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PD-type vs PID-type Fuzzy Controllers*

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abstract

It has been widely acclaimed and verified that a fuzzy controller can achieve a better performance, e.g. a shorter rise-time and a smaller overshoot, over conventional linear controllers, such as a linear PD controller. Generally speaking, a fuzzy controller considering the *error* and the *change rate of error* of the output variable can be regarded as a nonlinear PD controller. It is our intention in this paper to show that the PD-type fuzzy controller is deficient in structure and can be improved by incorporating the third factor, the *summation of the output errors*. The proposed PID-type fuzzy controller is capable of achieving zero steady-state error with very little overshoot for different reference points, which is a very difficult task for a fixed PD-type fuzzy controllers. Several examples are shown to illustrate the weakness of the PD-type fuzzy controllers and the improvements by adding the term of summation-of-errors in the premise of the rule-base.

1 Introduction

Many successful applications of fuzzy control, especially in the process control areas[6,7,8], have been reported since Mamdani's first steam engine control in 1974[5]. In particular, a number of commercial implementations by Japan, e.g. the detection of the load and the control of the washing cycle of a washing machine, the focusing of the video camera, etc, have caught the eyes of the world in recent years.

The basic idea of fuzzy control is to make use of the knowledge and experience from the experts to form a rule-base with linguistic *if-then* rules. Then proper control actions are provided by the rule-base which can be considered as an emulation of the behavior of the human operators. In the premise of the rule-base, the *error* and the *change rate of error* are usually adopted. Ying et al.[9] has shown that such a PD-type fuzzy controller is actually very similar to a linear PD controller. Buckley and Ying[1,2] also showed a limit theorem for linear fuzzy control rules stating that the defuzzified output of a fuzzy controller is approximately the same as a PD controller for sufficiently large number of rules. However, a very important topic which has not been fully discussed in the literatures is the reason why a fuzzy controller can perform so well in the industrial applications.

We believe that the answer lies in the fact that a fuzzy controller is, almost without exceptions, designed and constructed to handle a specific situation for a specific system, i.e. it is often designed for a predetermined reference point. However, as we shall show in section 2, such PD-type fuzzy controllers have difficulties in reducing its output steady-state errors for different reference points as in the case of the linear PD controllers. We shall then show that the problem can be satisfactorily solved by including the *summation of errors* in the premise of the rule-base.

The organisation of the paper is as follows. The basic structure and operations of a fuzzy controller are introduced in section 2. The problems of a PD-type as well as a PI-type fuzzy controllers are discussed in section 3. The proposed PID-type fuzzy controller is described in section 4 with examples to demonstrate its capability of solving the problems of the steady-state errors and the overshoots in its transient response. Finally, the conclusions are summarised in the last section.

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2 Format and Operations of Fuzzy Control

An introduction to fuzzy control will be given in the following to show the general structure of the fuzzy control systems. To simplify the notation, we shall discuss a rule-base with only two rules and each rule has only two state variables, E and CE, and one action variable U such as:

if E is A_1, and CE is B_1, then U is U_1
if E is A_2, and CE is B_2, then U is U_2

where A_i, B_i, and U_i are linguistic terms such as *Positive Small* or *Negative Large*. (As a matter of fact, the state variables are usually corresponding to the *error* and the *change rate of error* of the output variable.) It can be easily extended to systems with more state and action variables. The basic operating procedures of a fuzzy controller can be summarised as:

Fuzzification

Suppose that the premise part of the control rules takes inputs from the sensor readings which are usually real numbers. These real-valued sensor measurements, e.g. $x_1{}^0$ and $x_2{}^0$, are matched to their corresponding fuzzy variables by finding the matching membership values, such as:

$$\mu_{A_1}(x_1{}^0), \mu_{B_1}(x_2{}^0), \mu_{A_2}(x_1{}^0), \mu_{B_2}(x_2{}^0) \tag{1}$$

Fuzzy Reasoning

For all the control rules in the rule base, we derive the truth values(or *strengths*) of each rule in the premise by forming the conjunctions of the matching membership values:

$$\mu_1 = \mu_{A_1}(x_1{}^0) \wedge \mu_{B_1}(x_2{}^0) \qquad (\text{rule} - 1) \tag{2}$$

$$\mu_2 = \mu_{A_2}(x_1{}^0) \wedge \mu_{B_2}(x_2{}^0) \qquad (\text{rule} - 2) \tag{3}$$

The output of each control rule from its action part is represented by the fuzzy sets C_1' and C_2' and is calculated by:

$$C_1' = \mu_1 C_1(u) = \mu_1 \times C_1(u) \qquad (\text{rule} - 1) \tag{4}$$

$$C_2' = \mu_2 C_2(u) = \mu_2 \times C_2(u) \qquad (\text{rule} - 2) \tag{5}$$

Then the combined result of the inferences forms a fuzzy set C' as:

$$C' = C_1' \cup C_2' \tag{6}$$

where the $\cup$ operation is generally the "max" function.

Defuzzification Procedure

The purpose of the defuzzification process is to transform the output of the inferences, which is a fuzzy set, to a real number so that it could be used to control a process, i.e. we need to find a real number to represent a fuzzy set. Many algorithms were proposed and the most commonly adopted method(called the Center-of-Area operator) is to find the value corresponding to the center of area of the membership function of C' as follows:

$$u' = \frac{\int C'(u)u\,du}{\int C'(u)du} \tag{7}$$

where u' is the real-valued output.

3 Deficiencies of PD-type and PI-type Fuzzy Controllers

Most of the applications of fuzzy control adopt the error and the change rate of error of the output in the premise of the rule-base as shown in section 2. It has also been proved that such a fuzzy controller is approximately equal to a linear PD controller for sufficiently large number of rules. Although we usually find that a fuzzy controller is capable of handling difficult control problems, it DOES behave similarly to a linear PD controller in general.

It is well-known from the textbooks[3,4] that a linear PD controller has the properties of being:

- more stable, and with
- larger steady-state error.

While a linear PI controller is likely to be:

- less stable, less damped and with
- zero steady-state error.

Theoretically, a linear PD controller can only achieve a very small steady-state error and can not achieve a zero steady-state as the PI controller does. The same phenomenon can also be observed in the fuzzy controllers as shown in the followings.

For a second-order plant $G_1(s) = \frac{1}{s^2+10s+25}$, a PD-type and a PI-type fuzzy controllers were implemented for the reference point of $r = 1$ and their responses are shown in Figure 1. We can see that the PD-type response has a larger steady-state error of 0.0612 than the PI-type response's 0.004 and the PI-type response is less damped with an overshoot of 0.3153. For a different reference point, e.g. $r = 2$, the output responses maintain similar characteristics except with larger steady-state errors(0.1042 for PD-type and 0.0061 for PI-type) and overshoot(0.7864 for PI-type) respectively as shown in Figure 2.

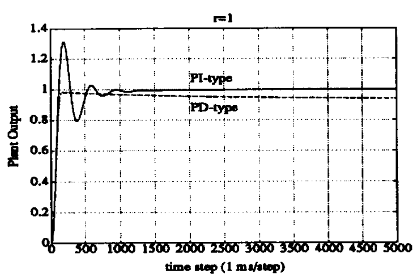

Figure 1: Output of PDFC and PIFC for $r = 1$

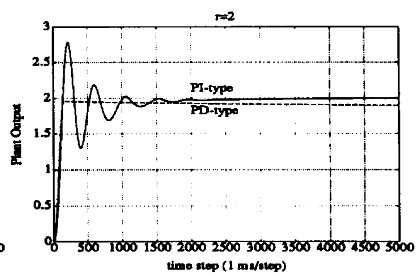

Figure 2: Output of PDFC and PIFC for $r = 2$

Therefore, it is clear that a PD-type (or PI-type) fuzzy controller will perform well only around the reference point where it is designed for. In most industrial applications, a fuzzy controller is usually designed for a specific task and its inflexibility to adapt to different reference points does not degrade its functionality much. However, for some other cases where different reference points are required, it would be better if a fuzzy controller can also be employed successfully and that is the topic we will discuss in the next section.

4 PID-type Fuzzy Controllers

In conventional control theory, a PID controller is used to take the advantages of both PD and PI controllers and to avoid their shortcomings[3,4]. It is hence natural to relate the solution to the field of fuzzy control. The study of the behavior of the PID-type fuzzy controller which use the information of the *error*, the *change rate of error*, and the *summation of errors* in the premise of the rule-base, confirms our speculation of a similar result to the linear PID controller.

For the second-order transfer function $G_1(s)$ discussed in section 2, a PID-type fuzzy controller is constructed and tested with the reference points $r = 1$ and $r = 2$. The results are shown in Figure 3 with a steady- state error of 0.0034 and an overshoot of 0.0222 for $r = 1$, and in Figure 4 with a steady-state error of 0.0082 and an overshoot of 0.1670 for $r = 2$.

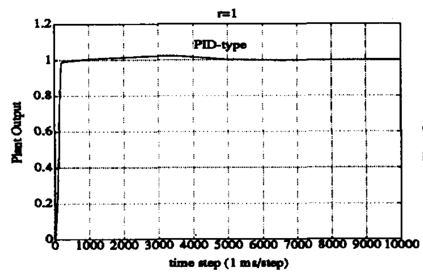

Figure 3: Output of PIDFC for $r = 1$

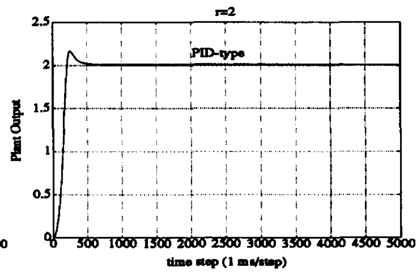

Figure 4: Output of PIDFC for $r = 2$

Comparing with Figure 1 and Figure 2, the PID-type fuzzy controller demonstrate a much better performance in terms of the overshoots and the steady-state errors. It is therefore our suggestion that a fuzzy controller should be designed based on the structure with the third factor, i.e. the summation of errors, included.

Moreover, several more examples were tested to further verify the afore-mentioned results with:

- $G_2(s) = \frac{1}{s^2+6s+50}$,
- $G_3(s) = \frac{1}{s^3+20s^2+15s+30}$,
- $G_4(s) = \frac{1}{s^2-s+1}$
- $G_5(s) = \frac{1}{s^2-s-1}$.

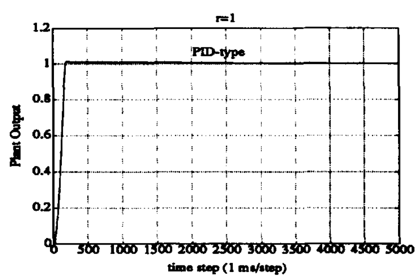

Figure 5: Output of PIDFC for $G_2(s)$

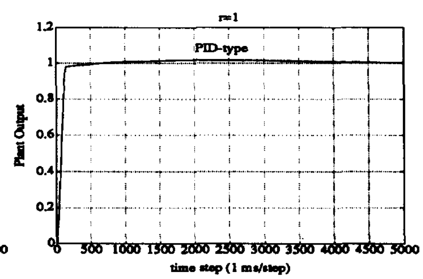

Figure 6: Output of PIDFC for $G_3(s)$

5 Conclusions

The PD-type fuzzy controller is the most commonly seen format in the area of fuzzy control. However, it has been shown to have similarities to the linear PD case and will have a larger steady-state error for reference points other than the one which is designed for. Following the conventional solution to such a problem, i.e. the PID controller, it will be beneficial to include the ***summation of errors*** in the premise of the rule-base to form the so called PID-type fuzzy controllers. Our simulation results show that a PID-type

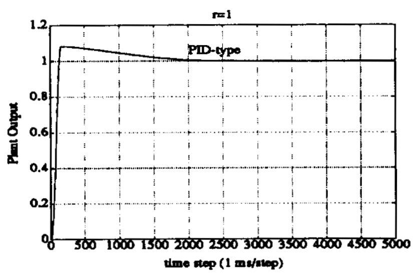

Figure 7: Output of PIDFC for $G_4(s)$

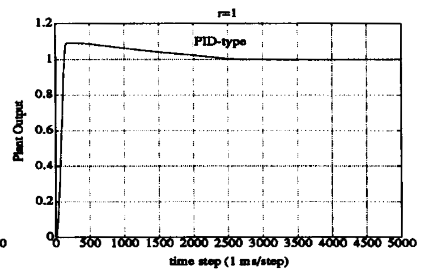

Figure 8: Output of PIDFC for $G_5(s)$

fuzzy controller is able to achieve a very small steady-state error and overshoot for different reference points for a number of different systems.

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