Performance characteristics of pulse tube refrigerators

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Received 19 October 1992; revised 18 March 1993

In the present study experiments were carried out to investigate the performance characteristics of pulse tube refrigerators. It was found that the cool-down time t_c during the transient or start-up period is dominated by the time constant of the pulse tube wall τ_{pt} and that the dynamics of a basic pulse tube (BPT) refrigerator approaches that of a first-order system. For steady state operation, the cold-end temperature T_L was found to vary with τ_{pt}, and the cooling load Q_L increases monotonically with increasing τ_{pt}. This indicates that heat pumped by the gas from the cold to the hot end increases with decreasing h_{pt} (i.e. less energy exchange between the gas and wall). The process of heat storage or release of the pulse tube wall is thus shown to have a negative effect on the performance of a BPT refrigerator. It was thus found experimentally that the gas compression/expansion process inside the pulse tube, which is similar to a Brayton cycle but lies between isothermal and adiabatic, can explain the performance of BPT refrigerators. The present experiment also shows that the performance of a pulse tube refrigerator at transient and steady states is mainly dominated by the time constant of the pulse tube wall τ_{pt}.

Keywords: pulse tube refrigerators; thermodynamics; performance

The performance of a basic pulse tube (BPT) refrigerator (*Figure 1*) has been studied experimentally or analytically by many researchers[1–12]. It was thought that the surface heat pumping effect occurs in a pulse tube by interaction between gas displacement along the wall surface, energy change in the gas, heat transfer from gas to wall and heat conduction along the wall, as a result of the periodic change of pressure of the gas[7–9]. This effect was considered to be the major mechanism by which heat is pumped from the cold to hot end and a refrigeration effect generated in BPT refrigerators.

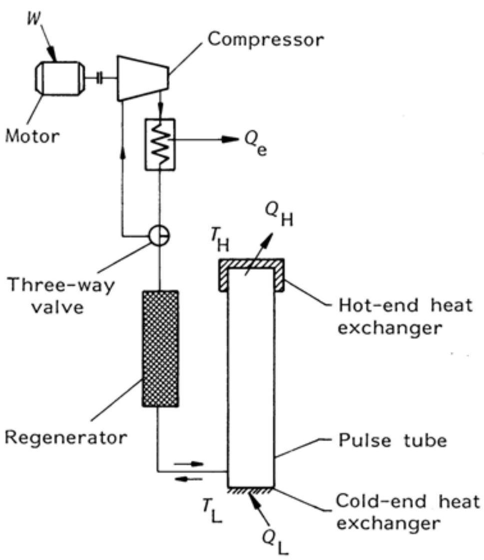

Figure 1 Basic pulse tube refrigerator

Gifford and Longsworth[7–9] assumed that heat transfer from gas to wall is mainly by gas conduction during quiescent periods (periods without gas flow). Rea[12] and Colangelo *et al.*[13] assumed that convective heat transfer from gas to wall during flow periods is a controlling mechanism. Huang *et al.*[14] further modified the analytical model of Colangelo *et al.*[13] and developed a system simulation scheme to predict the performance of a BPT refrigerator.

Recent studies[6], however, indicate that the surface heat pumping effect seems to be minimal and is not the major mechanism for producing the refrigeration effect in orifice pulse tube (OPT) refrigerators. Instead, the refrigeration effect of an OPT is generated by the phase shift and attenuation of pressure, temperature and mass flowrate waves across each component of the refrigerator.

In practice, the gas compression/expansion process inside the pulse tube, which is very similar to a Brayton cycle, may be the overriding thermodynamic process in BPT refrigerators. The gas elements inside the pulse tube are compressed during pressure charging periods. The compression process will lie between isothermal and adiabatic since there is energy exchange between gas and wall and mixing of adjacent gas elements. Therefore, compression energy is converted into heat so that a gas

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temperature gradient is built up from the cold to the hot end along the pulse tube and heat is given out to the surroundings at the hot-end exchanger. The heat exchange between gas and wall and heat conduction along the wall from the hot to the cold end may have a negative effect on BPT performance.

The performance of a BPT refrigerator at cyclically steady and start-up (transient) periods involves transient heat transfer processes. From the viewpoint of system dynamics, the performance of a pulse tube refrigerator can be characterized by the time constants of each component (e.g. regenerator and pulse tube). The relationship between the time constants and the performance of BPT refrigerators was thus studied experimentally and the surface heat pumping mechanism of BPT refrigerators is examined.

Experimental set-up

Several BPT refrigerators were designed for the experiment. The pulse tube was made from a stainless steel pipe 19 mm in diameter, 0.4 mm thick and 30 cm long. A 7 cm long water jacket was designed to connect to the downstream end of the pulse tube as the hot end heat exchanger. The regenerator was made from a stainless steel tube 19 mm in diameter and 0.4 mm thick. Iron and copper beads of different sizes were put into the regenerator tube to make different regenerators for the experiments. There are 11 different combinations of BPT refrigerators in the present study. The specifications are shown in *Table 1*. The BPT refrigerator was put in a vacuum chamber evacuated to $< 10^{-3}$ torr* (approximately) in order to reduce heat leakage during measurement of the cooling load.

Dried air with a $-60°C$ dew point and at 11 atm† was used as the working fluid. For simplicity, the three-way valve was replaced by two solenoidal valves which were controlled by an SSR relay and a wave form generator. The operating frequency of the BPT refrigerator was varied by adjusting the wave form generator. The BPT refrigerator was operated between $P_H = 11$ atm and $P_L = 1$ atm (open to the atmosphere at discharge periods).

Table 1 Regenerator specifications

Cooler number	Regenerator material	Bead size (mm)	Total mass, M_{reg}(g)
1	Iron	1.0	25
2	Iron	1.0	100
3	Iron	1.0	150
4	Iron	1.0	150
5	Iron	1.0	300
6	Copper	1.47	100
7	Copper	1.47	150
8	Copper	1.47	225
9	Copper	1.47	300
10	Copper	1.47	350
11	Copper	1.47	450

*1 torr = 133.322 Nm^{-2}
†1 atm = 101 325 N m^{-2}

T-type thermocouples were used to measure the temperature at various parts of the BPT refrigerator. The instantaneous pressure drop across the regenerator was measured by two pressure transducers (PG-20KU and PH-20KA, Kyowa Co.) for evaluating the mean mass flowrate and the heat transfer coefficients in the regenerator and pulse tube. The pressure drop *versus* mass flowrate of the regenerator at steady flows was measured separately prior to the refrigerator performance test. The results were used to derive a correlation using Darcy's law and in the calculation of instantaneous mass flowrate from the measurements of instantaneous pressure drop. This calculation of instantaneous mass flowrate is based on the quasi-steady assumption of Darcy's law which holds for a low operating frequency.

The performance of BPT refrigerators at the cyclically steady and start-up periods involves transient heat transfer processes. The performance of a pulse tube refrigerator can then be characterized by the time constant of the regenerator τ_{reg} and the pulse tube wall τ_{pt}, both of which are defined based on half-cycle mean properties.

The measured data were used to calculate the time constant of the regenerator τ_{reg} which is defined as

$$\tau_{reg} = \frac{M_{reg} C_{reg}}{h_{reg} A_s} \tag{1}$$

where M_{reg} and C_{reg} are, respectively, the total mass and specific heat of the regenerator, and h_{reg} is the gas heat transfer coefficient inside the regenerator which was calculated using the correlation derived by Colangelo *et al.*[13]

$$St\ Pr^{2/3} = 0.71 Re^{-0.41} \tag{2}$$

where: $St = h_{reg}/(\rho C_p \bar{u}_{reg})$; $Re = \rho d_h \bar{u}_{reg}/\mu$; $d_h = 2\varepsilon d_p/3(1-\varepsilon)$; $\bar{u}_{reg}$ is the half-cycle mean velocity of gas inside the regenerator; d_p is the diameter of the regenerator beads; A_s is the total surface area of the regenerator which is estimated by the equation[15] $A_s = 6(1-\varepsilon)V_R/d_p$; ε is the porosity; V_R is the total regenerator volume; and ρ and C_p are, respectively, the gas density and specific heat.

The time constant of the pulse tube wall τ_{pt} is defined as

$$\tau_{pt} = \frac{M_{pt} C_{pt}}{h_{pt} A_{pt}} \tag{3}$$

where M_{pt} and C_{pt} are, respectively, the total mass and specific heat of the pulse tube wall, and h_{pt} is the heat transfer coefficient between the gas and wall which was calculated using the correlation of Dittus and Boelter[16]

$$Nu = 0.023 Re_{pt}^{0.8} Pr^{0.3} \tag{4}$$

where: $Nu = h_{pt} d_{pt}/k$; $Re_{pt} = \rho d_{pt} \bar{u}_{pt}/\mu$; $\bar{u}_{pt}$ is the half-cycle mean velocity of gas inside the pulse tube; d_{pt} is the inside diameter of the pulse tube; and A_{pt} is the gas-wall surface area of the pulse tube.

Test results

Transient performance

The performance of a BPT refrigerator during the start-up period involves transient heat transfer processes. The dynamic response of a BPT refrigerator can thus be represented by the cool-down time t_c, which is defined as the time period for the cold-end temperature T_L to reach a steady value. t_c was defined in the present study as the 90% settling time of a dynamic system.

The test results for the 11 pulse tube refrigerators are listed in *Table 2*. The last column in *Table 2* is the radiative heat leakage to the outside surface of the pulse tube Q_k which is estimated from the measured data. Since most of the heat leakage is absorbed near the cold end (the coldest part), Q_k can approximate the cooling load, i.e. $Q_L \approx Q_k$.

It is seen from *Table 2* that τ_{reg} values are in the range 0.1–0.4 s and are much smaller than τ_{pt} (by an order of four). Hence, the dynamic response of a BPT refrigerator is apparently dominated by the pulse tube wall dynamics which can be represented by the s-plane pole $1/\tau_{pt}$. The pulse tube pole $1/\tau_{pt}$ is much closer to the origin than the regenerator pole $1/\tau_{reg}$. Hence, the dynamic response of the regenerator can be neglected. *Figure 2* shows that t_c increases monotonically with τ_{pt} and follows the relation

$$t_c \approx 2.0\tau_{pt} \tag{5}$$

It is known that for a first-order system

$$\tau_{90} = 2.3\tau_1 \tag{6}$$

where τ_1 is the time constant and τ_{90} is the 90% settling time of a first-order system. The BPT refrigerator is thus experimentally shown to approach a first-order dynamic system with a time constant τ_{pt} since Equation (5) is very close to Equation (6) if we let $\tau_1 = \tau_{pt}$.

Steady state performance

Under steady state operation, the wall temperature in a BPT refrigerator stays approximately constant due to wall damping effects. However, the transport processes inside the refrigerator are transient and cyclical in form. Therefore, the time constants of the pulse tube wall, the regenerator matrix and the heat exchangers play important parts in the steady state performance.

Figure 3a shows that, for various BPT refrigerators, the cold-end temperature T_L at various heat leakage rates (or cooling loads) varies with τ_{pt} and that a minimum T_L exists. It has been pointed out[13,14] that convective heat transfer in the pulse tube is one of the controlling mechanisms of a BPT refrigerator. A higher h_{pt} (i.e. low τ_{pt}) means that more heat is absorbed by the pulse tube wall during the heat pumping process from the cold to the hot end. Therefore, the cold-end temperature T_L (at various Q_L values) decreases with increasing τ_{pt}.

It is seen that T_L reaches a minimum and then starts to increase. This is due to the heat balance between convective and conductive heat transfer at the wall. Since the

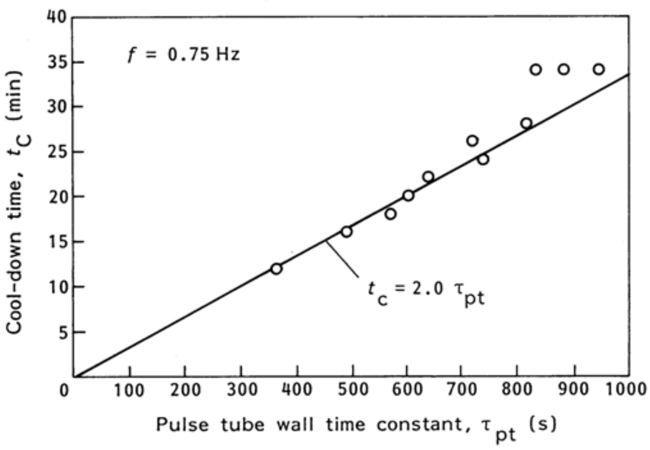

Figure 2 Transient performance results (cool-down time t_c *versus* τ_{pt})

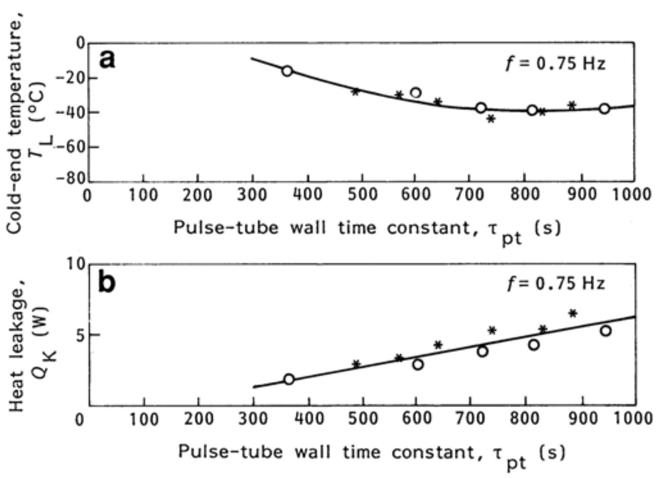

Figure 3 Steady state performance results: (a) T_L *versus* τ_{pt}; (b) Q_k *versus* τ_{pt}. Regenerator material: ○, iron beads (1.0 mm diameter); *, copper beads (1.47 mm diameter)

Table 2 Test results of pulse tube coolers

Cooler number	h_{reg} (W m^{-2}°C^{-1})	h_{pt} (W m^{-2}°C^{-1})	τ_{reg}(s)	τ_{pt}(s)	T_L (°C)	Cool-down time, t_c(min)	Mass ratio, M_{reg}/M_{pt}	Heat leakage, Q_k (W)
1	5360.6	224.2	0.107	362.9	−15.8	12	0.54	1.83
2	3698.3	134.9	0.155	603.0	−29.3	20	2.16	2.85
3	3229.1	112.7	0.177	722.1	−38.1	26	3.24	3.77
4	2894.0	99.5	0.198	817.6	−39.4	28	4.32	4.22
5	2707.6	86.2	0.211	943.5	−38.1	34	6.48	5.24
6	3635.9	166.4	0.212	489.1	−27.3	16	2.16	2.78
7	3260.5	142.7	0.237	570.1	−30.0	18	3.24	3.32
8	3000.3	126.8	0.257	641.6	−34.6	22	4.86	4.21
9	2616.1	109.9	0.295	740.4	−43.4	24	6.48	5.30
10	2376.3	97.5	0.325	834.9	−39.5	34	7.56	5.36
11	382.2	92.1	0.324	883.2	−36.6	34	9.73	6.45

heat absorption capability of the pulse tube wall decreases with decreasing h_{pt} (or increasing τ_{pt}), the effect of wall heat conduction from the hot to the cold end will not be negligible when the heat absorption capability of the wall via gas convection keeps decreasing (τ_{pt} increasing). Therefore T_L starts to rise again.

The phenomenon of a minimum T_L may also be due to the heat leakage Q_k ($\approx Q_L$). At higher T_L, only a small amount of Q_k will be conducted back to the cold end, since the temperature gradient from the hot to the cold end along the pulse tube wall is smaller. Most of Q_k absorbed by the wall will be convectively transferred to the gas inside the pulse tube and is pumped down to the hot end. Q_k is shown in *Figure 4* to decrease with increasing T_L. Hence, at lower values of T_L, T_L will reach a point at which Q_k is large enough, so that an additional wall heat conduction effect from the hot to the cold end will be present, which causes T_L to begin to rise again.

The heat leakage Q_k ($\approx Q_L$) is shown to increase monotonically with τ_{pt} as shown in *Figure 3b*. This indicates that the heat pumped by the gas inside the pulse tube from the cold to the hot end increases with decreasing h_{pt} (i.e. less energy exchange between gas and wall). The process of heat storage/release at the pulse tube wall is thus shown experimentally to have a negative effect on the performance of a BPT refrigerator. This can also be seen from the energy balance diagram of a BPT refrigerator shown in *Figure 5*. The cyclically-mean heat flows follow the energy balance relations below, neglecting wall conduction in the regenerator

$$Q_L = \langle Q_c \rangle - \langle Q_{pt} \rangle - \langle Q_r \rangle \tag{7}$$

$$Q_L = Q_H - \langle Q_r \rangle \tag{8}$$

where $\langle\ \rangle$ represents the cyclically-mean properties. For regenerators with high efficiency (> 0.99), $\langle Q_r \rangle = 0$. Therefore

$$Q_L = \langle Q_c \rangle - \langle Q_{pt} \rangle = Q_H \tag{9}$$

where $\langle Q_{pt} \rangle$ is the heat conduction along the pulse tube wall, which is in the opposite direction to the heat pumped by the gas $\langle Q_c \rangle$.

If more heat is absorbed by the pulse tube wall or a high heat transfer rate between the gas and wall (i.e. with high h_{pt}) exists during the heat pumping process, $\langle Q_c \rangle$ will be reduced and the wall conduction $\langle Q_{pt} \rangle$ will increase. Therefore, the cooling load Q_L is reduced.

It has been mentioned previously that the gas compression/expansion process inside the pulse tube, which is similar to a Brayton cycle, may be the major thermodynamic process in BPT refrigerators. The gas elements inside the pulse tube are compressed during pressure charging periods. The compression process actually lies between isothermal and adiabatic since there is an energy exchange between the gas and wall and mixing between adjacent gas elements. Therefore, compression energy is converted into heat so that the gas temperature gradient is build up from the cold to the hot end along the pulse tube such that heat can be rejected to the ambient atmosphere at the cold-end exchanger. Heat exchange between the gas and wall and heat conduction along the wall from the hot to the cold end may have a negative effect on BPT performance. This mechanism can explain the present experimental results.

Figures 2–4 show that the transient and steady performances of BPT refrigerators are mainly affected by the pulse tube wall dynamics, regardless of regenerator packing materials. The regenerator acts as a heat storage/release element in BPT refrigerators. The dynamic response of a regenerator should be fast enough so that heat can be completely stored in or released from the regenerator material during gas charging or discharging periods. The test results of τ_{reg} presented in *Table 2* show that $\tau_{reg} \ll \tau(= 1/f = 1.3\dot{3}\,s)$, where τ and f are the cyclic period and frequency of the valve. In addition, the experimental results show that $\tau_{reg} \ll \tau_{pt}$.

For fixed operating conditions (pressure and valve frequency f), the performance of a BPT refrigerator is also affected by the design of the regenerator, pulse tube, cold-end exchanger and hot-end exchanger. In practical designs, $\tau_{pt} > \tau_H > \tau_L > \tau_{reg}$ since the pipe wall at the hot- and cold-end exchangers is relatively smaller than the pulse tube wall. The dynamics of the pulse tube wall thus dominates BPT refrigerator dynamics. This may also explain the transient performance results shown in *Figure 2*.

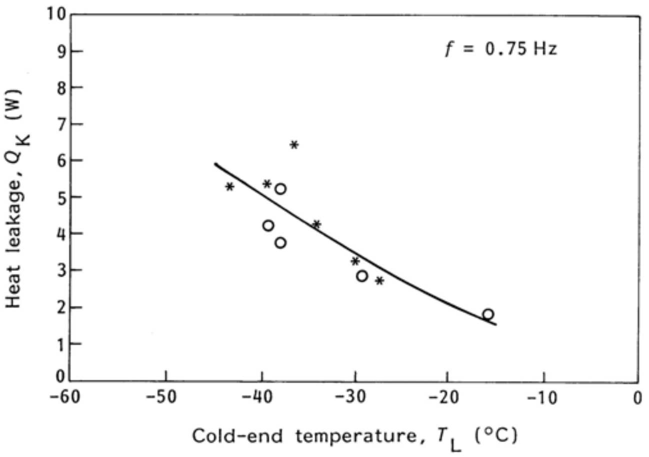

Figure 4 Steady state performance results (Q_k *versus* T_L). Regenerator material: ○, iron beads (1.0 mm diameter) ∗, copper beads (1.47 mm diameter)

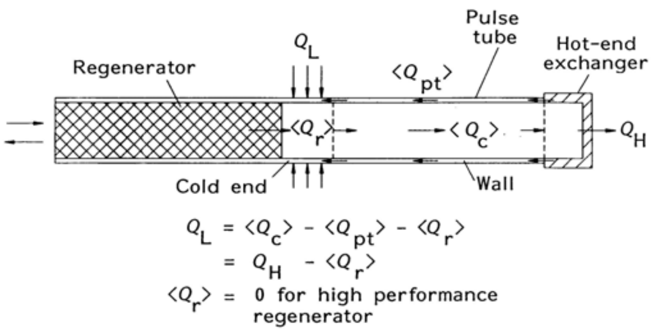

Figure 5 Schematic diagram of heat flows in BPT refrigerator

Discussion and conclusions

An experiment was carried out in the present study to investigate the system performance characteristics of pulse tube refrigerators. Both the transient and steady state performances of pulse tube refrigerators involve transient heat transfer processes. Thus, from the viewpoint of system dynamics, the performance of a pulse tube refrigerator can be characterized by the time constant of the regenerator τ_{reg} and the pulse tube wall τ_{pt}, both of which are defined based on half-cycle mean properties. Experiments were carried out in the present study on pulse tube refrigerators with different regenerators to determine τ_{reg} and τ_{pt}. It was found that the cool-down time t_c during the transient or start-up period is dominated by the time constant of the pulse tube wall τ_{pt}. The dynamics of BPT refrigerators is experimentally shown to approach that of a first-order system.

For steady state operations, the cold-end temperature T_L varies with τ_{pt}. The effect of τ_{reg} is shown to be negligible since $\tau_{reg} \ll \tau_{pt}$ for a well designed pulse tube refrigerator. This indicates that the design of a pulse tube refrigerator requires a regenerator with $\tau_{reg} \ll \tau_{pt}$ and appropriate design of the pulse tube wall such that τ_{pt} and T_L can reach an optimum value. Since τ_{pt} results mainly from convective heat transfer between the gas and wall and the thermal capacitance of the wall, the process of heat storage or release by the pulse tube wall is thus shown to have a negative effect on the performance of BPT refrigerators.

It is found that the gas compression/expansion process inside the pulse tube, which is similar to a Brayton cycle and lies between isothermal and adiabatic, can explain the experimental results. Convective heat exchange between the gas and pulse tube wall and heat conduction along the wall from the hot to the cold end can retard the build-up of a temperature gradient so that heat rejection at the hot-end exchanger is reduced. Hence, the cooling load is reduced as well.

The performance of a BPT refrigerator depends on the design of the regenerator, pulse tube, and cold and hot exchangers. The cold-end temperatures T_L obtained in the present experiments did not reach very low values. This is due to hardware problems. First of all, air is used as the working fluid and the dew point of air in the experiments was not controlled well below $-120°C$ (≈ 150 K). Air purification was another problem, which caused contamination of the regenerators. In addition, poor machining of the pulse tube causes a mixing loss during the gas compression process inside the pulse tube. However, none of these affect the aforementioned conclusions obtained in the present study. The higher cold-end temperature T_L obtained in the present experiment can reduce the effect of heat conduction along the regenerator, which was ignored in the discussions. Thus the present conclusions will not be changed.

Acknowledgement

The present study was supported by the National Science Council of the Republic of China, Taiwan, through Grant No. NSC78-0401-E002-27.

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