行政院國家科學委員會專題研究計畫成果報告

廣義平面負荷作用下，位於層狀土壤上基礎結構的承載力之有限元極限分析

A FEM Limit Analysis of Bearing Capacity of Foundations on Layered Soils under General Planar Loads

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一、中文摘要

廣義平面負荷作用下，位於層狀土壤上基礎結構的承載力之有限元極限分析本研究計畫旨在利用一有限元素極限分析法，以嚴謹探討一位於層狀土壤上之基礎結構的承載力，而此基礎結構承受一廣義平面負荷。本研究計畫的成果可做為一般土木工程的基礎設計，道路工程的鋪面設計，或飛機場的跑道設計估算承載力的重要依據，以補一般工程經驗公式或文獻上探討的不足。

如眾所周知，簡單問題的承載力估算可參考一些經驗公式，以做為保守設計的依據，但複雜問題往往不是經驗公式所能涵蓋。另一方面，雖然文獻以對承載力已有廣泛研究，然而大多局限於垂直、正中型的負荷探討，或僅考慮廣義負荷作用在均勻土壤層時的情況。本研究計畫利用非線性規畫來描述處理此類非線性問題。配合有限元素法的運用，將原本連續性的非線性塑力問題離散成矩陣與向量的表示形式。最後以一有效的計算法來迭代求解最佳化問題的非線性方程式。

關鍵詞：極限分析，上限解，有限元素法，層狀土壤，廣義平面載荷。

Abstract

The project aims to rigorously study the bearing capacity of foundations on layered soils under general planar loads by employing a finite-element limit analysis. The study is expected to benefit the design of foundations, pavements and the prediction of aircraft ground operation by providing rigorous upper bound solutions.

In literature, most efforts were made to estimate the bearing capacity corresponding to vertical loads. However, few studies were concerned with inclined, eccentric loading conditions and, if any, focused only on problems involving homogeneous media due to the difficulties in assuming an appropriate failure mechanism or the computation while dealing with layered media under general planar loads. In this project, we applied the upper bound theorem of limit analysis to give reliable upper bound solutions to the concerned problems. First, we treat the problem by using nonlinear programming method. Followed, the problem was converted to a matrix-vector form by a finite-element discretization. Finally, a general algorithm is utilized to seek the least upper bound to approximate the true solution by solving the resulting optimization problem.

Keywords: limit analysis, upper bound, finite element method, bearing capacity, layered soils, general planar loads

二、緣由與目的

當土壤結構發生降伏失效時，所能承受的外在極限負荷稱為承載力。在一般土木工程的基礎設計，道路工程的鋪面設計，或飛機場的跑道設計皆涉及到承載力的估算，以做為設計階段的重要依據。工程上所面對的問題，乃是來自地震作用、車輛或飛機等動態操作所引致而呈傾斜、偏心型式的負荷，而此廣義性負荷是由呈層狀分佈的土壤結構來承受。數學上來說，這是塑性力學上的非線性問題。由於此類問題的失效模式不易由工程經驗研判，而數值方法亦牽涉到非線性所關聯的一些複雜問題，因此完整而嚴謹的數值分析架構亟需建立。本研究計畫的目的在於利用非線性規畫以最佳化方式來處理承載力問題，以期發展一嚴謹、可靠的承載力數值分析架構，以補足一般工程經驗公式或文獻上探討的不足。如眾所周知，簡單問題的承載力估算可參考一些經驗公式，以做為保守設計的依據，但複雜問題往往不是經驗公式所能涵蓋。另一方面，雖然文獻以對承載力已有廣泛研究，然而大多局限於垂直、正中型的負荷探討，或僅考慮廣義負荷作用在均勻土壤層時的情況。本研究計畫利用非線性規畫來描述處理此類非線性問題。配合有限元素法的運用，將原本連續性的非線性塑力問題離散成矩陣與向量的表示形式。最後以一有效的計算法來迭代求解最佳化問題的非線性方程式。

三、方法

The mathematical modeling of our general problem are depicted as follows:

A. Lower Bound Formulation

The problem maximizes the load factor under constraints of equilibrium equations, static boundary conditions and the von Mises yield criterion is

maximize $\lambda(\sigma)$

subject to $\nabla\cdot\sigma+\bar{f}=0$ in D

$$\sigma\cdot\bar{n}=\lambda\bar{t} \quad \text{on } \delta D_{s1}$$

$$\sigma\cdot\bar{n}=\bar{q}_s \quad \text{on } \delta D_{s2}$$

$$\|\sigma\|_v\le c \quad \text{in D}$$

where

$$\delta D_s=\delta D_{s1}+\delta D_{s2}$$

$$\|\sigma\|_v=\sqrt{\frac{1}{2}S_{ij}S_{ij}}$$

and $\bar{f}$ is body force, $\bar{t}$ is the scalable distribution of the traction vector induced by footing on δD_{s1} with the load factor λ being the lower bound functional $\lambda(\sigma)$, $\bar{q}_s$ is the internal pressure applied on the boundary δD_{s2}.

The constrained problem is to seek the maximum load factor λ representing the action of a footing. However, if the maximum load factor λ =0, then we attain the collapse condition corresponding to the loading. This problem can also be interpreted as a statement of an elasticity problem if the convex constraint condition on stress tensor, . $\|\sigma\|_v\le c$, is not violated. Further, this problem is a convex programming problem. To resort to displacement- or velocity-based finite-element methods, we transform the lower bound formulation to the upper bound formulation as below.

B. Upper Bound Formulation

Equilibrium equations can be expressed alternatively in a weak form as

$$\int_D \bar{u}\cdot(\nabla\cdot\sigma+\bar{f})dA=0$$

Integration by parts gives

$$\int_{\partial D_t}\overline{u}\cdot\sigma\cdot\overline{n}-\int_D\sigma:\nabla\overline{u}dA+\int_D\overline{u}\cdot\overline{f}dA=0$$

After some manipulation, we may rewrite the above equation as

$$\lambda(\int_{\partial D_{s1}}\overline{u}\cdot\overline{t}dS)=$$
$$\int_D\sigma:\dot{\varepsilon}dA-\int_D\overline{u}\cdot\overline{f}dA-\int_{\partial D_{s2}}\overline{u}\cdot\overline{q}_s dS$$

Where $\dot{\varepsilon}$ is the strain rate matrix. Since $\overline{u}$ appears homogeneously and linearly in the above equation, we can scale $\overline{u}$ to normalize the equations such that

$$\int_{\partial D_{t1}}\overline{u}\cdot\overline{t}dS=1 \quad \text{on} \quad \delta D_{s1}$$

Now by aid of the generalized Holder inequality(Yang, 1991a), we have the inequlity in plasticity

$$|\sigma:\dot{\varepsilon}|\le\|\sigma\|_{(p)}\|\dot{\varepsilon}\|_{(d)}$$

where $\|\cdot\|_{(d)}$ is the dual norm in relation to the primal norm $\|\cdot\|_{(p)}$.

Thus it is not hard to show that $\lambda(\sigma)$ is bounded above by $\overline{\lambda}(\overline{u})$ and our problem can be reiterated as the upper bound formulation in the form of a constrained minimization problem as

Minimize $\overline{\lambda}(\overline{u})$

Subject to

$$\overline{\lambda}(\overline{u})=\int_D c\sqrt{(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y})^2+(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x})^2}dA$$
$$-\int_D\overline{u}\cdot\overline{f}dA-\int_{\partial D_{s2}}\overline{u}\cdot\overline{q}_s dS$$

$$\int_{\partial D_{s1}}\overline{u}\cdot\overline{t}dS=1 \quad \text{on} \quad \delta D_{s1}$$

$\nabla\cdot\overline{u}=0$ in D

Kinematic B.C.s on δD_k

四、算例

本計畫所完成的部份算例，與文獻14，15 比較的結果皆相當符合。有關結果請參閱文獻 25。

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