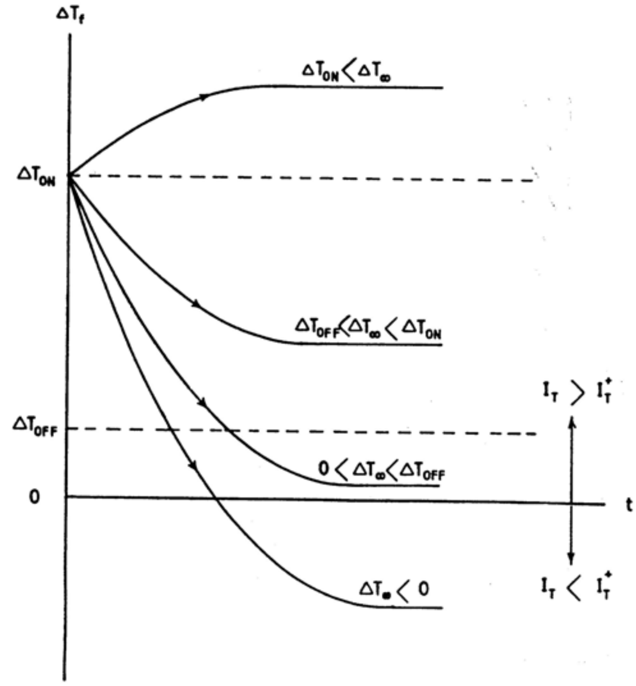

Fig.2. Temperature response curves in pump ON mode.

where ΔT_∞ is the steady-state temperature defined as

$$\Delta T_\infty \equiv \frac{F_R(\tau\alpha)_e}{GC_p}(I_T - I_T^+). \tag{11}$$

Equation (10) indicates that there are four types of temperature responses $\Delta T_f(t)$ depending upon the values of $I_T - I_T^+$ and ΔT_∞, as shown in Fig.2. For $\Delta T_\infty > \Delta T_{OFF}$, which includes two cases: (i) $\Delta T_{ON} < \Delta T_\infty$ and (ii) $\Delta T_{OFF} < \Delta T_\infty < \Delta T_{ON}$, the temperature response $\Delta T_f(t)$ will never hit the constant ΔT_{OFF} line. That is, the pump will never be turned off in this mode of operation.

For $\Delta T_\infty < \Delta T_{OFF}$, Which includes two cases: (i) $0 < \Delta T_\infty < \Delta T_{OFF}$ and (ii) $\Delta T_\infty < 0$, the temperature response curve will hit the constant ΔT_{OFF} line and the pump will be switched off. That is, the pump operation will be changed from ON mode to OFF mode and the pump cycling may occur.

From the above discussion, it can be concluded that, a sufficient condition for limit cycle to occur in the pump ON mode is

$$I_T < \frac{GC_P\Delta T_{OFF}}{F_R(\tau\alpha)_e} + \frac{F_RU_L}{F_R(\tau\alpha)_e}(T_i - T_a). \tag{12}$$

Equation (12) can be represented graphically by the lower half plane of the ON-line in the I_T vs. $T_i - T_a$plot with slope $F_RU_L/F_R(\tau\alpha)_e$ and intercept $GC_P\Delta T_{OFF}/F_R(\tau\alpha)_e$, as shown in Fig.3.

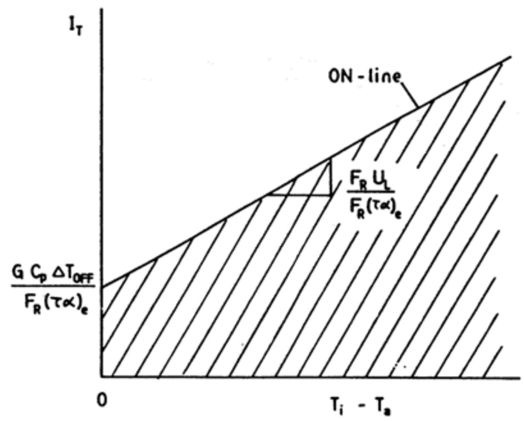

Fig.3. Limit cycle region in pump ON mode.

Pump OFF Mode

Assuming that the irradiation I_T is steady (i.e. I_T= constant). For constant T_{init}, T_a, and E, the time solution of Eq.(7) is

$$\Delta T_z(t) = \Delta T_{0\infty} + (\Delta T_{OFF} - \Delta T_{0\infty})e^{-t/\tau_o}, \tag{13}$$

where $\Delta T_{0\infty}$ is the steady-state temperature at $\dot{m} = 0$ defined as

$$\Delta T_{0\infty} \equiv E(I_T - I_0^+). \tag{14}$$

There are four types of temperature response curves $\Delta T_z(t)$ depending on the values of $I_T - I_0^+$ and $\Delta T_{0\infty}$. See Figure 4. For $\Delta T_{0\infty} < \Delta T_{ON}$, which includes three cases: (i) $0 < \Delta T_{0\infty} < \Delta T_{ON}$; (ii) $0 < \Delta T_{0\infty} < \Delta T_{OFF}$; (iii) $\Delta T_{0\infty} < 0$ (i.e. $I_T < I_0^+$), the temperature curves $\Delta T_z(t)$ will never hit the constant ΔT_{ON} line.

For $\Delta T_{0\infty} > \Delta T_{ON}$, the temperature response curve will hit the constant ΔT_{ON} line. This means that the pump will be switched on again in this operating mode. Thus, a sufficient condition for limit cycle to occur in the pump OFF mode is

$$I_T > \frac{\Delta T_{ON}}{E} + \frac{1}{E}(T_{init} - T_a). \tag{15}$$

Equation (15) can be represented graphically by the upper half plane of the OFF-line in the I_T *vs.* $T_{init} - T_a$ plot with slope $1/E$ and intercept $\Delta T_{ON}/E$, as shown in Fig.5.

Limit Cycling of Pump

Assuming that the two temperature sensors of

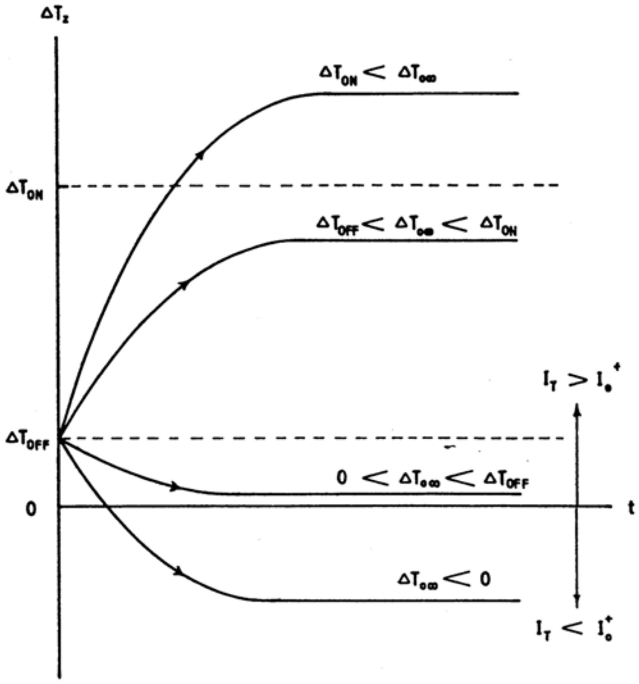

Fig.4. Temperature response curves in pump OFF mode.

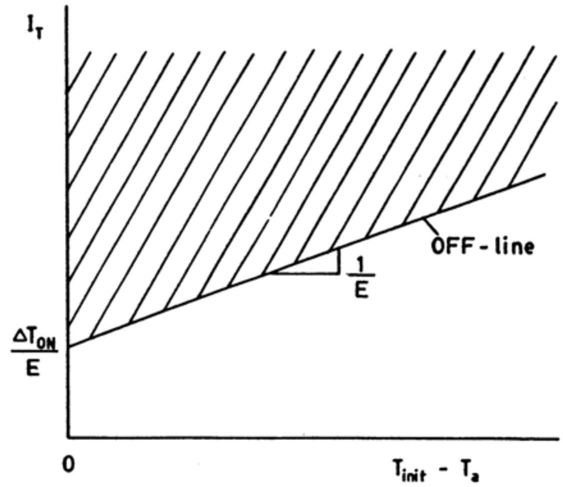

Fig.5. Limit cycle region in pump OFF mode.

the DT controller are installed at the inlet and outlet of the collector. For simplification, it was also assumed that at the instant of pump switching off (i.e. the beginning of the pump OFF mode), the initial mean water temperature inside the collector T_{init} equals approximately the inlet temperature in the pump ON mode, i.e. $T_{init} = T_i$. This is true since the temperature rise at the collector is usually a few degree higher at a steady flow. Thus, the coordinates $T_i - T_a$ and $T_{init} - T_a$, as shown in Figs.3 and 5 can be combined together.

Furthermore, it was assumed that the mean stagnant water temperature inside the collector in the pump OFF mode, T_z, can be represented by the temperature measured at the collector outlet. In practical applications, this assumption can hold if the water stagnant temperature T_z was replaced by the temperature measured at the collector outlet in the identification of the collector dynamic model at zero flow, as in Eq.(6). That is, the collector dynamic model can be defined with respect to the stagnant temperature measured at the collector outlet.

It was also assumed that, at the instant of pump switching on (i.e. the beginning of the pump ON mode), the exit water temperature T_e equals the instantaneous mean stagnant water temperature T_z right before pump switching on. Therefore, the control signal ΔT is continuous across the pump ON and OFF modes and the DT controller signal ΔT can be written as

$$\Delta T = \begin{cases} \Delta T_f = T_e - T_i, & \text{if pump is ON.} \\ \Delta T_z = T_z - T_{init} = T_z - T_i, & \text{if pump is OFF.} \end{cases} \tag{16}$$

Therefore, by combining Figs. 3 and 5, we can construct a phase space using the operating conditions I_T and $T_i - T_a$ as the coordinates to show the region of limit cycle. The sufficient condition for limit cycle is given as

$$\frac{\Delta T_{ON}}{E} + \frac{1}{E}(T_i - T_a) < I_T < \frac{GC_p \Delta T_{OFF}}{F_R(\tau\alpha)_e} + \frac{F_R U_L}{F_R(\tau\alpha)_e}(T_i - T_a). \tag{17}$$

The following four cases may occur depending on the magnitude of the intercepts and the slopes of each operating line. But, the pump limit cycling may occur only for Cases 1 through 3

Case 1

$$\frac{GC_p \Delta T_{OFF}}{F_R(\tau\alpha)_e} > \frac{\Delta T_{ON}}{E} \text{ and } \frac{F_R U_L}{F_R(\tau\alpha)_e} > \frac{1}{E}$$

Case 2

$$\frac{GC_p \Delta T_{OFF}}{F_R(\tau\alpha)_e} > \frac{\Delta T_{ON}}{E} \text{ and } \frac{F_R U_L}{F_R(\tau\alpha)_e} < \frac{1}{E}$$

Case 3

$$\frac{GC_p \Delta T_{OFF}}{F_R(\tau\alpha)_e} < \frac{\Delta T_{ON}}{E} \text{ and } \frac{F_R U_L}{F_R(\tau\alpha)_e} > \frac{1}{E}$$

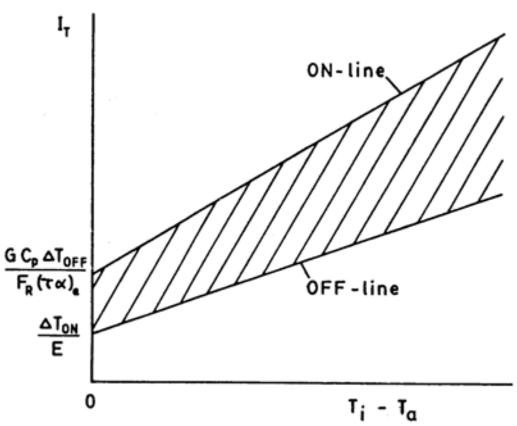

Fig.6. Limit cycle region for Case 1.

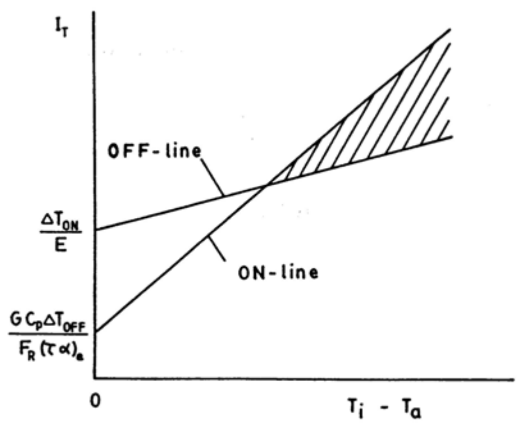

Fig.8. Limit cycle region for Case 3.

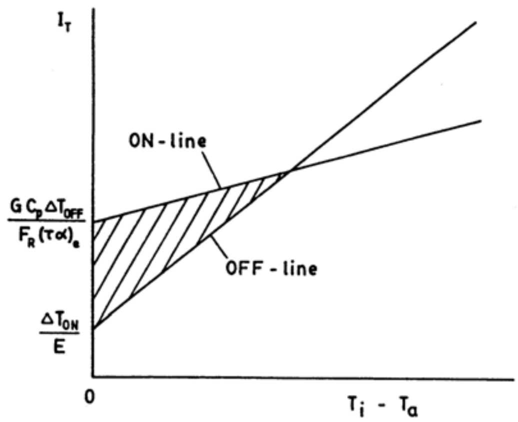

Fig.7. Limit cycle region for Case 2.

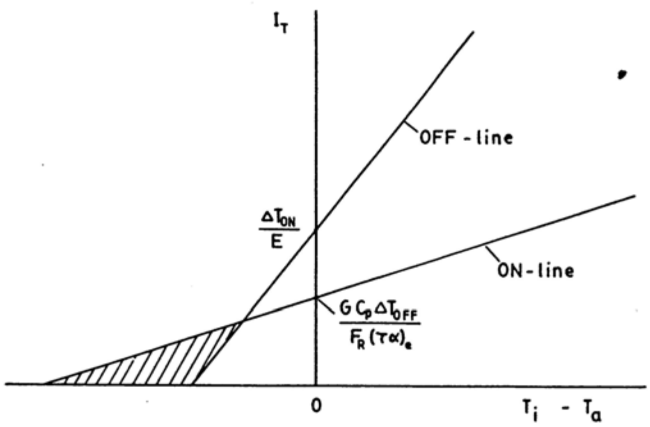

Fig.9. Limit cycle region for Case 4.

Case 4

$$\frac{GC_p\Delta T_{OFF}}{F_R(\tau\alpha)_e} < \frac{\Delta T_{ON}}{E} \text{ and } \frac{F_R U_L}{F_R(\tau\alpha)_e} < \frac{1}{E}$$

In cases 1, 2, and 3, the two cross-hatched half planes shown in Figs. 3 and 5 will construct an intersection on the phase plane which represents the limit cycle region. As shown in Figs. 6–8, the pump limit cycle will occur as the operating point falls in this intersection.

For case 4, the intersection between the ON-line and OFF-line for limit cycle to occur is outside the normal operating range, i.e. $T_i - T_a < 0$. See Figure 9. Since the operating point will never fall in the left half plane with $T_i < T_a$ in a normal performance of a solar system, the pump limit cycle will not occur in Case 4.

SYSTEM SIMULATION

It should be noted that the present analysis of limit cycle phenomenon described previously is a static analysis. That is, assuming constant I_T, T_i, T_a and $F_R(\tau\alpha)_e$ etc. In practice, the ON-line in the pump ON mode may vary with time. Thus, a transient analysis by simulation is required in order to understand the performance of a solar system with a Bang-Bang controller.

The present system simulation was carried out for a solar hot water system with a storage tank, a collector, and a Bang-Bang controller as shown in Fig.1. It is also assumed that there is no hot water withdraw, and no make-up water feed from and to the tank during daytime period. The sensors which actuates the Bang-Bang controller are usually installed inside the header tube of the collector and the bottom of the tank to pick up the input signals of the DT controller.

In addition to the collector dynamic model, the dynamic models of tank and connecting pipes have to be taken into account in the system simulation. In fact, the water temperature in the tank is non-uniform due to thermal stratification effect and the dynamic model of the tank will be a distributed-parameter system. The bottom-to-top temperature response will exhibit an infinite- order or high-order effect which makes the control system design extremely difficult.

Similarly, the dynamic model of the connecting pipes will also be a distributed-parameter system if the heat loss along the flow direction was taken into account. Hence, the high-order effect in the outlet-to-inlet temperature response will be present. It can be realized that the DT controller design will become extremely complicated if the distributed-parameter models of the tank and the connecting pipes were used.

In practice, the thermal stratification in the tank in a forced-circulation solar hot water systems can be destroyed by the large momentum of a high pumped flow, e.g. 0.02 kg/m^2s which has been frequently used. Hence, the water temperature in the tank can be assumed to be approximately uniform (i.e. T_t=constant) and the bottom temperature $T_b = T_t$. That is, a lumped tank model was used in the present study.

In addition, the heat loss in the connecting pipes is also small if the pipes were insulated well. Therefore, the collector inlet temperature equals the tank mean temperature in the pump ON and OFF modes, i.e. $T_i = T_b = T_t$.

Furthermore, it was assumed that no hot water draw-off is allowed during daytime, for simplification. Hence, the energy balance for the tank can be written as

$$M_t C_p \frac{dT_t}{dt} = \gamma \dot{m} C_p (T_o - T_t), \tag{18}$$

where

$$\gamma = \begin{cases} 1, & \text{if pump is ON,} \\ 0, & \text{if pump is OFF,} \end{cases}$$

$$T_o = \begin{cases} T_e, & \text{if pump is ON,} \\ T_z, & \text{if pump is OFF,} \end{cases}$$

and M_t is the total mass of water in the tank and $T_t = T_b = T_i$.

Similar to the previous analysis, assuming that the mean stagnant water temperature inside the collector in the pump OFF mode T_z can be represented by the temperature measured at the collector outlet. It was also assumed that T_{init} equals approximately the inlet temperature T_i in the pump ON mode, i.e. $T_{init} = T_i$. And, the exit water temperature T_e equals the mean stagnant water temperature T_z at the instant of pump on. Thus, the input signal of the DT controller ΔT is continuous across the pump ON and OFF modes and can be represented by Eq.(16).

Table 1. Pump cycling in a clear day.

ΔT_{ON}, °C	times cycled	efficiency, %
2	6	59.68
3	4	59.67
4	3	59.65
5	2	59.57
6	2	59.58
7	1	59.40
8	1	59.39
9	1	59.38
10	1	59.37
11	1	59.36
12	1	59.35

The pump is controlled by the signal defined as $\Delta T (= T_o - T_i)$. In the pump ON mode, $T_o = T_e$ and as $\Delta T_f \geq \Delta T_{ON}$ (the setting point in the Bang-Bang controller), the pump is turned on and the steady-flow model, Eq.(1), is used. ΔT_f will decrease with decreasing irradiation and increasing tank temperature. As ΔT_f decreases to $\Delta T_f \leq \Delta T_{OFF}$, a built-in fixed point of Bang-Bang controller, the pump is turned off and afterwards the collector dynamic model for zero flow, Eq.(7), should be used. In this case, the definition of ΔT has to be changed from ΔT_f to ΔT_z which is defined as $\Delta T_z = T_z - T_{init}$, where T_{init} is the initial mean water temperature inside the collector at the instant of pump off and was assumed to be equal T_i.

The pump will be turned on again whenever $\Delta T_z \geq \Delta T_{ON}$. Since the irradiation varies during a day, a simulation can be performed by solving Eqs.(1), (6) and (18) numerically using the fourth order Runge-Kutta method.

Consider a solar water heating system with tank size of 200 liters (i.e. M_t=200 kg), collector surface area A_c=2.56 m^2, collector heat transfer parameters: $F_R(\tau\alpha)_n$ = 0.81, $F_R U_L$=8.98 $W/m^{2\circ}C$, b_o = 0.33, τ_c=1.26 min, τ_o=15.5 min, E=0.082 $m^{2\circ} C/W$ (1982), collector tilted angle = 25° facing south, flowrate G=0.02 kg/m^2. If a typical commercial DT controller with fixed ΔT_{OFF} at 0.833°C (1.5°F) is used, the numerical solution then can be found for various ΔT_{ON} settings.

Figure 10 presents the variations of temperature difference across the collector ΔT_f and tank temperature $T_t (= T_i)$ for ΔT_{ON} = 3°C in a clear day. It can be seen that the oscillation of the collector exit temperature and the pump cycling occur only at late afternoon as the incident solar radiation becomes low. The number of times of pump cycling changes with

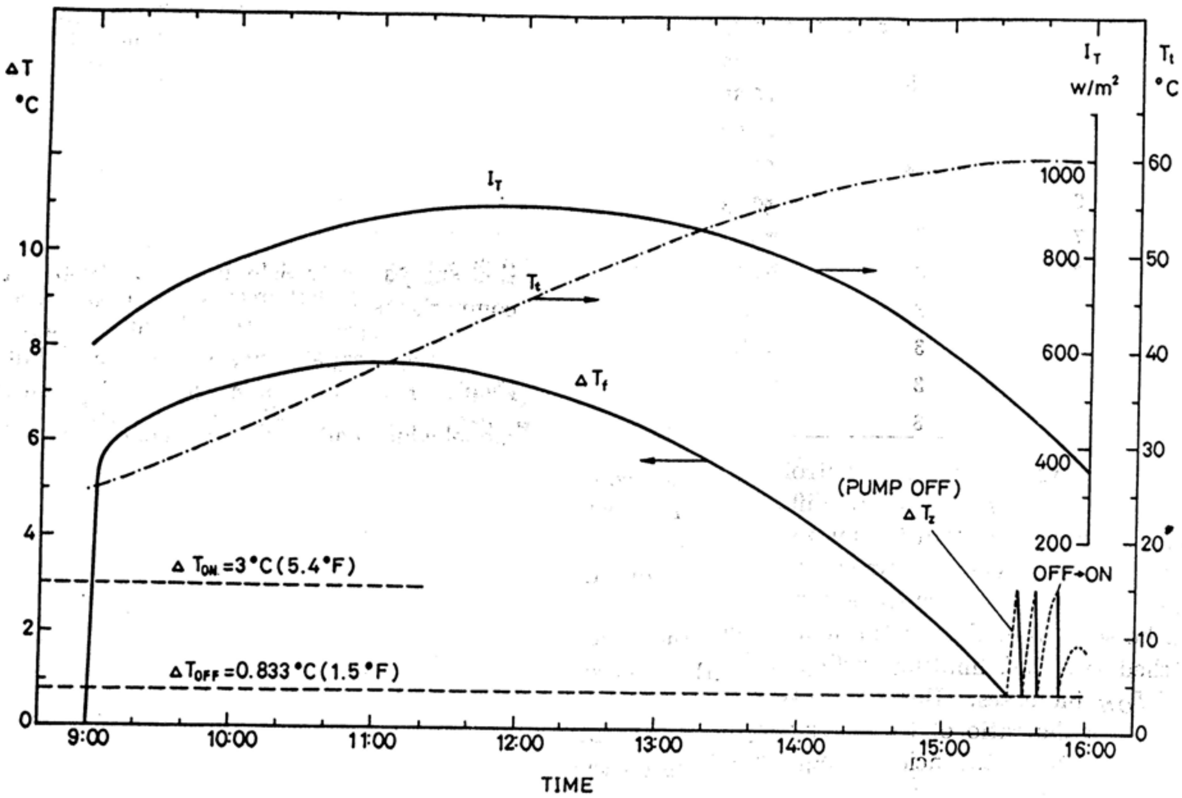

Fig.10. Transient thermal performance in a clear day.

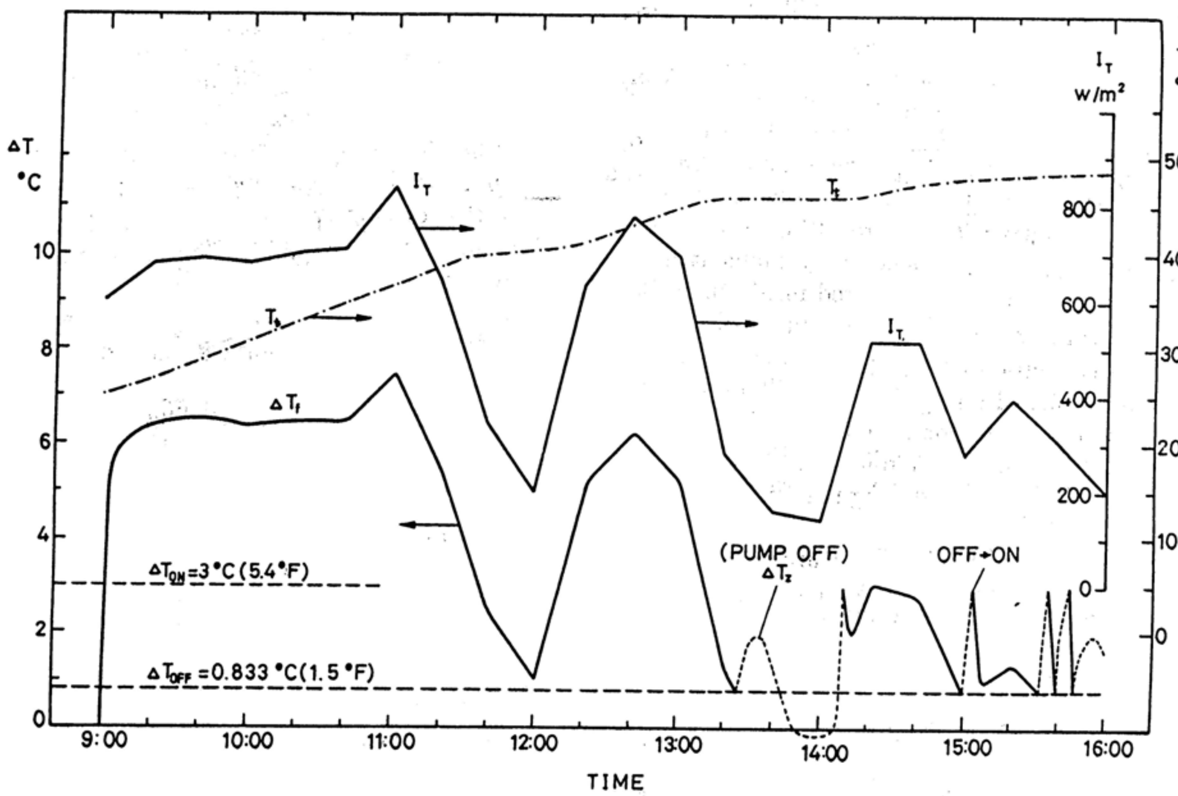

Fig.11. Transient thermal performance in a cloudy day.

Table 2. Pump cycling in a cloudy day.

ΔT_{ON}, °C	times cycled	efficiency, %
2	7	60.46
3	5	60.35
4	4	60.20
5	4	60.15
6	3	59.83
7	3	59.73
8	3	59.62
9	3	59.50
10	3	59.38
11	3	59.23
12	3	59.04

the settings ΔT_{ON} in the DT controller. Several computations were carried out for different ΔT_{ON} settings. Table 1 shows that the pump cycling decreases with increasing ΔT_{ON} settings, but the overall efficiency of solar system changes very little. This can also be seen from Figs. 6 through 8 that the cross-hatched area (i.e. limiting cycling region) decreases as ΔT_{ON} increases. Here, the overall efficiency is defined as the ratio of the energy collected during daytime to the total incident solar radiation on the inclined collector surface.

Figure 11 presents the thermal performance in a cloudy day with more rough variation of solar radiation intensity. It can be seen that temperature oscillation and pump cycling are more rough. The temperature rise across the collector is shown to vary proportionally with solar irradiation during the periods of pump on. When the temperature difference drops to ΔT_{OFF}, the pump is shut down at 13:25 PM. And the temperature fluctuations at zero flowrate (ΔT_z) shown as the dotted line are resulting from cooling of the water inside the collector by ambient air or heating by the residual heat stored inside the collector structural material. When the fluid temperature inside the collector raises such that $\Delta T_z \geq \Delta T_{ON}$, the pump is turned on again. An oscillation of ΔT_f takes place right after pump-on, at 14:10 and 15:05 PM (Fig.11). These are attributed to the energy storage effect of the collector during the transient periods of pump ON/OFF.

Table 2 shows that temperature oscillation and pump cycling also decreases with increasing ΔT_{ON} settings and the overall energy collecting efficiency decreases slightly.

DISCUSSION AND CONCLUSION

A transient thermal performance analysis for solar hot water systems with differential-temperature or Bang-Bang controller was carried out in the present study by using a variable-structure collector model. Two semi-empirical collector dynamic models were used: (1) first-order Klein's model using Hottel-Whillier-Bliss relation (1974), Eq.(1), is used to describe the thermal performance when pump is ON; and (2) Huang and Lu's first-order dynamic model (1982) is used when pump is OFF, instead of the modified Hottel-Whillier-Bliss model for zero-flow condition suggested by Schiller *et al.* (1980). The direct comparisons of the present method with Schillers' is difficult because of the difficulties in estimating the collector capacitance defined by Schiller *et al.* (1980). However, Huang and Lu's semi-empirical model which had been verified experimentally is more reliable for the transient thermal analysis. Furthermore, the occurrence of pump limit cycle can be clearly illustrated by using the phase plane diagram developed in the present study.

The oscillations of the collector exit temperature and the pump cycling were observed from the simulation of a solar system. It also indicates that as Schiller *et al.* (1980) cited, the setting point for ΔT_{ON} only has a minimal effect on the energy collecting efficiency as long as it is not very high such that the pump does not turn on until late in the morning. The present results have shown a clear picture on the system behavior of a solar hot water system with DT controller.

It should be emphasized here that the present results are based on a control system analysis which uses a lumped tank model and ignores the heat loss of the connecting pipes. The ignorance of thermal stratification effect in the tank will under-estimate the limit cycle phenomenon. This is due to the fact that, the consideration of thermal stratification in the dynamic model of the tank will lead to a distributed system which is extremely complicated. For simplification, a lumped model is used in the present analysis as a first order approximation though it may lead to some errors.

In practice, the pump cycling can also be aflected by the setting of ΔT_{OFF} and various types of irradiation pattern. A study for an optimum DT controller tunning of ΔT_{ON} and ΔT_{OFF} using a similar phase plane analysis but with second-order collector dynamic models is still underway. In addition to obtain an optimum energy efficiency, the optimum tunning can be done by using a long-term simulation result and the maximum durable pump cycling frequency. The effects of irradiation pattern are thus taken into account. All of these will be published later.

NOMENCLATURE

A_c	collector area, m^2
b_o	collector angle modifier coefficient
C_p	specific heat of water, $J/kg°C$
E	equilibrium constant of collector at $\dot{m}=0$, $m^2°C/W$
G	mass flowrate per unit collector area, kg/m^2s
$F_R(\tau\alpha)_e$	effective collector absorption-transmission coefficient
$F_R(\tau\alpha)_n$	collector absorption-transmission coefficient at solar noon
F_RU_L	collector performance parameter, $W/m^2°C$
I_0^+	critical solar irradiation on tilted surface defined in equation(9), W/m^2
I_T	solar irradiation on tilted surface, W/m^2
I_T^+	critical solar irradiation on tilted surface defined in equation(5), W/m^2
$\dot{m}$	mass flowrate, kg/s
M_t	total mass of water in tank, kg
t	time, s
T_a	ambient temperature, $°C$
T_b	tank bottom temperature, $°C$
T_e	collector exit temperature at steady flow, $°C$
T_i	collector inlet temperature, $°C$
T_{init}	initial mean water temperature in pump OFF mode, $°C$
T_t	tank temperature, $°C$
T_z	mean stagnant water temperature (i.e. at $\dot{m}=0$) inside collector, $°C$
ΔT_f	$=T_e-T_i$, $°C$
ΔT_{OFF}	pump OFF setting point of DT controller, $°C$
ΔT_{ON}	pump ON setting point of DT controller, $°C$
ΔT_z	$=T_z-T_{init}$, $°C$
ΔT_∞	steady-state temperature in pump ON mode defined in equation (11), $°C$
$\Delta T_{0\infty}$	steady-state temperature in pump OFF mode defined in equation (14), $°C$
τ_c	collector time constant at $\dot{m}\neq 0$
τ_o	collector time constant at $\dot{m}=0$
γ	DT controller signal
θ	solar incident angle to the collector slope, *degree*

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在開關控制器控制下的太陽能熱水系統暫態能

黃秉鈞

國立台灣大學機械工程學系教授

摘 要

本研究主要利用太陽能集熱器的可變結構模式，來分析在開關控制器控制下的太陽能熱水系統的暫態性能。在泵開啓狀態時，太陽能集熱器的動態模式採用 Hottel-Whillier-Bliss 的一階模式來分析其操作性能。在泵關閉狀態時，太陽能集熱器的動態模式則採用 Huang & Lu 的模式來分析操作性能。本研究利用相平面圖來清楚地顯示泵的極限循環 (limit cycle) 現象，並以系統模擬來研究太陽能熱水系統的暫態性能。研究結果發現，當太陽能輻射量的變動較大，或水槽水溫較高時，泵的週期開關振盪會變得較頻繁。此種泵的開關振盪可藉由調高泵控制器的設定點 ΔT_{ON} 來降低發生的頻率。本研究也發現，泵的週期開關振盪對太陽能熱水系統的集熱效率影響並不大。