模糊數學意義下需課稅廠商的利潤

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The Profit of the Manufacturer with Tax in the Fuzzy Sense based on Decomposition Theorey

by

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Abstract

We consider two kinds of profits of the manufacturer with tax in the fuzzy sense. The first case is that both the price and the cost are in the fuzzy sense but not the demand. The second case is that the demand is in the fuzzy sense but not the rest. In both case we all obtain the fuzzy profit with tax for the manufacturer. Also we can get the membership functions of the fuzzy profits and the conditions that the manufcturer applied to calculate his fuzzy profit.

Keywords : Economics, membership functions, fuzzy profit.

1. Introduction

In [3, 5], we considered the consumer surplus and producer surplus in the fuzzy sense. In [4, 6], we considered the best price of two or three mutually complementary merchandises in the fuzzy sense. [2] considered the optimal revenue in the fuzzy sense. [7] considered ecnomic principle on profit in the fuzzy sense.In this paper , we consider the profit of the manufacturer with tax in the fuzzy sense.

Let the demand function be $p = a - bx, 0 \leq x \leq \frac{a}{b}$. And the cost function be $C(x) = s + qx + rx^2, x \geq 0$. In the monopolist market, the total revenue of the manufacturer is $R(x) = ax - bx^2$. And let \$t be the tax paid for each unit sold. Hence the profit is

$$\begin{aligned} \pi(x) &= R(x) - C(x) - tx \\ &= -s + (a - q - t)x - (b + r)x^2, \quad 0 \leq x \leq \frac{a}{b} \end{aligned}$$

The maximum profit can be found by Property 2. But in the perfect competitive market, we shall consider the following two cases.

In section 2, we consider the case that both the price and the cost are in the fuzzy sense. i.e.,

$$\begin{aligned} \widetilde{p} &= \widetilde{a} - \widetilde{b}x \\ \widetilde{C}(x) &= \widetilde{s} + \widetilde{q}x + \widetilde{r}x^2 \end{aligned}$$

where $\widetilde{a}$, $\widetilde{b}$, $\widetilde{s}$, $\widetilde{q}$, $\widetilde{r}$ are fuzzy numbers. Then we have the fuzzy profit

$$\widetilde{\pi}(x) = -\widetilde{s} + x\widetilde{a} - x\widetilde{q} - tx - x^2\widetilde{b} - x^2\widetilde{r}.$$

Thus, we have Property 3.

In section 3, we consider the case that the demand x is in the fuzzy sense.Then we get the fuzzy profit

$$\widetilde{\pi}(x) = -s + (a - q - t)\widetilde{x} - (b + r)\widetilde{x}^2.$$

By using the α -cut method, we can get the membership function $\mu_{\widetilde{\pi}}(z)$ of $\widetilde{\pi}$ and its centroid, which is used as the estimated value of the profit in the fuzzy sense. By using this value, we can calculate the profit of the manufacturer in the fuzzy sense. In section 4, we give some examples. Section 5 is the discussion.

2. The profit of the manufacturer with tax in the fuzzy sense for fuzzy price, fuzzy cost and crisp demand.

From [1], [8] we have the followings: Let $\widetilde{U} = (u_1, u_2, u_3), u_1 < u_2 < u_3$ be a triangular fuzzy number (abbreviated as fuzzy number) with membership function

$$\mu_{\widetilde{U}}(u) = \begin{cases} \frac{u-u_1}{u_2-u_1}, & u_1 \leq u \leq u_2 \\ \frac{u_3-u}{u_3-u_2}, & u_2 \leq u \leq u_3 \\ 0, & \text{elsewhere} \end{cases} \tag{1}$$

and the centroid is

$$M_U = \frac{\int_{-\infty}^{\infty} u\mu_{\widetilde{U}}(u)du}{\int_{-\infty}^{\infty} \mu_{\widetilde{U}}(u)du} = \frac{1}{3}(u_1 + u_2 + u_3)$$

By [1] and [8],

Property 1. Let $\widetilde{U} = (u_1, u_2, u_3)$, $\widetilde{V} = (v_1, v_2, v_3)$ be two fuzzy numbers. By transformation $y = k_1u + k_2v + k_3$ we have $\widetilde{y} = k_1\widetilde{U} + k_2\widetilde{V} + k_3$

1. If $(k_1 \geq 0) \wedge (k_2 > 0)$ or $(k_1 > 0) \wedge (k_2 \geq 0)$, then

   $\widetilde{y} = (k_1u_1 + k_2v_1 + k_3, k_1u_2 + k_2v_2 + k_3, k_1u_3 + k_2v_3 + k_3)$

2. If $(k_1 \geq 0) \wedge (k_2 < 0)$ or $(k_1 > 0) \wedge (k_2 \leq 0)$, then

   $\widetilde{y} = (k_1u_1 + k_2v_3 + k_3, k_1u_2 + k_2v_2 + k_3, k_1u_3 + k_2v_1 + k_3)$

3. If $(k_1 \leq 0) \wedge (k_2 > 0)$ or $(k_1 < 0) \wedge (k_2 \geq 0)$, then

$$\widetilde{y} = (k_1u_3 + k_2v_1 + k_3, k_1u_2 + k_2v_2 + k_3, k_1u_1 + k_1v_3 + k_3)$$

4. If $(k_1 \leq 0) \wedge (k_2 < 0)$ or $(k_1 < 0) \wedge (k_2 \leq 0)$, then

$$\widetilde{y} = (k_1u_3 + k_2v_3 + k_3, k_1u_2 + k_2v_2 + k_3, k_1u_1 + k_2v_1 + k_3)$$

Now we consider the demand function

$$p = a - bx, \quad 0 \leq x \leq \frac{a}{b} \tag{2}$$

where x is demand and p is price and the cost function is

$$C(x) = s + qx + rx^2, \quad 0 \leq x \tag{3}$$

where a, b, s, q, r are positive known and satisfy $0 < a - q - t < \frac{2a(b+r)}{b}$, t : tax paid per unit sold. By (2), we have the total revenue is $R(x) = xp = ax - bx^2$. So we have the profit of the manufacturer

$$\begin{aligned} \pi(x) &= R(x) - C(x) - tx \\ &= -s + (a - q - t)x - (b + r)x^2, \quad 0 \leq x \leq \frac{a}{b} \end{aligned} \tag{4}$$

In the monopolistic market, we can find the sale price p, sales amount x, which makes the maximum profit reached. Since $\pi'(x) = a - q - t - 2(b + r)x$, set $\pi'(x) = 0$, we have $x = \frac{a-q-t}{2(b-r)}$ $(\equiv d_*)$. Then $p = a - bd_*$ is the corresponding price such that maximum profit is reached. It is obvious that $d_* \in [0, \frac{a}{b}]$.

Property 2. In crisp case, (2) is the demand function and (3) is the cost function. When sales amount is $x = d_*$, price is $p = a - bd_*$, then the maximum profit is

$$\pi(d_*) = -s + \frac{(a - q - t)^2}{4(b + r)}$$

In a perfect competitive market, corresponding to the same demand x, price may not always equal to $p(\equiv a - bx)$. It varies a little. Same thing happens to $C(x)$. It varies too. Therefore we fuzzify a, b, s, q, r and have

$$\begin{aligned}
&\widetilde{a} = (a - w_{11}, a, a + w_{12}), \quad \widetilde{b} = (b - w_{21}, b, b + w_{22}), \\
&\widetilde{s} = (s - w_{31}, s, s + w_{32}), \quad \widetilde{q} = (q - w_{41}, q, q + w_{42}), \\
&\widetilde{r} = (r - w_{51}, r, r + w_{52})
\end{aligned} \tag{5}$$

where $0 < w_{11} < a$, $0 < w_{21} < b$, $0 < w_{31} < s$, $0 < w_{41} < q$, $0 < w_{51} < r$; $0 < w_{i2}, i = 1, 2, 3, 4, 5$; and $w_{ij}, i = 1, 2, 3, 4, 5; j = 1, 2$ are determined by the decision maker.

The centroids of $\widetilde{a}$, $\widetilde{b}$, $\widetilde{s}$, $\widetilde{q}$, $\widetilde{r}$ are $M_a = a + \frac{1}{3}(w_{12} - w_{11})$, which is the estimated value of a in the fuzzy sense, and when $w_{11} = w_{12}, M_a = a$. Similarly, $M_b = b + \frac{1}{3}(w_{22} - w_{21})$, which is the estimated value of b in the fuzzy sense, and when $w_{21} = w_{22}, M_b = b$. $M_s = s + \frac{1}{3}(w_{32} - w_{31})$, and when $w_{31} = w_{32}, M_s = s$. $M_q = q + \frac{1}{3}(w_{42} - w_{41})$, and when $w_{41} = w_{42}, M_q = q$. $M_r = r + \frac{1}{3}(w_{52} - w_{51})$, and when $w_{51} = w_{52}, M_r = r$. So that the fuzzy price is $\widetilde{p} = \widetilde{a} - \widetilde{b}x$ and the fuzzy cost is $\widetilde{C}(x) = \widetilde{s} + \widetilde{q}x + \widetilde{r}x^2$. By (4), the fuzzy profit is $\widetilde{\pi}(x) = -\widetilde{s} + \widetilde{a}x - \widetilde{q}x - tx - \widetilde{b}x^2 - \widetilde{r}x^2$.

Since $x > 0$, by Property 1, we have the following fuzzy numbers:

$$\begin{aligned}
\widetilde{p} =&(a - w_{11} - (b + w_{22})x, a - bx, a + w_{12} - (b - w_{21})x) \\
\widetilde{C}(x) =&(s - w_{31} + (q - w_{41})x + (r - w_{51})x^2, s + qx + rx^2, \\
&s + w_{32} + (q + w_{42})x + (r + w_{52})x^2) \\
\widetilde{\pi}(x) =&(S_1, S_2, S_3) \text{ where} \\
S_1 =& -(s + w_{32}) + (a - w_{11})x - (q + w_{42})x - tx - (b + w_{22})x^2 - (r + w_{52})x^2 \\
S_2 =& -s + ax - qx - tx - bx^2 - rx^2 = \pi(x) \\
S_3 =& -(s - w_{31}) + (a + w_{12})x - (q - w_{41})x - tx - (b - w_{21})x^2 - (r - w_{51})x^2
\end{aligned}$$

The centroids of $\widetilde{p}$, $\widetilde{C}(x)$, $\widetilde{\pi}(x)$ are

$$M_p(x) = a - bx + \frac{1}{3}[w_{12} - w_{11} - (w_{22} - w_{21})x], \quad 0 \le x \le \frac{a}{b} \tag{6}$$

$$M_C(x) = s + qx + rx^2 + \frac{1}{3}[w_{32} - w_{31} + (w_{42} - w_{41})x + (w_{52} - w_{51})x^2], \quad 0 \leq x \tag{7}$$

$$\begin{aligned} M_\pi(x) =& \frac{1}{3}(S_1 + S_2 + S_3) \\ =& \pi(x) + \frac{1}{3}[(w_{31} - w_{32}) + (w_{12} - w_{11} - w_{42} + w_{41})x \\ & - (w_{22} - w_{21} + w_{52} - w_{51})x^2], \quad 0 \leq x \leq \frac{a}{b} \end{aligned} \tag{8}$$

That is $M_\pi(x) = M_p(x)x - M_C(x) - tx$. Here $M_p(x)$ is the estimated value of the price of the manufacturer in the fuzzy sense when demand is x. i.e. the demand function in the fuzzy sense; $M_C(x)$ is the estimated value of the cost of the manufacturer in the fuzzy sense. $M_\pi(x)$ is the estimated value of the profit of the manufacturer in the fuzzy sense when demand is x, i.e. the profit function in the fuzzy sense. When $w_{i1} = w_{i2}, i = 1, 2, 3, 4, 5$; $M_p(x) = p$, $M_C(x) = C(x)$, $M_\pi(x) = \pi(x)$, that is, in the fuzzy sense is the same as in crisp case. When $w_{ij} = 0$, $\forall i = 1, 2, 3, 4, 5; j = 1, 2$; in fuzzy case turns to be in crisp case.

$$\begin{aligned} \frac{d}{dx}M_\pi(x) =& a - q - t + \frac{1}{3}(w_{12} - w_{11} - w_{42} + w_{41}) \\ & - 2[b + r + \frac{1}{3}(w_{22} - w_{21} + w_{52} - w_{51})]x \end{aligned}$$

Let $d^* = \frac{3(a-q-t)+w_{12}-w_{11}-w_{42}+w_{41}}{2(3b+3r+w_{22}-w_{21}-w_{52}-w_{51})}$, then we have the following property.

Property 3 If $\widetilde{p} = \widetilde{a} - \widetilde{b}x$, $\widetilde{C}(x) = \widetilde{s} + \widetilde{q}x + \widetilde{r}x^2$, and $0 < d^* < \frac{a}{b}$, then the profit reaches its maximum $M_\pi(d^*)$ at sales amount $x = d^*$ and price $M_p(d^*)$.

3. The profit of the manufacturer in the fuzzy sense for fuzzy demand.

As in section 2, when the demand function is (2): $p = a - bx, 0 \leq x \leq \frac{a}{b}$ and the cost function is (3): $C(x) = s + qx + rx^2, 0 \leq x$; we restrict our condition $0 < a - q - t < \frac{2a(b+r)}{b}$, i.e. $d_* = \frac{a-q-t}{2(b+r)} \in (0, \frac{a}{b})$. Then by Property 2, the maximum profit is $\pi(d_*) = -s + \frac{(a-q-t)^2}{4(b+r)}$.

In a perfect competitive market, for the same price p, the demand varies around $x (= \frac{a-p}{b})$ a little. So we have fuzzy number $\widetilde{x} = (x-\Delta_1,\ x,\ x-\Delta_2), 0 < \Delta_1 < x, 0 < \Delta_2$; Δ_1, Δ_2 are determined by the decision maker, and the membership function of $\widetilde{x}$ is

$$
\mu_{\widetilde{x}}(y) = \begin{cases} \frac{y-x+\Delta_1}{\Delta_1}, & x-\Delta_1 \leq y \leq x \\ \frac{x-\Delta_2-y}{\Delta_2}, & x \leq y \leq x+\Delta_2 \\ 0, & \text{elsewhere} \end{cases} \tag{9}
$$

From (4), we have the fuzzy profit

$$
\widetilde{\pi} = -s + (a-q-t)\widetilde{x} - (b+r)\widetilde{x}^2 \tag{10}
$$

We proceed to find out the membership function of $\widetilde{\pi}$ by the α- cut method as the following.

Let the α-cut of $\widetilde{x}$ be $x(\alpha) = \{y|\mu_{\widetilde{x}}(y) \geq \alpha\} = [x_L(\alpha),\ x_U(\alpha)]$, where $x_L(\alpha)$, $x_U(\alpha)$ are the left, right hand side point of this α-cut respectively.

From (9), we have

$$
\begin{aligned}
x_L(\alpha) &= (x-\Delta_1) + \Delta_1\alpha > 0 \\
x_U(\alpha) &= (x+\Delta_2) - \Delta_2\alpha = x + (1-\alpha)\Delta_2 > 0
\end{aligned}
$$

Hence by Property 1, the α-cut of $\widetilde{x}^2$ is $[x_L(\alpha)^2, x_U(\alpha)^2]$. Since $0 < a-q-t < \frac{2a(b+r)}{b}$, so by (10) and Property 1, for each $\alpha \in [0,1]$, the α-cut of π is $[\pi_L(\alpha), \pi_U(\alpha)]$ and

$$
\begin{aligned}
\pi_L(\alpha) = &- s + (a-q-t)x_L(\alpha) - (b+r)x_U(\alpha)^2 \\
= &- s + (a-q-t)x - (b+r)x^2 \\
&- [(a-q-t)\Delta_1 + 2(b+r)\Delta_2 x](1-\alpha) \\
&- (b+r)\Delta_2^2(1-\alpha)^2
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
\pi_U(\alpha) =& - s + (a - q - t)x_U(\alpha) - (b + r)x_L(\alpha)^2 \\
=& - s + (a - q - t)x - (b + r)x^2 \\
& + [(a - q - t)\Delta_2 + 2(b + r)\Delta_1 x](1 - \alpha) \\
& - (b + r)\Delta_1^2(1 - \alpha)^2
\end{aligned} \tag{12}
$$

Remark 1: In section 3, after fuzzify the demand into $\widetilde{x}$, the tax in the left hand point of the α-cut of the fuzzy profit $\widetilde{\pi}$ is $t(x - \Delta_1)$, and the tax in the right hand point of the α-cut of $\widetilde{\pi}$ is $t(x + \Delta_2)$, $\frac{1}{2}[t(x - \Delta_1) + t(x + \Delta_2)] = tx + \frac{1}{2}(\Delta_2 - \Delta_1)$ is the estimate of tax in the fuzzy sense, which is different from the tax tx in the crisp case. In section 2, the tax in the profit $M_\pi(x)$ in the fuzzy sense (8) is tx which is same as in the crisp case.

Insert Fig. 1 here here.

Let
$$
\begin{aligned}
A(x) &= (a - q - t)\Delta_1 + 2(b + r)\Delta_2 x \quad (> 0) \\
B &= (b + r)\Delta_2^2 \quad (> 0) \\
D(x) &= (a - q - t)\Delta_2 + 2(b + r)\Delta_1 x \quad (> 0) \\
E &= (b + r)\Delta_1^2 \quad (> 0)
\end{aligned}
$$

Then, for each $\alpha \in [0, 1]$, (11), (12) becomes

$$
\pi_L(\alpha) = \pi(x) - A(x)(1 - \alpha) - B(1 - \alpha)^2 \tag{13}
$$

$$
\pi_U(\alpha) = \pi(x) + D(x)(1 - \alpha) - E(1 - \alpha)^2 \tag{14}
$$

For each $\alpha \in [0, 1]$, let $\pi_L(\alpha) = y$, then (13) becomes

$$
B(1 - \alpha)^2 + A(x)(1 - \alpha) + y - \pi(x) = 0
$$

Solve for $1 - \alpha$: Then either

$$
1 - \alpha = \frac{-A(x) - \sqrt{A(x)^2 - 4B(y - \pi(x))}}{2B} < 0, \quad \text{if } y \leq \frac{A(x)^2 + 4B\pi(x)}{4B}
$$

which can not happen since $1-\alpha>0$. or

$$1-\alpha=\frac{-A(x)+\sqrt{A(x)^2+4B\pi(x)-4By}}{2B}, \quad \text{if } y\le\frac{A(x)^2+4B\pi(x)}{4B} \tag{15}$$

Since $1-\alpha\in[0,1]$, therefore $0\le\frac{-A(x)+\sqrt{A(x)^2+4B\pi(x)-4By}}{2B}\le 1$ Thus y is in the region $A(x)\le\sqrt{A(x)^2+4B\pi(x)-4By}\le A(x)+2B$ and $y\le\frac{A(x)^2}{4B}+\pi(x)$. i.e.

$$\pi(x)-A(x)-B\le y\le\pi(x) \quad \text{and} \quad y\le\frac{A(x)^2}{4B}+\pi(x)$$

$$\text{i.e.} \quad \pi(x)-A(x)-B\le y\le\pi(x) \tag{16}$$

Let $\pi_U(\alpha)=y$, then (14) bec5mes

$$E \qquad (1-\alpha)^2-D(x)(1-\alpha)+y-\pi(x)=0$$

Solve for $(1-\alpha)$, then $1-\alpha=\frac{D(x)-\sqrt{D(x)^2-4E(y-\pi(x))}}{2E}$ or $1-\alpha=\frac{D(x)-\sqrt{D(x)^2-4E(y-\pi(x))}}{2E}$ if $y\le\frac{D(x)^2+4E\pi(x)}{4E}$.

$$(1^\circ).\ \text{If } 1-\alpha=\frac{D(x)-\sqrt{D(x)^2-4E(y-\pi(x))}}{2E} \tag{17}$$

since $0\le 1-\alpha\le 1$, i.e. $0\le\frac{D(x)-\sqrt{D(x)^2-4E(y-\pi(x))}}{2E}\le 1$, then y is in the region

$$\sqrt{D(x)^2-4Ey+4E\pi(x)}\le D(x), \tag{18}$$

$$\text{and} \quad D(x)-2E\le\sqrt{D(x)^2+4E\pi(x)-4Ey}, \quad \text{and} \quad y\le\frac{D(x)^2}{4E}+\pi(x) \tag{19}$$

$D(x)>0$, from (18), $\pi(x)\le y$. Since $D(x)-2E=(a-q-t)\Delta_2+2(b+r)\Delta_1(x-\Delta_1)>0$, by (19), $y\le D(x)+\pi(x)-E$. Since $D(x)+\pi(x)-E<\frac{D(x)^2}{4E}+\pi(x)$, so we have

$$\pi(x)\le y\le D(x)+\pi(x)-E \tag{20}$$

$$(2^\circ).\ \text{If} \quad 1-\alpha=\frac{D(x)+\sqrt{D(x)^2+4E\pi(x)-4Ey}}{2E}$$

since $1-\alpha \in [0,1]$, i.e. $0 \le \frac{D(x)+\sqrt{D(x)^2+4E\pi(x)-4Ey}}{2E} \le 1$, so

$$\sqrt{D(x)^2+4E\pi(x)-4Ey} \le 2E - D(x) \quad \text{and} \quad y \le \pi(x) + \frac{D(x)^2}{4E} \tag{21}$$

Since $D(x) - 2E > 0$, so (21) cannot hold. From (15), (16), (17) and (20), we have

$$\mu_{\tilde{\pi}}(y) = \begin{cases} 1 - \frac{-A(x)+\sqrt{A(x)^2+4B\pi(x)-4By}}{2B}, & \text{if } \pi(x) - A(x) - B \le y \le \pi(x) \\ 1 - \frac{D(x)-\sqrt{D(x)^2-4E\pi(x)-4Ey}}{2E}, & \text{if } \pi(x) \le y \le D(x) + \pi(x) - E \\ 0, & \text{elsewhere} \end{cases} \tag{22}$$

When $\pi(x) - A(x) - B \le y \le \pi(x)$

$\frac{d}{dy}\mu_{\tilde{\pi}}(y) = \frac{1}{[A(x)^2+4B\pi(x)-4By]^{\frac{1}{2}}} > 0$, $\quad \frac{d^2}{dy^2}\mu_{\tilde{\pi}}(y) = \frac{2B}{[A(x)^2+4B\pi(x)-4By]^{\frac{3}{2}}} > 0$

When $\pi(x) \le y \le \overline{U}(x)$,

$\frac{d}{dy}\mu_{\tilde{\pi}}(y) = \frac{-1}{[D(x)^2+4E\pi(x)-4Ey]^{\frac{1}{2}}} < 0$, $\quad \frac{d^2}{dy^2}\mu_{\tilde{\pi}}(y) = \frac{-2E}{[D(x)^2+4E\pi(x)-4Ey]^{\frac{3}{2}}} < 0$

Hence we have Fig. 2.

Insert Fig. 2 here.

$$\text{Let} \quad W_1(p,q;a,b) = \int_p^q \sqrt{a-by}\,dy = \frac{2}{3b}[(a-bp)^{\frac{3}{2}} - (a-bq)^{\frac{3}{2}}] \tag{23}$$

$$\begin{aligned} W_2(p,q;a,b) = \int_p^q y\sqrt{a-by}\,dy &= \frac{2a}{3b^2}[(a-bp)^{\frac{3}{2}} - (a-bq)^{\frac{3}{2}}] \\ &+ \frac{2}{5b^2}[(a-bq)^{\frac{5}{2}} - (a-bp)^{\frac{5}{2}}] \end{aligned} \tag{24}$$

From (22), (24), (25) use centroid to defuzzify, we have

$$\begin{aligned} P &= \int_{-\infty}^{\infty} \mu_{\tilde{\pi}}(y)dy \\ &= (1 + \frac{A(x)}{2B})(A(x)+B) + (1 - \frac{D(x)}{2E})(D(x) - E) \end{aligned}$$

$$- \frac{1}{2B} W_1(\pi(x) - A(x) - B, \pi(x); A(x)^2 + 4B\pi(x), 4B)$$
$$+ \frac{1}{2E} W_1(\pi(x), D(x) - \pi(x) - E; D(x)^2 + 4E\pi(x), 4E) \tag{25}$$

$$\begin{aligned} Q &= \int_{-\infty}^{\infty} y\mu_{\widetilde{\pi}}(y)dy \\ &= \frac{1}{2}(1 + \frac{A(x)}{2B})[\pi(x)^2 - (\pi(x) - A(x) - B)^2] \\ &+ \frac{1}{2}(1 - \frac{D(x)}{2E})[(D(x) + \pi(x) - E)^2 - \pi(x)^2] \\ &- \frac{1}{2B} W_2(\pi(x) - A(x) - B, \pi(x); A(x)^2 + 4B\pi(x), 4B) \\ &+ \frac{1}{2E} W_2(\pi(x), D(x) + \pi(x) - E; D(x)^2 + 4E\pi(x), 4E) \end{aligned} \tag{26}$$

That is the centroid of $\mu_{\widetilde{\pi}}(y)$ (in (22)). From (25), (26) we have

$$\frac{Q}{P} \equiv M_{\pi}(x) \qquad \text{(say)} \tag{27}$$

$M_{\pi}(x)$ is the profit function in the fuzzy sense. Thus we have

Property 4. Let the demand function be $p(x) = a - bx, 0 \leq x \leq \frac{a}{b}$; the cost function be $C(x) = s + qx + rx^2, x \geq 0$; and the profit function be $\pi(x) = R(x) - C(x) - tx = -s + (a - q - t)x - (b + r)x^2, 0 \leq x \leq \frac{a}{b}$. If we fuzzify the "$x$" (in (4)) into (9), then we have

(1°). The fuzzy profit $\widetilde{\pi}$ is

$$\widetilde{\pi} = -s + (a - q - t)\widetilde{x} - (b + r)\widetilde{x}^2$$

(in (10))

(2°). The membership function of $\widetilde{\pi}$ is

$$\mu_{\widetilde{\pi}}(y) = \begin{cases} 1 - \frac{-A(x) + \sqrt{A(x)^2 + 4B\pi(x) - 4By}}{2B}, & \text{if } \pi(x) - A(x) - B \leq y \leq \pi(x) \\ 1 - \frac{D(x) - \sqrt{D(x)^2 + 4E\pi(x) - 4Ey}}{2E}, & \text{if } \pi(x) \leq y \leq D(x) + \pi(x) - E \\ 0, & \text{elsewhere} \end{cases}$$

(in (22))

(3°). The estimate of the profit function in the fuzzy sense is

$$M_\pi(x) = \frac{Q}{P} \qquad (\text{ in } (28))$$

where P and Q are of the form (25), (26) respectively.

4. Example

Let the demand function be $p(x) = 100 - 2x$, $0 \le x \le 50$, and the cost function be $C(x) = 10 + 2x + 0.5x^2$, $0 \le x$, and the tax $t = 2$. Then the revenue is $R(x) = xp(x) = 100x - 2x^2$ and the profit is $\pi(x) = R(x) - C(x) - 2x = -10 + 96x - 2.5x^2$. i.e., $a = 100, b = 2, s = 10, q = 2, r = 0.5$

[A] In the crisp case of Property 2,

$$\pi'(x) = 96 - 5x \overset{\text{let}}{=} 0, \Longrightarrow d_* = \frac{96}{5} = 19.2, \text{ price } P_* = 61.6$$

$$\pi(d_*) = \pi(19.2) = 911.6$$

[B] In the fuzzy case of Property 3,

Let $\quad w_{11} = 1, w_{12} = 3, w_{21} = 0.5, w_{22} = 1, w_{31} = 1, w_{32} = 0.5$

$\quad w_{41} = 2, w_{42} = 1, w_{51} = 1, w_{52} = 1.5$

By Property 3,

$$d^* = \frac{3(a-q-t) + w_{12} - w_{11} - w_{42} + w_{41}}{2(3b + 3r + w_{22} - w_{21} + w_{52} - w_{51})} = \frac{291}{17} = 17.12 \quad \in [0, 50]$$

$$r_d = \frac{d^* - d_*}{d_*} \times 100\% = -10.833\%$$

Hence the best price in the fuzzy sense is

$$M_p(d^*) = a - bd^* + \frac{1}{3}[w_{12} - w_{11} - (w_{22} - w_{21})d^*] = 63.57$$

$$\frac{M_p(d^*) - P_*}{P_*} \times 100\% = 3.20\%$$

And the maximum profit is

$$
\begin{aligned}
M_{\pi}(d^*) =& \pi(d^*) + \frac{1}{3}[(w_{31} - w_{32}) + (w_{12} - w_{11} - w_{42} + w_{41})d^* \\
& - (w_{22} - w_{21} + w_{52} - w_{51})d^{*2}] \\
& = 820.37
\end{aligned}
$$

$$
\frac{M_{\pi}(d^*) - \pi(d_*)}{\pi(d_*)} \times 100\% = -10.01\%
$$

[C] In the fuzzy case of Property 4 for fuzzy demand.

$$
\pi(x) = -10 + 96x - 2.5x^2
$$

From Property 4 $M_{\pi}(x)$ and let $r_{\pi}(x) = \frac{M_{\pi}(x) - \pi(x)}{\pi(x)} \times 100\%$, we have the following results.

Insert Table 1 about here

5. Discussion

[A] In Property 3, if $w_{j1} = w_{j2}, j = 1, 2, 3, 4, 5$; i.e., the fuzzy numbers $\widetilde{a}, \widetilde{b}, \widetilde{s}, \widetilde{q}, \widetilde{r}$ in (5) are all isosceles, then $d^* = d_*$. That is, Property 3 turns to be Property 2.

[B] In Property 3, if $w_{j1} > w_{j2}$ for $j = 2, 4, 5$ and $w_{12} > w_{11}$, then $d_* < d^*$. On the other hand, if $w_{j1} < w_{j2}$ for $j = 2, 4, 5$ and $w_{12} < w_{11}$, then $d_* > d^*$.

[C] In Property 3, if $w_{j1} > w_{j2}$ for $j = 2, 3, 4, 5$ and $w_{12} > w_{11}$, then $M_{\pi}(x) > \pi(x)$. On the other hand , if $w_{j1} < w_{j2}$ for $j = 2, 3, 4, 5$ and $w_{12} < w_{11}$, then $M_{\pi}(x) < \pi(x)$.

Table 1. The numerical examples of Property 4.

Δ_1	Δ_2	x	$M_\pi(x)$	$\pi(x)$	$r_\pi(x)(\%)$
1.0	0.5	17.45	901.71	903.94	-0.25
1.0	0.5	18.45	908.79	910.19	-0.15
1.0	0.5	19.45	910.87	911.44	-0.06
1.0	0.5	20.45	907.95	907.69	0.03
1.5	0.5	17.45	899.47	903.94	-0.49
1.5	0.5	18.45	907.39	910.19	-0.31
1.5	0.5	19.45	910.30	911.44	-0.13
1.5	0.5	20.45	908.22	907.69	-0.06
0.5	1.0	17.45	904.62	903.94	0.07
0.5	1.0	18.45	910.04	910.19	-0.02
0.5	1.0	19.45	910.45	911.44	-0.11
0.5	1.0	20.45	905.87	907.69	-0.20
0.5	1.5	17.45	905.30	903.94	0.15
0.5	1.5	18.45	909.88	910.19	-0.03
0.5	1.5	19.45	909.46	911.44	-0.22
0.5	1.5	20.45	904.04	907.69	-0.40
1.0	1.5	17.45	903.37	903.94	-0.06
1.0	1.5	18.45	908.79	910.19	-0.15
1.0	1.5	19.45	909.20	911.44	-0.25
1.0	1.5	20.45	904.62	907.69	-0.34
1.5	1.0	17.45	900.46	903.94	-0.39
1.5	1.0	18.45	907.54	910.19	-0.29
1.5	1.0	19.45	910.30	911.44	-0.13
1.5	1.0	20.45	908.22	907.69	0.06
0.5	3.5	17.45	904.87	903.93	0.10
0.5	3.5	18.45	906.10	910.19	-0.45
0.5	3.5	19.45	902.34	911.44	-1.00
0.5	3.5	20.45	893.57	907.69	-1.56
3.5	0.5	17.45	887.46	903.94	-1.82
3.5	0.5	18.45	898.69	910.19	-1.26
3.5	0.5	19.45	904.92	911.44	-0.72
3.5	0.5	20.45	906.15	907.69	-0.17

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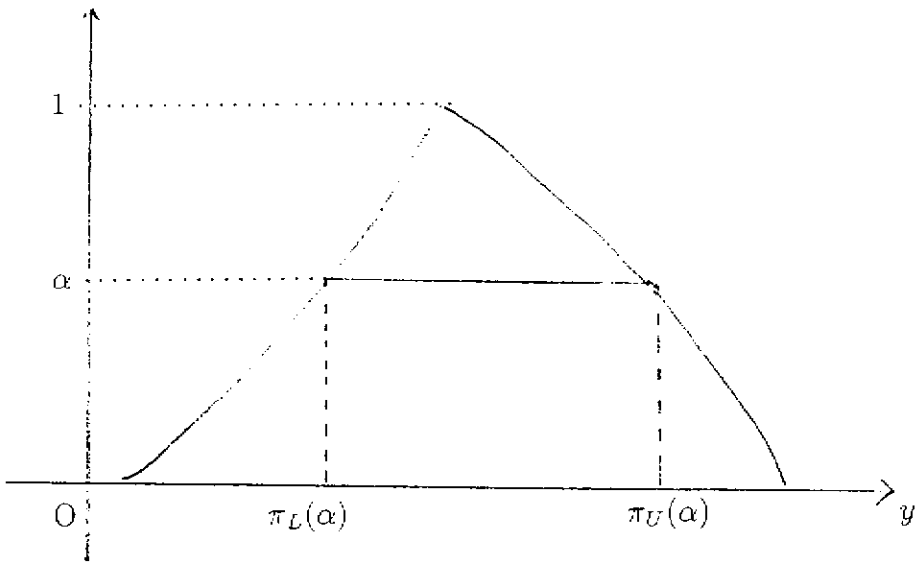

Fig. 1 . The α-cut of the fuzzy set $\widetilde{\pi}$

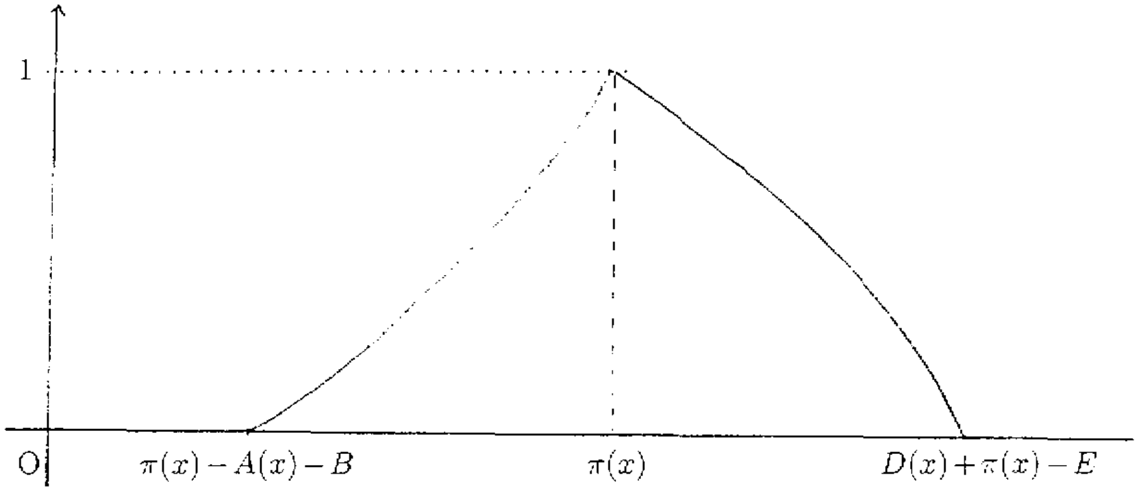

Fig.2 . Graph of $\mu_{\tilde{\pi}}(y)$ in (22)