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Pricing Patents under Knowledge-Based Economy:

Real Option Approach

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中文摘要

專利權之持有人可選擇立即將具專利技術商品化或延後進入市場，因此實質權之架構來評估專利權之價值。本文另以跳躍擴散模式來推演競爭者加入對專利權價值的影響，結果不但可以解決 Takalo and Kanninen(2000)之問題，也可驗證 Sarkar(2000)之假說：即投資與不確定性具正向關係。另外本文亦可推論出專利權之保護政策應以提高專利權價值為目標，而非著眼於對新種技術提供最大保護。

關鍵詞：專利權、實質選擇權。

Abstract

The problem of pricing patents is a sequential investment decision. The patent holder can decide to launch new product immediately, or delay to the market introduction. The value of patent option is composed of the patent fee and the lost option value of deferring to patented new product. The numerical results in this paper illustrate the impact of rivalry entry on the value of patent option. Our numerical results not only fill up an open gap found by Takalo and Kanninen (2000), but also enhance the insight in the paper of Sarkar (2000) that there may be a positive relation between uncertainty and investment. Finally, our model suggests that a policy for the

purpose of patent protection should make the value of patent option higher, rather than focus on the maximum protection on new production or technology.

Keywords: Patent , Real options.

1 、 Introduction

In this paper we apply real option approach to evaluate patents and demonstrate a policy implication for the patent protection. The real option approach is extensively applied to the investment decision in recent years, because the traditional capital budgeting analysis doesn't take into account the existence of both market uncertainties and management flexibilities.

The patent holder has an option to launch the new product immediately, or postpone market introduction. Hence, the patent is similar to the compound option as illustrated in Geske (1979). Pakes (1986) applies the concept of compound option approach to investigating the patent renewal decision. The patent renewal fee indeed includes both the current returns, which accrues to the patent in the coming year, and the option to pay the renewal fee in the following period. In other words, the value of option is embedded in the patent renewal fee.

Recently Lambrecht (2000) treats the patent as a strategic real option and explores the problem of sleeping patent. Reiss (1998) treats the patent as a perpetual American call option, and analyzes the problems of patenting and commercializing new product¹. The major reason for the existence of patents is to encourage the firms engaging in the development of new product or technology for promoting technical progress. Takalo and Kanninen (2000) argue that the patents may actually lead the innovators to postponing market introduction of new product or technology. Actually, the methodology that adopted by these authors had been studied earlier by Dixit and Pindyck (1994). Although this paper follows that methodology to explore the problem of pricing patents, we furthermore present the numerical results, unlike the above authors, by using the *Newton-Raphson method*. Our numerical results not only fill a gap in the paper of Takalo and Kanninen, but also enhance the insight in the paper of Sarkar (2000) that there may be a positive relation between uncertainty and investment.

2.The Basic Model

¹ The patent gives its owner the right to invest in the project, the decision to invest has no deadline. As suggested by Lambrecht (2000), since patents are granted for a term of 17 years in the United States, patents can be treated as a perpetual American call option.

The patent holder has the exclusive right (but not the obligation) to launch the product innovation under patent protection at some specified investment cost K . When an innovator has received a patent on a product, he can immediately decide to commercialize the product innovation, or delay to implement the market introduction. Let x_t denote a random profit generated by the project at time $t \in [0, \infty]$, and x_t evolves exogenously according to the following geometric Brownian motion

$$dx_t = (r - d)x_t dt + \sigma x_t dW_t \quad (1)$$

and, by Ito's lemma, we then have

$$x_t = x_0 \exp\left\{\left[(r - d) - \frac{1}{2}\sigma^2\right]t + \sigma W_t\right\} \quad (2)$$

where r is a risk-free rate standing for the expected growth rate of x_t , d measures the opportunity cost for delaying implementation of the project. d is also known as the dividend yield or convenience yield on the value of underlying asset in the financial term. Hence $(r - d)$ represents net expected growth rate of x_t . $\sigma > 0$ is a volatility parameter or instantaneous standard deviation, and W_t follows a standard Brownian motion whose increment, dW_t , is normally distributed with mean zero and standard deviation dt .

The value of the project is the expected present value of random profit x_t under the world of risk-neutral. According to equation (2), the value of project $V_0(x)$ is as follows

$$V_0(x) = \int_0^\infty E(x_t) e^{-rt} dt = \int_0^\infty x_0 e^{(r-d)t} e^{-rt} dt = \frac{x_0}{d} \quad (3)$$

Thus the present value of project is simply proportional to the initial profit x_0 .

The patent holder has the right to delay the commercialization of the product innovation, thus, the patent option can be viewed as a perpetual American call option. The innovator's investment decision is equivalent to deciding under what conditions it is optimal to implement the project, which is an optimal stopping problem. We will denote the value of the patent option by $F(x)$. The value of the patent option from exercising at time t is $V(x_t) - K$, we want to maximize its expected present value

$$F(x) = \sup_t E\{[V(x_t) - K]e^{-rt}\} \quad (4)$$

where t is called the stopping time, and $\sup$ is taken for the all possible stopping time t . Since the patent option gives its owner the right to launch the product innovation, the value of the patent option cannot be negative, then, equation (4) can be rewritten as

$$F(x) = \max\{0, \sup_t E\{[V(x_t) - K]e^{-rt}\}\} \quad (5)$$

Equation (5) says that if $\max\{0, \sup_t E\{[V(x_t) - K]e^{-rt}\}\} = 0$

which creates the possibility of a sleeping patent, the innovator never commercializes

the product innovation.

The stopping time t^* that is optimally chosen so as to maximize the equation (4) is called the optimal stopping time t^* . If the optimal stopping time t^* satisfies equation (4), then equation (4) can be expressed as

$$F(x^*) = \sup_t E\{e^{-rt} [V(x_t) - K]\} = E\{e^{-rt^*} [V(x^*) - K]\} \quad (6)$$

where x^* is called the critical value, which partitions the set of all (x_t, t) into the mutually exclusive sets, one is so called

continuation region, $C = \{(x_t, t) : F(x^*) > V(x_t - K)\}$,

and the complementary set is called stopping region $S = \{(x_t, t) : F(x^*) = V(x_t) - K\}$. x^* is known as the optimal stopping boundary.

If the optimal stopping time t^* satisfies

$$t^* = \inf\{t \geq 0, x_t \geq x^*\}$$

then t^* is called the smallest optimal stopping time, hence the first time that (x_t, t) leaves the continuation region is the solution to equation (6). Though we can compute the expectations of all stopping time points that satisfy equation (6), and take the maximum of them as the solution, our methodology however, draws on the Bellman *Principle of Optimality* in the continuous time dynamic programming framework²

In the continuation region, the value of the patent option satisfies the following Bellman equation

$$rF(x)dt = E[dF(x)] \quad (7)$$

Using Ito's lemma to expand $dF(x)$, we have

$$dF(x) = F'(x)dx + \frac{1}{2}F''(x)(dx)^2 \quad (8)$$

where the primes denote the derivatives with respect to x . Substituting equation (1) into equation (8), and using the fact that $E(dW_t) = 0$, then the expected capital gain $E[dF(x)]$ can be written as

$$E[dF(x)] = (r - \sigma^2)x F'(x) + \frac{1}{2}\sigma^2 x^2 F''(x) \quad (9)$$

Combining equation (9) and equation (7), we have the following second-order ordinary differential equation in the value of the patent option $F(x)$

$$\frac{1}{2}\sigma^2 x^2 F''(x) + (r - \sigma^2)x F'(x) - rF(x) = 0 \quad (10)$$

To solve for $F(x)$, we need the following three boundary conditions

$$\lim_{x \rightarrow 0^+} F(x) = 0 \quad (11-1)$$

$$\lim_{x \rightarrow x^*} F(x) = V(x^*) - K \quad (11-2)$$

² For a rigorous exposition of the methodology, please refer to Dixit and Pindyck (1994).

³ For the detailed solution of the technique, please refer to Dixit and Pindyck (1994), and Shimko (1992).

$$\lim_{x \rightarrow x^*} F'(x) = V'(x^*) \quad (11-3)$$

Equation (11-1) says that if the profit of project x goes to zero, then the patent stays there (i.e., sleeps) forever. The value of the patent option would be worthless. Equation (11-2) is the value-matching condition, which simply states that when the critical value x^* is touched, then the innovator pays the sunk cost K to invest in the project for getting the payoff of the project. Equation (11-3) is known as the smooth-pasting condition, it implies that the slope of the value function $F(x)$ is continuous. Equation (11-2) and (11-3) jointly refer to the smooth fit conditions. Equation (10) and (11-1)—(11-3) constitute the problem of free boundary in the mathematical sense.

The solution to equation (10), hence the value of the patent option, has the form of³

$$\begin{aligned} F(x) &= A V_0(x)^a \quad \text{for } x < x^*, \\ &= V_0(x) - K \quad \text{for } x \geq x^*. \end{aligned} \quad (12)$$

Substituting equation (12) into (11-2) and (11-3), then we have

$$x^* = \frac{a}{a-1} dK, \quad (13-1)$$

$$\text{and, from equation (3)} \quad V_0(x^*) = \frac{a}{a-1} K. \quad (13-2)$$

$$A = [(V_0(x^*) - K) / V_0(x^*)]^a \quad (14)$$

Equation (13-1) or (13-2) is the optimal stopping boundary, which gives the closed-form solution to the investment decision. The power term a in equation (14) is the positive root that satisfies the following quadratic equation

$$\frac{1}{2} s^2 a(a-1) + (r-d)a - r = 0 \quad (15)$$

$$\text{and} \quad a = \frac{(s^2/2) - (r-d) + \sqrt{[(r-d) - (s^2/2)]^2 + 2rs^2}}{s^2} > 1$$

3. The Valuation Model of Patent Option

In the case of rival entry, we consider the following jump-diffusion process⁴

$$dx_t = (r-d)x_t dt + \sigma x_t dW_t - q x_t dq_t \quad (16)$$

where dq_t is a Poisson process with a constant jump intensity parameter λ ⁵, and

$$\begin{aligned} dq_t &= 0 && \text{with probability } 1 - \lambda dt \\ &= 1 && \text{with probability } \lambda dt \end{aligned}$$

dq_t and dW_t are independent, $E(dq_t dW_t) = 0$ for all t .

The jump-diffusion process in equation (16) is composed of two parts, one is continuous process part given by $(r-d)x_t dt + \sigma x_t dW_t$, and the other is a discontinuous jump process part given by $-q x_t dq_t$, where $q \in [0,1]$ is the percentage reduction in

⁴ Following Merton (1976), we assume that the jump risk is diversifiable, hence, not priced with a risk premium.

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x_t ascribing to a concurrent entrance of competitors. There is a small probability $/dt$ that x_t will drop to $(1-q)x_t$ at some small time interval dt when Poisson event occurs. Stadler (1991) has shown that the expected time of rival innovation decreases with an increasing hazard rate, which implies that the constant hazard rate $/dt$ can reasonably be regarded as a measure of the intensity of rivalry.

In the case of jump occurrence, the value of the project $V(x)$ can be expressed as follows⁶

$$V(x) = \frac{x_0}{/ + d} \quad (17)$$

In contrast to equation (3), this is just the jump adjusted growth rate of the perpetual profit. Compared with the situation without competitors entry, the value of the project will be eroded facing rival entry, thus, $V(x)$ can be expressed as a fraction of $V(x_0)$, i.e.,

$$V(x) = \frac{d}{/ + d} V(x_0) \quad (18)$$

It is easy to see that $\frac{d}{/ + d} < 1$, and $V(x)$ is a decreasing function of $/$. In other words, the higher value of intensity of rivalry $/$, the lower the value of the project $V(x)$ will be as compared with the case of no rival entry.

Let $F_I(x)$ be the value of the patent option, it follows that the expected capital gain $E[dF_I(x)]$ will be changed to

$$E[dF_I(x)] = \frac{1}{2} \sigma^2 x^2 F_I''(x) dt + (r - d)x F_I'(x) dt - / \{F_I(x) - F_I[(1-q)x]\} dt \quad (19)$$

The term $F_I(x) - F_I[(1-q)x]$ represents the impact of rival entry on the value of the patent option. Combining equation (19) and equation (7), we obtain the following second-order ordinary differential equation for the value of the patent option

$$\begin{aligned} & \frac{1}{2} \sigma^2 x^2 F_I''(x) + (r - d)x F_I'(x) \\ & - (/ + r)F_I(x) + /F_I[(1-q)x] = 0 \end{aligned}$$

Together with the three boundary conditions expressed in the previous section, the value of the patent option can be solved as

$$\begin{aligned} F_I(x) &= A V(x)^r \quad \text{for } x < x_I^* \\ &= V(x) - K \quad \text{for } x \geq x_I^* \end{aligned} \quad (20)$$

and the critical trigger value is

$$x_I^* = \frac{r}{r-1} (/ + d) K \quad (21-1)$$

hence, according to equation (18)

$$V(x_I^*) = \frac{r}{r-1} K \quad (20-2)$$

$$A = [V(x_I^*) - K] / V(x_I^*)^r \quad (21)$$

⁶ The proof is available from the authors upon request.

where r is the positive solution that satisfies the following nonlinear equation

$$\frac{1}{2}S^2 r(r-1) + (r-d)r - (r+1) + 1(1-q)^r = 0 \quad (22)$$

The value of r can be found by numerical method. The numerical results will be presented in section 4.

In what follows, we will briefly discuss the timing problem of marketing the new product, using the approach analogous to Reiss (1998), Takalo and Kanninen (2000). When the innovator has completed the product innovation, he faces two mutually exclusive decisions. The first is whether to patent the new product or not. We consider the following jump-diffusion process, which is different from equation (16)

$$dx_t = (r-d)x_t dt + Sx_t dW_t - x_t dq_t \quad (23)$$

Equation (23) describes the situation where innovator loses the all the possible values of the project, when the competitors develop some close substitutes. Let $F_p(x)$ denote the value of the option to patent the new product, which satisfies the following ordinary

$$\text{differential equation} \quad \frac{1}{2}S^2 x^2 F_p''(x) + (r-d)x F_p'(x) - (1+r)F_p(x) = 0 \quad (24)$$

We need the following boundary conditions, which are rather different from those boundary conditions in section 2, to find $F_p(x)$

$$\lim_{x \rightarrow 0^+} F_p(x) = 0 \quad (25-1)$$

$$\lim_{x \rightarrow x_p^*} F_p(x) = F_I(x_p^*) - P \quad (25-2)$$

$$\lim_{x \rightarrow x_p^*} F_p'(x) = F_I'(x_p^*) \quad (25-3)$$

where P denotes the patent fee, and x_p^* is the critical trigger point for the option to patent the new product. The option to patent new product will be exercised if $x \geq x_p^*$, otherwise the innovator will keep the option to patent the new product alive. Hence the solution to equation (24) is given by

$$\begin{aligned} F_p(x) &= A_p V(x)^b & \text{for } x < x_p^* \\ &= V(x) - P & \text{for } x \geq x_p^* \end{aligned} \quad (26)$$

Now b satisfies the following quadratic equation

$$\frac{1}{2}S^2 b(b-1) + (r-d)b - (r+1) = 0 \quad (27)$$

then

$b = 1/2 - (r-d)/S^2 + \sqrt{[(r-d)/S^2 - 1/2]^2 + 2(r+1)/S^2}$ Equation (25-3) is the value-matching condition which says that the following equation holds at the critical trigger value, x_p^*

$$F_I(x_p^*) = F_p(x_p^*) + P \quad (28)$$

When the innovator exercises the option to patent the new product, the effective cost of the patent is composed of the patent fee P and the lost option value of delaying to patent $F_p(x_p^*)$, as stated in equation (28).

Here we have two critical trigger points, which are x_p^* and x_I^* . If $x_p^* < x_I^*$, then, according equation (20), we have $F_I(x_p^*) = A_I V(x_p^*)^r$. In this case, the innovator

obtains the patent option, and waits for a better time for actual investments. When $x_p^* \geq x_I^*$, the best policy to the innovator is to acquire the patent option, and invests in the project immediately. The payoff is $F_I(x_p^*) = V(x_p^*) - K$.

4. The Results of Numerical Approximation

We use the *Newton-Raphson method* to compute the values of r in equation (22) at two different volatility parameters $S = 0.2$ and $S = 0.3$. For simplicity, we set $r = d = 0.04$ and $K = 1$. We choose three different values of β under each of the volatility parameters to simulate the impact of rival entry on the value of r and $V(x_I^*)$. The numerical results are, reported in Tables 1 and 2.

Table 1

The numerical results with volatility parameter $\sigma = 0.2$ under three different values of β

$\beta = 0.1$				$\beta = 0.5$				$\beta = 1.0$			
θ	ρ	$V(x_I^*)$	A_p	θ	ρ	$V(x_I^*)$	A_p	θ	ρ	$V(x_I^*)$	A_p
0.1	2.33	1.75	0.20	0.1	3.72	1.37	0.11	0.1	5.39	1.23	0.08
0.2	2.61	1.62	0.18	0.2	4.83	1.26	0.09	0.2	6.95	1.17	0.06
0.3	2.83	1.55	0.16	0.3	5.35	1.23	0.08	0.3	7.48	1.15	0.05
0.5	3.08	1.48	0.14	0.5	5.67	1.21	0.11	0.5	7.71	1.15	0.05
0.9	3.19	1.46	0.14	0.9	5.72	1.21	0.07	0.9	7.73	1.15	0.05

Table 2

The numerical results with volatility parameter $\sigma = 0.3$ under three different values of β

$\beta = 0.1$				$\beta = 0.5$				$\beta = 1.0$			
θ	ρ	$V(x_I^*)$	A_p	θ	ρ	$V(x_I^*)$	A_p	θ	ρ	$V(x_I^*)$	A_p
0.1	1.73	2.37	0.31	0.1	2.40	1.71	0.20	0.1	3.26	1.44	0.13
0.2	1.88	2.41	0.27	0.2	3.08	1.48	0.14	0.2	4.37	1.30	0.10
0.3	2.01	1.99	0.25	0.3	3.51	1.40	0.12	0.3	4.92	1.26	0.08
0.5	2.26	1.79	0.21	0.5	3.89	1.35	0.11	0.5	5.27	1.23	0.08
0.9	2.33	1.75	0.20	0.9	4.00	1.33	0.11	0.9	5.33	1.23	0.08

The numerical results in Tables 1 and 2 are consistent with the option pricing theory, which states that a higher value of volatility parameter will increase the option value.

We also find that a higher value of volatility parameter β leads to a higher critical trigger value $V(x_I^*)$. These findings are also consistent with the real option theory, which suggests that the higher volatility will increase the value of option to wait. But, if we fix the value of volatility parameter S , the critical trigger level $V(x_I^*)$ will be lower, the greater the level of the intensity parameter β . This means that when we consider both the values of S and β together, the conclusion that higher volatility will increase the value of option to wait cannot be reached unambiguously.

5. Conclusion

The patent holder has an option to launch new product immediately, or delay the market introduction. Jump-diffusion process is applied to price the patent when rivalry entry is allowed. The numerical results show that the scopes of patent

protection and the intensity of rivalry entry will affect the investment decision of an innovator. We also find that the value of volatility has a negative relation with the elasticity of patent option. For policymakers, it is important to realize that maximizing the value of the patent option is more meaningful than maximizing the protection for new technology. In other words, partial protection may be better for innovation and investments than full protection.

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