

行政院國家科學委員會專題研究計畫 成果報告

保險公司殖利率曲線變動風險的免疫策略 研究成果報告(精簡版)

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行政院國家科學委員會專題研究計畫成果報告

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中文摘要

本篇文章首先展示傳統的利率變動免疫策略並無法排除殖利率曲線變動的風險，本文進而提出目標規劃法來控制殖利率曲線變動的風險。在和傳統(Tzeng et al., 2000)的利率變動免疫策略所用的方法比較之後，本文證明目標規劃法在盈餘管理上非常顯著地降低了殖利率曲線變動的風險。

關鍵字：資產負債管理, 免疫策略, 殖利率曲線變動

Abstract

This paper first demonstrates that the immunization strategy proposed by the traditional immunization strategy fails to protect the surplus of an insurance company against yield curve shift risks. Using goal programming, we propose a new method of immunization. The results of our simulation show that, compared to the traditional immunization strategy (Tzeng et al., 2000), the goal programming proposed in this paper can significantly reduce the risks of yield curve shifts against an insurance company's surplus.

Key words: asset and liability management, immunization strategy, yield curve shift.

Introduction

The early literature (Grove, 1974; Bierwag, 1987; and Reitano, 1991 and 1992) recommended that insurance companies can

set the duration of assets equal to the asset/liability ratio times the duration of liabilities to immunize the interest-rate risk against an insurance company's surplus. However, most of the research since has shared a common assumption that the shift of the yield curve is parallel. To recognize the non-parallel shifts of the curve, (Tzeng, et al., 2000) extended the traditional immunization strategy to the case where interest rates follow a stochastic process. However, their strategy immunizes only the risk of a shift of the current interest rate; and this shift has almost no impact on the long-term yield, since they assumed that the underlined stochastic process of the interest rates is unchanged. Thus, (Tzeng et al., 2000)'s strategy can be applied to only a limited set of non-parallel yield curve shifts.

(Reitano, 1996) proposed a mean-variance approach to immunize an insurer's surplus against a stochastic shift of the yield curve. While this idea is intriguing, in practice it may be difficult to find the required information for implementing this strategy. To calculate the mean and variance of the insurer's surplus under a stochastic yield curve shift, (Reitano, 1996) assumed that the insurance company knows the distribution of this shift. However, it be unrealistic to assume that this information is known in real practice.

In this paper, we intend to propose an immunization strategy against unknown yield curve shifts. Unlike (Reitano, 1996), we assume that insurance companies do not know the distribution of the stochastic yield curve shift. The mechanism of our method

relies on the insurance company forming a strategy to immunize parameter changes characterized by the yield curve. (Wang and Tzeng, 2003) proposed a method for an insurance company to immunize the parameter risks of the stochastic process. However, the change of the stochastic process is unobservable. Likewise, the parameter changes in the stochastic process are unobservable. Unlike (Wang and Tzeng, 2003) focus on unobservable parameter changes in the stochastic process, our paper focuses on the shift of the yield curve, which is generally observable by the insurance companies.

Thus, we first demonstrate that (Tzeng et al., 2000)'s strategy cannot immunize the risk of a general yield curve shift. Furthermore, unlike (Tzeng et al., 2000)'s focus on the current interest rate change under a stochastic process, we follow (Reitano, 1996)'s idea of coping directly with the shift of the yield curve. We then propose a goal-programming algorithm to form the strategy against parameter changes of the yield curve. The results of our simulation show that, compared to (Tzeng et al., 2000), the goal programming proposed in this paper can significantly reduce the yield curve shift risks against an insurance company's surplus.

Model

Assume that the yield curve function $r(i)$ is as follows,¹

$$r(i) = c_0 + c_1i + c_2i^2 + c_3i^3 + c_4i^4 + c_5i^5, \quad (1)$$

where i is the maturity period and $i = 1, \dots, n$, $r(i)$ is the yield rate of i maturity period, and c_0, c_1, c_2, c_3, c_4 , and c_5 are the parameters.

¹ The setting of the yield curve function is just for the sake of simplicity. Many models are proposed to sketch the yield curve function, e.g., (McCulloch, 1975), (Vasicek and Fong, 1982), (Nelson and Siegel, 1987), and (Svensson, 1994). The main idea of this paper can be applied to other yield curve functions.

Assume that $CI(i)$ and $CO(i)$ are the cash inflows and outflows of an insurance company at i maturity period. Thus, as evaluated by the net present value, the assets and liabilities of an insurance company are then equal to

$$\begin{aligned} A &= \sum_{i=0}^n CI(i)P(i) \\ L &= \sum_{i=0}^n CO(i)P(i) \end{aligned} \quad (2)$$

where $P(i) = \exp(-r(i)i)$, as a discount factor. Let S denote an insurance company's surplus, and S is then equal to

$$S = A - L \quad (3)$$

We characterize movement of the yield curve shift as parameter changes in Equation (1). To immunize the risks of the yield curve shifting under an insurance company's surplus management, we propose the goal programming method as follows,

$$\begin{aligned} &\min_{CI(i)} d \\ \text{s.t. } &\left| \frac{\partial S}{\partial c_0} \right| \leq dw_{c_0}, \\ &\left| \frac{\partial S}{\partial c_1} \right| \leq dw_{c_1}, \\ &\left| \frac{\partial S}{\partial c_2} \right| \leq dw_{c_2}, \\ &\left| \frac{\partial S}{\partial c_3} \right| \leq dw_{c_3}, \\ &\left| \frac{\partial S}{\partial c_4} \right| \leq dw_{c_4}, \\ &\left| \frac{\partial S}{\partial c_5} \right| \leq dw_{c_5}, \\ &S = A - L, \\ &\sum_{i=0}^j (CI(i) - CO(i)) \frac{P(i)}{P(j)} \geq K, j = 1, \dots, n, \\ &\text{and} \\ &CI(i) \geq 0, i = 1, \dots, n. \end{aligned} \quad (4)$$

where d is the minimum distance of the surplus change between the goal and the final outcome of insurance company—i.e., the risk position the insurance company has to bear²;

² The purpose of insurance company is trying to

w_{c_0} , w_{c_1} , w_{c_2} , w_{c_3} , w_{c_4} , and w_{c_5} are the weights of the parameter risks; and K is the insurance company's minimum cash flow holding at j maturity period.

The first six constraints in Equation (4) control the impact on the surplus of each parameter change. It should be noted that insurance companies can set a smaller weight for any parameter if they are more concerned about a specific parameter change. Following (Tzeng et al., 2000), we assume that an insurance company may need to fulfill the minimum solvency margins and may have borrowing constraints. The last two constraints in Equation (4) represent the solvency and borrowing constraints for insurance companies.

Conclusion

In this paper, we proposed to use goal programming to form an immunization strategy against risks of yield curve shifts. The method in our paper could be easier to implement in real practice than that in (Reitano, 1996) because it copes with observable changes of the yield curve and requires no information about the distribution of stochastic yield curve shifts. Moreover, the results of our simulation show that, compared to (Tzeng et al., 2000), the goal programming proposed in this paper can significantly reduce risks of yield curve shifts against an insurance company's surplus.

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immunize the yield curve shift risks---i.e., minimize the maximum surplus change of the insurance company while the parameters changed.

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研究生助理 鐘正阜

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☒出席國際學術會議心得報告及發表之論文各一份

☐國際合作研究計畫國外研究報告書一份

中 華 民 國 96 年 10 月 22 日

行政院國家科學委員會補助國內專家學者出席國際學術會議報告

96 年 5 月 8 日

附件三

報告人姓名	曾郁仁	服務機構及職稱	台灣大學財務金融系 教授
時間 會議 地點	2007/04/20~2007/04/22 美國賓州州立大學	本會核定 補助文號	NSC 95-2416-H-002-043
會議 名稱	(中文) 2007 風險理論研討會 (英文) 2007 Risk Theory Seminar		
發表 論文 題目	(中文) 保險市場的隱藏後悔及優勢選擇 (英文) Hidden Regret and Advantageous Selection in Insurance Markets		
報告內容應包括下列各項： 一、參加會議經過 Risk Theory Seminar 是學術界關於風險與保險研究精英色彩相當濃厚的會議。Risk Theory Seminar 一年舉辦一次，今年四月二十日到四月二十二日在美國賓州州立大學舉行。此次會議共有有 10 篇論文發表，包括許多國際知名學者的論文。 二、與會心得 個人於會議中發表一篇論文，個人的論文從從與會學者處得到很實質的幫助。最重要的是個人透過此次會議與多位國際重要學者交換研究心得並商討合作計畫。 三、攜回資料名稱及內容 個人出席此次論文發表會，我同時帶回許多進行中的研究論文，這些論文包括許多進行中關於風險與保險的尖端研究，我也非常願意與國內其他學者分享這些資料。			

Hidden Regret and Advantageous Selection in Insurance Markets*

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March 16, 2007

Abstract

We examine insurance markets in which there are two types of customers: those who regret suboptimal decisions and those who don't. In this setting, we characterize the equilibria under hidden information about the type of customers and hidden action. We show that both pooling and separating equilibria can exist. Furthermore, there exist separating equilibria that predict a positive correlation between the amount of insurance coverage and risk type, as in the standard economic models of adverse selection, but there also exist separating equilibria that predict a negative correlation between the amount of insurance coverage and risk type, i.e. advantageous selection.

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1 Introduction

The markets for annuities, long-term care insurance, and Medigap insurance have become increasingly important for societies whose population is aging, as e.g. in the US and Europe. Surprisingly, the demand for these insurance products is very low (see e.g. Mitchell, et al., 1999, Brown and Finkelstein, 2004) which might put a huge burden on future generations. Whether public or private insurance provision is more efficient depends on the underlying inefficiencies in these markets, a large part of which is due to asymmetric information. Interestingly, those markets exhibit contrasting characteristics with respect to the relation between insurance coverage and risk type.

Rothschild and Stiglitz (1976) show in their classical adverse selection model in insurance markets that, in equilibrium—if it exists—lower risk individuals self-select into contracts which offer lower insurance coverage. The model thus predicts a positive correlation between the amount of insurance coverage and claim frequency. Similarly, economic models of moral hazard predict this positive relation: individuals with higher insurance coverage reduce their investments in risk-mitigating measures and thereby are of higher risk type. The empirical evidence, however, of the relation between insurance coverage and claim frequency, is mixed. In the markets for acute care health insurance and annuities the empirical evidence is consistent with the prediction of adverse selection and moral hazard models (see e.g. Cutler and Zeckhauser, 2000, Mitchell, et al., 1999, Finkelstein and Porteba, 2004). In contrast, a negative relationship between insurance coverage and claim frequency exists in the markets for term-life insurance, long-term care, and Medigap insurance (see e.g. Cawley and Philipson, 1999, Finkelstein and McGarry, 2006, Fang, et al., 2006).

de Meza and Webb (2001) argue that a negative relationship between insurance coverage and risk type—which they term *advantageous selection*—can be explained by hidden heterogeneity of individuals’ degree of risk aversion. They show that there exist equilibria in which high risk-averse individuals both purchase more insurance coverage and invest more in risk-mitigating measure—thereby becoming lower risk types—than less risk-averse individuals. The empirical evidence, however, on the sign of the negative relationship between degree of risk aversion and risk type is mixed. Finkel-

stein and McGarry (2006) find evidence in the long-term care insurance market that is consistent with advantageous selection, i.e. more risk averse individuals are more likely to purchase long-term care insurance and less likely to enter a nursing home. In contrast, Cohen and Einav (2006) and Fang, et al. (2006) find the opposite in automobile and Medigap insurance: risk type is positively correlated with risk aversion.

In this paper, we propose hidden heterogeneity in degrees of anticipatory regret as an alternative reason for a negative relationship between insurance coverage and risk type. Regret is interpreted as the anticipated disutility incurred from an ex-ante choice that turns out to be ex-post suboptimal and individuals make their decision by trading off the maximization of expected utility of wealth against the minimization of expected disutility from anticipated regret. The latter is modeled as a second attribute to the utility function that depends on the difference in utilities of wealth levels derived from the foregone best alternative and derived from the actual choice.

Our intuition suggests that individuals who consider ex-ante a disutility associated with ex-post regret, i.e. with having foregone a better alternative, might both purchase less insurance coverage under actuarially fair prices and invest less in risk-mitigating measures than individuals who do not consider regret. We examine the existence and type of equilibria when insurers can neither observe this preference heterogeneity nor investment behavior in risk-mitigating measures. We are particularly interested in deriving conditions under which separating equilibria exist that are consistent with advantageous selection. The interesting and challenging feature of anticipatory regret is that foregone alternatives, and thus the menu of insurance contracts offered, impact individual welfare which could imply the existence of pooling equilibria.

Regret theory was initially developed by Bell (1982) and Loomes and Sugden (1982) and has been shown in both the theoretical and experimental literature to explain individual behavior. More recently, the impact of regret on decision making has been examined in different scenarios. Braun and Muermann (2004) and Muermann, et al. (2006) show that regret moves individuals away from extreme decisions, i.e. regret leads to more (less) insurance coverage if insurance is relatively expensive (cheap) and, similarly, regret leads financial investors to buy more (less) risky

stocks if the equity risk premium is relatively high (low). In a dynamic setting, Muermann and Volkman (2006) show that anticipatory regret and pride can cause investors to sell winning stocks and hold on to losing stocks, i.e. it can explain behavior that is consistent with the disposition effect. Regret preferences have also been applied to asset pricing and portfolio choice in an Arrow-Debreu economy (Gollier and Salanié 2006), to currency hedging (Michenaud and Solnik 2006), and to first price auctions (Filiz and Ozbay 2006).

In this paper, we contribute to the literature above by considering the equilibrium effects under asymmetric information in a market in which both types of investors coexist, those that consider anticipated regret in their decision-making, and those that do not.

In the following section, we introduce the model and derive properties of indifference curves as those will be used for our graphical analysis of equilibria in Section 3. In Section 4 we derive comparative statics of model parameters with respect to the existence and type of equilibria. We conclude in Section 5.

2 Model Approach

The model will focus on two types of individuals: those that regret suboptimal decisions, type R individuals, and those that do not, type N individuals. Let λ be the fraction of type R individuals in the population. Both types are endowed with initial wealth w and face a potential loss of size L with initial probability p_0 . Individuals can invest in self-protection at a disutility $f_i \in \{0, F\}$, $i = N, R$, to reduce the probability of a loss from $p(0) = p_0$ to $p(F) = p_F < p_0$. Type N individuals maximize expected utility with respect to an increasing, concave utility function $u(\cdot)$. For type R individuals, we follow Bell (1982) and Loomes and Sugden (1982) by implementing the following two-attribute utility function to incorporate regret in preferences

$$u_R(W) = u(W) - g(u(W^{\max}) - u(W)). \quad (1)$$

Type R individuals thus maximize expected utility with respect to the utility function $u_R(\cdot)$.¹ The first attribute is the utility derived from the final level of wealth, W , and is thus equivalent to the utility of type N individuals. The second attribute accounts for the fact that the individual considers regret in his decision-making. Regret depends on the difference between the utility of wealth, $W^{\max}$, the individual could have obtained with the foregone best alternative (FBA) and the utility of actual final wealth, W . The function $g(\cdot)$ measures the disutility incurred from regret and we assume that $g(\cdot)$ is increasing and convex with $g(0) = 0$. This assumption is supported in the literature (Thaler, 1980, Kahneman and Tversky, 1982) and has recently found experimental support by Bleichrodt et al., 2006.

Insurers are risk-neutral and offer insurance contracts which are specified by the amount of insurance coverage, I , and the premium rate, c , per dollar of coverage. We assume that there is asymmetric information about both preferences and actions. That is, whether or not a specific individual regrets his decision and whether or not he invests in risk-mitigating measures is private information to the individual. The insurer only knows the distribution of the two types of individuals, N and R , in the population; that is, the insurer knows the parameter λ .

2.1 Investing in self-protection

The gains in expected utility for type i individuals, $\Delta_i(I, c)$, $i = N, R$, from investing in self-protection under an insurance contract (I, c) is

$$\Delta_N(I, c) = (p_0 - p_F) (u(w - cI) - u(w - L + (1 - c)I)) - F \quad (2)$$

¹This two-attribute utility function is consistent with the axiomatic foundation of regret developed by Sugden (1993) and Quiggin (1994).

for type N individuals and

$$\begin{aligned}
\Delta_R(I, c) &= \Delta_N(I, c) - p_F g(u(W_L^{\max}) - u(w - L + (1 - c)I) + F) \\
&\quad + p_0 g(u(W_L^{\max}) - u(w - L + (1 - c)I)) \\
&\quad - (1 - p_F) g(u(W_{NL}^{\max}) - u(w - cI) + F) + (1 - p_0) g(u(W_{NL}^{\max}) - u(w - cI))
\end{aligned} \tag{3}$$

where $W_L^{\max}$ and $W_{NL}^{\max}$ are the wealth levels under the FBA in the Loss and No-Loss state, respectively. As investing in self-protection only has ex-ante value to the insured, it is never optimal from an ex-post point of view to have invested in self-protection. In the No-Loss state, the FBA is thus to not have invested in self-protection and to not have bought insurance coverage, i.e. $W_{NL}^{\max} = w$. In the Loss state, the FBA is to have bought the contract with the highest net coverage $(1 - \tilde{c})\tilde{I} = \arg \max_{(I, c)} (1 - c)I$ amongst the set of contracts offered, i.e. $W_L^{\max} = w - L + (1 - \tilde{c})\tilde{I}$. Let $X = (\tilde{I}, \tilde{c})$ denote the insurance contract with the highest net insurance coverage amongst the set of contracts offered. Therefore

$$\begin{aligned}
\Delta_R(I, c) &= \Delta_N(I, c) - p_F g\left(u\left(w - L + (1 - \tilde{c})\tilde{I}\right) - u(w - L + (1 - c)I) + F\right) \\
&\quad + p_0 g\left(u\left(w - L + (1 - \tilde{c})\tilde{I}\right) - u(w - L + (1 - c)I)\right) \\
&\quad - (1 - p_F) g(u(w) - u(w - cI) + F) + (1 - p_0) g(u(w) - u(w - cI)). \tag{4}
\end{aligned}$$

The gains from investing in self-protection is larger for the type N individuals if the cost of investing in self-protection, F , is high enough. More precisely, (4) implies that $\Delta_N(I, c) > \Delta_R(I, c)$ if and only if F satisfies the following inequality

$$\begin{aligned}
&p_F g\left(u\left(w - L + (1 - \tilde{c})\tilde{I}\right) - u(w - L + (1 - c)I) + F\right) + (1 - p_F) g(u(w) - u(w - cI) + F) \\
> &p_0 g\left(u\left(w - L + (1 - \tilde{c})\tilde{I}\right) - u(w - L + (1 - c)I)\right) + (1 - p_0) g(u(w) - u(w - cI)). \tag{5}
\end{aligned}$$

To demonstrate the existence of equilibria in which there exists a negative relation between insurance coverage and risk type, i.e. *advantageous selection*, we assume that the cost of investing in self-protection, F , is high enough such that type R individuals will not find it optimal to invest in self-protection under any contract, i.e. we assume a level of F such that $\Delta_R(I, c) < 0$ for all I and c with $p_F \leq c \leq p_0$.

2.2 Demand for insurance

Braun and Muermann (2004) have shown that type R individuals “hedge their bets” by avoiding extreme decisions. That is, type R individuals purchase more (less) insurance coverage than type N individuals if it is optimal for type N individuals to purchase very little (a lot of) insurance coverage. This implies that type R individuals value insurance coverage relatively more (less) than type N individuals if an insurance contract offers very little (a lot of) coverage.

2.3 Graphical analysis

We will use diagrams to analyze the existence of equilibria. In all diagrams, the x-axis represents the individuals’ level of final wealth in the No-Loss state, $W_{NL} = w - cI$, whereas the y-axis denotes the individuals’ level of final wealth in the Loss state, $W_L = w - L + (1 - c)I$. The individuals’ endowment point is $(w - L, w)$ and labeled A . $\mathbf{P}_F$, $\bar{\mathbf{P}}$, and $\mathbf{P}_0$ denote the actuarially fair pricing lines with respect to the premium rates $c = p_F$, $c = \bar{p} = \lambda p_F + (1 - \lambda)p_0$, and $c = p_0$, respectively.

Type N individuals. The levels of expected utility for type N individuals investing and not investing in self-protection under contract (I, c) are

$$EU_N(I, c, F) = p_F u(W_L) - (1 - p_F) u(W_{NL}) - F$$

and

$$EU_N(I, c, 0) = p_0 u(W_L) - (1 - p_0) u(W_{NL}).$$

The slope of type N individuals' indifference curve are

$$\frac{dW_L}{dW_{NL}}|_{EU_N(I,c,F)} = -\frac{1-p_F}{p_F} \frac{u'(W_{NL})}{u'(W_L)}$$

and

$$\frac{dW_L}{dW_{NL}}|_{EU_N(I,c,0)} = -\frac{1-p_0}{p_0} \frac{u'(W_{NL})}{u'(W_L)}.$$

The slope of the locus of contracts under which type N individuals are indifferent between investing and not investing in self-protection, i.e. for which $\Delta_N(I, c) = 0$, is given by

$$\frac{dW_L}{dW_{NL}}|_{\Delta_N(I,c)=0} = \frac{u'(W_{NL})}{u'(W_L)}.$$

The line of those contracts is thus increasing and below the 45° line as $0 < \frac{dW_L}{dW_{NL}}|_{\Delta_N(I,c)=0} < 1$. Furthermore, for any premium rate c there exists a unique level of coverage $\bar{I}(c) < L$ such that $\Delta_N(\bar{I}(c), c) = 0$. Since $\frac{\partial \Delta_N(I,c)}{\partial W_L} < 0$ we conclude that it is optimal for types N individuals to not invest in self-protection under all contracts that are above the line of contracts for which $\Delta_N(\bar{I}(c), c) = 0$. For all contracts below this line, it is optimal for type N individuals to invest in self-protection. Note, that for all contracts with $\Delta_N(\bar{I}(c), c) = 0$, indifference curves are kinked with a steeper slope below than above as

$$\left| \frac{dW_L}{dW_{NL}}|_{EU_N(I,c,F)} \right| > \left| \frac{dW_L}{dW_{NL}}|_{EU_N(I,c,0)} \right|.$$

Type R individuals. The level of expected utility of type R individuals not investing in self-protection under contract (I, c) is

$$EU_R(I, c, 0) = EU_N(I, c, 0) - p_0 g\left(u\left(w - L + (1 - \tilde{c}) \tilde{I}\right) - u(W_L)\right) - (1 - p_0) g(u(w) - u(W_{NL})).$$

Note that the expected utility of type R individuals and therefore the shape of the indifference curves depends upon the contract $X = (\tilde{I}, \tilde{c})$ that offers the highest net insurance coverage. The slope of type R individuals' indifference curve is

$$\frac{dW_L}{dW_{NL}} \Big|_{EU_R(I, c, 0)} = -\frac{1-p_0}{p_0} \frac{u'(W_{NL})}{u'(W_L)} \frac{1+g'(u(w)-u(W_{NL}))}{1+g'\left(u\left(w-L+(1-\tilde{c})\tilde{I}\right)-u(W_L)\right)} < 0. \quad (6)$$

The second derivative of type R individuals' indifference curve is given by

$$\begin{aligned} & \frac{d^2 W_L}{dW_{NL}^2} \Big|_{EU_R(I, c, 0)} \\ = & -\left(\frac{dW_L}{dW_{NL}} \Big|_{EU_R(I, c, 0)}\right)^2 \frac{u''(W_L)}{u'(W_L)} - \frac{1-p_0}{p_0} \frac{u''(W_{NL})}{u'(W_L)} \frac{1+g'(u(w)-u(W_{NL}))}{1+g'\left(u\left(w-L+(1-\tilde{c})\tilde{I}\right)-u(W_L)\right)} \\ & + \left(\frac{dW_L}{dW_{NL}} \Big|_{EU_R(I, c, 0)}\right)^2 u'(W_L) \frac{g''\left(u\left(w-L+(1-\tilde{c})\tilde{I}\right)-u(W_L)\right)}{1+g'\left(u\left(w-L+(1-\tilde{c})\tilde{I}\right)-u(W_L)\right)} \\ & + \frac{1-p_0}{p_0} \frac{(u'(W_{NL}))^2}{u'(W_L)} \frac{g''(u(w)-u(W_{NL}))}{1+g'\left(u\left(w-L+(1-\tilde{c})\tilde{I}\right)-u(W_L)\right)} \\ > & 0. \end{aligned}$$

Type R individuals' indifference curve are thus also both decreasing and convex.

Comparison of indifference curves between types. We next compare the slopes of the indifference curves of type R and type N individuals with contracts under which type N individuals invest in self-protection. The indifference curve of type R individuals are flatter than the one of type N individuals if and only if

$$\frac{1-p_0}{p_0} \frac{1+g'(u(w)-u(W_{NL}))}{1+g'\left(u\left(w-L+(1-\tilde{c})\tilde{I}\right)-u(W_L)\right)} \leq \frac{1-p_F}{p_F}. \quad (7)$$

At the endowment point $A = (w-L, w)$ condition (7) is satisfied and the indifference

curve of type R individuals are thus flatter than the one of type N individuals. The effect of

increasing the amount of coverage I at the same premium rate c on the left-hand side of condition (7) is

$$\begin{aligned} & \frac{\partial}{\partial I} \left(\frac{1 + g'(u(w) - u(W_{NL}))}{1 + g'(u(w - L + (1 - \tilde{c})\tilde{I}) - u(W_L))} \right) = \frac{cu'(W_{NL})g''(u(w) - u(W_{NL}))}{1 + g'(u(w - L + (1 - \tilde{c})\tilde{I}) - u(W_L))} \\ & + \frac{(1 - c)u'(W_L)g''(u(w - L + (1 - \tilde{c})\tilde{I}) - u(W_L))(1 + g'(u(w) - u(W_{NL})))}{\left(1 + g'(u(w - L + (1 - \tilde{c})\tilde{I}) - u(W_L))\right)^2} \\ & > 0. \end{aligned}$$

This implies that, for a given premium rate c and contract $X = (\tilde{I}, \tilde{c})$, condition (7) can only switch once at the unique level of insurance coverage $\hat{I} = \hat{I}(c, X)$. We thus conclude that for low levels of coverage the indifference curve of type R individuals are flatter than the one of type N individuals whereas for high levels of coverage the indifference curve of type R individuals can be steeper than the one of type N individuals. This is consistent with the result of Braun and Muermann (2004) who show that type R individuals value insurance coverage relatively more (less) than type N individuals if an insurance contract offers very little (a lot of) coverage. Valuing insurance coverage relatively more (less) implies a flatter (steeper) indifference curve.

At the level $I = \hat{I}(c, X)$, the indifference curves of type R and type N individuals have the same slope, i.e. condition (7) is satisfied with equality which implies

$$\begin{aligned} & \frac{d^2 W_L}{dW_{NL}^2} \Big|_{\overline{EU_R(I, c, 0)}} \\ & = \frac{d^2 W_L}{dW_{NL}^2} \Big|_{\overline{EU_N(I, c, F)}} + \left(\frac{dW_L}{dW_{NL}} \Big|_{\overline{EU_R(I, c, 0)}} \right)^2 u'(W_L) \frac{g''(u(w - L + (1 - \tilde{c})\tilde{I}) - u(W_L))}{1 + g'(u(w - L + (1 - \tilde{c})\tilde{I}) - u(W_L))} \\ & + \frac{1 - p_0}{p_0} \frac{(u'(W_{NL}))^2}{u'(W_L)} \frac{g''(u(w) - u(W_{NL}))}{1 + g'(u(w - L + (1 - \tilde{c})\tilde{I}) - u(W_L))} \\ & > \frac{d^2 W_L}{dW_{NL}^2} \Big|_{\overline{EU_N(I, c, F)}}. \end{aligned}$$

The indifference curve of type R individuals at $I = \hat{I}(c, X)$ are thus more convex than the one of type N individuals for all premium rates c and contracts X .

Changing the foregone best alternative. An interesting feature of regret is that preferences depend upon foregone alternatives. In particular, insurance companies can change preferences of type R individuals by offering a contract X with higher net insurance coverage $(1 - \tilde{c})\tilde{I}$. The impact of increasing net insurance coverage of the foregone best alternative on the slope of the indifference curve of R types is

$$\frac{\partial}{\partial (1 - \tilde{c})\tilde{I}} \left(\frac{dW_L}{dW_{NL}} \Big|_{EU_R(I, c, 0)} \right) = - \frac{\partial}{\partial (1 - \tilde{c})\tilde{I}} \left(\frac{1 + g'(u(w) - u(W_{NL}))}{1 + g'(u(w - L + (1 - \tilde{c})\tilde{I}) - u(W_L))} \right) > 0.$$

This implies that the indifference curves of types R individuals become flatter at any contract (I, c) . The intuition is that increasing net insurance coverage of the foregone best alternative increases the regret in the Loss-state and thereby makes coverage relatively more valuable to type R individuals. This implies that offering a contract with a higher net insurance coverage increases the level of coverage $I = \hat{I}(c, X)$ at which condition (7) switches.

3 Equilibrium Analysis

We have shown that type R individuals might be both less willing to invest in self-protection and prefer less insurance coverage than type N individuals. These results suggests that there can be equilibria in which there exists a negative relation between insurance coverage and risk type, i.e. *advantageous selection*. We consider the following game between insurers and individuals:

Stage 1 Insurers make binding offers of insurance contracts specifying coverage I and premium rate c .

Stage 2 Individuals choose either a contract from the set of contracts offered or no contract. If the same contract is offered by two insurers, individuals toss a fair coin.

Stage 3 Individuals choose whether or not to invest in self-protection.

We will consider the existence and type of pure-strategy, subgame-perfect Nash equilibria. As examined above, regret introduces two interesting features. First, types R individuals value insurance relatively more (less) than type N individuals if the level of coverage offered is small (large). Second, the foregone best alternative and thus the expected utility of type R individuals depends on the set of contracts offered. An insurance company could thus strategically offer two contracts: one contract, contract X , is only offered to change the expected utility of type R individuals and the other contract serves the purpose of attracting customers given the shift in preferences of type R individuals. We can restrict our strategies to those where the “preference-shifting” contract X offers a higher net coverage than the other contracts offered as only then type R preferences shift. In our equilibrium analysis, we thus have to carefully examine those strategies.

3.1 Pooling equilibria

In this section, we examine the existence of pooling equilibria as a function of the level of coverage offered and via graphical analysis. The main result of this section is that, contrary to Rothschild and Stiglitz (1976) and de Meza and Webb (2001), a pooling equilibrium can exist. As defined above, contract $(\hat{I}(\bar{p}, X), \bar{p})$ denotes the contract with premium rate $\bar{p}$ under which the indifference curve of type R individuals have the same slope than the one of type R individuals and contract $(\bar{I}(\bar{p}), \bar{p})$ denotes the contract with premium rate $\bar{p}$ under which type N individuals are indifferent between investing and not investing in self-protection, i.e. $\Delta_N(\bar{I}(\bar{p}), \bar{p}) = 0$. As shown above, both contracts are unique.

Proposition 1 *Suppose that the cost of investing in self-protection, F , is high enough such that $\Delta_R(I, c) < 0$ for all I and c with $p_F \leq c \leq p_0$.*

1. *If $\hat{I}(\bar{p}, X) \geq \bar{I}(\bar{p})$ for some contract X , then there exists no pooling equilibrium.*

2. If $\hat{I}(\bar{p}, X) < \bar{I}(\bar{p})$ for $X = (L, c')$ where c' is implicitly defined by $EU_N(\bar{I}(\bar{p}), \bar{p}, F) = EU_N(L, c', 0)$, then the contract $(\bar{I}(\bar{p}), \bar{p})$ is the unique pooling equilibrium if and only if

$$(a) \quad EU_R(\bar{I}(\bar{p}), \bar{p}, 0) > EU_R(I, p_0, 0) \text{ for all } I$$

$$(b) \quad EU_N(\bar{I}(\bar{p}), \bar{p}, F) > EU_N(I, \bar{p}, F) \text{ for all } I \text{ that satisfy } EU_R(I, \bar{p}, 0) > EU_R(\bar{I}(\bar{p}), \bar{p}, 0)$$

$$(c) \quad EU_N(\bar{I}(\bar{p}), \bar{p}, F) > EU_N(I, p_0, 0) \text{ for all } I \text{ with } \Delta_N(I, p_0) < 0$$

Proof. First note that no pooling equilibrium exists under which neither type R nor type N invest in self-protection. Type N individuals prefer full coverage and type R individuals prefer partial coverage as shown in Braun and Muermann (2004), i.e. for any pooling contract (I, p_0) there exist a contract to which either type R or type N individuals deviate. We can thus restrict our analysis to all contracts $(I, \bar{p})$ with $I \leq \bar{I}(\bar{p})$.

1. Suppose $\hat{I}(\bar{p}, X) > \bar{I}(\bar{p})$ for some contract X . This implies for any pooling contract $B = (I, \bar{p})$ we must have $I < \hat{I}(\bar{p}, X)$. We have shown above that for all $I < \hat{I}(\bar{p}, X)$ the indifference curve of type R individuals are flatter than the one of type N individuals, i.e. (7) is satisfied (see Figure 1). This implies that no pooling equilibrium exist under those contracts as a contract with slightly less coverage and a potentially different premium rate (contract D in Figure 1) attracts type N individuals but not type R individuals. Note that contract D does not change preferences of type R individuals as it offers lower net indemnity than contract B . The intuition behind this result is that for low levels of coverage $I < \hat{I}(\bar{p}, X)$ type R individuals value insurance coverage relatively more than type N individuals and can thus not be attracted by such contracts. This is equivalent to the proof in Rothschild and Stiglitz (1976) who show that under any pooling contract there exist contracts that attract low-risk types but not high-risk types as high-risk types value insurance coverage relatively more.

Suppose $\hat{I}(\bar{p}, X) = \bar{I}(\bar{p})$. Contract $B = (\hat{I}(\bar{p}, X), \bar{p})$ in Figure 2 cannot be a pooling equilibrium as the indifference curve of type R individuals at $I = \hat{I}(\bar{p}, X)$ are more convex than the one of type N individuals. This implies that there exists a contract with slightly less coverage

and a potentially different premium rate which attracts type N individuals but not type R individuals (contract D in Figure 2). Again, contract D does not change preferences of type R individuals as it offers lower net indemnity than contract B .

2. Suppose $\hat{I}(\bar{p}, X) < \bar{I}(\bar{p})$ with $X = (L, c')$ where c' is defined as above. Contract X (see Figure 5) is the contract with the highest net insurance coverage such that neither type R nor type N individuals will prefer X over $(\bar{I}(\bar{p}), \bar{p})$. As argued above, any contract $B = (I, \bar{p})$ with $I < \hat{I}(\bar{p}, X)$ cannot be a pooling equilibrium. Equivalently to above, the contract $B = (\hat{I}(\bar{p}, X), \bar{p})$ is also not a pooling equilibrium (see Figure 3).

For any contract $B = (I, \bar{p})$ with $\hat{I}(\bar{p}, X) < I < \bar{I}(\bar{p})$, the indifference curve of type R individuals is steeper than the one of type N individuals, i.e. type N individuals value insurance coverage relatively more than type R individuals (see Figure 4). A contract offering slightly more coverage and a potentially different premium rate (contract D in Figure 4) attracts type N individuals but not type R individuals. Note, however, that the introduction of contract D does not change the preferences of type R individuals as contract X offers higher net insurance coverage than contract D . Thus, no pooling equilibria $B = (I, \bar{p})$ exist with $\hat{I}(\bar{p}, X) \leq I < \bar{I}(\bar{p})$.

Now let's examine contract $B = (\bar{I}(\bar{p}), \bar{p})$ (see Figure 5). Since $\hat{I}(\bar{p}, X) < \bar{I}(\bar{p})$, the indifference curve of type R individuals is steeper at B than the one of type N individuals. This implies that there does not exist any contract (I, c) with $\Delta_N(I, c) > 0$ that attracts type N individuals but not type R individuals. Condition 2a implies that any contract (I, c) with $\Delta_N(I, c) < 0$ must offer a rate $c < p_0$ to attract type N individuals and thereby make negative profits. Condition 2b ensures that no other contract $(I, \bar{p})$ on the price line $\bar{\mathbf{P}}$ attracts both types of individuals. Last, condition 2c implies that no contract (I, p_0) on the price line $\mathbf{P}_0$ attracts type R individuals. Therefore, $B = (\bar{I}(\bar{p}), \bar{p})$ constitutes a pooling equilibrium under those conditions.

■

In the pooling equilibrium, type R individuals value insurance coverage relatively less than type N individuals, i.e. the amount of insurance coverage must be relatively high. Offering less coverage would be relatively more attractive to type R individuals and, under the conditions above, yield negative expected profits. Offering more coverage would induce type N individuals to not invest in self-protection and also imply negative expected profits.

3.2 Separating equilibria

In this section, we examine the existence and type of separating equilibria. We assume that each contract offered and chosen by individuals must earn non-negative expected profits. We thus do not allow for cross-subsidization between types as it is examined, for example, in Miyazaki (1977) and Crocker and Snow (1985).

Under the assumption that type R individuals do not invest in self-protection, the contract chosen by type R individuals in equilibrium is priced at the rate $c = p_0$ and offers the optimal amount of coverage $I_R^* = I_R^*(p_0, X)$, given contract X that offers the highest net insurance coverage. Let us denote this contract by $R = (I_R^*(X), p_0)$. As shown by Braun and Muermann (2004) that the optimal amount of coverage at a fair rate is less than full coverage, i.e. $I_R^*(p_0, X) < L$ for all X . As optimal amount of coverage depends on contract X , three contracts might be offered in separating equilibria: contract N and R chosen by types N and R individuals, respectively, and the “preference-changing” contract X which is not chosen by any type of individual. In equilibrium, contract X must offer the highest net insurance coverage such that neither type chooses the contract. If a contract X' with lower net insurance coverage and thus $R(X') = (I_R^*(p_0, X'), p_0)$ is offered then offering contract X with higher net insurance coverage than X' together with contract $R(X) = (I_R^*(p_0, X), p_0)$ attracts type R individuals as the optimal amount of insurance coverage is increasing in the net insurance coverage of the foregone best alternative in the Loss-state.

In the following proposition we show that there exists a separating equilibrium under which both types do not invest in self-protection and both types receive the optimal amount of coverage given the rate $c = p_0$.

Proposition 2 *Suppose that the cost of investing in self-protection, F , is high enough such that $\Delta_R(I, c) < 0$ for all I and c with $p_F \leq c \leq p_0$. Then the two contracts $N = (L, p_0)$ and $R = (I_R^*(p_0, X), p_0)$ constitute a separating equilibrium if and only if $EU_N(L, p_0, 0) > EU_N(I, \bar{p}, F)$ for all $I \leq \bar{I}(\bar{p})$ that satisfy $EU_R(I, \bar{p}, 0) \geq EU_R(I_R^*(p_0, X), p_0, 0)$.*

Proof. Figure 6 illustrates the equilibrium. In this scenario, the “preference-changing” contract X with the highest net insurance coverage coincides with contract N . The additional condition outlined in the proposition assures that no pooling contract can attract both types of individuals while making zero expected profits. ■

In the separating equilibrium outlined above, both types of individuals do not invest in self-protection but purchase different amounts of insurance coverage. The empirical prediction under this scenario is thus that both types are of identical risk-type and that type R individuals purchase less insurance coverage than type N individuals. The equilibrium contracts only separate different preference type individuals rather than risk type as in Rothschild and Stiglitz (1976). Note that the premium rates for both types are the same. Thus, this type of equilibrium predicts that the premium rate and coverage will be insignificant related in empirical studies.

3.2.1 Advantageous selection

In the following proposition, we show that there exist and characterize separating equilibria that predict a negative relationship between insurance coverage and risk type, i.e. *advantageous selection*. As above, we denote the amount of coverage $\bar{I} = \bar{I}(c)$ under which type N individuals are indifferent between investing and not investing in self-protection, i.e. under which $\Delta_N(\bar{I}(c), c) = 0$.

Proposition 3 *Suppose that the cost of investing in self-protection, F , is high enough such that $\Delta_R(I, c) < 0$ for all I and c with $p_F \leq c \leq p_0$. Then the three contracts N , R , and X constitute a separating equilibrium with advantageous selection if and only if under one of the following two scenarios:*

1. $N = (\bar{I}(p_F), p_F)$, $X = (L, c)$, and $R = (I_R^*(p_0, X), p_0)$ where

- (a) c satisfies $EU_N(\bar{I}(p_F), p_F, F) = EU_N(L, c, 0)$ and $c \geq p_0$
 - (b) $EU_R(I_R^*(p_0, X), p_0, 0) \geq EU_R(\bar{I}(p_F), p_F, 0)$
2. $N = (\bar{I}(c_N), c_N)$, $X = (L, c)$, and $R = (I_R^*(p_0, X), p_0)$ where
- (a) c_N satisfies $EU_N(\bar{I}(c_N), c_N, F) = EU_R(I_R^*(p_0, X), p_0, 0)$
 - (b) c is the maximum rate that satisfies both $EU_N(\bar{I}(c_N), c_N, F) \geq EU_N(L, c, 0)$ and $EU_R(I_R^*(p_0, X), p_0, 0) \geq EU_R(L, c, 0)$, and $c \geq p_0$
 - (c) $EU_R(\bar{I}(\bar{p}), \bar{p}, 0) \leq EU_R(I_R^*(p_0, X), p_0, 0) \leq EU_R(\bar{I}(p_F), p_F, 0)$
 - (d) $EU_N(\bar{I}(c_N), c_N, F) \geq EU_N(I, p_0, F)$ where I satisfies $EU_R(I_R^*(p_0, X), p_0, 0) = EU_N(I, p_F, F)$
 - (e) $EU_N(\bar{I}(c_N), c_N, F) \geq EU_N(I, \bar{p}, F)$ for all $I \leq \bar{I}(\bar{p})$ that satisfy $EU_R(I, \bar{p}, 0) \geq EU_R(I_R^*(p_0, X), p_0, 0)$

Proof.

1. Figure 7 illustrates the equilibrium. Condition 1b represents the property that the indifference curve of type R individuals through contract R crosses the locus of contracts $\Delta_N(I, c) = 0$ above the line $\mathbf{P}_F$. Contract N is then the best contract for type N individuals among the set of contracts on $\mathbf{P}_F$ that are not preferred by type R individuals over contract R . Contract $X = (L, c)$ is the contract with highest net coverage that is not preferred by either type over their respective equilibrium contract. The condition $c \geq p_0$ assures that contract (L, p_0) does not attract type N individuals.
2. Figure 8 illustrates this equilibrium. Condition 2a defines contract N . Condition 2c represents the property that the indifference curve of type R individuals through contract R crosses the locus of contracts $\Delta_N(I, c) = 0$ below the line $\mathbf{P}_F$ but above the line $\bar{\mathbf{P}}$. Conditions 2d and 2e assure that type N individuals do neither prefer contract N' nor prefer any pooling contract $(I, \bar{p})$ that would also be taken by type R individuals over contract N .

■

The key factor that allows for separating equilibria with advantageous selection is the fact that type R individuals prefer partial coverage at a fair rate. Type N individuals can separate themselves from type R individuals with more insurance coverage since they value insurance coverage relatively more at the fair rate. Both equilibrium 1 and equilibrium 2 in the above proposition have interesting features.

In equilibrium 1, the presence of type R individuals in the market does not cause any negative externality on type N individuals. The equilibrium contracts $N = (\bar{I}(p_F), p_F)$ and $R = (I_R^*(p_0, X), p_0)$ are identical to the equilibrium contract under hidden action if there were only one type of customers in the market. Equilibrium 1 is in fact the only equilibrium with that feature.

In equilibrium 2, insurance companies make strictly positive expected profits with contract N while they break even with contract R . However, this is true only under pure separation and also due to the fact that we do not allow for cross-subsidies between types. This implies that there exist semi-separating equilibria under the conditions outlined in equilibrium 2 in which a certain fraction of type R individuals choose contract N . The maximum fraction of those types is determined by the break-even condition of insurance companies for contract N .

3.2.2 Adverse selection

In this section, we characterize the separating equilibrium that predicts a positive relationship between insurance coverage and risk type, i.e. *adverse selection*.

Proposition 4 *Suppose that the cost of investing in self-protection, F , is high enough such that $\Delta_R(I, c) < 0$ for all I and c with $p_F \leq c \leq p_0$. Then the three contracts $N = (I_N, p_F)$, $R = (I_R^*(p_0, X), p_0)$, and $X = (L, c)$ constitute a separating equilibrium with adverse selection if and only if*

1. (a) I_N satisfies $EU_N(I_N, p_F, F) = EU_R(I_R^*(p_0, X), p_0, 0)$

- (b) c is the maximum rate that satisfies both $EU_N(I_N, p_F, F) \geq EU_N(L, c, 0)$ and $EU_R(I_R^*(p_0, X), p_0, 0) \geq EU_R(L, c, 0)$, and $c \geq p_0$
- (c) $EU_R(I_R^*(p_0, X), p_0, 0) \leq EU_R(\bar{I}(p_F), p_F, 0)$
- (d) $EU_N(I_N, p_0, F) \geq EU_N(\bar{I}(c_N), c_N, F)$ where c_N satisfies $EU_R(I_R^*(p_0, X), p_0, 0) = EU_R(\bar{I}(c_N), c_N, 0)$
- (e) $EU_N(\bar{I}_N, p_F, F) \geq EU_N(I, \bar{p}, F)$ for all $I \leq \bar{I}(\bar{p})$ that satisfy $EU_R(I, \bar{p}, 0) \geq EU_R(I_R^*(p_0, X), p_0, 0)$

Proof. Figure 9 illustrates this equilibrium. Condition 1a defines contract N . Condition 1c precludes equilibrium 1 with advantageous selection in Proposition 3. Conditions 1d and 1e assure that type N individuals do neither prefer contract N' nor prefer any pooling contract $(I, \bar{p})$ that would also been taken by type R individuals over contract N . ■

Under the conditions outlined in the above proposition, the “advantageously selecting” contract $N = (\bar{I}(c_N), c_N)$ in Proposition 3 under equilibrium 2 is relatively too expensive such that type N individuals prefer not to self-select into the contract with higher coverage but rather self-select into the contract $N = (I_N, p_F)$ which offers partial coverage at their respectively fair rate $c = p_F$.

4 Comparative Statics

In this section, we discuss comparative statics of the model with regard to the types of equilibrium examined in Section 3. In our model, the type of market equilibrium varies with cost of investing in self-protection, F , the intensity of regret of type R individuals, as measured by the convexity of g , and the fraction of type R individuals in the population, λ .

4.1 Cost of investment in self-protection

If the cost of investing in self-protection, F , is extremely high or low both types of individuals optimally do not invest or invest in self-protection. Individuals are therefore heterogeneous only

regarding their preferences but not regarding their risk type. Thus, type N individuals optimally obtain full coverage, whereas type R individuals optimally obtain partial coverage, as shown by Braun and Muermann (2004).

Case 1 *For very small (high) levels of F , it is optimal for both type R and type N individuals to invest (to not invest) in self-protection and the unique equilibrium is a separating equilibrium in which both types receive the optimal amount of coverage at the rate $c = p_F$ ($c = p_0$), i.e. full coverage for type N and partial coverage for type R individuals.*

Suppose that the cost of investing in self-protection is in a range such that we obtain equilibria under which type R individuals do not invest in self-protection but type N individuals do. As the cost F increases, the set of contracts under which it is optimal for type N individuals to invest in self-protection decreases, i.e. the locus of contracts $\Delta_N(I, c) = 0$ shifts down (see Figure 10 with $F_1 < F_2 < F_3$). We then derive the following comparative statics.

Case 2 *Suppose that the cost of investing in self-protection, F , is high enough such that $\Delta_R(I, c) < 0$ for all I and c with $p_F \leq c \leq p_0$.*

1. *For low levels of F (e.g. F_1 in Figure 10), condition 1b in Proposition 3 is satisfied - i.e. type R indifference curve crosses $\Delta_N(I, p_0) = 0$ line above the pricing line $\mathbf{P}_F$ - and a separating equilibrium with advantageous selection as in Figure 7 is obtained.*
2. *For medium levels of F (e.g. F_2 in Figure 10), condition 2c in Proposition 3 is satisfied - i.e. type R indifference curve crosses $\Delta_N(I, p_0) = 0$ line below the pricing line $\mathbf{P}_F$ - and a separating equilibrium with advantageous selection as in Figure 8 is obtained.*
3. *For high levels of F (e.g. F_3 in Figure 10), either a separating equilibrium as in Figure 9 or a pooling equilibrium as in Figure 5 is obtained.*

4.2 Intensity of regret

We measure the intensity of regret by the convexity of the function g . From the slope of type R individuals' indifference curve (see equation 6) we deduce that the more convex the g function is the steeper the indifference curves of type R individuals are. Furthermore, a higher convexity of the function g implies a lower level of optimal insurance coverage for type R individuals.² Figure 11 illustrates the comparative statics with respect to the convexity of g (g_3 is more convex than g_2 which is more convex than g_1 - thus $R_3 < R_2 < R_1$).

Case 3 Suppose that the cost of investing in self-protection, F , is high enough such that $\Delta_R(I, c) < 0$ for all I and c with $p_F \leq c \leq p_0$.

1. For highly convex functions g (e.g. g_3 in Figure 11), condition 1b in Proposition 3 is satisfied - i.e. type R indifference curve crosses $\Delta_N(I, p_0) = 0$ line above the pricing line $\mathbf{P}_F$ - and a separating equilibrium with advantageous selection as in Figure 7 is obtained.
2. For medium levels of convexity of g (e.g. g_2 in Figure 11), condition 2c in Proposition 3 is satisfied - i.e. type R indifference curve crosses $\Delta_N(I, p_0) = 0$ line below the pricing line $\mathbf{P}_F$ - and a separating equilibrium with advantageous selection as in Figure 8 is obtained.
3. For low levels of convexity of g (e.g. g_1 in Figure 11), either a separating equilibrium as in Figure 9 or a pooling equilibrium as in Figure 5 is obtained.

4.3 Fraction of type R individuals in the population

In Rothschild and Stiglitz (1976), even the separating equilibrium does not exist if the fraction of high risk type individuals in the population is too low. The reason behind this non-existence result is that a pooling contract not only attracts high risk individuals but also low risk individuals as the pooling premium rate is just slightly above the fair premium rate for low risk individuals. The same

² Alternatively, Braun and Muermann (2004) propose a "regret coefficient" k in the utility function of type R individuals such that $u_R(W) = u(W) - kg(u(W^{\max}) - u(W))$. They show that the higher the regret coefficient k the lower the optimal amount of insurance coverage under a fair premium.

reasoning applies to the separating equilibrium with adverse selection in Figure 9 if the fraction λ of type R individuals is too low. Then both types of individuals are better off under a pooling contract. This pooling contract, however, does satisfy condition 2b in Proposition 1, which implies that it cannot be an equilibrium. Thus, as in Rothschild and Stiglitz (1976), there does not exist any equilibrium.

Interestingly, the same result does not hold under the conditions for the existence of a separating equilibrium with advantageous selection.

Lemma 1 *The existence of the separating equilibrium with advantageous selection as in Figure 7 does not depend on the level of λ .*

Proof. Note that all conditions in 1 of Proposition 3 are independent of λ . Furthermore, for any level of λ , no pooling contract attracts type N individuals (see Figure 7). ■

5 Conclusion

Economic models of moral hazard and adverse selection predict a positive correlation between the amount of insurance coverage individuals purchase and their claim frequency. The empirical evidence on this prediction is mixed. In some markets, e.g. in the annuity and health insurance market, such positive correlation is confirmed, whereas in other markets, e.g. in the life and long-term care insurance market, the opposite relation holds. In this paper, we propose heterogeneous, hidden degrees of aversion towards anticipatory regret as a rationale for self-selection in insurance markets. In our equilibrium analysis, we have shown that both pooling and separating equilibria can exist. Furthermore, there exist separating equilibria of both types, advantageous and adverse selection. We have characterized the conditions for each type of equilibrium and examined the comparative statics with respect to the model's parameter. Understanding the reasons behind advantageous and adverse selection is highly relevant for the design of governmental policies aimed at reducing inefficiencies due to informational asymmetries. This is particularly crucial for societies with aging

populations as the markets for annuities, long-term care insurance, and Medigap insurance become increasingly important for them.

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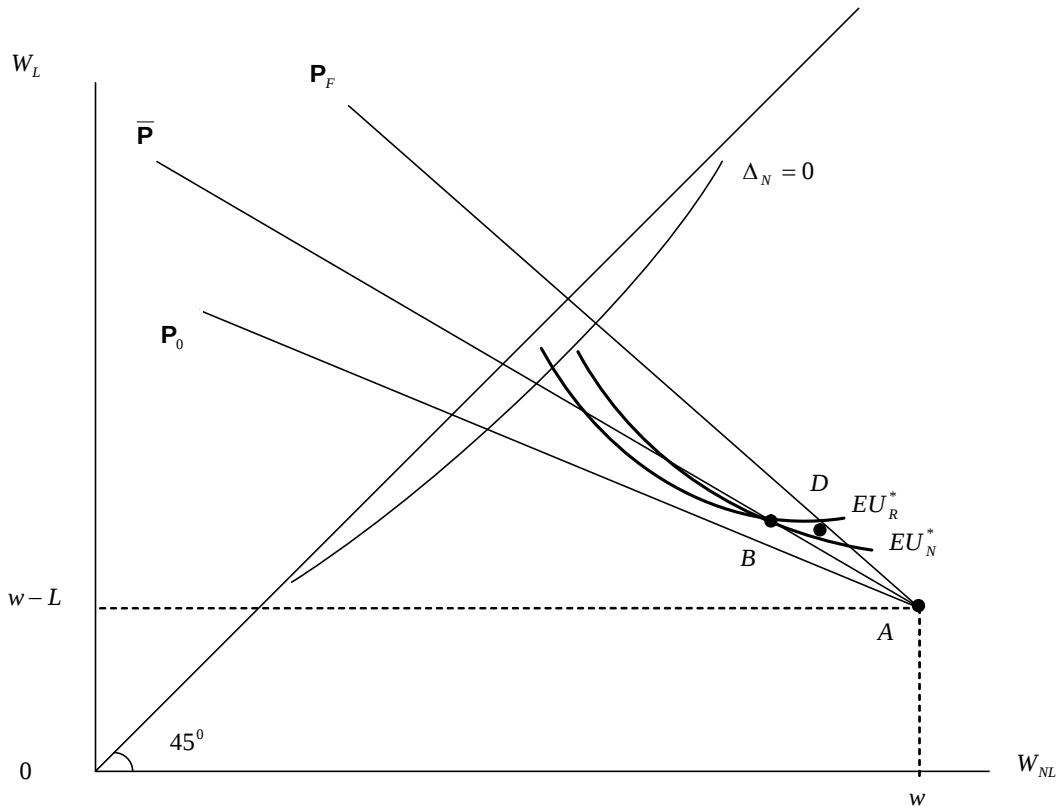


Figure 1: No Pooling Equilibrium for $I < \hat{I}$: type N individuals invest in self-protection, whereas type R individuals do not. Indifference curve of type R individuals is flatter at B than that of type N individuals. Contract D attracts type N individuals but not type R individuals.

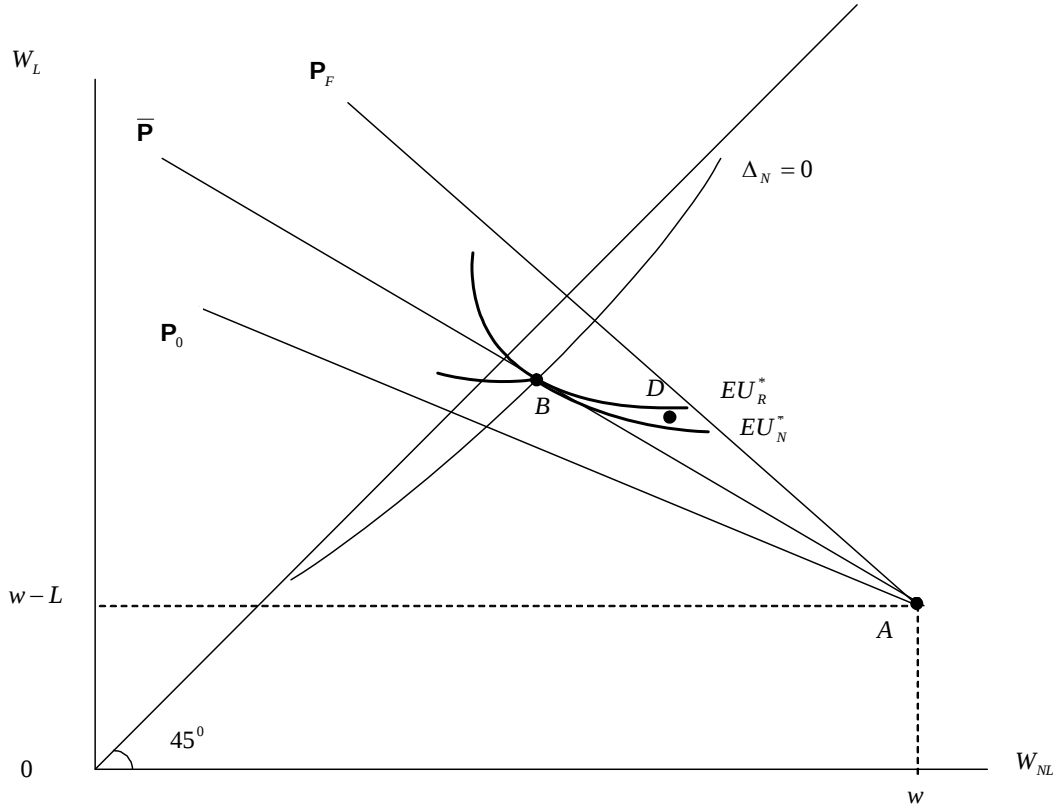


Figure 2: No Pooling Equilibrium for $I = \hat{I} = \bar{I}$: type N individuals invest in self-protection, whereas R type individuals do not. Indifference curve of type R and type N individuals have the same slope but are more convex at B . Contract D attracts type N individuals but not type R individuals.

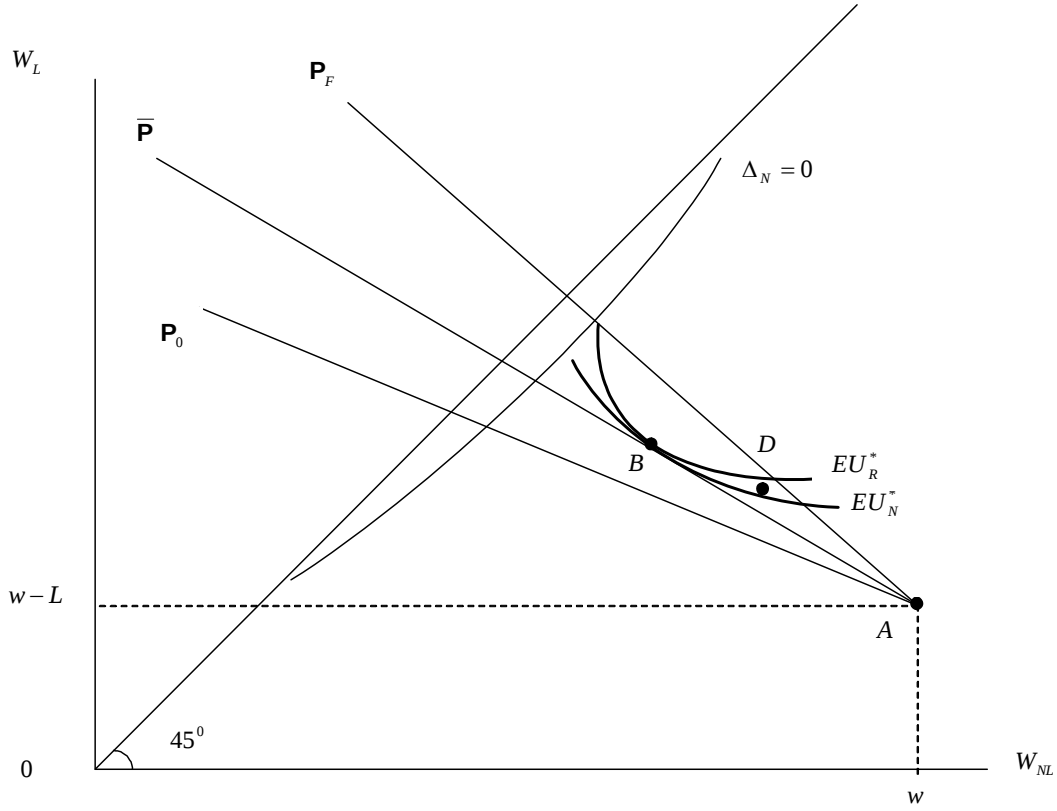


Figure 3: No Pooling Equilibrium for $I = \hat{I} < \bar{I}$: type N individuals invest in self-protection, whereas type R individuals do not. Indifference curve of type R and type N individuals have the same slope but are more convex at B . Contract D attracts type N individuals but not type R individuals.

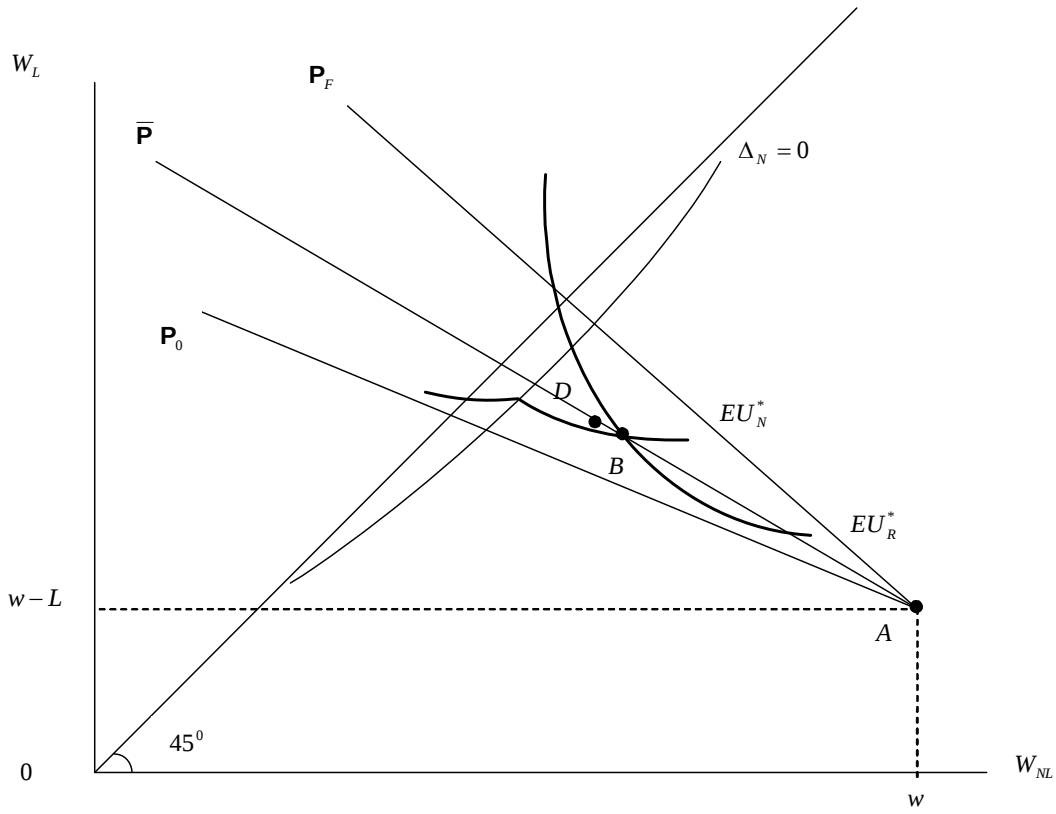


Figure 4: No Pooling Equilibrium at $\hat{I} < I < \bar{I}$: type N individuals invest in self-protection, whereas type R individuals do not. Indifference curve of type R individuals is steeper at B than that of type N individuals. Contract D attracts type N individuals but not type R individuals.

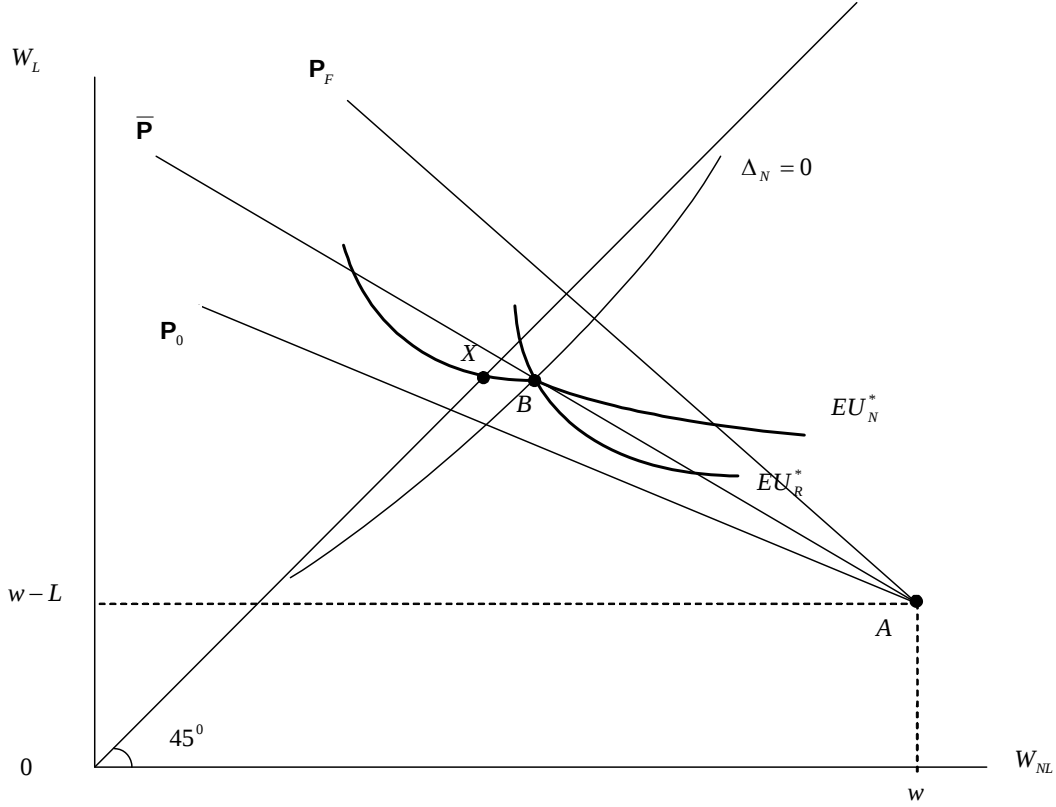


Figure 5: Pooling Equilibrium $(\bar{I}, \bar{p})$: type N individuals are indifferent between investing and not investing in self-protection under B , type R individuals do not invest in self-protection. Indifference curve of type R individuals is steeper at B than that of type N individuals and type R prefer contract B over any contract on P_0 . Note that contract D cannot be offered to attract type N individuals and induces them to not invest in self-protection and the company offering D would make losses.

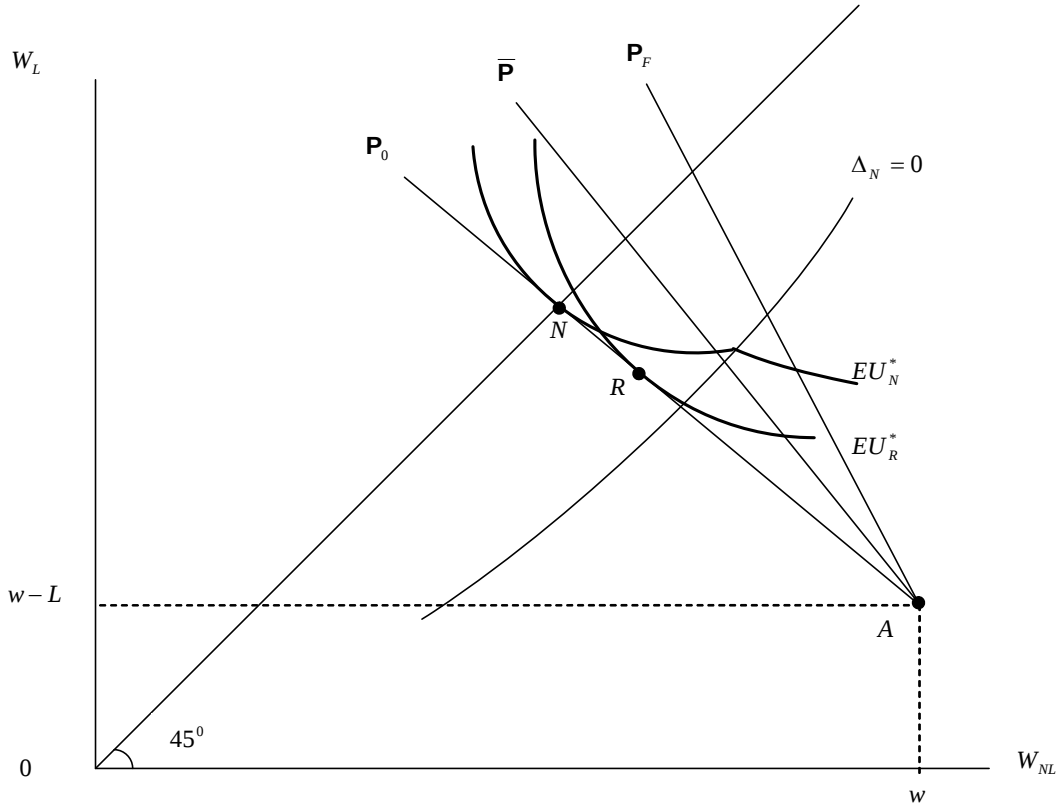


Figure 6: Separating Equilibrium: both types of individuals do not invest in self-protection and receive their respectively optimal amount of insurance coverage.

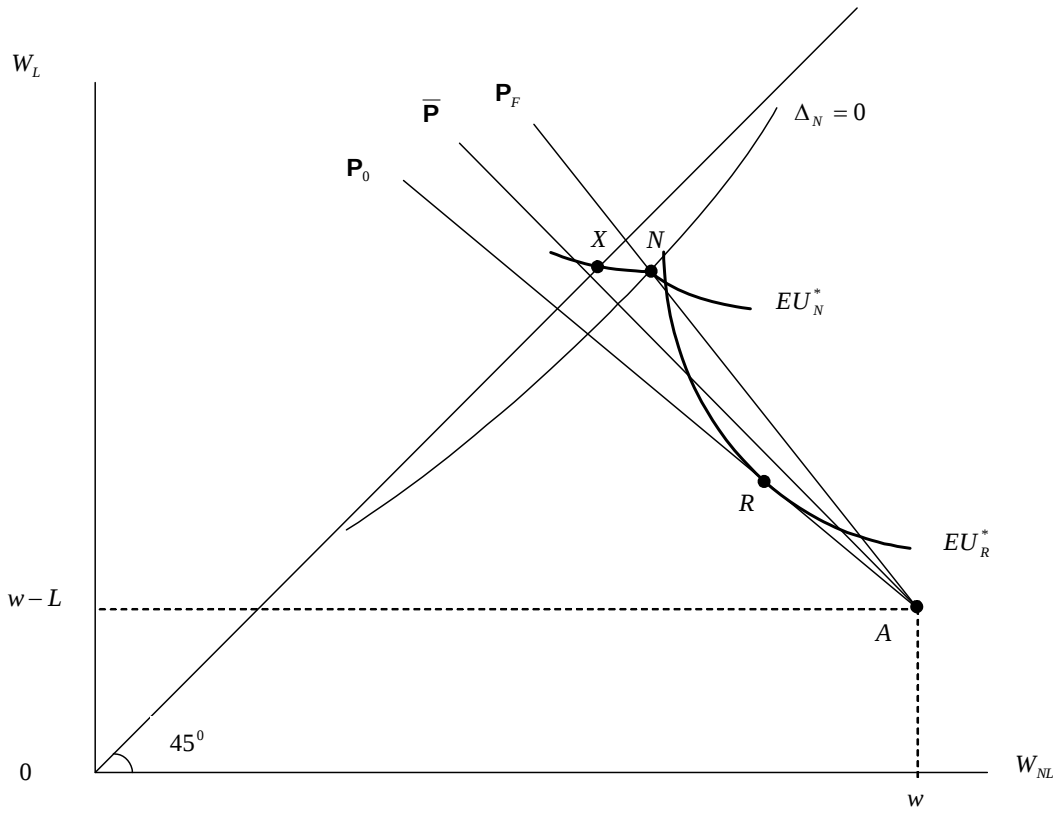


Figure 7: Separating Equilibrium with advantageous selection 1: type N individuals invest in self-protection, whereas type R individuals do not. Type N individuals obtain more insurance coverage than type R individuals.

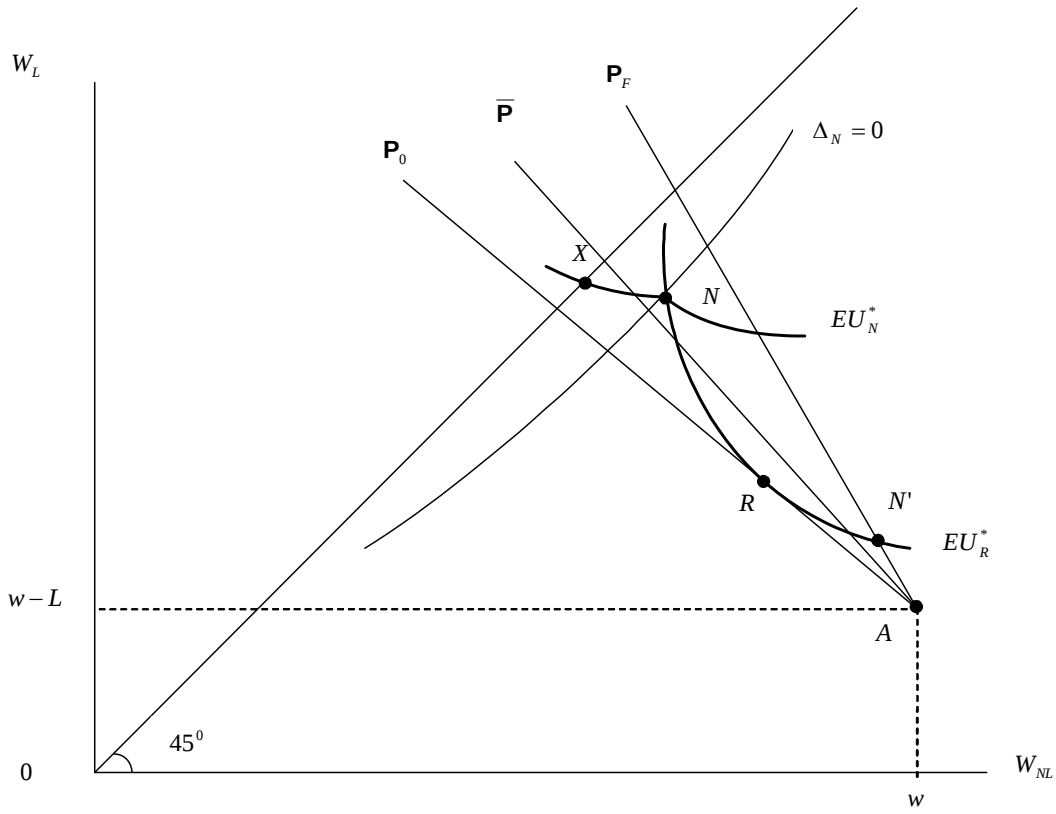


Figure 8: Separating Equilibrium with advantageous selection 2: type N individuals invest in self-protection, whereas type R individuals do not. Type N individuals obtain more insurance coverage than type R individuals.

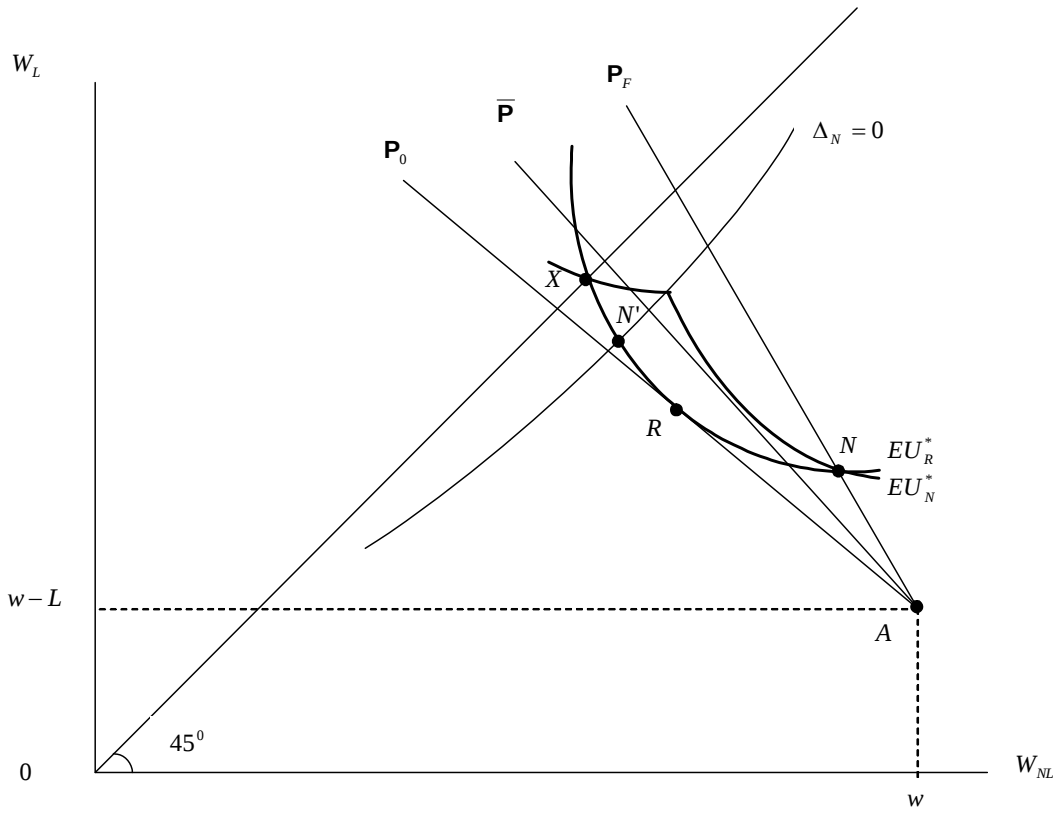


Figure 9: Separating Equilibrium with adverse selection: type N individuals invest in self-protection, whereas type R individuals do not. Type N individuals obtain less insurance coverage than type R individuals.

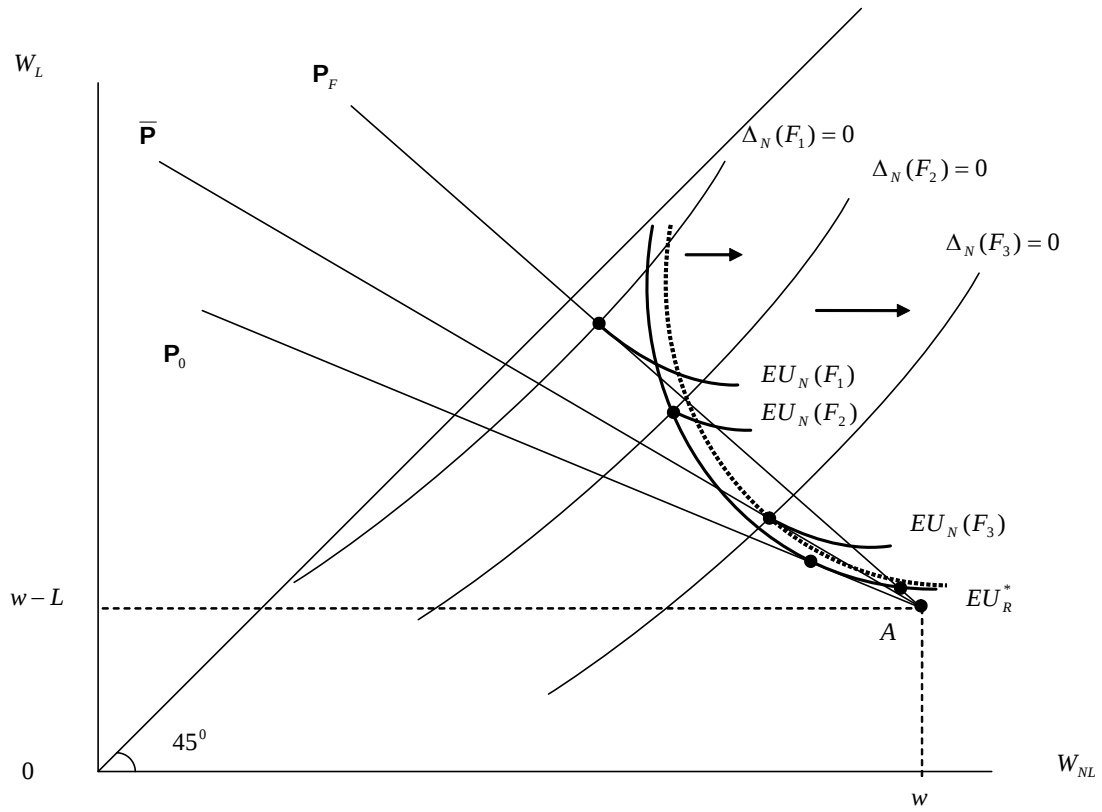


Figure 10: Comparative statics with respect to F - $F_1 < F_2 < F_3$.

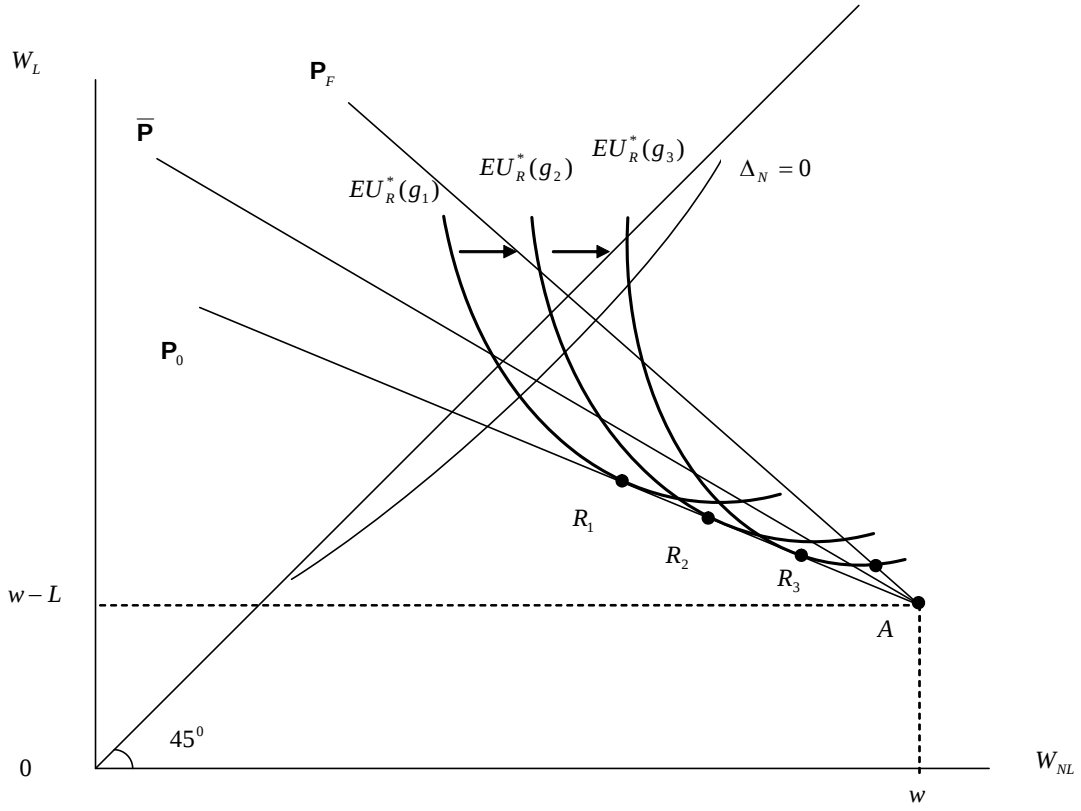


Figure 11: Comparative statics with respect to intensity of regret - g_3 is more convex than g_2 which is more convex than g_1 .