Drop behaviour of random early detection with discrete-time batch Markovian arrival process

Y.-C. Wang, J.-A. Jiang and R.-G. Chu

Abstract: A matrix-analytic approach is applied to analyse both the long-term and the short-term drop behaviour of a router with an RED scheme. The bursty nature of packet drop is examined by means of conditional statistics with respect to alternating congested and non-congested periods. All of the four related performance measures are derived by conditional statistics, including the long-term drop probability, and the three short-term measures comprising average length of a congested period, average length of a non-congested period, and the conditional packet drop probability during a congested period.

1 Introduction

TCP congestion control is receiving increased attention due to its functions in terms of Internet stability and robust bandwidth resources management on an end-to-end per-connection basis [1]. The essence of this congestion control scheme is that a TCP sender adjusts its sending rate according to the probability of packets being dropped in the Internet router. In traditional implementations of router buffer management, the packets are dropped when the buffer becomes full, a mechanism referred to as 'drop-tail'. In order to overcome the synchronisation of TCP session problems encountered in 'drop-tail', the RED buffer management mechanism has been proposed [2].

So far, much research attention has been focused on the performance and behaviour of RED congestion control schemes. Firoiu and Borden [3] proposed a method for configuring an RED congestion control scheme, based on a model of RED as the feedback control system with TCP sessions. Chiu and Jain [4] formulated a set of basic principles for the additive-increase and multiplication-decrease congestion avoidance to increase efficiency. In [5], Kelly, Maulloo and Tan showed that a network deploying the additive-increase and multiplicative-decrease congestion avoidance tends to have a distribution rate according to proportional fairness. Vojnovic *et al.* [6] showed that in a network employing additive-increase and multiplication-decrease, the source rates tend to be distributed in order to maximise the objective function of fairness. In [7], Misra, Gong, and Towsley used jump process-driven stochastic differential equations to model the interactions of TCP sessions and RED routers in the network setting. In order to reduce the analysis complexity of the RED mechanism, some studies have chosen to bypass the interactions between TCP sessions and RED mechanisms, instead constructing a simple queueing model for RED mechanism. For example, May *et al.* [8] proposed a simple analytic model with Poisson input process for RED, and used these models to quantify the properties of RED. Wang [9] proposed the MAP/M/1/K queueing system with an RED scheme to derive the loss information of the RED mechanism. Although all of the above papers explore the characteristics of a TCP congestion control scheme, they neglect the quality of service problem in the Internet. In contrast, this paper considers this problem and investigates the drop behaviour of the router with an RED scheme.

We propose a complicated discrete-time queueing model for the router with an RED scheme, and use this model to analyse the drop behaviour of RED. The packet stream is considered to follow a discrete-time batch Markovian arrival process (D-BMAP) [10, 11], and the queueing model of the router with an RED scheme can be modelled as D-BMAP/D/1/K. With a threshold level set in the RED scheme, the drop behavior of the D-BMAP/D/1/K queueing system with the RED scheme is characterised by examining the conditional statistics in a congested period and in a non-congested period through two hypothesised discrete-time absorbing Markov chains. Formulas are derived to explore the distributions of the lengths of a congested period and a non-congested period. In addition, the distribution of the number of packets dropped during a congested period and the long-term packet drop rate are evaluated.

2 Traffic model

In [11], it is shown that traffic with certain bursty features can be qualitatively modelled by a generic Markovian arrival process, called the batch Markovian arrival process (BMAP). BMAP is a generalisation of the batch Poisson process which allows for non-exponential inter-arrival times of batches, while still preserving an underlying Markovian structure. It is a point process with group arrivals generated at the transition epochs of a particular type of m-state Markov process. Many familiar processes such as MMPP, PH-renewal and MAP can be considered as special cases of BMAP [10, 11]. The application of D-BMAP is proposed in

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Y.-C. Wang is with the Department of Electrical Engineering, National Taipei University of Technology, No. 1, Sec. 3, Chung-Hsiao E. Rd., Taipei 106, Taiwan, Republic of China

J.-A. Jiang is with the Department of Bio-Industrial Mechatronics Engineering, National Taiwan University, No. 1, Sec. 4, Roosevelt Rd., Taipei 106, Taiwan, Republic of China

R.-G. Chu is with Lee Ming Institute of Technology, No. 2, Li-Chuan Rd., Liming Village, Taishan, Taipei 243, Taiwan, Republic of China

[12] to model video sources. The arrival processes discussed in this paper are assumed to be D-BMAPs since time is assumed to be slotted.

A D-BMAP can be described by a special type of discrete-time Markov chain. Let $\{(N(n), J(n))\}_{n\geq 0}$ be a discrete-time Markov chain with two-dimensional state space $\{(l, j)|l\geq 0, 1\leq j\leq m\}$ and transition probability matrix

$$\begin{bmatrix} \boldsymbol{D}_0 & \boldsymbol{D}_1 & \boldsymbol{D}_2 & \boldsymbol{D}_3 & \cdots \\ 0 & \boldsymbol{D}_0 & \boldsymbol{D}_1 & \boldsymbol{D}_2 & \cdots \\ 0 & 0 & \boldsymbol{D}_0 & \boldsymbol{D}_1 & \cdots \\ & & & \cdot & \cdots \end{bmatrix}$$

where $N(n)$ stands for a counting variable, $J(n)$ represents an auxiliary state or phase variable, and $\boldsymbol{D}_i$ are non-negative $m\times m$ matrices whose entries are between 0 and 1, called parameter matrices. The transition probability from state (l, j) to state $(l+i, j')$, which corresponds to the arrival of a batch of size i, is the (j, j')th entry $(D_i)_{j,j'}$ of the $m\times m$ matrix $\boldsymbol{D}_i$. $(D_i)_{j,j'}$ may depend on phases j and j'. The sum of all parameter matrices

$$\boldsymbol{D} = \sum_{i=0}^{\infty} \boldsymbol{D}_i \qquad (1)$$

is an $m\times m$ stochastic matrix which is the transition probability matrix of the underlying Markovian structure $\{J(n)\}_{n\geq 0}$ with respect to the D-BMAP. $(\boldsymbol{I}-\boldsymbol{D}_0)$ is assumed to be non-singular such that the sojourn time at any state of the state space $\{(l, j)|l\geq 0, 1\leq j\leq m\}$ is finite with probability 1, thus guaranteeing that the process never terminates. The fundamental arrival rate λ of this D-BMAP can then be defined as

$$\lambda = \boldsymbol{\pi}\left(\sum_{i=1}^{\infty} i\boldsymbol{D}_i\right)\boldsymbol{e} \qquad (2)$$

where $\boldsymbol{\pi}$ is the stationary probability vector of $\boldsymbol{D}$ in (1), i.e. $\boldsymbol{\pi D}=\boldsymbol{\pi}$, $\boldsymbol{\pi e}=1$, and $\boldsymbol{e}$ is assumed in this paper to be the all-1 column vector with the designated dimension.

3 Drop behaviour of RED

To characterise the real packet drop behaviour of an RED, it is not adequate to examine only the long-term packet drop probabilities. For example, a packet stream may experience the dropping of a string of consecutive packets followed by bursty arrivals, though the long-term packet drop probability is small. This phenomenon makes the traffic source suffer from a significant quality of service degradation in that time period. Therefore, in light of the high correlation among consecutive packet arrivals in high speed networks, it is necessary to study the packet loss behaviour during a short-term interval, i.e. the conditional packet loss behaviour as well as during long-term intervals, in order to characterise the real packet drop behaviour of an RED queueing system.

As demonstrated in the previous section, traffics will be modelled by D-BMAPs. Determining the characterising parameter matrices for a D-BMAP is, of course, an essential problem. This obstacle is not dealt with here; and a large class of variable bit rate (VBR) sources and their superpositions have already been studied in [12].

3.1 RED queueing system

In this Section, the proposed basic model is described, and used to examine the related loss information. We consider a single server queue with a buffer size K. With the RED buffer management scheme, incoming packets are dropped with a probability that is an increasing function of the queue size k. A drop probability is defined by two parameters, min_{th} and max_{th}, e.g.

$$q_k = \begin{cases} 0 & k < min_{th} \\ \dfrac{k - min_{th}}{max_{th} - min_{th}} & min_{th} \leq k < max_{th} \\ 1 & k \geq max_{th} \end{cases}$$

The RED buffer management scheme is shown in Fig. 1. Note that there is no particular reason for choosing $max_{th}<K$ in this case, hence we let $max_{th}=K$.

With time-slotted and service time assumed to be constant for each packet, an RED queue with finite buffer capacity K (packets) can be modelled by a D-BMAP/D/1/K queue. Consider the embedded Markov chain $\{(L(n), J(n))\}_{n\geq 0}$ of the queueing system, which can be described as a particular type of semi-Markov process where the state jumps regularly at a constant slot time [13, pp. 220–221]. This is considered in the state space $\{0,1,\ldots, K\}\times\{1,2,\ldots, m\}$, where $L(n)$ and $J(n)$ denote the buffer occupancy and the phase of the D-BMAP respectively at the end of the nth time slot. For convenience, the queueing system is said to be at a level j if its buffer occupancy is equal to j. Under the RED scheme with a threshold min_{th} to indicate a congestion level of the buffer occupancy, the embedded Markov chain now has an irreducible transition probability matrix of the following block form:

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{D}_0 & \boldsymbol{D}_1 & \cdots & \boldsymbol{D}_{min_{th}-1} & \boldsymbol{D}_{min_{th}} & \cdots & \boldsymbol{D}_{K-1} & \sum_{i=K}^{\infty}\boldsymbol{D}_i \\ \boldsymbol{D}_0 & \boldsymbol{D}_1 & \cdots & \boldsymbol{D}_{min_{th}-1} & \boldsymbol{D}_{min_{th}} & \cdots & \boldsymbol{D}_{K-1} & \sum_{i=K}^{\infty}\boldsymbol{D}_i \\ 0 & \boldsymbol{D}_0 & \cdots & \boldsymbol{D}_{min_{th}-2} & \boldsymbol{D}_{min_{th}-1} & \cdots & \boldsymbol{D}_{K-2} & \sum_{i=K-1}^{\infty}\boldsymbol{D}_i \\ 0 & 0 & \cdots & \boldsymbol{D}_{min_{th}-3} & \boldsymbol{D}_{min_{th}-2} & \cdots & \boldsymbol{D}_{K-3} & \sum_{i=K-2}^{\infty}\boldsymbol{D}_i \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{D}_1 & \boldsymbol{D}_2 & \cdots & \boldsymbol{D}_{K-min_{th}-3} & \sum_{i=K-min_{th}-2}^{\infty}\boldsymbol{D}_i \\ 0 & 0 & \cdots & \boldsymbol{E}_0(q_{min_{th}}) & \boldsymbol{D}_1(q_{min_{th}}) & \cdots & \boldsymbol{D}_{K-min_{th}} - 2(q_{min_{th}}) & \sum_{i=K-min_{th}-1}^{\infty}\boldsymbol{D}_i(q_{min_{th}}) \\ 0 & 0 & \cdots & 0 & \boldsymbol{E}_0(q_{min_{th}+1}) & \cdots & \boldsymbol{D}_{K-min_{th}} - 1(q_{min_{th}+1}) & \sum_{i=K-min_{th}}^{\infty}\boldsymbol{D}_i(q_{min_{th}+1}) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & \boldsymbol{D}_1(q_{K-1}) & \sum_{i=2}^{\infty}\boldsymbol{D}_i(q_{K-1}) \\ 0 & 0 & \cdots & 0 & 0 & \cdots & \boldsymbol{E}_0(q_K) & \sum_{i=1}^{\infty}\boldsymbol{D}_i(q_K) \end{bmatrix} \qquad (3)$$

where $\boldsymbol{E}_0(q_j) \equiv \boldsymbol{D}_0 + \sum_{i=1}^{\infty} q_j\boldsymbol{D}_i$, $\boldsymbol{D}_i(q_j) \equiv (1-q_j)\boldsymbol{D}_i$, $min_{th}\leq j\leq K$. Each block is of dimension $m\times m$ and corresponds to the transition from one buffer level to another buffer level. The first min_{th} columns of the $(K+1)\times(K+1)$ block transition probability matrix $\boldsymbol{Q}$ in (3) correspond to the transitions to the non-congestion buffer levels from 0 to $min_{th}-1$ where no packets will be dropped. The last $K-min_{th}+1$ columns of the block matrix

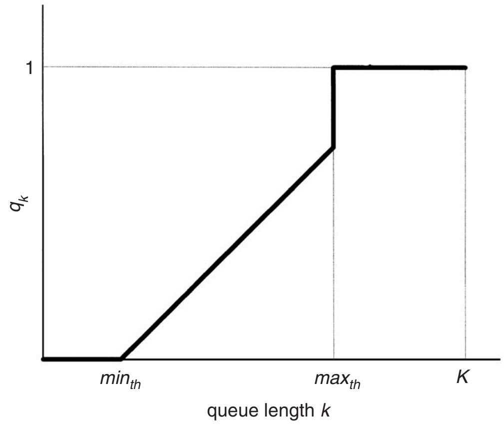

Fig. 1 *RED buffer management scheme*

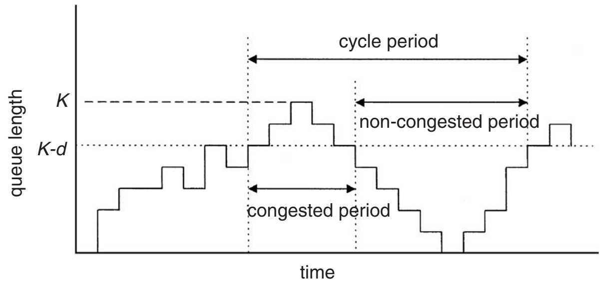

Fig. 2 *Sample path of buffer occupancy in an RED queueing system with buffer capacity K and threshold $min_{th} = K-d$*

$\boldsymbol{Q}$ correspond to the transitions to the congestion buffer levels from min_{th} to K, where incoming packets will be discarded. The incoming packets will be dropped due to buffer overflow, as indicated in the last column of the block matrix $\boldsymbol{Q}$.

3.2 Long-term packet drop probabilities

Let $\bar{\boldsymbol{x}} = [\boldsymbol{x}_0, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_K]$ with $\boldsymbol{x}_k = [x_{k,1}, x_{k,2}, \ldots, x_{k,m}]$ $\forall k$, be the steady-state probability vector of the queueing system, i.e. $\bar{\boldsymbol{x}}\boldsymbol{Q} = \bar{\boldsymbol{x}}$ and $\bar{\boldsymbol{x}}\boldsymbol{e} = 1$. Let *Drop* denote the number of packets dropped during a time slot due to the RED scheme, with only long-term packet drop probability considered as significant. Now, the expected value $E[Drop]$ of *Drop* can be evaluated as

$$\begin{aligned} E[Drop] =& \boldsymbol{x}_0 \sum_{i=1}^{\infty} iD_{K+i}e \\ &+ \sum_{k=1}^{min_{th}-1} \boldsymbol{x}_k \left(\sum_{i=1}^{\infty} iD_{K-k+1+i} \right) \boldsymbol{e} \\ &+ \sum_{k=min_{th}}^{K} \boldsymbol{x}_k \left(\sum_{i=1}^{\infty} iq_k D_i \right) \boldsymbol{e} \\ &+ \sum_{k=min_{th}}^{K} \boldsymbol{x}_k \left(\sum_{i=1}^{\infty} i(1-q_k) D_{K-k+1+i} \right) \boldsymbol{e} \end{aligned} \quad (4)$$

by considering the last $K - min_{th}$ columns of the block matrix $\boldsymbol{Q}$ in (3). Consequently, the long-term packet drop probability, denoted by P_{drop}, is

$$P_{drop} = \frac{E[Drop]}{\lambda} \quad (5)$$

where λ is the fundamental arrival rate of the packet stream as in (2).

3.3 Two hypothesised Markov chains

As shown in Fig. 2, the level of buffer occupancy of a queueing system passes through alternating congested and non-congested periods. A congested period is indicated by a period of bursty arrivals. To study the short-term loss behaviour of an RED queue during a bursty period, we decompose the state space $\{0,1,\ldots, K\} \times \{1,2,\ldots, m\}$ into two subsets

$$\begin{aligned} \Omega_{nc} &= \{0, 1, \ldots, min_{th} - 1\} \times \{1, 2, \ldots m\} \\ \Omega_c &= \{min_{th}, min_{th} + 1, \ldots, K\} \times \{1, 2, \ldots, m\} \end{aligned}$$

according to the congestion buffer level min_{th}. With this partition of the state space, the transition probability matrix $\boldsymbol{Q}$ in (3) of the embedded Markov chain of the queueing system can be partitioned as follows

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{U}_{nc} & \boldsymbol{A}_{nc,c} \\ \boldsymbol{A}'_{c,nc} & \boldsymbol{U}_c \end{bmatrix} \quad (6)$$

where

$$\boldsymbol{U}_{nc} = \begin{bmatrix} \boldsymbol{D}_0 & \boldsymbol{D}_1 & \cdots & \boldsymbol{D}_{min_{th}-2} & \boldsymbol{D}_{min_{th}-1} \\ \boldsymbol{D}_0 & \boldsymbol{D}_1 & \cdots & \boldsymbol{D}_{min_{th}-2} & \boldsymbol{D}_{min_{th}-1} \\ 0 & \boldsymbol{D}_0 & \cdots & \boldsymbol{D}_{min_{th}-3} & \boldsymbol{D}_{min_{th}-2} \\ 0 & 0 & \cdots & \boldsymbol{D}_{min_{th}-4} & \boldsymbol{D}_{min_{th}-3} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{D}_0 & \boldsymbol{D}_1 \end{bmatrix}$$

$$\boldsymbol{A}_{nc,c} = \begin{bmatrix} \boldsymbol{D}_{min_{th}} & \boldsymbol{D}_{min_{th}+1} & \cdots & \boldsymbol{D}_{K-1} & \sum_{i=K}^{\infty} \boldsymbol{D}_i \\ \boldsymbol{D}_{min_{th}} & \boldsymbol{D}_{min_{th}+1} & \cdots & \boldsymbol{D}_{K-1} & \sum_{i=K}^{\infty} \boldsymbol{D}_i \\ \boldsymbol{D}_{min_{th}-1} & \boldsymbol{D}_{min_{th}} & \cdots & \boldsymbol{D}_{K-2} & \sum_{i=K-1}^{\infty} \boldsymbol{D}_i \\ \boldsymbol{D}_{min_{th}-2} & \boldsymbol{D}_{min_{th}-1} & \cdots & \boldsymbol{D}_{K-3} & \sum_{i=K-2}^{\infty} \boldsymbol{D}_i \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{D}_1 & \boldsymbol{D}_2 & \cdots & \boldsymbol{D}_{K-min_{th}-3} & \sum_{i=K-min_{th}-2}^{\infty} \boldsymbol{D}_i \end{bmatrix}$$

$$\boldsymbol{A}'_{c,nc} = \begin{bmatrix} 0 & 0 & \cdots & 0 & \boldsymbol{E}_0(q_{min_{th}}) \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{U}_c = \begin{bmatrix} \boldsymbol{D}_1(q_{min_{th}}) & \boldsymbol{D}_2(q_{min_{th}}) & \cdots & \boldsymbol{D}_{K-min_{th}-2}(q_{min_{th}}) & \sum_{i=K-min_{th}-1}^{\infty} \boldsymbol{D}_i(q_{min_{th}}) \\ \boldsymbol{E}_0(q_{min_{th}+1}) & \boldsymbol{D}_1(q_{min_{th}+1}) & \cdots & \boldsymbol{D}_{K-min_{th}-1}(q_{min_{th}+1}) & \sum_{i=K-min_{th}}^{\infty} \boldsymbol{D}_i(q_{min_{th}+1}) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{E}_0(q_K) & \sum_{i=1}^{\infty} \boldsymbol{D}_i(q_K) \end{bmatrix}$$

The matrices $\boldsymbol{U}_{nc}$, $\boldsymbol{A}_{nc,c}$, $\boldsymbol{A}'_{c,nc}$ and $\boldsymbol{U}_c$ are transition probability submatrices governing transitions from Ω_{nc} into itself, from Ω_{nc} into Ω_c, from Ω_c into Ω_{nc} and from Ω_c into itself, respectively.

To study the distribution of the length of a non-congested period, we consider those times for the queueing system that start from a state in the non-congested state subspace Ω_{nc} and enter a state in the congestion state subspace Ω_c for the first time (i.e. no other states in Ω_c have been visited before). We use $\tau^{nc,c}_{(i,j),(i',j')}$ to denote such a time from a state (i, j) in Ω_{nc} to a state (i', j') in Ω_c.

To investigate the distribution of $\tau^{nc,c}_{(i,j),(i',j')}$, a hypothesised Markov chain is defined by modifying the stochastic transition matrix $\boldsymbol{Q}$ of the embedded Markov chain as

$$\boldsymbol{Q}^{nc} = \begin{bmatrix} \boldsymbol{U}_{nc} & \boldsymbol{A}_{nc,c} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \tag{7}$$

where $\boldsymbol{U}_{nc}$ and $\boldsymbol{A}_{nc,c}$ are in (6) and $\boldsymbol{0}$ and $\boldsymbol{I}$ are the all-zero and the identity matrices with the designated dimensions respectively. Futhermore, since the level of buffer occupancy in the original queueing system starts at $min_{th}-1$ in every non-congested period (except possibly the first one), the initial probability vector of this hypothesised Markov chain is assumed to be $[\tilde{\boldsymbol{\alpha}}, \tilde{\boldsymbol{0}}]$, where $\tilde{\boldsymbol{\alpha}}$ is a row vector of the form $[\bar{\boldsymbol{0}}, \bar{\boldsymbol{0}}, \ldots, \bar{\boldsymbol{0}}, \boldsymbol{\alpha}_{min_{th}-1}]$ in which $\boldsymbol{\alpha}_{min_{th}-1} = [\boldsymbol{\alpha}_{min_{th}-1,1}, \ldots, \boldsymbol{\alpha}_{min_{th}-1,m}]$, such that $\boldsymbol{\alpha}_{min_{th}-1}\boldsymbol{e} = 1$. Here, $\tilde{\boldsymbol{0}}$ and $\bar{\boldsymbol{0}}$ are all-zero row vectors with the designated dimensions. With this setup, Ω_c consists of the entire absorbing states in this hypothesised Markov chain, and $\tau^{nc,c}_{(i,j),(i',j')}$ is the absorbing time.

Let $p^{nc,c}_{(i,j),(i',j')}(n) = Pr\left\{\tau^{nc,c}_{(i,j),(i',j')} = n\right\}$, $n = 1, 2, \ldots$, be the probability distribution of the absorbing time $\tau^{nc,c}_{(i,j),(i',j')}$. However, this distribution may be defective, i.e. $\sum_{n=1}^{\infty} p^{nc,c}_{(i,j),(i',j')}(n) < 1$. Let $\boldsymbol{P}_{nc,c}(n)$ be an $|\Omega_{nc}| \times |\Omega_c|$ matrix with its $((i, j), (i', j'))$th entry to be $p^{nc,c}_{(i,j),(i',j')}(n)$ for $(i, j) \in \Omega_{nc}$ and $(i', j') \in \Omega_c$, for all $n \geq 1$. Thus the sequence of matrices $\boldsymbol{P}_{nc,c}(n)$ represents the distributions of the entire absorbing times of the hypothesised Markov chain $\boldsymbol{Q}^{nc}$. Then from (7), we have

$$\boldsymbol{P}_{nc,c}(k) = \boldsymbol{U}^{k-1}_{nc}\boldsymbol{A}_{nc,c},\ k = 1, 2, \ldots.$$

Let $\bar{\boldsymbol{P}}_{nc,c}(z)$ be the generating function of probability matrix $\boldsymbol{P}_{nc,c}(k)$, i.e. $\bar{\boldsymbol{P}}_{nc,c}(z) = \sum_{k=1}^{\infty} \boldsymbol{P}_{nc,c}(k)z^k$. Then we have

$$\bar{\boldsymbol{P}}_{nc,c}(z) = z(\boldsymbol{I} - z\boldsymbol{U}_{nc})^{-1}\boldsymbol{A}_{nc,c} \tag{8}$$

It can be seen that $\bar{\boldsymbol{P}}_{nc,c}(z)|_{z=1}$ is a stochastic matrix, i.e.

$$(\boldsymbol{I} - \boldsymbol{U}_{nc})^{-1}\boldsymbol{A}_{nc,c}\boldsymbol{e} = \boldsymbol{e} \tag{9}$$

Similarly, to study the distribution of the length of a congested period, we consider times for the queueing system starting from a state in Ω_c and transiting to a state in Ω_{nc} for the first time (i.e. no other states in Ω_{nc} have been visited before). We also use $\tau^{c,nc}_{(i,j),(i',j')}$ to denote such a time from a state (i, j) in Ω_c to a state (i', j') in Ω_{nc}. By the nature of the investigated queueing system, the first state visited in Ω_{nc} should be at the level $min_{th}-1$. Thus, only the times $\tau^{c,nc}_{(i,j),(K-d-1,j')}$ which end at level $i = min_{th}-1$ are examined.

Now, we can set up another hypothesised Markov chain by modifying the stochastic transition matrix $\boldsymbol{Q}$ of the embedded Markov chain of the original queueing system as

$$\boldsymbol{Q}^{c} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{A}_{c,nc} & \boldsymbol{U}_c \end{bmatrix} \tag{10}$$

where $\boldsymbol{A}_{c,nc} = [(\boldsymbol{E}_0(q_{min_{th}}))^t, 0, \ldots, 0]^t$ (a truncation of $\boldsymbol{A}'_{c,nc}$ in (6), and where $\boldsymbol{U}_c$ is same as in (6). Note that the states at a level less than $min_{th}-1$ have been neglected. The initial probability vector of $\boldsymbol{Q}^c$ is supposed to be $[\bar{\boldsymbol{0}}, \tilde{\boldsymbol{\beta}}]$, where $\tilde{\boldsymbol{\beta}}$ is a row vector of the form $[\beta_{min_{th}}, \beta_{min_{th}+1}, \cdots, \beta_K]$, in which $\beta_i = [\beta_{i,1}, \ldots, \beta_{i,m}]$ Such that $\Sigma^K_{i=min_{th}}\beta_i\boldsymbol{e} = 1$. Thus, each state at level $min_{th}-1$ is an absorbing state in this hypothesised Markov chain and $\tau^{nc,c}_{(i,j),(min_{th}-1,j')}$ is the absorbing time.

Let $p^{c,nc}_{(i,j),(min_{th}-1,j')}(n)$, $n = 1, 2, \ldots$, be the probability distribution of the absorbing time $\tau^{c,nc}_{(i,j),(min_{th}-1,j')}$, which may also be defective. And let $\boldsymbol{P}_{c,nc}(n)$, $n = 1, 2, \ldots$, be the sequence of $|\Omega_c| \times m$ matrices which represents the distributions $p^{c,nc}_{(i,j),(min_{th}-1,j')}(n)$ of the entire absorbing times of the hypothesised Markov chain $\boldsymbol{Q}^c$ discussed above. Then by (10), we have

$$\boldsymbol{P}_{c,nc}(k) = \boldsymbol{U}^{k-1}_{c}\boldsymbol{A}_{c,nc}, k = 1, 2, \ldots$$

Let $\bar{\boldsymbol{P}}_{c,nc}(z)$ be the generating function of probability matrix $\boldsymbol{P}_{c,nc}(k)$, i.e. $\bar{\boldsymbol{P}}_{c,nc}(z) = \Sigma_{k=1}^{\infty}P_{c,nc}(k)z^k$. Then we have

$$\bar{\boldsymbol{P}}_{c,nc}(z) = z(\boldsymbol{I} - z\boldsymbol{U}_c)^{-1}\boldsymbol{A}_{c,nc} \tag{11}$$

It can also be seen that $\bar{\boldsymbol{P}}_{nc,c}(z)|_{z=1}$ is a stochastic matrix, i.e.

$$(\boldsymbol{I} - \boldsymbol{U}_c)^{-1}\boldsymbol{A}_{c,nc}\boldsymbol{e} = \boldsymbol{e} \tag{12}$$

Note that the inverse matrices in (8) and (9) and in (11) and (12) are available since both $\boldsymbol{U}_{nc}$ and $\boldsymbol{U}_c$ are substochastic matrices.

3.4 Absorbing probability vectors $\boldsymbol{\alpha}_{min_{th}-1}$ and $\tilde{\boldsymbol{\beta}}$

As stated in the previous subsection, the proposed queueing system passes through alternating congested and non-congested periods. The behaviour of the queueing system during a non-congested period can be fully described by the hypothesised Markov chain $\boldsymbol{Q}^{nc}$ with the initial probability vector $[\bar{\boldsymbol{0}}, \ldots, \bar{\boldsymbol{0}}, \boldsymbol{\alpha}_{min_{th}-1}, \tilde{\boldsymbol{0}}]$. Similarly, its behaviour during a congested period can be fully described by another hypothesised Markov chain $\boldsymbol{Q}^c$ with the initial probability vector $[\bar{\boldsymbol{0}}, \tilde{\boldsymbol{\beta}}]$. In a steady state, the probability vector $\boldsymbol{\alpha}_{min_{th}-1}$ is just the absorbing probability vector of the hypothesised Markov chain $\boldsymbol{Q}^c$ with the initial probability vector $[\bar{\boldsymbol{0}}, \tilde{\boldsymbol{\beta}}]$, i.e.

$$\boldsymbol{\alpha}_{min_{th}-1} = \sum_{k=1}^{\infty} \tilde{\boldsymbol{\beta}}_{c,nc}(k) = \tilde{\boldsymbol{\beta}}(\boldsymbol{I} - \boldsymbol{U}_c)^{-1}\boldsymbol{A}_{c,nc} \tag{13}$$

by substituting z with 1 into (11). And, the probability vector $\tilde{\boldsymbol{\beta}}$ is just the absorbing probability vector of the hypothesised Markov chain $\boldsymbol{Q}^{nc}$ with the initial probability vector $(\tilde{\boldsymbol{\alpha}}, \tilde{\boldsymbol{0}})$, i.e.

$$\tilde{\boldsymbol{\beta}} = \sum_{k=1}^{\infty} \tilde{\boldsymbol{\alpha}}\boldsymbol{P}_{nc,c}(k) = \tilde{\boldsymbol{\alpha}}(\boldsymbol{I} - \boldsymbol{U}_{nc})^{-1}\boldsymbol{A}_{nc,c} \tag{14}$$

by substituting z with 1 into (8).

From (13) and (14), recursive formulas can be obtained to calculate the two absorbing probability vectors $\boldsymbol{\alpha}_{min_{th}-1}$ and $\tilde{\boldsymbol{\beta}}$ iteratively by assigning an arbitrary probability vector to $\boldsymbol{\alpha}_{min_{th}-1}$ or to $\tilde{\boldsymbol{\beta}}$ at first. The existence and uniqueness of $\boldsymbol{\alpha}_{min_{th}-1}$ and $\tilde{\boldsymbol{\beta}}$ can be deduced from the fact that the original embedded Markov chain is positive recurrent. In fact, let $\boldsymbol{S}_{nc}$ be the submatrix of $(\boldsymbol{I}-\boldsymbol{U}_{nc})^{-1}$ that consists of the last m rows in $(\boldsymbol{I}-\boldsymbol{U}_{nc})^{-1}$. Then by (13) and (14), we have

$$\begin{aligned} \boldsymbol{\alpha}_{min_{th}-1} &= \boldsymbol{\alpha}_{min_{th}-1}(\boldsymbol{S}_{nc}\boldsymbol{A}_{nc,c}(\boldsymbol{I} - \boldsymbol{U}_c)^{-1}\boldsymbol{A}_{c,nc}), \\ \boldsymbol{\alpha}_{min_{th}-1}\boldsymbol{e} &= 1 \end{aligned} \tag{15}$$

and

$$\tilde{\boldsymbol{\beta}} = \tilde{\boldsymbol{\beta}}((\boldsymbol{I} - \boldsymbol{U}_c)^{-1}\boldsymbol{A}_{c,nc}\boldsymbol{S}_{nc}\boldsymbol{A}_{nc,c}),\ \tilde{\boldsymbol{\beta}}\boldsymbol{e} = 1 \tag{16}$$

which indicate that the absorbing probability vectors $\boldsymbol{\alpha}_{min_{th}-1}$ and $\tilde{\boldsymbol{\beta}}$ are stationary probability vectors of $\boldsymbol{S}_{nc}\boldsymbol{A}_{nc,c}(\boldsymbol{I}-\boldsymbol{U}_c)^{-1}\boldsymbol{A}_{c,nc}$ and $(\boldsymbol{I}-\boldsymbol{U}_c)^{-1}\boldsymbol{A}_{c,nc}\boldsymbol{S}_{nc}\boldsymbol{A}_{nc,c}$, respectively.

Since the absorbing probability vectors $\boldsymbol{\alpha}_{min_{th}-1}$ and $\tilde{\boldsymbol{\beta}}$ respectively summarise the behaviour of the queueing system during a congested and a non-congested period, the original queueing system can be described as an alternating renewal process in which a cycle consists of a congested period and a non-congested period.

In the following two subsections, we evaluate the average lengths of congested and non-congested periods as well as the conditional high-priority packet loss probability during a congested period.

3.5 Average lengths of non-congested and congested periods

Let L_{nc} and L_c be the lengths of non-congested and congested periods in the queueing system, respectively. It is clear that L_{nc} and L_c are the life-spans of the two hypothesised Markov chains $\boldsymbol{Q}^{nc}$ and $\boldsymbol{Q}^c$ before absorption, respectively. From the definitions of $\boldsymbol{P}_{nc,c}(k)$ and $\boldsymbol{P}_{c,nc}(k)$, $k = 1, 2, \ldots$, in Section 3.3, the probability distributions of L_{nc} and L_c are $P\{L_{nc} = k\} = \tilde{\boldsymbol{\alpha}}\boldsymbol{P}_{nc,c}(k)\boldsymbol{e}$ and $P\{L_c = k\} = \tilde{\boldsymbol{\beta}}\boldsymbol{P}_{c,nc}(k)\boldsymbol{e}$ respectively, for all $k = 1, 2, \ldots$ The respective probability generating functions of L_{nc} and L_c are

$$\begin{aligned}\bar{P}_{L_{nc}}(z) &= \tilde{\boldsymbol{\alpha}}\bar{\boldsymbol{P}}_{nc,c}(z)\boldsymbol{e} = z\tilde{\boldsymbol{\alpha}}(\boldsymbol{I} - z\boldsymbol{U}_{nc})^{-1}\boldsymbol{A}_{nc,c}\boldsymbol{e}\\ \bar{P}_{Lc}(z) &= \tilde{\boldsymbol{\beta}}\bar{\boldsymbol{P}}_{c,nc}(z)\boldsymbol{e} = z\tilde{\boldsymbol{\beta}}(\boldsymbol{I} - z\boldsymbol{U}_c)^{-1}\boldsymbol{A}_{c,nc}\boldsymbol{e}\end{aligned}$$

by (8) and (11). Now the average lengths of non-congested and congested periods are

$$\begin{aligned}E[L_{nc}] &= \frac{\partial}{\partial z}\bar{P}_{L_{nc}}(z)|_{z=1} = \tilde{\boldsymbol{\alpha}}(\boldsymbol{I} - \boldsymbol{U}_{nc})^{-2}\boldsymbol{A}_{nc,c}\boldsymbol{e}\\ E[L_c] &= \frac{\partial}{\partial z}\bar{P}_{L_c}(z)|_{z=1} = \tilde{\boldsymbol{\beta}}(\boldsymbol{I} - \boldsymbol{U}_c)^{-2}\boldsymbol{A}_{c,nc}\boldsymbol{e}\end{aligned}$$

and by (9), (12) and $\tilde{\boldsymbol{\alpha}}(\boldsymbol{I} - \boldsymbol{U}_{nc})^{-1} = \boldsymbol{\alpha}_{min_{th}-1}\boldsymbol{S}_{nc}$, we have

$$E[L_{nc}] = \boldsymbol{\alpha}_{min_{th}-1}\boldsymbol{S}_{nc}\boldsymbol{e} \tag{17}$$

$$E[L_c] = \tilde{\boldsymbol{\beta}}(\boldsymbol{I} - \boldsymbol{U}_c)^{-1}\boldsymbol{e} \tag{18}$$

3.6 Packet drop probability during a congested period

To investigate the packet drop behaviour during a congested period, the submatrix $\boldsymbol{U}_c$ in (10) is decomposed as

$$\boldsymbol{U}_c = \boldsymbol{U}_c(0) + \sum_{l=1}^{\infty}\boldsymbol{U}_c(\ell)$$

where

$$\boldsymbol{U}_c(0) = \begin{bmatrix} \boldsymbol{D}_1(q_{min_{th}}) & \boldsymbol{D}_2(q_{min_{th}}) & \cdots & \boldsymbol{D}_{K-min_{th}-2}(q_{min_{th}}) & \boldsymbol{D}_{K-min_{th}-1}(q_{min_{th}}) \\ \boldsymbol{D}_0 & \boldsymbol{D}_1(q_{min_{th}+1}) & \cdots & \boldsymbol{D}_{K-min_{th}}(q_{min_{th}-1}) & \boldsymbol{D}_{K-min_{th}}(q_{min_{th}+1}) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{D}_0 & \boldsymbol{D}_1(q_k) \end{bmatrix}$$

$$\boldsymbol{U}_c(\ell) = \begin{bmatrix} 0 & 0 & \cdots & 0 & \boldsymbol{D}_{K-min_{th}-1+\ell}(q_{min_{th}}) \\ q_{min_{th}+1}\boldsymbol{D}_l & 0 & \cdots & 0 & \boldsymbol{D}_{K-min_{th}+\ell}(q_{min_{th}+1}) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & q_k\boldsymbol{D}_\ell & \boldsymbol{D}_{1+\ell}(q_K) \end{bmatrix}, \quad \forall \ell \geq 1$$

The matrix $\boldsymbol{U}_c(0)$ consists of the probabilities of events which make state transitions within Ω_c without any packet drops. However, the matrix $\boldsymbol{U}_c(\ell), \ell \geq 1$, consists of the probabilities of events which make state transitions within Ω_c with the drop of ℓ packets. Similarly, the submatrix $\boldsymbol{A}_{nc,c}$ in (7) can also be considered as

$$\boldsymbol{A}_{nc,c} = \boldsymbol{A}_{nc,c}(0) + \sum_{l=1}^{\infty}\boldsymbol{A}_{nc,c}(\ell)$$

where

$$\boldsymbol{A}_{nc,c}(0) = \begin{bmatrix} \boldsymbol{D}_{min_{th}} & \boldsymbol{D}_{min_{th}+1} & \cdots & \boldsymbol{D}_{K-1} & \boldsymbol{D}_K \\ \boldsymbol{D}_{min_{th}} & \boldsymbol{D}_{min_{th}+1} & \cdots & \boldsymbol{D}_{K-1} & \boldsymbol{D}_K \\ \boldsymbol{D}_{min_{th}-1} & \boldsymbol{D}_{min_{th}} & \cdots & \boldsymbol{D}_{K-2} & \boldsymbol{D}_{K-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{D}_2 & \boldsymbol{D}_3 & \cdots & \boldsymbol{D}_{K-min_{th}-3} & \boldsymbol{D}_{K-min_{th}-2} \end{bmatrix}$$

$$\boldsymbol{A}_{nc,c}(\ell) = \begin{bmatrix} 0 & 0 & \cdots & 0 & \boldsymbol{D}_{K+\ell} \\ 0 & 0 & \cdots & 0 & \boldsymbol{D}_{K+\ell} \\ 0 & 0 & \cdots & 0 & \boldsymbol{D}_{K-1+\ell} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \boldsymbol{D}_{K-min_{th}-2+\ell} \end{bmatrix}, \quad \forall \ell \geq 1$$

The matrix $\boldsymbol{A}_{nc,c}(0)$ consists of the probabilities of events which make a transition from a state in Ω_{nc} to a state in Ω_c without any packet drop; and the matrix $\boldsymbol{A}_{nc,c}(\ell), \ell \geq 1$, consists of the probabilities of events which make such a transition with the drop of ℓ packets.

Note that the behaviour of the RED queueing system during a congested period can be described by the hypothesised Markov chain $\boldsymbol{Q}^c$. For a state (i, j) in Ω_c, let $p_{(i,j),c}(k, \ell)$ be the probability that the state of the hypothesised Markov chain $\boldsymbol{Q}^c$ enters (i, j) with a total of ℓ packets dropped after k transitions. Let $\boldsymbol{p}_c(k, \ell)$ be an $|\Omega_c|$-vector whose (i, j)th entry is $\boldsymbol{p}_{(i,\ j),c}(k, \ell)$. The initial vector $\boldsymbol{p}_c(0, \ell)$ can be determined by the behaviour of the queueing system during a previous non-congested period as

$$\boldsymbol{p}_c(0,\ \ell) = \left(\sum_{k=1}^{\infty} \tilde{\boldsymbol{\alpha}} \boldsymbol{U}_{nc}^{k-1}\right) \boldsymbol{A}_{nc,c}(\ell)$$
$$= \tilde{\boldsymbol{\alpha}}(\boldsymbol{I} - \boldsymbol{U}_{nc})^{-1} \boldsymbol{A}_{nc,c}(\ell),\ \forall \ell \geq 0 \qquad (19)$$

In addition, the vector $\boldsymbol{p}_c(k, \ell)$, $k \geq 1$, $\ell \geq 0$, can be obtained recursively by

$$\boldsymbol{p}_c(k,\ \ell) = \sum_{i=0}^{\ell} \boldsymbol{p}_c(k-1,\ \ell - i) \boldsymbol{U}_c(i) \qquad (20)$$

Let $p_{drop,c}(k, \ell) = P_r\{L_c = k,\ Drop_c = \ell\}$ be the probability that there is a total of ℓ packets dropped during a congested period of length k. Then,

$$p_{drop,c}(k,\ \ell) = \boldsymbol{p}_c(k-1,\ \ell) \boldsymbol{A}_{c,nc} \boldsymbol{e},$$
$$\forall k \geq 1,\ \ell \geq 0 \qquad (21)$$

The generating function $\bar{p}_{drop,c}(z_1, z_2)$ of $p_{drop,c}(k, \ell)$, i.e. $\bar{p}_{drop,c}(z_1, z_2) = \sum_{k=1}^{\infty} \sum_{\ell=0}^{\infty} p_{drop,c}(k,\ \ell) z_1^k z_2^l$ is

$$\bar{p}_{drop,c}(z_1,\ z_2) = z_1 \bar{\boldsymbol{p}}_c(z_1,\ z_2) \boldsymbol{A}_{c,nc} \boldsymbol{e} \qquad (22)$$

by (21), where $\bar{\boldsymbol{p}}_c(z_1, z_2) \equiv \sum_{k=0}^{\infty} \sum_{\ell=0}^{\infty} \boldsymbol{p}_c(k,\ \ell) z_1^k z_2^\ell$ is the generating function of $\boldsymbol{p}_c(k, \ell)$. From (19) and (20), we have

$$\boldsymbol{p}_c(z_1,\ z_2) = \tilde{\boldsymbol{\alpha}}(\boldsymbol{I} - \boldsymbol{U}_{nc})^{-1} \boldsymbol{A}_{nc,c}(z_2)$$
$$\times (\boldsymbol{I} - z_1 \boldsymbol{U}_c(z_2))^{-1} \qquad (23)$$

where $\boldsymbol{A}_{nc,c}(z_2) = \sum_{\ell=0}^{\infty} \boldsymbol{A}_{nc,c}(\ell) z_2^\ell$ and $\boldsymbol{U}_c(z_2) = \sum_{\ell=0}^{\infty} \boldsymbol{U}_c(\ell) z_2^\ell$. Now the average total number of packets lost during a congested period, denoted as $E[Drop_c]$, can be calculated as

$$E[Drop_c] = \frac{\partial}{\partial z_2} \bar{p}_{loss,c}(z_1,\ z_2)|_{z_1=1, z_2=1}$$
$$= \alpha_{K-d-1} \boldsymbol{S}_{nc} \left(\sum_{\ell=1}^{\infty} \ell \boldsymbol{A}_{nc,c}(\ell)\right) \boldsymbol{e}$$
$$+ \tilde{\boldsymbol{\beta}}(\boldsymbol{I} - \boldsymbol{U}_c)^{-1} \left(\sum_{\ell=1}^{\infty} \ell \boldsymbol{U}_c(\ell)\right) \boldsymbol{e} \qquad (24)$$

by (22), (23), (14), (12) and $\tilde{\boldsymbol{\alpha}}(\boldsymbol{I} - \boldsymbol{U}_{nc})^{-1} = \boldsymbol{\alpha}_{K-d-1} \boldsymbol{S}_{nc}$. Consequently, the packet drop probability $P_{drop,c}$ during a congested period can be obtained by

$$P_{drop,c} = \frac{E[Drop_c]}{\lambda E[L_c]} \qquad (25)$$

where $E[L_c]$ is the average length of a congested period in (18) and λ is the fundamental arrival rate of the arrival traffic.

4 Numerical results

In this Section, we investigate and discuss the numerical results from a router with an RED scheme. The time is slotted such that the unit time is equal to the packet transmission time, which is equal to (536 bytes)/(C Mbit/s), where C Mbit/s is the link capacity of the backbone router with an RED scheme. The numerical results are computed by the algorithm developed in the previous section.

In this paper, the arrival process has the mean rate μ Mbit/s, the standard deviation of the rate σ Mbit/s, and the autocovariance function of the rate $r(\tau) = \sigma^2 e^{-a\tau}$. By the methods proposed in [12] to model the arrival process by a D-BMAP, the underlying Markovian structure for the traffic is assumed to be an m-state birth-and-death process, where each of the m states (i.e. phases) corresponds to a level in the uniform quantisation of the rate, from 0 to $m-1$, with the transition probability matrix $\boldsymbol{D} = \boldsymbol{B}(m-1;1)$, where

$$\boldsymbol{B}(k;\ n) = \begin{bmatrix} 1-kp & kp & 0 & \cdots & 0 & 0 \\ q & 1-q-(k-1)p & (k-1)p & \cdots & 0 & 0 \\ 0 & 2q & 1-2q-(k-2)p & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & kq & 1-kq \end{bmatrix}$$

in which $p = a/[1 + (n/k)(\mu^2/\sigma^2)]$ and $q = a/[1 + (k/n)(\sigma^2/\mu^2)]$. The sequences $\{\boldsymbol{D}_i\}_{i \geq 0}$ of parameter matrices for the packet traffic are

$$\boldsymbol{D}_i = \begin{bmatrix} a_i(0;\ \phi(m-1)) & 0 & \cdots & 0 \\ 0 & a_i(1;\ \phi(m-1)) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_i(m-1; \phi(m-1)) \end{bmatrix} \boldsymbol{D},\ \forall i \geq 0$$

respectively, where

$$a_i(k; \phi(r)) = \binom{k}{i} \phi(r)^i (1 - \phi(r))^{k-i}$$

with $\phi(r) = (\mu/r + \sigma^2/\mu)/C$. Note that $\boldsymbol{D}_i = 0$ for all $i \geq m$.

In this example, the arrival process has the mean rate $\mu = 35.1$ Mbit/s, the standard deviation of the rate $\sigma = 5.19$ Mbit/s, and the autocovariance function of the rate $r(\tau) = 5.19^2 e^{-3.9\tau}$. In this study, we have selected $m = 10$ such that the highest level of rate of the birth-and-death underlying Markovian structure corresponds to the peak rate of the traffic. The capacity C Mbit/s of the server is adjusted such that the queue will have different load conditions $\rho = \mu/C$. The buffer capacity K is taken to be 30 due to the limitation of the software used in our personal computer.

The combination of a TCP mechanism and drop-tail buffer management scheme induces a loss event at a router that involves many packets. If these packets belong to different TCP sessions, these sessions will simultaneously experience loss, decrease their rates/windows in synchrony, and then tend to stay synchronised [14]. This phenomenon presents the so-called synchronisation of multiple TCP sessions. The RED buffer management scheme will also spread out packet drops, breaking the synchronisation pattern which will occur with drop-tail. The findings from Fig. 3 indicate that the drop probability between RED and drop-tail is very close under heavy load conditions. This shows that RED not only resolves the synchronisation problem but also has the same loss performance as the drop-tail scheme under heavy loads.

When TCP applies additive-increase and multiplicative-decrease mechanisms [3, 4, 6], the $E[L_{nc}]$ represents the additive-increase period and the $E[L_c]$ represents a multiplicative-decrease period. The rate behaviour of TCP with additive-increase and multiplicative-decrease mechanisms

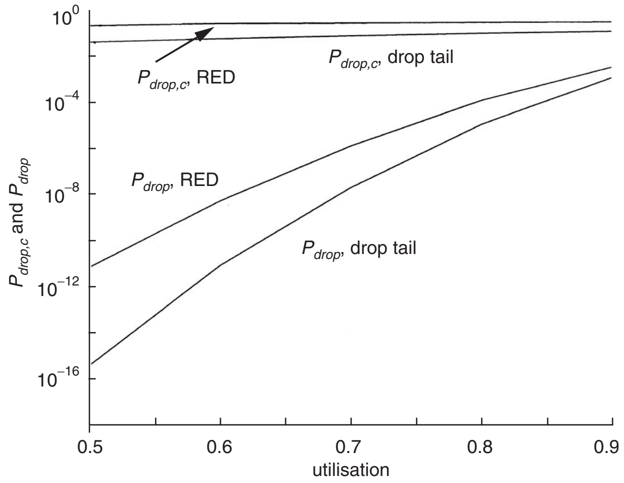

Fig. 3 *Drop probability for drop-tail and RED with* $min_{th}=20$

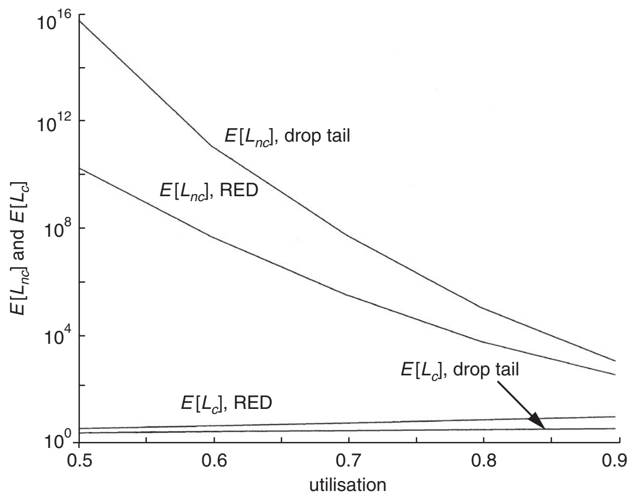

Fig. 4 *Average lengths of non-congested and congested periods for drop-tail and RED with* $min_{th}+1=20$

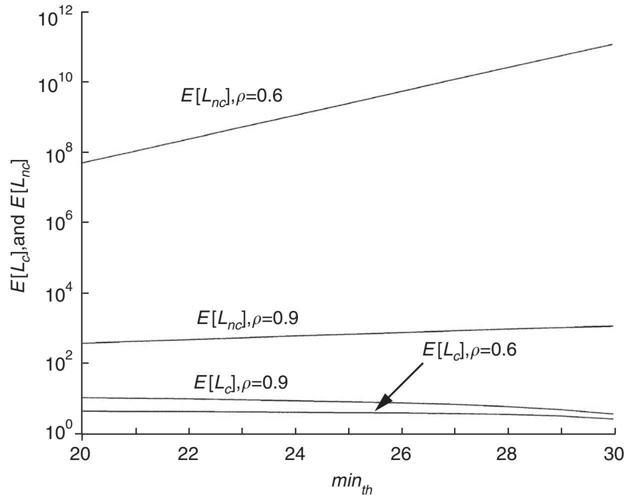

Fig. 5 *Average lengths of non-congested and congested periods*

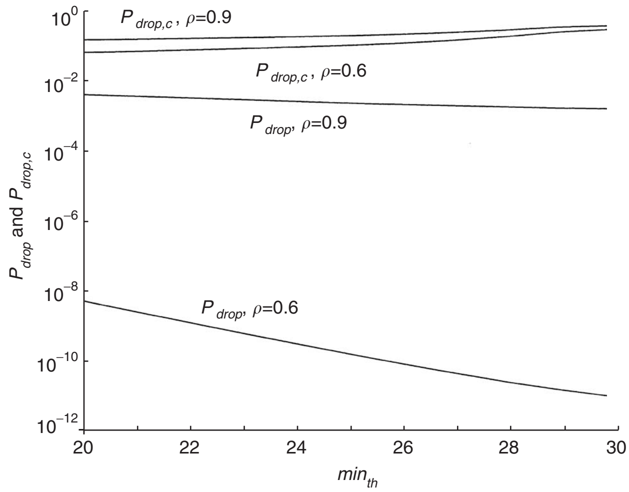

Fig. 6 *Comparison of short-term and long-term packet drop probabilities*

oscillates between the additive-increase period and multiplicative-decrease period [7, 15].

The results in Fig. 4 show that the rate oscillation behaviour of RED is better than that of drop-tail when TCP applies the additive-increase and multiplication-decrease mechanisms.

Figure 5 summarises the numerical results of the average lengths of a congested period $E[L_c]$ and a non-congested period $E[L_{nc}]$ with respect to the threshold value min_{th}. The chosen load values $\rho=0.6$ and $\rho=0.9$ correspond to a moderate and a heavy load condition, respectively. As expected, the average length $E[L_{nc}]$ of a non-congested period is much longer than that of a congested period under a moderate load condition. In addition, $E[L_{nc}]$ decreases as min_{th} decreases, and this decay is more pronounced in a moderate load condition than in a heavy load condition. On the other hand, the average length $E[L_c]$ of a congested period seems to be insensitive to the threshold setting in any load condition.

In Fig. 6, we can find that the short-term packet drop probability $P_{drop,c}$ during a congested period is higher than its long-term counterpart P_{drop}, confirming our general expectations. Note that the short-term packet drop probability $P_{drop,c}$ in a moderate load condition is as high as that in a heavy load condition. This confirms our intuition that the short-term packet drop probability during a congested period is a significant performance measure of a router with an RED scheme in both moderate and heavy load conditions. It can also be concluded that in a moderate-to-heavy load condition, the short-term drop probability $P_{drop,c}$ increases as the threshold value min_{th} increases. Although the long-term drop probability P_{drop} decreases as min_{th} increases, the decay speed is far slower than that of the short-term drop probability $P_{drop,c}$.

5 Conclusions

A matrix-analytic approach has been applied to investigate the drop behaviour of a router with a RED scheme. The bursty nature of packet drop has been examined by means of conditional statistics with respect to alternating congested and non-congested periods, and the long-term packet drop probabilities have been evaluated. By the conditional statistics, all of the four related performance measures were derived, including a long-term drop probability P_{drop}, and the three short-term measures, comprising the average length $E[L_c]$ of a congested period, the average length $E[L_{nc}]$ of a non-congested period, and the conditional packet drop probability $P_{drop,c}$ during a congested period. We used this RED queueing model to quantify the benefits from using RED. The results show that the drop probability between RED and drop-tail is very close under heavy load conditions. The findings also show that the rate oscillation behaviour of RED is better than that of the drop-tail scheme. This implies that an RED scheme can reduce the required buffer capacity in the router when TCP applies the additive-increase and multiplicative-decrease mechanisms.

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