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地表通量之監測與推估（1/2）

Estimation and Monitoring of surface fluxes

計畫編號：NSC 96-2745-M-002-002

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Abstract

Soil heat flux may play an important role in surface energy balance. In this study, we examined the performances of two methods for predicting soil heat flux from single layer time series data of soil temperature. The first one is the traditional method, which an analytical solution of soil heat flux can be obtained by assuming the surface soil temperature varies sinusoidally. The second one is the connection between surface soil temperature and soil heat flux derived by half order derivative/integral, and is based on a simple model of heat transfer described by a one-dimensional diffusion equation with a constant heat diffusivity. Good agreements between measured and predicted soil heat fluxes were found for both methods. However, it was shown that the half order derivative method has a better capability to capture flux accuracy and trend than the traditional method for long-term soil heat flux estimation.

1. Introduction

In a well-watered and full vegetation covering surface, in fact, soil heat flux may be the same order as sensible heat flux. When the vegetation is becoming senescence, soil heat flux may be the same order as latent heat flux. Moreover, there is a considerable portion of solar radiation which contributes to the soil surface heat flux during the early growth period (*Kustas and Daughtry, 1990*). The ratio of the soil heat flux to the net radiation is found to be 0.5 and 0.3 for dry soils and wet soil respectively (*Idso et al., 1975*) . Based on many research about the Land-atmosphere interaction, the surface soil heat flux, therefore, becomes a vital impact factor in the energy balance system.

The influence of soil heat flux on chemical reactions and microclimate are self-evident. Thus, both for the plant growth models and meteorological models, the soil heat flux plays an important rule in the energy balance between air and soil (*van Loon, Bastings and Moors, 1998*). The determinations of the human-induced greenhouse gas (radiative) forcing and energy budgets in climate model demonstrate that the energy balance and heat flux are also significant factors (*Delworth and Knutson, 2000*). And the acquirement of soil heat flux information should be more applicable so that we could modify the climate model development with more detailed soil heat flux data (*Beltrami et al., 2006*).

As the soil ground heat flux is one of the important parts of the energy balance at the interface between land and atmosphere, the method possess the most appropriate approximations with fewer limitations is the first thing to be figured out. Many methods have been built to do the indirect measurements of the ground heat flux, and most of the indirect measurements need that the soil temperature profile be measured at the same time. It is not convenient and it may cost a lot while doing large area measurement and conducting high frequency sampling.

As a matter of fact, solutions to the one-dimensional diffusion equation with a constant diffusivity can exhibit the behavior of soil temperature in space and time. The analytical solutions derived under the condition, however, which has many restrictions, and do not represent real soil environment very well. There are still many numerical methods for solving the equation and simulating the real condition, but numerical methods are difficult to understand the behavior of the system (*Campbell and Norman, 2000*). This paper compares two different methods of forecasting soil heat flux. One is to evaluate the analytical solution of soil heat flux when assume that the surface soil temperature varies sinusoidally (*Carslaw and Jaeger, 1986; Campbell and Norman, 2000*). Another is to simulate the diffusion equation with half order derivative/ integral method (*Wang and Bras,1998; Wang and Bras,1999; Wesson et al. 2001*). Both methods get the simplicity of indirect measurements to estimate soil heat flux with only single-level temperature. This study provides real environmental scenario to quantitatively make careful assessment of the capability of these two methods. It is the first to collect long-term, continuous, supporting observation data, which are necessary for the validation and assessment.

2. Theory

2.1. Traditional method

According to the thought of heat conduction in solid (*Carslaw and Jaegr,* 1986), the soil properties are assumed to be uniform over entire soil profile, and the soil temperature varies sinusoidally. Given these assumptions the temperature can be formed as a function of time t (sec) and depth z (m):

$$T(z,t) = T_{ave} + A(0)\exp(-z/D)\sin\left[\omega(t-t_0) - z/D\right] \quad (1)$$

where $A(0)$ (^{o}C) is the amplitude of the temperature fluctuations; T_{ave} (^{o}C) is the average temperature over a temperature circle(one day); t_0 (sec) is the phase shift. And ω (1/sec) and D (m) represent for the angular frequency and damping depth, respectively, which are calculated as:

$$\omega = \frac{2\pi}{\tau} \quad (2)$$

$$D = \sqrt{\frac{2D_0}{\omega}} \quad (3)$$

where τ is the time over a temperature circle; $D_0 = \frac{k}{\rho_s c_s}$ (m^2 s^{-1}) is the thermal diffusivity and k (W m^{-1} K^{-1}) is the thermal conductivity, ρ_s (Mg m^{-3}) is the soil density, c_s (J g^{-1} K^{-1}) is the soil specific heat, and $\rho_s c_s$ (MJ m^{-3} K^{-1}) is the volumetric heat capacity. In equation (1), damping depth shows two important concepts about the hypothetical phenomenon of soil temperature. First, the amplitude of temperature variation attenuating with depth depends on the damping depth. Also, the damping depth affects the temperature circle shifting with depth. In a real soil environment, the physical behaviors of soil temperature have similar characteristics mentioned above.

In order to obtain the soil heat flux, Fourier's law for heat transport is used to compute the heat flow passing through a soil layer to the next one. The Fourier's law for heat transport is described as:

$$G = -k\frac{dT}{dz} \tag{4}$$

where G is the soil heat flux (W m^{-2}); T (^{o}C)is the soil temperature and z (m) is the depth below the ground surface. Apply equation (1) to the Fourier's law and set $z = 0$, the surface soil heat flux can be computed from:

$$G(0,t) = \frac{\sqrt{2}A(0)k\left[\sin\left[\omega(t-t_0)\right]+\pi/4\right]}{D} \tag{5}$$

Equation (5) indicates that soil heat flux also varies sinusoidally with time and its mean equals to zero. When apply the traditional method, only single layer time series data of soil temperature and soil properties are needed for estimating soil heat flux.

2.2. Half order derivative/integral method

In addition to the Fourier's law, applying the continuity equation to construct the diffusion equation is required, which can depict physical phenomenon of heat transfer completely. The continuity equation is shown as:

$$\rho_s c_s \frac{\partial T}{\partial t} = -\frac{\partial G}{\partial z} \tag{6}$$

The right hand side and left hand side present their physical meanings, the rate of heat storage in a soil layer and rate of change of soil heat flux, respectively.

Combining the Fourier's law and continuity equation and assuming the thermal diffusivity D_0 is constant derive:

$$\frac{\partial T}{\partial t} = D_0\frac{\partial^2 T}{\partial z^2} \tag{7}$$

Practically, one-dimensional heat transfer over vertically homogeneous soil can be described by Equation (7) with a constant diffusivity. For solving the differential equation (7), defining the reasonable boundary conditions is necessarily. Thereafter, the variation of soil temperature far below the ground surface is smaller than the variation of soil temperature near the ground surface and the initial soil temperature profile is uniform are defined as two boundary conditions for the differential equation (7). Therefore, the boundary condition for the soil temperature can be presented as:

$$T = T_0, \qquad \text{for } t = 0, z < 0 \tag{8}$$

$$T = T_0, \qquad \text{for } t > 0, z \to -\infty \tag{9}$$

In Equation (8), the initial soil temperature profile uniformly distribute through every layers, the assumption implies that the iterative computation of soil heat flux is started when the soil heat flux is zero. Equation (9) is the other boundary condition for solving the differential equation (7) and it means that the variation of soil temperature is nearly to zero when the depth exceeds the damping depth.

Moreover, the process for acquiring analytical solution of equation (7) is tedious and has many limitations. Use of the half order time derivative method (*Wang and Bras*, 1998) facilitates the derivation. Main advantage of the method is to establish a relationship linking the soil heat flux to the soil temperature with minimum limitations. When the heat transfer mechanism is delineated by equation (7)-(9), the half order time derivative method derives that the vertical gradient of temperature can be a function of weighted average of soil temperature time series according to the concept of fractional calculus. Then, the approximation model used to estimate the soil heat flux G by the soil temperature T at any depth can be built (Appendix A). The result is as follows:

$$\begin{aligned} G(z,t) &= D_0 \frac{\partial}{\partial z} T(z,t) \\ &= \sqrt{\frac{D_0}{\pi}} \int_0^t \frac{dT(z,s)}{\sqrt{t-s}} \end{aligned} \tag{10}$$

where s is the integration variable. As the equation (10) represents, only measuring single layer time series data of soil temperature at certain depth and the soil property information are required to estimate soil heat flux at the same depth.

2.3. Application

Assessing the applicability of these two methods with continuous, long-term measured information, therefore, are trustworthy. In the process of verifying the methods, no matter which method has been chosen to use, the soil temperature T always is the dominant required measured variable. And both of these two methods also need the soil properties to be the input parameters, such as the thermal conductivity k, the soil density ρ_s and soil specific heat c_s.

According to the basic assumption of the traditional methods, the soil temperature varies sinusoildally, which can be approximately determined by the amplitude of the temperature fluctuations A(0) when daily maximum and minimum soil temperature are picked from real soil temperature time series. Moreover, in order to get the more appropriate sinusoidal fitting for the temperature time series, the phase shift must be taken into account.

The completeness of the soil temperature time series information has considerable influence on the modeling results, especially when the half order derivative/intergral method is applied. The shortage of temperature time series information leads to disrupt integral computation. In other words, the more complete temperature time series information can be collected, the more accurate estimated soil heat flux can be evaluated. Furthermore, when the half order derivative/integral method is operating, the starting time point of the integration should coincide with the time when soil heat flux equals to zero. In fact, it is difficult to determine the initial time point of integration, because the exact time point when the soil heat flux equals zero remains unknown without measured soil heat flux data. A proper way to deal with this problem is to set the starting time of integration as the beginning of night when the soil heat flux approximately approach to zero (not exact equal to zero). Therefore, the approximation still has its own satisfactory accuracy, which can be elucidated in this study.

3. Experiments

The measurements of the soil heat flux and the soil temperature were used to assess the performance of these two methods —Traditional method and Half order derivative/integral method. Also, the measurements were conducted for this study through whole year 2002 and the experiment site was located in fertilized grassland in Co. Cork in southern Ireland (Latitude: 52.14°N, Longitude: 8.66°W). In the experiment site, the dominant plant species was perennial ryegrass, and the grassland type was moderately high quality pasture and meadow. Besides, the grass height in the experiment site varied from 0.1 to 0.2 meter. Additionally, the soil properties are essential parameters, and some soil property information is summarized in Table 1 and Table 2.

Different from other complicated experiments for determining soil heat flux, this study only set up single level equipment to do the indirect measurement. The soil heat flux plate (HUKSELFFLUX) was buried parallel to the top of the soil surface at the depth of 5 centimeters and the temperature probe was buried at the same depth near the soil heat flux plat. Also, the net radiometer was erected to collect the net radiation data for confirming the processes of energy transport. The soil heat flux, soil temperature and net radiation data were collected every 30 minutes by a datalogger during whole study year 2002.

4. Results and discussion

When estimate the soil heat flux by the traditional method and half order derivative/intergral method, the two parameters — volumetric heat capacity $\rho_s c_s$ and thermal conductivity k, are required which depend on the thermal properties. The volumetric heat capacity is therefore computed as the sum of the heat capacities of the three parts of soil components — minerals, water and organic matter (*Zhang et al., 2007*):

$$\rho_s c_s = \phi_m \rho_m c_m + \theta \rho_w c_w + \phi_o \rho_o c_o \tag{11}$$

where ϕ_m (m^3/m^3) is the volume fraction of minerals, ϕ_o (m^3/m^3) is the volume fraction of organic matter; and θ (m^3/m^3) is the volumetric soil moisture. During the year 2002, the measured volumetric soil moisture θ ranged from 0.22 to 0.61, the volume fraction of organic matters ϕ_o was small to be negligible and the volume fraction of minerals was 0.4. When the heat capacities of minerals and water were 2.31 (MJ m^{-3}K^{-1}) and 4.81 (MJ m^{-3}K^{-1}) respectively, the volumetric heat capacity $\rho_s c_s$ can be derived to be 1.84 to 3.47 (MJ m^{-3} K^{-1}).

The other required input parameter — thermal conductivity k, is reasonably estimated to be 1.1 (W m^{-1} K^{-1}) by using the first 50％ soil temperature and soil heat flux data and applying the half order derivative/integral method to minimize the root mean squire error (RMSE) between observed soil heat flux G_o and estimated soil flux G_e. The mathematical presentation can be written as:

$$Minimize: \left\{ \frac{1}{N} \sum \left(G_e - G_o \right)^2 \right\}^{1/2} \tag{12}$$

where N is the quantities of data. Remember that G_e contains a variable k and the goal is to search for the optimum k that can minimize the RMSE.

4.1. traditional method

There was no appropriate equipment for measuring surface temperature precisely and successively until now. Therefore, only the soil temperature at the depth 5 centimeters can be measured in this study. When complete soil properties information was collected, the traditional method and half order derivative/intergral method were pushed to estimate the soil heat flux just at a certain depth due to the lack of integral soil surface temperature. But heat flux at the energy exchange interface — soil surface, is more important in the energy balance system. Although the surface temperature may be estimated by the linear extrapolation of the soil temperature at deeper layers and can also be used to estimate the surface soil heat flux, the estimated surface temperature still has a bias against the real surface soil temperature. For the soil temperature is not a linear function of depth and has a phase shift with depth, solving this problem through the relationship between the surface soil heat flux Gs and the soil heat flux at deeper layer Gz is a better alternative. The difference between the surface soil heat flux Gs and the soil heat flux Gz (estimated heat flux) at a certain depth z is caused by the heat storage term S within the layer. It can be derived from equation (6) and expressed as

$$G_s = G_z + S \tag{13}$$

Where

$$S = \int_0^z \rho_s c_s \frac{\partial T}{\partial t} dz \tag{14}$$

Fig.1 shows the observed soil temperature time series at depth z equals to 5 centimeters through all the study year 2002. Follow the equation (5), (13) and (14), Fig.2 shows estimation versus observation of soil surface heat flux by the traditional method and the solid line is 1:1 and R^2=0.762 in Fig.2. The result is a good illustration to explain that the traditional method has acceptable ability to catch the flux accuracy and trend, although the soil temperature is fitted by a sinusoidal function instead of real soil temperature. For clarity, choose successive thirty days (day 70 to day 100) to plot the estimated and observed time series of soil surface heat flux in Fig.3. It reveals that the magnitude of soil heat flux transport upward is overestimated during nighttime (The value of soil heat flux is negative). This kind of error was caused, because the variation of soil temperature cannot be fitted well due to the assumption of the traditional method—soil temperature varies sinusoidally with time and depth. But the use of the traditional method has an advantage, it does not need successive and integral soil temperature data. Contrarily, only daily maximum and minimum soil temperature data are required. It is to mean that the lost of temperature data influences the results little by the traditional method and the results of estimated surface soil heat flux are good enough.

4.2. half order derivative/integral method

Apply the observed soil temperature data showed in Fig.1 to the equation (10) and follow the equation (13) and (14), Fig.4 shows the relationship between observed and estimated surface soil heat flux by the half order derivative/integral method. In Fig.4, The solid line is 1:1 and R^2=0.984. The result shows that use of the half order derivative/integral method has a better capability to predict soil surface heat flux than the traditional method. In fact, the iterative computation of soil heat flux cannot be exactly started at the time point that the real soil heat flux equals to zero. It means that the undetermined starting iterative time point may leads to error. In Fig.4, the integration is started only once according to the completeness of the soil temperature time series information. The soil heat flux nearly approach to zero at the beginning of night, because the absence of solar radiation in the nighttime and the intensive solar radiation in the daytime make the quantities of soil heat flux passes upward is much smaller than the quantities of soil heat flux passes downward. This is the reason why the starting iterative time has been chosen at the beginning of night. Furthermore, the lost of temperature data will increase the number of restarting iterative time point and reduce the accuracy by the half order derivative/integral method. It is to mean that accumulated errors depend on the number of restarting iterative time point. For clarity, the estimated and observed heat flux data within the same period (day 70 to day 100) as Fig.2 are plotted in Fig.5. It reveals that even though the half order derivative/integral method has uncertainties for predicting soil heat flux, it still keeps good precision.

There are some more information that cannot be detected in Fig.4 and Fig.5. Thus, the observed and estimated cumulative soil heat fluxes through the study year were used to examine the performance of half order derivative/integral method in detail and plotted in Fig.6. It shows that the cumulative values of estimated soil heat flux increased slowly during the first 75 days of the year, but the values in the real state of affairs decreased rapidly. And the estimated cumulative soil heat flux has quite good accuracy after the first 75 days. At the same time, the net radiation data also shows the same phenomenon as the observed cumulative soil heat flux. It is because the near ground air temperature is lower than soil temperature and it usually happens in colder weather. The results and discussion above show that the values of heat flux transport upward were underestimated by the half order derivative/integral method during the first 75 days of the year. After the first 75 days of the year, the estimation error of soil heat flux may be neutralized with increasing computational time because daily variation of soil heat flux alternately goes up to positive values (heat transports downward) and goes down to negative values (heat transports upward). Furthermore, we can conclude that the undetermined starting iterative time point indeed produces this error and the results are not so good as the heat transports upward more, but the error attenuates with computational time. Also, the daytime ratio of the soil heat flux (measured or estimated) to net radiation ranging from 0.2 to 0.5 agrees with the results proposed by Clothier et al. (1986) and Ogee et al. (2000). It indicates that measured data are reliable and estimated results are satisfactory.

Not only the soil heat flux but also the soil temperature can be estimated through the half order derivative/integral method. The soil temperature can be expressed as the integration of soil heat flux using the half order derivative/integral method adversely (*Wang and Bras, 1999*). With complete soil heat flux information, soil temperature can be computed by the equation below:

$$T_g(t) = T_0 + \frac{1}{\sqrt{\pi k_s C_s}} \int_0^t \frac{G(s)\,ds}{\sqrt{t-s}} \tag{15}$$

Follow the equation (15), Fig.7 shows the estimation versus observation of soil temperature through the study year. In Fig.7, The solid line is 1:1 and R^2=0.94. And Fig.8 shows the observed and estimated soil temperature time series. It is clear that although the trend of the estimated soil temperature is fairly good as the observed soil temperature, the soil temperature is considerably underestimated. It indicates that the estimation results of soil temperature are not as well as soil heat flux. When the first 75 days of the study year are skipped, Fig.9 presents the estimation versus observation of soil temperature. Also, Fig.10 shows the observed and estimated soil temperature time series. In Fig.9, The solid line is 1:1 and R^2=0.9384. Therefore, the results are better than the estimation including the first 75 days data. There are many reason can be observed to explain why the errors and bias exist. Theoretically, the undetermined starting time point for soil heat flux estimation and undetermined initial temperature T_0 may cause the errors. In other aspect, it makes the cumulated error increase continuously and let the bias between observed and estimated soil temperature become larger because the behavior of soil temperature is different from soil heat flux. Long-term estimations of soil heat flux and soil temperature by the half order derivative/integral method shows that both of these two estimations reveal different orders in errors, when the air temperature is higher than the soil temperature and the soil heat flux transport upward.

5. Conclusion

We examined the performances of the traditional method and the half order derivative/integral method for predicting soil heat flux. Also, we estimated the diurnal average and annual average soil thermal conductivity by the half order derivative/integral method. Based on our measurements and predictions, we have demonstrated the following:

（1） Different from other indirect measurements, both of these two methods only need single layer time series data of soil temperature. And it can reduce the cost of experiment and it is more convenient than other indirect measurements.

（2） Because the variation of soil temperature cannot be fitted well by the traditional method, the magnitude heat flux transport upward is overestimated and the magnitude heat flux transport downward is underestimated.

（3） In order to obtain a good estimation through the half order derivative/integral method, the successive and complete data information is required. In other words, the broken data information is the vital fact to reduce the accuracy. And the estimation error of soil heat flux can be eliminated with computational time.

（4） The half order derivative/integral method can be contrarily used to estimate soil temperature from the soil heat flux. Different from the soil heat flux, the estimation error of soil temperature cumulatively increases with time.

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Appendix A：The process for evaluating the soil heat flux by half order derivative/intergral method

The algorithm for the half order derivative/intergral method has been proposed by (*Wang and Bras*, 1998). First of all, define new variables $\tilde{z} = z / \sqrt{D_0}$ and $\Theta = T - T_0$, and make equation(7)(9)(10) transfer to:

$$\frac{\partial \Theta}{\partial t} = \frac{\partial^2 \Theta}{\partial \tilde{z}^2} \tag{A1}$$

$$\Theta = 0 \text{, for } t = 0 \text{, } \tilde{z} < 0 \tag{A2}$$

$$\Theta = 0 \text{, for } t > 0 \text{, } \tilde{z} \to -\infty \tag{A3}$$

We begin by taking the Laplace transform of both sides of the differential equation (A1) according to the definition of Laplace transform $L\left\{\frac{\partial}{\partial t} f(t)\right\}(s) = sF(s) - f(0)$, and the left hand side becomes $s\tilde{\Theta}(\tilde{z}, s) - \Theta(\tilde{z}, 0) = s\tilde{\Theta}(\tilde{z}, s)$ because of the initial condition (A2). Also, Laplace transform of the right hand side is still not changed comparing to the original form because the Laplace transform is based on the variable t.

$$s\tilde{\Theta}(\tilde{z}, s) = \frac{\partial^2 \tilde{\Theta}}{\partial \tilde{z}^2} \tag{A4}$$

where $\tilde{\Theta}$ is the Laplace transform of Θ and its definition is:

$$\tilde{\Theta}(\tilde{z}, s) = \int_0^\infty \exp(-st)\Theta(\tilde{z}, t)\, dt \tag{A5}$$

Now, setting $\tilde{\Theta} = e^{\lambda \tilde{z}}$ and applying it to (A4) turn out $\lambda = \pm\sqrt{s}$. It means that the general solution of equation(A4) is a linear combination:

$$\tilde{\Theta}(\tilde{z}, s) = A(s)\exp\left(\tilde{z}\sqrt{s}\right) + B(s)\exp\left(-\tilde{z}\sqrt{s}\right) \tag{A6}$$

where $A(s)$ and $B(s)$ are arbitrary functions of s and can be determined by the boundary conditions. $B(s)$ equals zero because of the boundary condition (A3) and then equation (A6) becomes:

$$\tilde{\Theta}(\tilde{z},s) = A(s)\exp\left(\tilde{z}\sqrt{s}\right) \tag{A7}$$

Differentiating both sides with respect to $\tilde{z}$ gives:

$$\frac{\partial}{\partial \tilde{z}}\tilde{\Theta}(\tilde{z},s) = \sqrt{s}A(s)\exp\left(\tilde{z}\sqrt{s}\right) \tag{A8}$$

and substituting (A7) into (A8) yields

$$\frac{\partial}{\partial \tilde{z}}\tilde{\Theta}(\tilde{z},s) = \sqrt{s}\tilde{\Theta}(\tilde{z},s) \tag{A9}$$

Depending on the equation derived by the fractional calculus (*Miller and Ross*, 1993):

$$L\left\{D^{v} f(t)\right\} = s^{v}F(s) - \sum_{k=0}^{m-1} s^{m-k-1}D^{k-m+v} f(0) \tag{A10}$$

,the right hand side of (A9) becomes

$$\sqrt{s}\tilde{\Theta}(\tilde{z},s) = L\left\{\frac{\partial^{\frac{1}{2}}}{\partial t^{\frac{1}{2}}}\Theta(\tilde{z},t)\right\} \tag{A11}$$

The last term of (A10) is eliminated by the boundary condition $f(0)=0$. Replace the right hand side of (A9) by (A11). Thus,

$$\frac{\partial}{\partial \tilde{z}}\tilde{\Theta}(\tilde{z},s) = L\left\{\frac{\partial^{\frac{1}{2}}}{\partial t^{\frac{1}{2}}}\Theta(\tilde{z},t)\right\} \tag{A12}$$

Inverting the Laplace transform of (A12) leads to

$$\frac{\partial}{\partial \tilde{z}}\Theta(\tilde{z},t) = \frac{\partial^{\frac{1}{2}}}{\partial t^{\frac{1}{2}}}\Theta(\tilde{z},t) \tag{A13}$$

Now, equation(A13) becomes

$$\frac{\partial}{\partial z}T(z,t)=\frac{1}{\sqrt{D_0}}\frac{\partial^{\frac{1}{2}}}{\partial t^{\frac{1}{2}}}\left[T(z,t)-T_0\right] \quad \text{(A14)}$$

by returning the new variables to the originals. Again, substituting a equation introduced by the fractional calculus (*Miller and Ross*, 1993)

$$\frac{d^\alpha f(t)}{dt^\alpha}=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t \frac{f(s)}{(t-s)^\alpha}ds \quad \text{(A15)}$$

for the right hand side of (A14) derives that

$$\frac{\partial}{\partial z}T(z,t)=\frac{1}{\sqrt{\pi D_0}}\int_0^t \frac{\partial T(z,s)}{\partial s}\frac{ds}{\sqrt{t-s}}$$

$$=\frac{1}{\sqrt{\pi D_0}}\int_0^t \frac{dT(z,s)}{\sqrt{t-s}} \quad \text{(A16)}$$

Finally, the prognostic result can be evaluated by applying the equation(A16) to the Fourier’s law

$$G(z,t)=D_0\frac{\partial}{\partial z}T(z,t)=\sqrt{\frac{D_0}{\pi}}\int_0^t \frac{dT(z,s)}{\sqrt{t-s}} \quad \text{(A17)}$$

In practice, transforming equation(A17) into discrete form is easier to start the integration. Equation (A17) also can be written down as

$$G(z,t)=\sqrt{\frac{D_0}{\pi}}\int_0^t \frac{\partial T(z,s)}{\partial s}\frac{ds}{\sqrt{t-s}} \quad \text{(A18)}$$

And its discrete form is

$$G=2\sqrt{\frac{D_0}{\pi}}\sum_{i=0}^{N}\frac{T_{i+1}-T_i}{t_{i+1}-t_i}\left[\sqrt{t_N-t_{i+1}}-\sqrt{t_N-t_i}\right] \quad \text{(A19)}$$

where N is the number of intervals .

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List of Figures

series by the half order derivative/intergral method through the study year. (17520 points)

Figure 9. Estimation versus observation of soil temperature by the half order derivative/intergral method from day 75 to day 365 (The solid line is 1:1 and R^2=0.9384)(13923 points).

Figure 10. Estimated and observed soil temperature time series by the half order derivative/intergral method from day 75 to day 365. (13923 points)

Table1. Experiment site information in Cork through the study year 2002.

Soil properties (0 – 10 centimeters)	
Vegetation	Humid grassland
Bulk density (g/cm^3)	1.3
Clay fraction	0.06
Organic content at soil surface (kg C/kg)	0.05
Soil moisture (water filled porosity)	0.5
Porosity (% volume)	70

Table2. Annual average soil properties in the experiment site.

Soil thermal conductivity ($W\ m^{-1}\ K^{-1}$)	1.07
Soil thermal diffusivity ($m^2\ s^{-1}$)	2.18×10^{-7}
Damping depth (m)	1.48

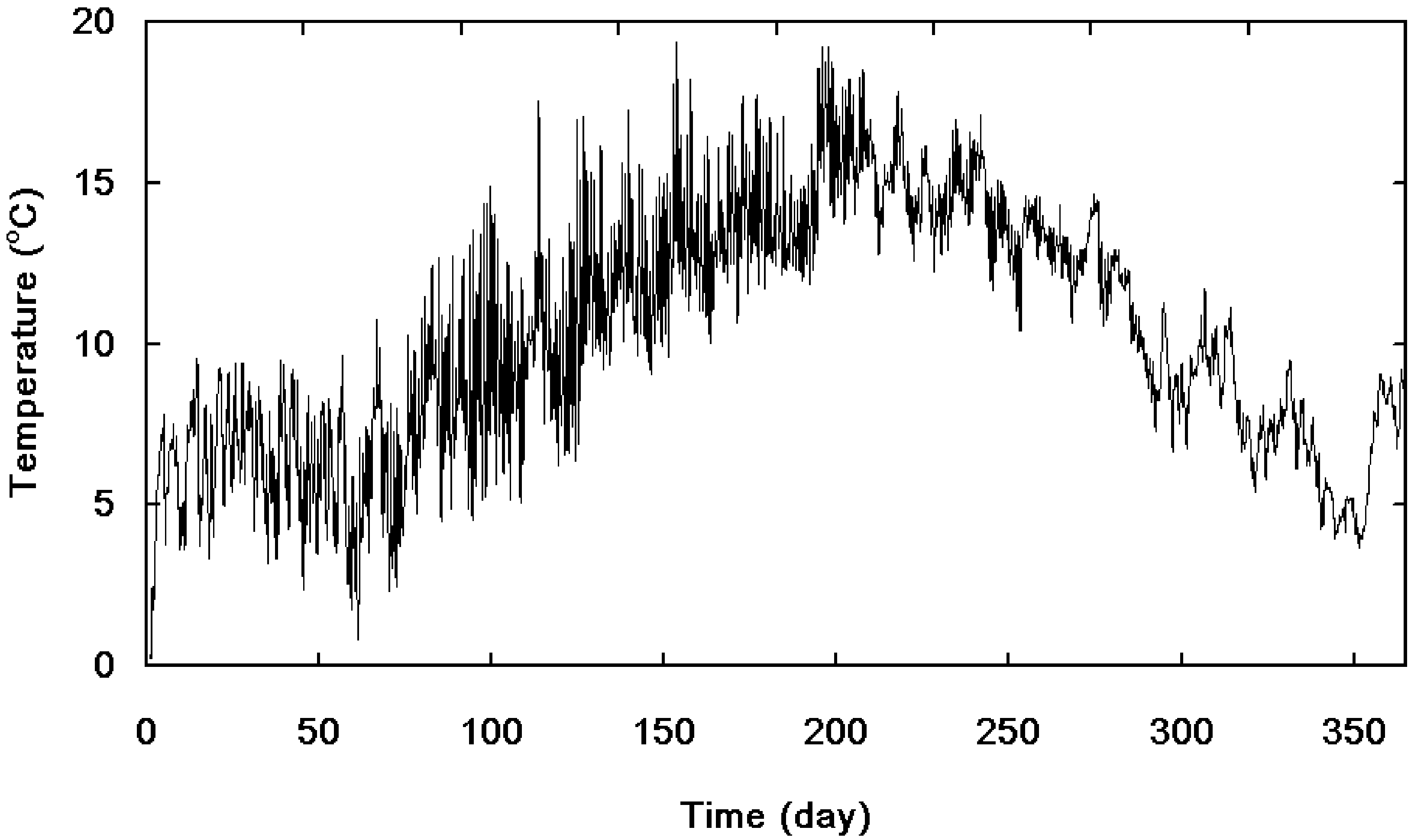


Fig.1

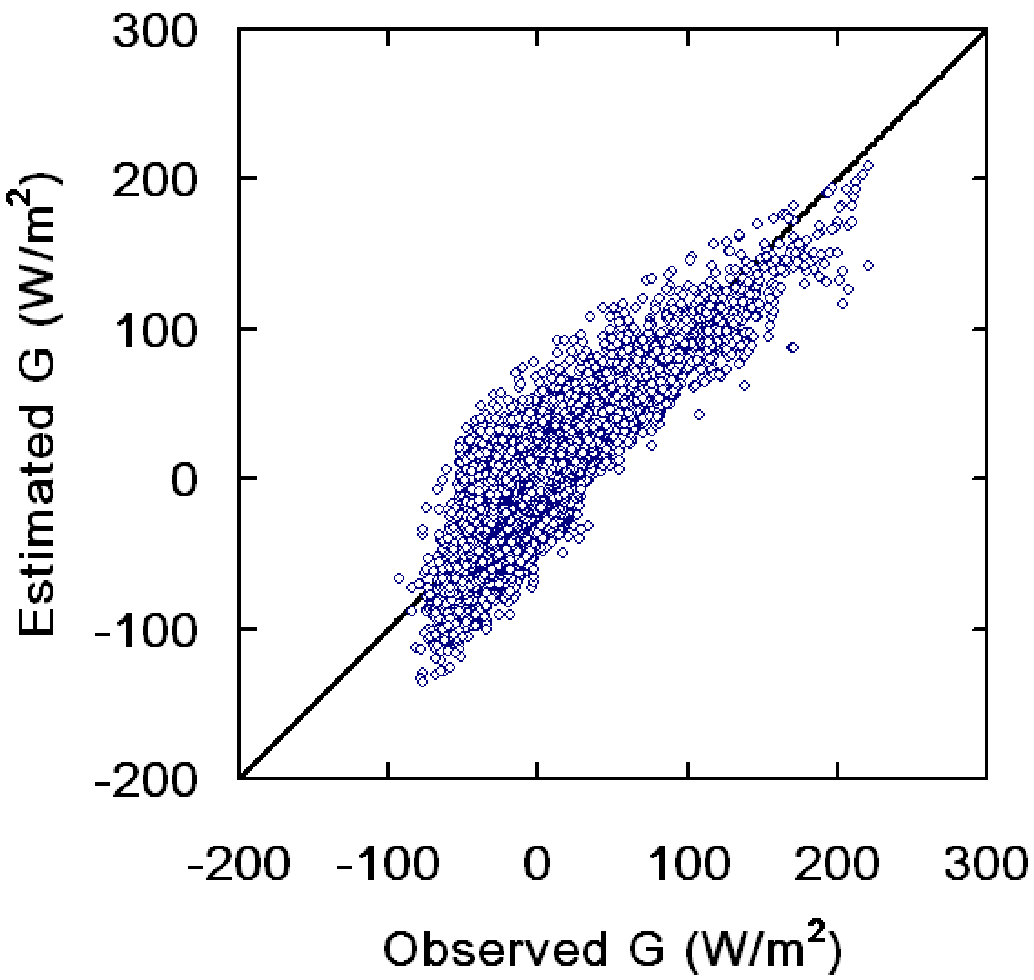

300
200
100
0
-100
-200
Estimated G (W/m2)
-200
-100
0
100
200
300
Observed G (W/m2)

Fig.2

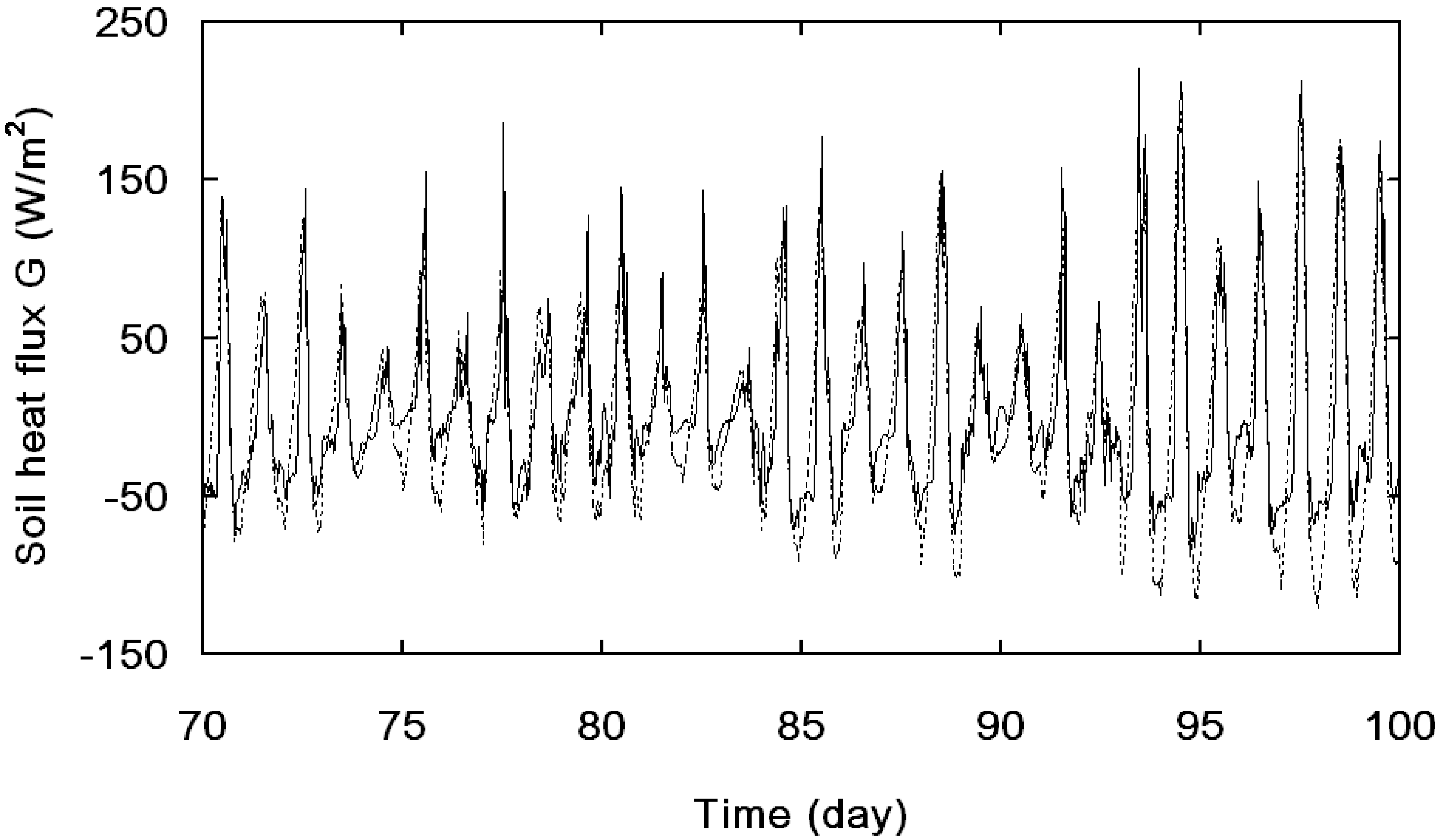

250
150
50
-50
-150
Soil heat flux G (W/m2)
70
75
80
85
90
95
100
Time (day)

Fig.3

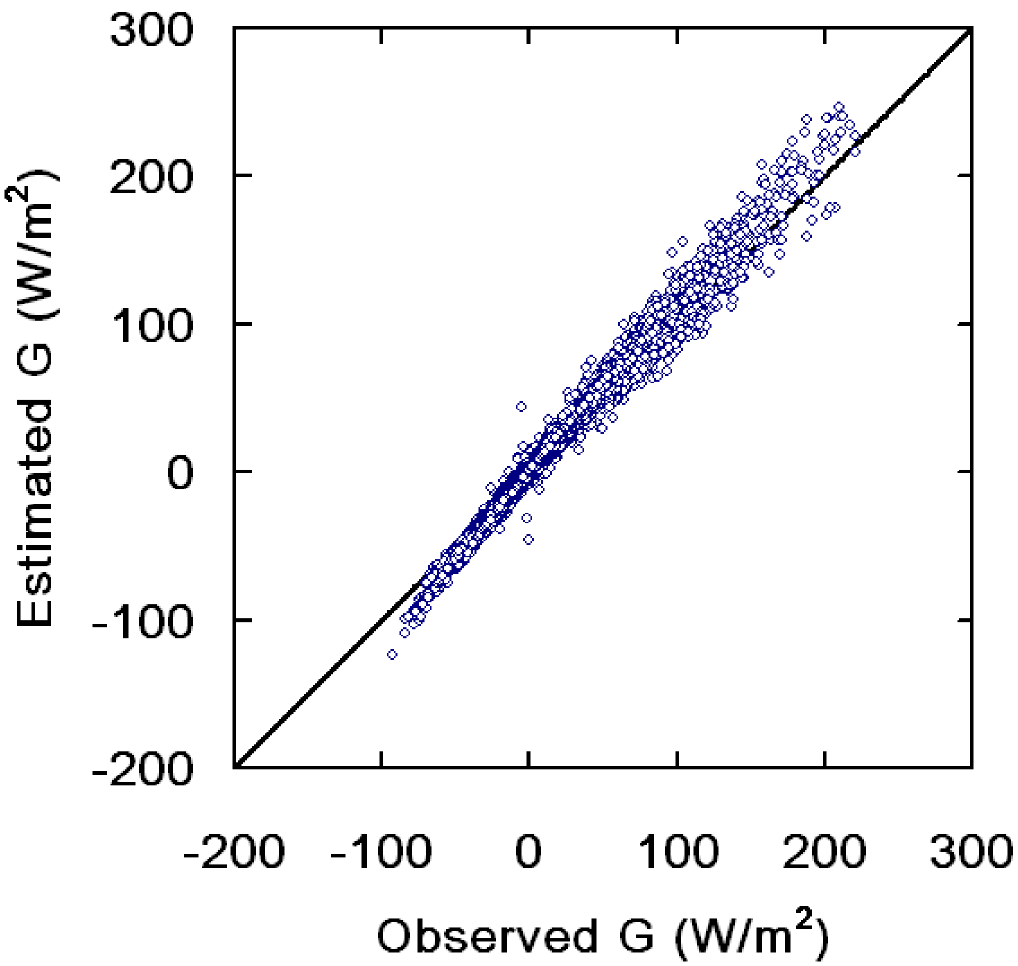

300
200
100
0
-100
-200
Estimated G (W/m2)
-200
-100
0
100
200
300
Observed G (W/m2)

Fig.4

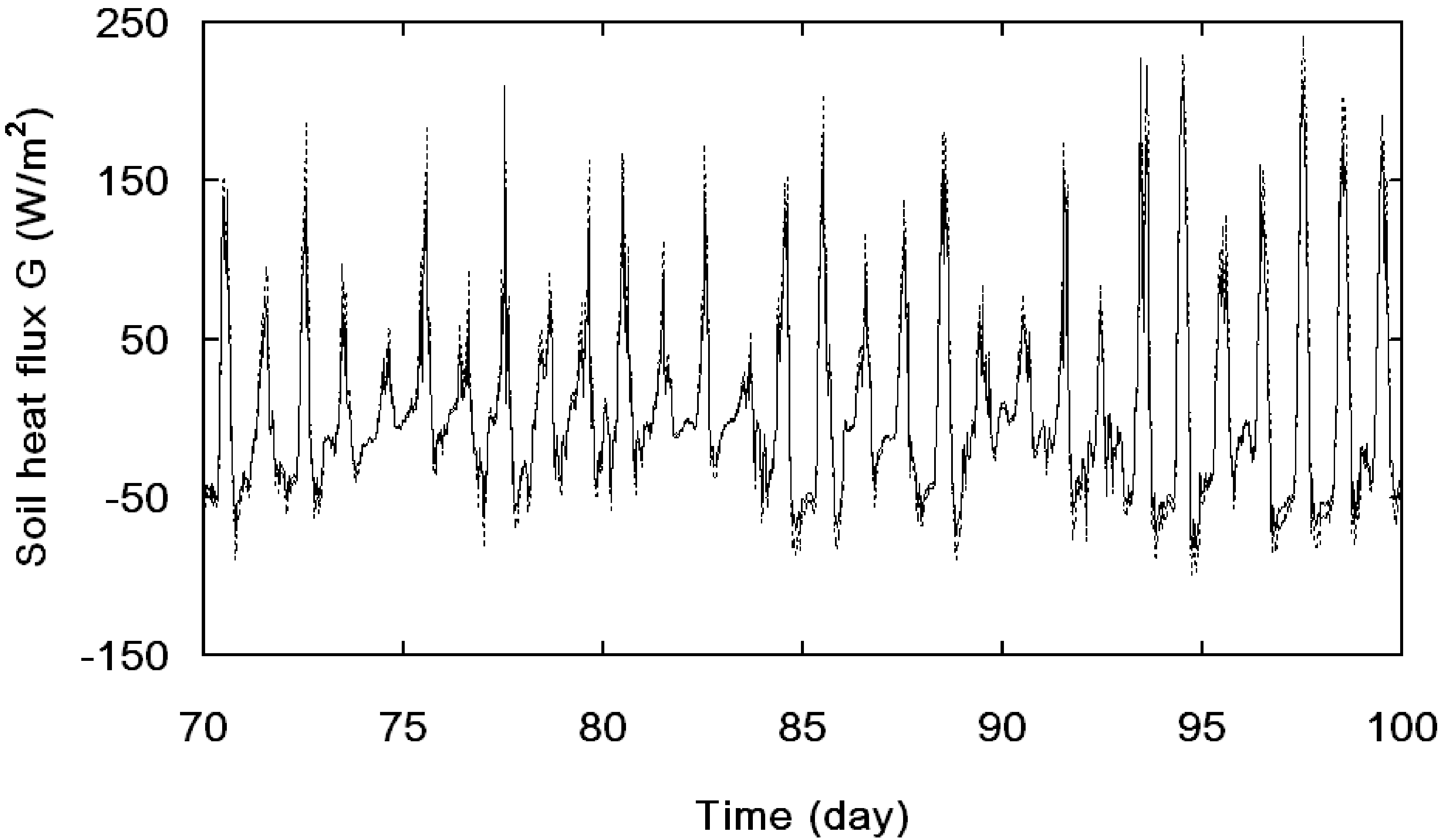

250
150
50
-50
-150
Soil heat flux G (W/m2)
70
75
80
85
90
95
100
Time (day)

Fig.5

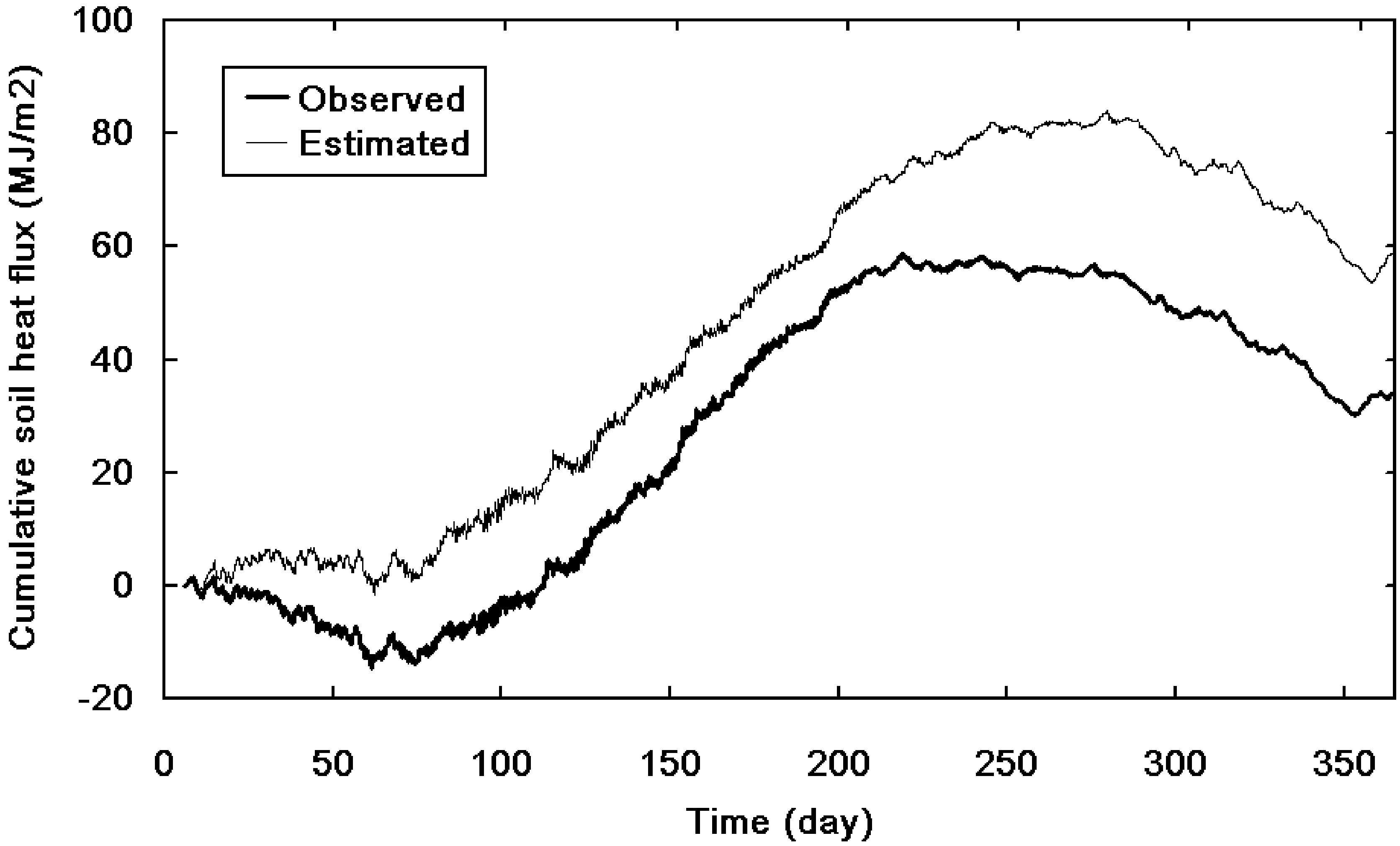

Observed
Estimated
Cumulative soil heat flux (MJ/m2)
100
80
60
40
20
0
-20
0
50
100
150
200
250
300
350
Time (day)

Fig.6

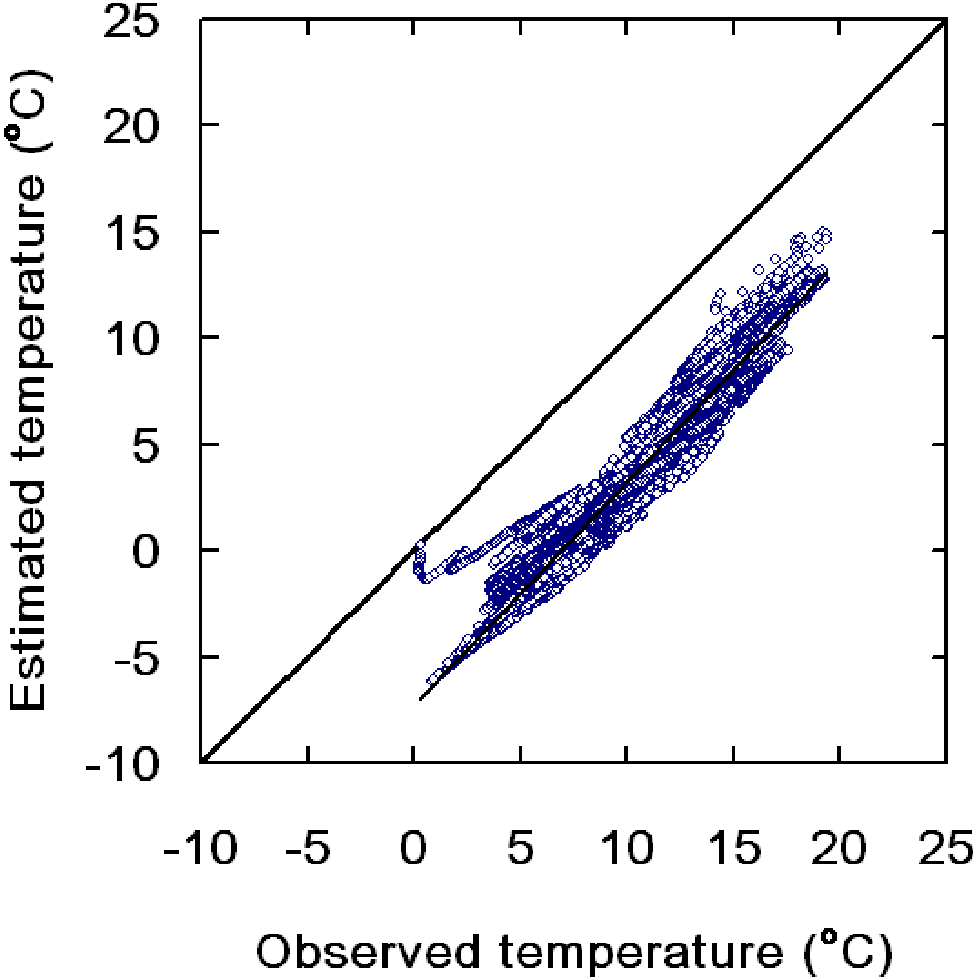

25
20
15
10
5
0
-5
-10
Estimated temperature (°C)
-10
-5
0
5
10
15
20
25
Observed temperature (°C)

Fig.7

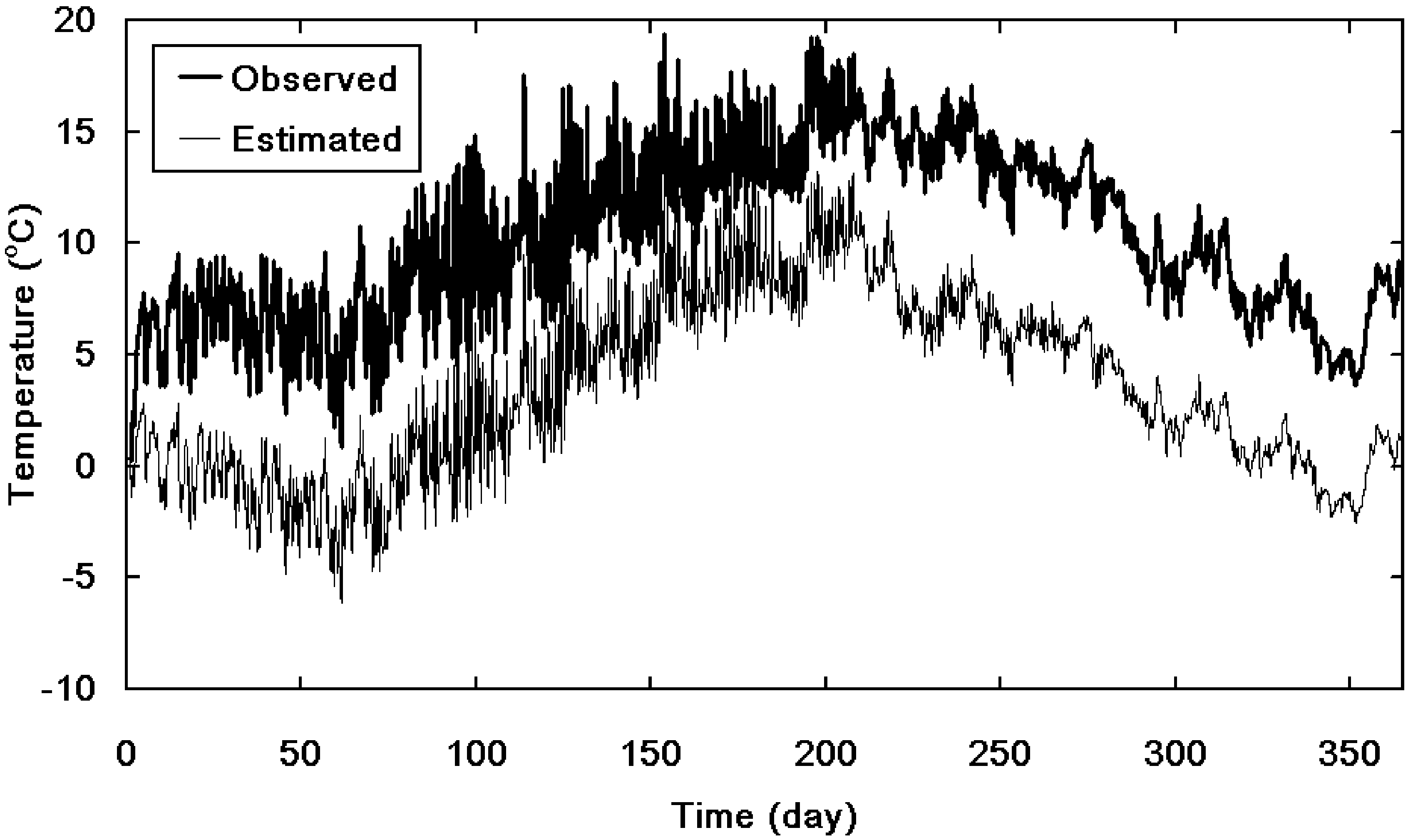

Observed
Estimated
20
15
10
5
0
-5
-10
Temperature (°C)
0
50
100
150
200
250
300
350
Time (day)

Fig.8

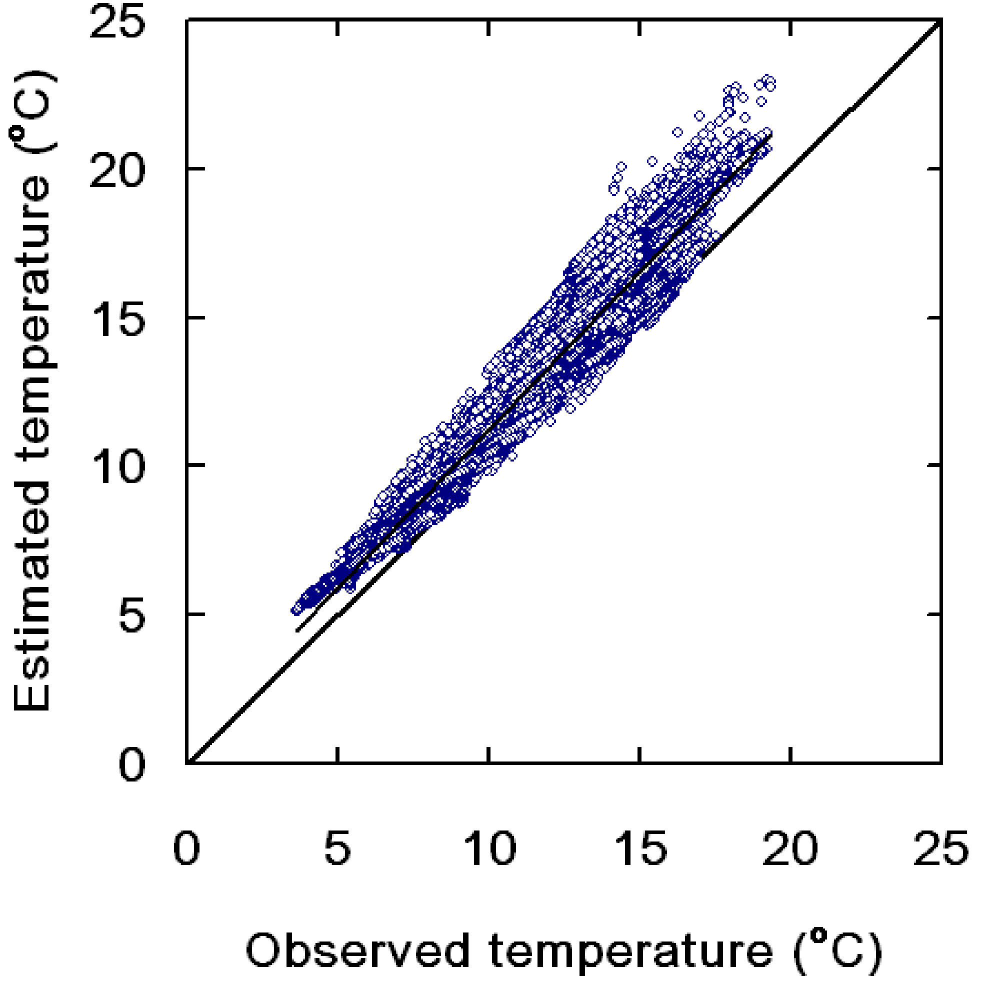
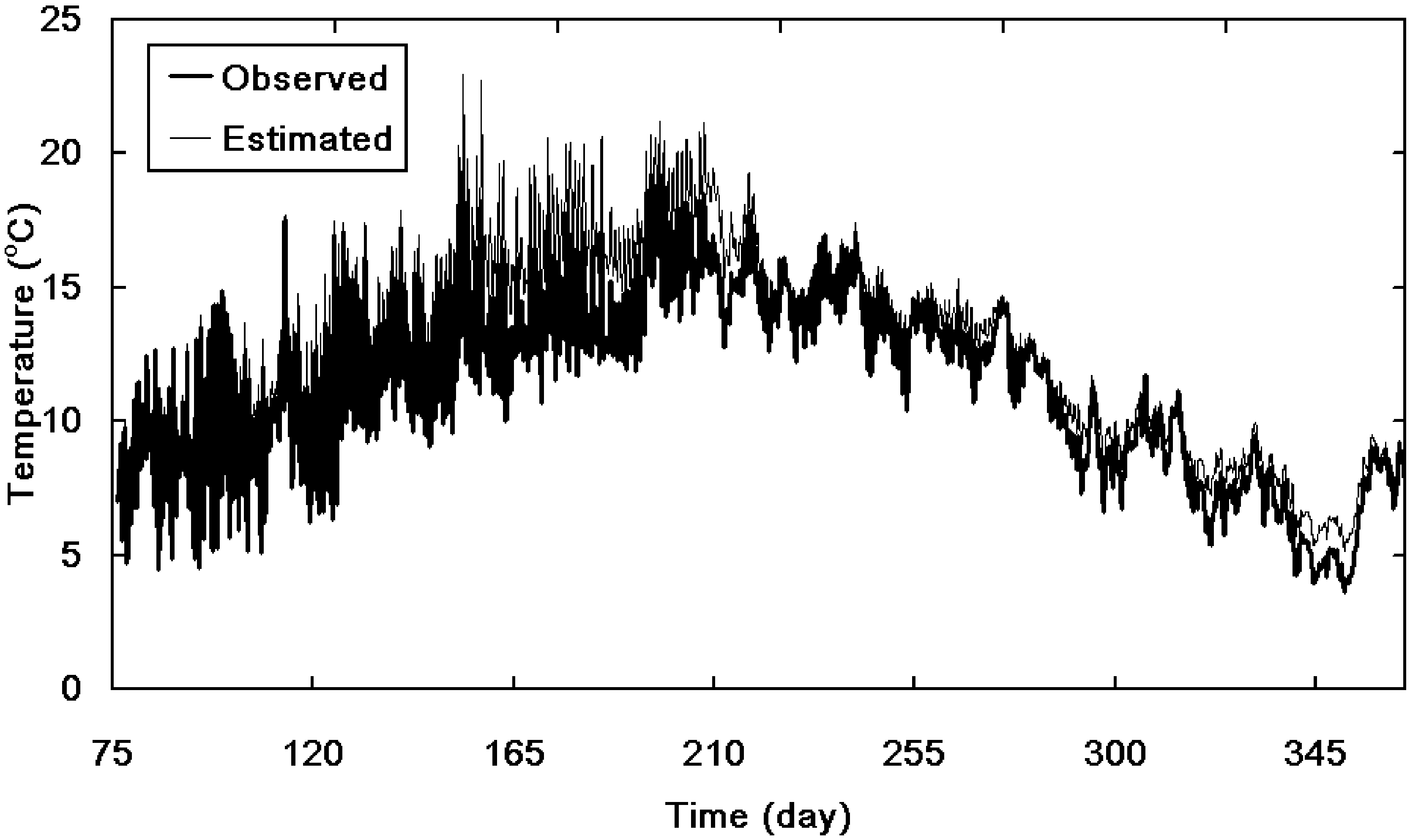

25
20
15
10
5
0
Estimated temperature (°C)
0
5
10
15
20
25
Observed temperature (°C)

Fig.9



Observed
Estimated
Temperature (°C)
Time (day)
25
20
15
10
5
0
75
120
165
210
255
300
345

Fig.10