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代數纖叢上的雙有理幾何(2/3)

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一、中文摘要

我們證明對於一般形的三維極小模型，若其奇點是高瑞斯坦且正典，則其上的第五正典映射是雙有理。

關鍵詞：線性系、重正典映射、傅利葉-向井變換、非正規流形。

Abstract

Let X be a complex projective minimal Gorenstein 3-fold of general type with canonical singularities. We prove that the 5-canonical map is birational onto its image..

Keywords: linear series, pluricanonical map Fourier-Mukai transform, irregular varieties.

二、緣由與目的

One main goal of algebraic geometry is to classify algebraic varieties. The successful 3-dimensional MMP has been attracting more and more mathematicians to the study of algebraic 3-folds. In this paper, we restrict our interest to projective minimal Gorenstein 3-folds X of general type where there still remain many open problems.

Denote by K_X the canonical divisor and $\Phi_m := \Phi_{mKX}$ the m-canonical map. There have been a lot of works along the line of the canonical classification. For instance, when X is a smooth 3-fold of general type with the pluri-genus $h^0(X, k K_X) \geq 2$, in [Ko], as an application to his research on higher direct images of dualizing sheaves, Kollár proved that Φ_m, with $m=11k+5$, is birational onto its image. This result was improved by Meng Chen [Ch] to include the cases m with $m\geq 5k+6$.

On the other hand, for 3-folds X of general type with $q(X)>0$, Kollár [Ko] first proved that Φ_{225} is birational.

Recently, a joint work with Hacon [CH] proved that Φ_m is birational for $m \geq 7$ by using the Fourier-Mukai transform. Moreover, Luo [Lu] has some results for 3-folds of general type with $h^2(O_X)>0$.

Now for minimal and smooth projective 3-folds, it has been established that Φ_m, $m \geq 6$ is a birational morphism onto its image after 20 year long research by Wilson in the year 1980, Benveniste [Be] in the year 1986 ($m\geq 8$), Matsuki [Ma] in the year 1986 ($m=7$), Meng Chen [Ch] in the year 1998 ($m= 6$) and independently by Lee [Le], in the years 1999 ($m=6$; and also the base point freeness of m-canonical system for $m\geq 4$). A very natural and well-known question arises:
Question: Let X be a minimal Gorenstein 3-fold of general type. Is Φ_5 birational onto its images?

One reason to account for this is that the non-birationality of the 4-canonical system for surfaces may happen when they have smaller p_g or K^2 (see Bombieri [Bo]), whence a naïve induction on the dimension would predict the non-birationality of the 5-canonical system on certain 3-folds with smaller invariants.

Nevertheless, there are also evidences supporting the birationality of Φ_5 for Gorenstein minimal 3-folds X of general type. For instance, one sees that $K^3 \geq 2$ for minimal and smooth X. So an analogy of Fujita's conjecture would predict that $|5K_X|$ gives a birational map. We recall that Fujita's conjecture (the freeness part) has been proved by Fujita, Ein-Lazarsfeld and Kawamta when dim $X\leq 4$.

三、結論與討論

Our main result is the following:
Theorem. Let X be a projective minimal Gorenstein 3-fold of general type with canonical singularities. Then the m-canonical map Φ_m is a birational morphism onto its image for all $m\geq 5$.
Example. The numerical bound "5" in Theorem is optimal.

There are plenty of supporting examples. For instance, let $f:V\rightarrow B$ be any fibration where V is a smooth projective 3-fold of general type and B a smooth curve.

Assume that a general fiber of f has the minimal model S with $K_S^2=1$ and $p_g(S)=2$. (For example, take the product.) Then Φ_4 is apparently not birational (see [Bo]).

The main technique involve is a partial resolution, together with the Kawamata Viehweg vanishing theorem. We remark that

by using the similar trick in [CH], one can prove a slightly weaker result with simpler argument for irregular threefolds.

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