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ORIGINAL ARTICLE

A heuristic moment-based framework for optimization design under uncertainty

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Abstract
To search an optimal design under uncertainty, this study proposes an effective framework that integrates the moment-based reliability analysis into a heuristic optimization algorithm. Integration of an equivalent single-variable performance function is an ideal concept to calculate the failure probability. However, such integration is often not available and is alternatively computed using the first four moments and a generalized moment-based reliability index is established, in which the Gaussian–Hermite integration and dimension reduction are implemented to enhance the effectiveness. To overcome the limited applicable range of moment-based approach, an adjustable optimization procedure is proposed, in which different reliability methods are performed depending on results of the constraint assessments. In addition, the ε level comparison is integrated into particle-swarm optimization to consider the constraint violation. Several literature studies are used to verify the accuracy of the proposed optimization framework including problems having linear, highly nonlinear, implicit probabilistic constraint functions with normal or non-normal variables and system-level reliability analysis. The effects of several parameters, such as the number of estimate point, the number of dimension, and the degree of uncertainty, are thoroughly investigated. Results indicating that tri-variate with seven points are able to provide a stable solution under a high degree of uncertainty.

Keywords Risk and reliability · Safety · Uncertainty · Probabilistic design · Optimization techniques

1 Introduction

Uncertainties in an engineering design problem are often inevitable; to ensure a greater performance, reliability-based design optimization (RBDO) is often adopted. A traditional RBDO is a nested double-loop approach in which the outer loop is the deterministic optimization, and the inner loop is the reliability analysis, which is often a barrier in application. To simplify the computational complexity of an RBDO problem, many algorithms have been proposed. For example, safety factor has been adopted in many design codes to take uncertainties into consideration. The safety factor approach, however, often fails to identify the relative importance among design parameters, because all uncertainties are represented by s single factor throughout the design [11]. Du and Chen [2] proposed the sequential optimization and reliability assessment (SORA) to sequentially conduct the deterministic optimization and inverse reliability analysis until the solution converges. Liao and Ha [10] incorporated a reliability analysis of mean value method with an SORA to further improve efficiency of an RBDO solution. Mean value method is a first-order Taylor's expansion using mean value as the expansion point, which is a simplified approach and is suitable for a problem without highly nonlinear performance functions. Li et al. [9] considered an SORA problem using hybrid uncertainty, which is described by randomness and fuzziness. This approach is specially designed for a problem with different cognitive levels to various uncertain parameters. Yi et al. [26] proposed a method that approximately estimated the most probable point (MPP) and probabilistic performance measure in reliability assessment to improve the efficiency of SORA. In their approach, evaluation of performance function in the deterministic optimization is not needed and, therefore, reducing the computational time. Raza and Liang [17] also utilized the similar concept to

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decouple the reliability analysis in a traditional double-loop RBDO optimization task. Liu and Zhang [13] utilized the decoupling strategy to develop an equivalent single-layer optimization model that is solved by the sequential quadratic programming (SQP) method. The benefit of this approach is similar to SORA, in which a decoupling strategy was developed to solve the nesting optimization problem, alleviating the computational burden.

Other than decoupling approach, Mansour and Olsson [14] adopted the response surface methodology in solving an RBDO problem, which is particularly useful for the industrial applications due to the use of the finite-element software. Shan and Wang [18] converted the double-loop RBDO into a single-loop procedure, in which a deterministic optimization problem is formulated and constrained by a reliable design space rather than the original deterministic feasible space. In balancing the computational cost and the accuracy of the reliability assessment, this study incorporates the methods of moments [13, 27, 28, 30, 31] as the reliability analysis procedure and particle-swarm optimization (PSO) for solving RBDO problems.

Many gradient-based optimization algorithms have been proposed in the literature [5–7, 22]. For a gradient-based approach (e.g., SQP), derivatives of the objective and constraint functions are needed to find the optimal direction and step in the next iteration. However, derivative calculations could be an obstacle for non-differentiable cases. Recently, heuristic optimization algorithm such as the PSO has been proposed and drawn many attentions [3, 5, 22]. Because derivatives are not needed in PSO, PSO is adopted this study. Similar to optimization, many algorithms have been proposed to estimate the failure probability for a given performance function. The first-order reliability analysis (FORM) and the Monte Carlo simulation (MCS) are two frequently approaches. FORM computes the failure probability through an optimization procedure, in which the goal is to find the minimal distance between the origin and the limit-state function in the standard normal and uncorrelated random variable space. The limit-state function is defined as $g=0$, where g is the performance function. The point on the limit-state corresponding to this minimum distance is called the MPP. The shortest distance is called reliability index (β) with a corresponding failure probability. MCS, on the other hand, is a sampling-based approach, relying on repeated random sampling to obtain the failure probability. If the failure probability is very small, which is often found in an engineering design problem, a large sample size is often required. Both FORM and MCS are easily to be integrated into the RBDO. However, the accuracy of the FORM and the computational cost of MCS are two major concerns if such approach is adopted.

This study incorporates the methods of moments with PSO to solve an RBDO problem. Failure probability is the integral of the performance function under a given probability density function (PDF). In reality, the PDF of a performance function is often not available. The first few central moments, calculated by point estimates in the standard normal space, are used to compute the probability of failure in the proposed RBDO approach. A generalized multivariate dimension-reduction method proposed by Xu and Rahman [25] is utilized to consider a performance function with multiple random variables. Details of the proposed algorithm are described in the following sections.

2 The reliability analyses used in the current study: methods of moments [29]

The failure probability (P_{F}) of a given performance (G) is the integration of

$$P_{\mathrm{F}} = P(Z = G(\mathbf{X}) \le 0) = \int_{G(\mathbf{X})\le 0} f_{\boldsymbol{x}}(\boldsymbol{x})\mathrm{d}x, \quad (1)$$

where $\mathbf{X}$ are the random variables considered and $\boldsymbol{f_x}$ is the PDF of $\mathbf{X}$. If the performance function can be represented by a PDF with a single variable of Z, the integration described in Eq. (1) can solved as described in the following equation:

$$\begin{cases} P_{\mathrm{F}} = P(Z \le 0) = P(\sigma_G Z_S + \mu_G \le 0) = P\left(Z_S \le -\dfrac{\mu_G}{\sigma_G}\right) \\ P_{\mathrm{F}} = P(Z_S \le -\beta_{2\mathrm{M}}) = \displaystyle\int_{-\infty}^{-\beta_{2\mathrm{M}}} f_{Z_s}(Z_s)\mathrm{d}z_s, \end{cases} \quad (2)$$

where Z_s is the standardization of Z, and σ_G and μ_G are standard deviation and mean value of the performance function, respectively. $\beta_{2\mathrm{M}}$ are the second moment reliability index. Similar to the Rosenblatt transformation, assuming that the CDFs of Z_s and standard normal (U) are equal, then one can establish the relationship of the standardized variable and standard normal variable using the first few moments of Z, as described in the following equation:

$$\begin{aligned} Z_s &= S(U, \mathbf{M}) \\ U &= S^{-1}(Z_s, \mathbf{M}), \end{aligned} \quad (3)$$

where S^{-1} is the inverse function of S and $\mathbf{M}$ is the first few moments of Z. A general moment-based reliability index can be derived, as described in the following equation:

$$\begin{aligned} F_{Z_s} &= \Phi(u) = \Phi[S^{-1}(z_s, \mathbf{M})] \\ P_{\mathrm{F}} = F_{Z_s}(-\beta_{2\mathrm{M}}) &= \Phi[S^{-1}(-\beta_{2\mathrm{M}}, \mathbf{M})] \\ \beta = -\Phi^{-1}(P_{\mathrm{F}}) &= -S^{-1}(-\beta_{2\mathrm{M}}, \mathbf{M}). \end{aligned} \quad (4)$$

Thus, the remaining tasks of reliability analysis are to find the inverse function S^{-1} and the moments of the performance function (**M**). The calculation of moments of the performance is first described below.

2.1 Point estimation and dimension reduction

Estimating moments for a performance function in a standard normal space is conceptually identical to Gaussian–Hermite integration. In this study, the kth moment of a performance function, $G(\mathbf{X})$, is estimated, as described in the following equation:

$$\int z(u)\phi(u)\mathrm{d}u = \sum_{j=1}^{m} P_j z(u_j), \tag{5}$$

where ϕ is the standard normal probability density function, u is standard normal variable, P_j and u_j are the weight and point, m is the number of point, and $z(u)$ is expressed in the following equation:

$$z(u) = \{G[T^{(-1)}(U)] - \mu_G\}^k, \tag{6}$$

where T is the Rosenblatt transformation function, and k is the kth moment under estimation. There are two different ways of standardizing the Hermite polynomials: the "probabilists' Hermite polynomials" and the "physicists' Hermite polynomials". These two definitions are related to each other through the following equation:

$$H_m(x) = 2^{\frac{m}{2}} H_{\mathrm{em}}\left(\sqrt{2}x\right), \tag{7}$$

where H_{m} is the physicists' Hermite polynomials and H_{em} is the probabilists' Hermite polynomials. The weights of the physicists' Hermite polynomials are given as follows:

$$w_j = \frac{2^{m-1} m! \sqrt{\pi}}{m^2 H_{m-1}^2(x_j)}. \tag{8}$$

Compared to the physicists' Hermite polynomials, the probabilists' Hermite polynomials are closer to Eq. (5) that is used to estimate moments for a performance function in a standard normal space in the current study. By comparing Eq. (5) and the integration using probabilists' Hermite polynomials, Eq. (9) can be derived:

$$P_j = \frac{1}{\sqrt{\pi}} w_j. \tag{9}$$

With Eqs. (7) and (9), the weights (P_j) in Eq. (5) can be derived and is described in the following equation:

$$P_j = \frac{m!}{m^2 H_{\mathrm{em}-1}^2(u_j)}, \tag{10}$$

where H_{em} is the probabilists' Hermite polynomials, and u_j is the abscissas of the Hermite polynomials.

For a function of many variables, a generalized multivariate dimension-reduction method proposed by Xu and Rahman [25] is adopted here. The multivariate approximation, G', consists of all terms of the Taylor series expansion of an N-dimensional, $G(\mathbf{X})$, that have no more than D variables ($D \le N$). That is, for $G(\mathbf{X}) = G(x_1, x_2, \ldots, x_N)$, a D-variate approximation G' is given in the following equation:

$$G' = \sum_{i=0}^{D} (-1)^i \binom{N - S + i - 1}{i} G_{D-i}, \tag{11}$$

where G_D defines a summation of terms that contain at most D variables, as given in the following equation:

$$G_D = \sum_{k_1<k_2<\cdots<k_D} G(0, \ldots, 0, x_{k_1}, 0, \ldots, 0, x_{k2}, 0, \ldots, 0, x_{k_D}, 0, \ldots, 0), \quad 0 \le D \le N. \tag{12}$$

After applying the D-variate approximation, the point estimate procedure for a single variable described in Eq. (5) is then used to compute the first fourth moments of $G(\mathbf{X})$, as shown in the following equation:

$$\begin{cases} \mu_G = \mu_{1G} \\ \sigma_G = \sqrt{\mu_{2G} - \mu_{1G}^2} \\ \alpha_{3G} = (\mu_{3G} - 3\mu_{2G}\mu_{1G} + 2\mu_{1G}^3)/\sigma_G^3 \\ \alpha_{4G} = (\mu_{4G} - 4\mu_{3G}\mu_{1G} + 6\mu_{2G}\mu_{1G}^2 - 3\mu_{1G}^4)/\sigma_G^4 \end{cases}, \tag{13}$$

where μ_{kG} (k=1, 2, 3, 4) refers to the kth moments about zero that is defined in the following equation:

$$\mu_{kG} = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \{G[T^{(-1)}(u)]\}^k \phi(u)\mathrm{d}u. \tag{14}$$

2.2 Third- and fourth-moment reliability methods

Following the assumption of Tichy [21], the standardized variable Z_s is described by the three-parameter lognormal distribution, the third-moment reliability index based on Eqs. (3) and (4) is provided in the following equation:

$$\beta_{3\mathrm{M}-L} = -\frac{\alpha_{3G}}{6} - \frac{3}{\alpha_{3G}} \ln\left(1 - \frac{1}{3}\alpha_{3G}\beta_{2M}\right), \tag{15}$$

where α_{3G} is the third dimensionless central moment, i.e., the skewness of $Z = G(\mathbf{X})$. Zhao and Ono [28] assumed that Z_s follows the 3P square normal distribution, the third-moment reliability index following Eqs. (3) and (4) can be expressed as

$$\beta_{3\mathrm{M}-S} = \frac{1}{\alpha_{3G}}\left(3 - \sqrt{9 + \alpha_{3G}^2 - 6\alpha_{3G}\beta_{2M}}\right). \tag{16}$$

Wang et al. [23] proposed another third-moment reliability index that has a wider applicable range and less limitation compared to that of Eqs. (15) and (16), as shown in Eq. (17), which is obtained by fitting the average value of β_{3M-L} and β_{3M-S}:

$$\beta_{3M-W} = \frac{1}{3}\beta_{2M}\left[2 + e^{\frac{1}{2}\alpha_{3G}\left(\beta_{2M}-\frac{1}{\beta_{2M}}\right)}\right]. \tag{17}$$

Equation (17) is implemented in the proposed RBDO when the fourth-moment method is not applicable. It is observed that the third-moment reliability index becomes undefined when β_{2M} is equal to zero. Nevertheless, it is very unusual to encounter such a case in a practical engineering problem. The aforementioned 3M approaches are approximate methods and the applicable range of α_{3G} (if $r = 2\%$, r is the allowable relative error between β_{3M-L} and β_{3M-S}) is described in Eq. (18) [31]:

$$-2.4/\beta_{2M} \le \alpha_{3G} \le 0.8/\beta_{2M}. \tag{18}$$

It is noted that the 3M method is more suitable for negative values of skewness, since the probability of failure is integrated in the left tail of the PDF. Based on the $U-Z_s$ transformation provided by Fleishman [4], as shown in the following equation:

$$Z_S = a_1 + a_2U + a_3U^2 + a_4U^3, \tag{19}$$

in which a_i $(i = 1, 2, 3, 4)$ are deterministic coefficients. The 4M reliability index can be established which is briefly described in the following. Assuming that the first four central moments of U and Z_s are the same, one can find a_2 and a_4 through the following equation:

$$\begin{aligned} 2A_1A_2 &= \alpha_{3G}^2 \\ 3A_1A_3 + 3A_4 &= \alpha_{4G}, \end{aligned} \tag{20}$$

where $A_i = h_i(a_2, a_4)$, h_i $(i = 1, 2, 3, 4)$ are polynomial functions. Once a_2 and a_4 are obtained, a_1 and a_3 can be solved using the following equation:

$$a_3 = -a_1 = \frac{\alpha_{3G}}{2A_2}. \tag{21}$$

The 4M reliability index based on Eqs. (3) and (4) is provided in the following equation:

$$\beta_{4M-C} = \frac{P}{D} - D + \frac{a_3}{3a_4}, \tag{22}$$

where

$$D = \sqrt[3]{\frac{\Delta - q}{2}}, \Delta = \sqrt{q^2 + 4p^3}, p = \frac{3a_2a_4 - a_3^2}{9a_4^2},$$

$$q = \frac{2a_3^3 - 9a_2a_4a_3}{27a_4^3} + \frac{a_1 + \beta_{2M}}{a_4}.$$

Similarly, an applicable range of α_{4G} is provided in Eq. (23) [30] which is used to determine whether the 4M reliability is applied or not in our proposed RBDO approach. Details are provided in “The proposed RBDO”:

$$2.9 + \alpha_{3G}^2 \le \alpha_{4G} \le 5.2 + \alpha_{3G}^2. \tag{23}$$

Zhao et al. [32] developed a complete expression of the fourth-moment normal transformation including six cases with different combinations of skewness and kurtosis. The applicable ranges are provided below:

$$\begin{cases} abs(1.9 - 0.0649\alpha_{3G} + 1.5988\alpha_{3G}^2) \le \alpha_{4G} \le abs(3.1 - 0.45\alpha_{3G} + 1.6111\alpha_{3G}^2) \text{ if } a_4 < 0 \\ abs(3.1 - 0.45\alpha_{3G} + 1.6111\alpha_{3G}^2) \le \alpha_{4G} \le 7 \text{ if } a_4 \ge 0 \end{cases}. \tag{24}$$

Equation (24) is used for the first three examples for comparison. For details, please refer Sect. 6.

2.3 Methods of moments for system reliability

In reality, a structure usually has multiple failure modes. The calculation of the failure probability for a system is necessary; however, it is generally not an easy task. If a moment-based reliability approach is adopted, one of the difficulties is in obtaining the mutual correlations among the failure modes. Several approaches have been proposed to lessen the computational burden of a system reliability problem. A moment-based method, in which the failure probability is evaluated without MCS and does not require the computation of the mutual correlations among failure modes, is adopted in the current study [27]. The performance function of a series system, G, is expressed as the minimum among each individual performance function, as indicated in the following equation:

$$G(\mathbf{X}) = \min[g_1, g_2, \ldots, g_k], \tag{25}$$

where g_i is the performance function of the ith failure mode. Similarly, the performance function of a parallel system, G, is expressed as the maximum among each individual performance function, as indicated in the following equation:

$$G(\mathbf{X}) = \max[g_1, g_2, \ldots, g_k]. \tag{26}$$

For a combined series–parallel system, the performance function of the system then becomes a combination of maximum and minimum of the component performance functions. Once the system performance function and its corresponding central moments are identified, the reliability can be determined, as described in Sect. 2.

3 The proposed optimization: PSO

PSO was proposed by Eberhart and Kennedy [3]. PSO is an algorithm operating on the basis of a large population of solutions and has drawn many attentions due to it is simple and easy to adopt. To execute a PSO, a group of random numbers was used to initialize the entire population, consisting of a set of individual particles. Particle movements are influenced by the optimal experience of an individual particle and that of the population. Weighted values are used to determine the degree of influence between the two. Random elements are also considered when determining the directions of the particle movements, giving particles a chance to leave local trends and preventing them from being trapped within local optimums. Many variations have been proposed for PSOs. The following briefly describes the PSO used in this study. A new particle is generated using Eq. (27), as shown below:

$$\vec{x}_i(t+1) = \vec{v}_i(t+1) + \vec{x}_i(t), \tag{27}$$

where $\vec{x}_i(t+1)$ denotes the position of the ith particle in the next iteration, $\vec{x}_i(t)$ denotes the position of the ith particle in the current iteration, and $\vec{v}_i(t+1)$ denotes the velocity of the ith particle in the current iteration. The position of a particle represents the values of that particle. The velocity of the ith particle is determined by the following equation:

$$\vec{v}_i(t+1) = w \times \vec{v}_i(t) + r_1 c_1(\vec{x}_{\mathrm{pBest}} - \vec{x}_i(t)) + r_2 c_2(\vec{x}_{\mathrm{gBest}} - \vec{x}_i(t)), \tag{28}$$

where w is the inertia factor, $\vec{v}_i(t)$ is the velocity at the previous iteration, r_i (i = 1–2) are random numbers between 0 and 1, and c_1 and c_2 are the cognition and social factors, respectively. $\vec{x}_{\mathrm{pBest}}$ is the particle position with the minimum objective value in the ith population and $\vec{x}_{\mathrm{gBest}}$ is the particle position with the minimum objective value in the entire population.

Various strategies in constraint handling for PSO have been proposed. Takahama et al. [20] proposed the ε-constrained PSO to improve the Deb's comparison rules, which is adopted in this study. If the constraint violation, $C(\mathbf{X})$, is less than a specified threshold, ε, the particle is considered feasible. The constrained violation used in ε-constrained PSO is given by Eqs. (29) and (30):

$$C(\mathbf{X}) = \max\left\{ \max_j \left\{0, l_j(\mathbf{X})\right\}, \max_j \left| h_j(\mathbf{X}) \right| \right\} \tag{29}$$

$$C(\mathbf{X}) = \sum_j \left\| \max\left\{0, l_j(\mathbf{X})\right\} \right\|^p + \sum_j \left\| h_j(\mathbf{X}) \right\|^{np}, \tag{30}$$

where l_j is the jth inequality constraint, h_j is the jth equality constraint, and np is a positive number. It is seen that the constraint violation is given by the maximum of all constraints or the sum of all constraints. The ε-constrained PSO uses a so-called "ε-level comparison" which is an order relation of fitness value and constraint violation [$f(\mathbf{X})$, $\phi(\mathbf{X})$] to determine the pBest and gBest. The ε-level comparison, $\leq_\varepsilon$, between (f_1, ϕ_1) and (f_2, ϕ_2) is defined by the following equation:

$$(f_1, \phi_1) \leq_\varepsilon (f_2, \phi_2) \Leftrightarrow \begin{cases} \text{if } \phi_1, \phi_2 \leq \varepsilon \text{ and } f_1 \leq f_2, \\ \text{if } \phi_1 = \phi_2 \text{ and } f_1 \leq f_2 \\ \text{if } \varepsilon \leq \phi_1 \leq \phi_2 . \end{cases} \tag{31}$$

Equation (31) indicates that the feasibility of $\mathbf{X}$ has higher priority than that of the fitness value.

4 The proposed RBDO

The proposed RBDO utilizes the second-, third-, or fourth-moment-based method with multivariate dimension-reduction technique (e.g., univariate, bi-variate, and tri-variate) as the reliability analysis procedure and the PSO as the optimizer, named PSO-4M–3M–2M hereafter. The primary goal of the algorithm is to achieve the necessary accuracy with reasonable cost. As described in "Third- and fourth-moment reliability methods", the third- and fourth-moment reliability indices are applicable in certain ranges. The reliability index (e.g., the second-, third-, or fourth moment-based) that gives better accuracy will be selected in our proposed RBDO. The accuracy of the moment-based reliability indices usually follows:

1. Within the applicable range, the fourth-moment reliability index, β_{4M}, is more accurate than other reliability indices.
2. The third-moment reliability index, β_{3M}, within the applicable range, is more accurate than the fourth-moment reliability index (β_{4M}) that is out of the applicable range.

Based on the above observation, the reliability analysis procedure of this study is illustrated in Fig. 1 as described below.

1. Calculate the central moments of the performance function using dimension-reduction method.

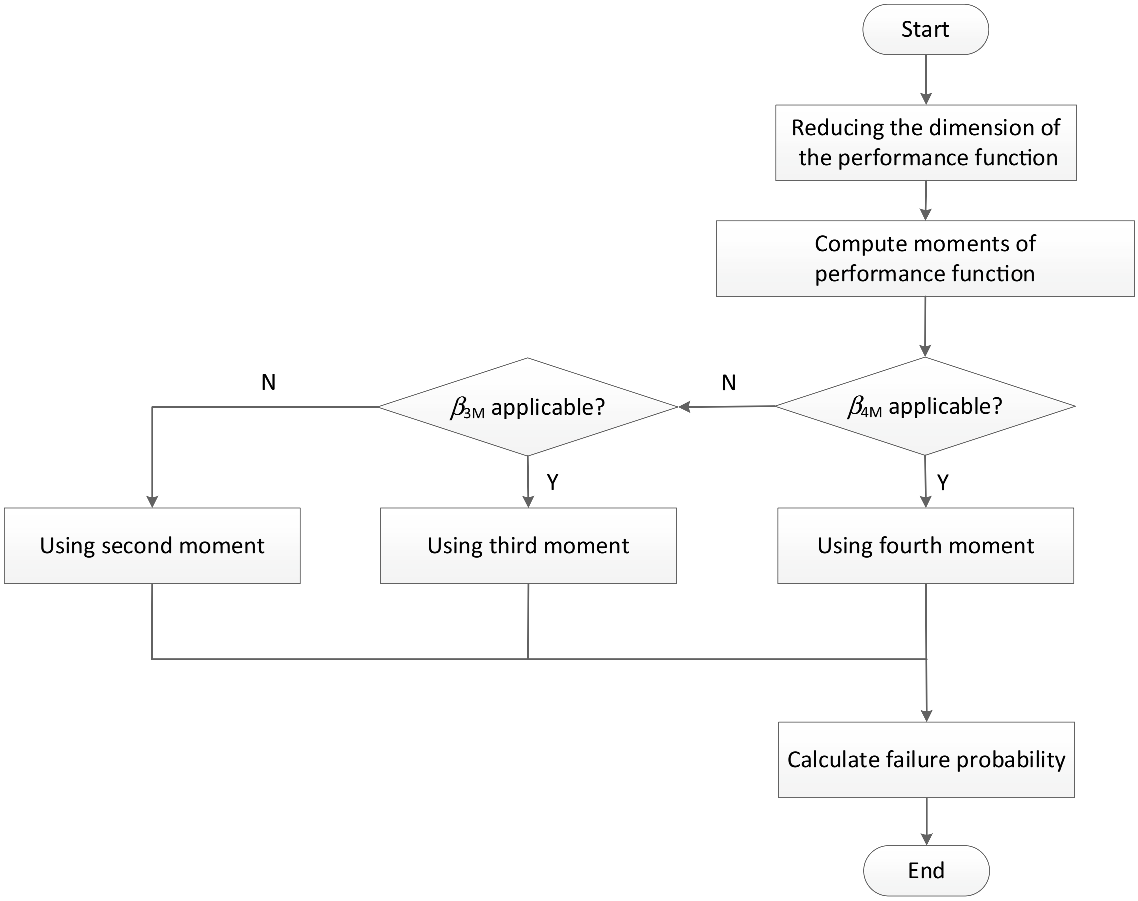

Fig. 1 Reliability analysis evaluation procedure in PSO-4M–3M–2M

2. Calculate the applicable range of the third- and fourth-moment reliability indices.
3. If the fourth-moment reliability index is within the applicable range, then it is used as the reliability index.
4. If the fourth-moment reliability index is out of applicable range, but the third reliability index is within applicable range, then the third-moment reliability index is used.
5. If the fourth- and third-moment reliability indices are not operable, then the second moment reliability index is used.

Figure 2 shows the detailed PSO evaluation process. For a given set of particles, the optimization process first computes particles' position and velocity randomly. The fitness value and reliability index (based on Fig. 1) is then calculated. The ε-constrained is used to update each particle's pBest and gBest. If the predefined iteration number is not met, then update the velocity and position of each particle using Eqs. (27) and (28) for the next iteration.

All the demonstrated RBDO problems are employed in Intel Core i7 CPU 870 @ 2.93 GHz. The proposed algorithm is coded in the Matlab 2013a version. The statistical toolbox is used here for the calculation of PDF and CDF of the different probability distributions. The SAP2000 is used to facilitate the structural analysis for the ten-bar truss problem, in which the Open Application Programming Interface (OAPI) is developed to allow a third party software (such as MATLAB, the optimizer in the current study) to access the SAP2000 functions.

5 Numerical examples

Several problems in the literature, as shown in Table 1, are used to examined the proposed RBDO algorithm. As indicated, different types of RBDO including linear and nonlinear performance functions, normal and non-normal variables, component and system reliability analyses are investigated.

5.1 Beam design

The objective of this optimization is to find the minimum weight or the minimum cross-sectional area of the cantilever

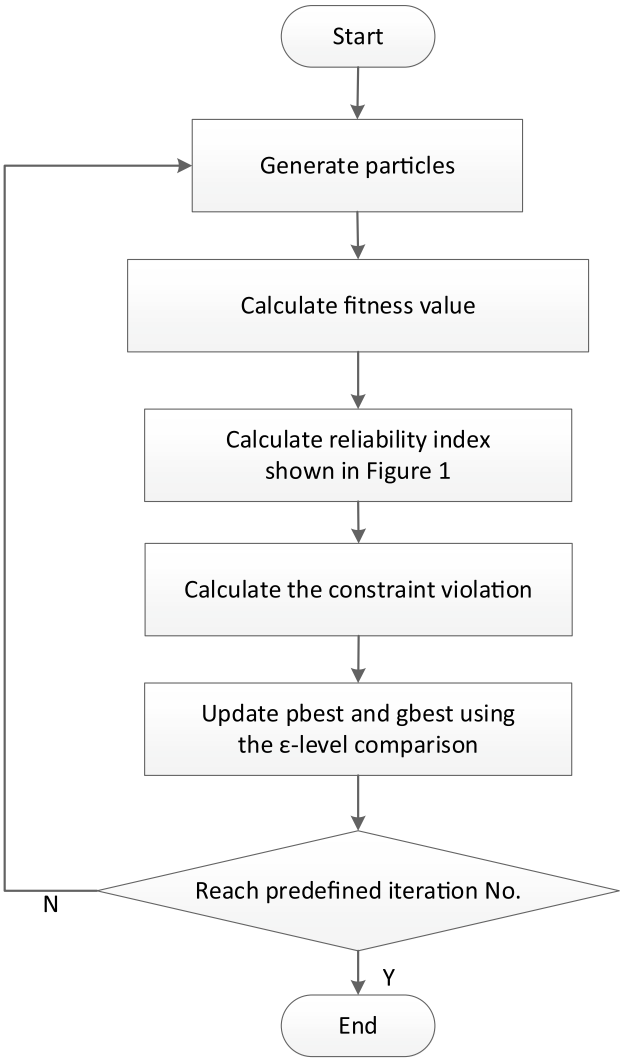

Fig. 2 Evaluation procedure of PSO in this study

beam, as shown in Fig. 3. The width (w) and height (t) of the beam are considered as deterministic design variables. Two loads (F_X, F_Y) at the free end, the modulus of elasticity (E), and the yield strength (R) are considered as random design parameters.

The mathematical formulation of the RBDO problem is given in the following equation:

$$\begin{aligned} \text{Min.}A &= wt \\ \text{s.t. Prob.}[G_i \le 0] &\le P_f^T \\ 0 \le w, \quad t &\le 10 \end{aligned}$$

$$G_1(\boldsymbol{d}, \boldsymbol{X}) = R - (600/(wt^2)F_Y + 600/(w^2t)F_X) \tag{32}$$

$$G_2(\boldsymbol{d}, \boldsymbol{X}) = D_0 - \frac{(4L^3)}{Ewt}\sqrt{((F_Y/t^2)^2 + (F_X/w^2)^2)},$$

where w (in.) is the width, t (in.) is the depth, A (in.^{2}) is the area, P_f^T is the target failure probability (i.e., 1.35×10^{-3}), R (psi) follows N(40,000, 2000) and is the allowable strength, F_X is an external horizontal force (lb) following N(500, 100), and F_Y is an external vertical load (lb) following N(1000, 100). L (span) is 100 in. D_o = 2.25 in. is the allowable tip displacement. E (psi) is the elastic modulus which follows $N(29 \times 10^6, 1.45 \times. 10^6)$. Please note that the two performance functions (G_1 and G_2) are separately considered (named case 1 and case 2). G_1 is linear, while G_2 is a nonlinear function. Nine-point estimation with three scenarios, univariate, bi-variate, and tri-variate dimension-reduction methods, is used in reliability analysis. 30 particles and 100 iterations are assigned in PSO, in which the values of c_1 and c_2 are set to be 2.0. The velocity limit is imposed and a linearly decreasing inertial parameter from 0.9 to 0.4 is used.

Results using PSO-4M–3M–2M algorithm are shown in Tables 2, 3, and 4. For linear and nonlinear performance functions consisting of normal random variables, the PSO-4M–3M–2M is able to find the optimal design. The mean of the ten MCS runs with 10^6 samples is used to calculate the relative error, which is described in the following equation:

$$\%\ \text{error} = \frac{|\bar{\beta}_{\text{MCS}} - \beta_{\text{4M - 3M - 2M}}|}{\bar{\beta}_{\text{MCS}}}, \tag{33}$$

where β_{MCS} is the mean reliability index of ten independent MCS runs and $\beta_{\text{4M–3M–2M}}$ is the reliability index obtained from the algorithm of PSO-4M–3M–2M. For both cases

Table 1 Summary of RBDO problems

Number	Description	Case	References
1	Beam design	Case 1: linear performance function Case 2: nonlinear performance function	Wu et al. [24], Qu and Haftka [15], Ramu et al. [16], Liao and Ivan [11]
2	Short column design	Case 1: normal Case 2: non-normal	Aoues and Chateauneuf [1]
3	Ten-bar truss	Implicit performance function	Shayanfar et al. [19]
4	2D-Mathematical problem	System reliability	Lee et al. [8], Liao and Lu [12]

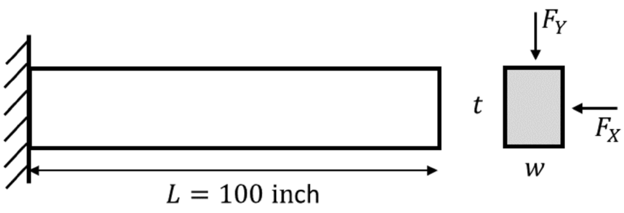

Fig. 3 Illustration of the cantilever beam

Table 3 Optimum designs of different RBDO algorithms (case 1)

	w (in.)	t (in.)	Area (in.^{2})	MCS
Wu et al. [24]	2.4484	3.8884	9.52036	0.00135
Qu and Haftka [15]	2.4526	3.8884	9.53669	0.00124
Ramu et al. [16]	2.4460	3.8920	9.51983	0.00134
Liao and Ivan [11]	2.4460	3.8922	9.52025	0.00137
PSO-4M–3M–2M	2.4502	3.8805	9.52030	0.00135

(i.e., the stress and displacement constraint), a decreasing trend in relative error is observed when a greater number of variates are used in dimension-reduction method. However, as expected, the function evaluation (FE) is increased in such approach. To be specific, the FE needed per reliability analysis is only 25 for univariate method, while it is 265 and 993 for the methods of bi-variate and tri-variate. With respect to the execution time of the univariate method, approximate three times and five times are needed for the methods of bi-variate and tri-variate.

From Tables 3 and 4, it is observed that the results of PSO-4M–3M–2M are similar to that of the previous studies. In general, the PSO-4M–3M–2M is able to find a better RBDO solution excepting the solution from Wu et al. [24]. Compared to the solutions of PSO-4M–3M–2M, earlier researches either have a bigger objective or a smaller objective but violating the probabilistic constraint, indicating that the scenario of using 4M–3M–2M approach is able to provide an accurate reliability index.

5.2 Short column design

A short column with rectangular cross section of dimensions b and h is subjected to normal axial force F and bi-axial bending moments M_1 and M_2. The limit-state function, calculated from the elastic–plastic constitutive law, is considered as the constraint function [G in Eq. (34)]. The mathematical formulation of the RBDO problem is then given in the following equation:

Table 4 Optimum designs of different RBDO algorithms (case 2)

	w (in.)	t (in.)	Area (in.^{2})	MCS
Wu et al. [24]	2.6999	3.4098	9.2061	0.0015813
Liao and Ha [10]	2.7210	3.3920	9.2296	0.0013509
PSO-4M–3M–2M	2.7021	3.4153	9.2285	0.0013498

$$
\begin{aligned}
&\text{Min.}A = \mu_b \mu_h \\
&\text{s.t. Prob.}[G(\boldsymbol{d},\boldsymbol{X}) \le 0] \le P_{\text{F}}^{T} \\
&0.5 \le \frac{\mu_b}{\mu_h} \le 2 \qquad (34) \\
&G(\boldsymbol{d},\boldsymbol{X}) = 1 - (4M_1)/(bh^2 f_y) - (4M_2)/(b^2 h f_y) - F^2/(bhf_y)^2,
\end{aligned}
$$

where b is the width, d is the depth, f_y is the yield strength, M_1 and M_2 are the bi-axial moments, F is the axial force, and P_f^T is the target failure probability (i.e., 1.35 $\times 10^{-3}$). To examine the suitability of the proposed PSO-4M–3M–2M for a problem with non-normal random variables, different types of PDF, such as Weibull and Gumbel, are considered, as indicated in Table 5. In addition, the influence of the variation from random variables (e.g., COV) is also examined, as shown in Table 5 (e.g., 0, 0.05, 0.1, and 0.15). Table 5 also illustrates other statistical properties such as mean and standard deviation of the random variables.

Table 6 displays optimization results of PSO-4M–3M–2M. As shown, the obtained optimal point is closer

Table 2 Summary of results for PSO-4M–3M–2M on beam design problem (nine points)

	Unit	Case 1: stress limit			Case 2: displacement limit		
		Univariate	Bi-variate	Tri-variate	Univariate	Bi-variate	Tri-variate
w	in.	2.4473	2.4386	2.4502	2.7140	2.7006	2.7021
t	in.	3.8902	3.9042	3.8855	3.3992	3.4158	3.4153
Area	in.^{2}	9.5205	9.5207	9.5203	9.2254	9.2246	9.2285
β_{MCS}	–	2.9980	2.9999	2.9996	2.9901	2.9944	2.9960
$\beta_{\text{4M–3M–2M}}$	–	3.0004	3.0007	3.0001	3.0004	3.0003	3.0000
Relative error	%	0.0799	0.0266	0.0166	1.1904	0.3979	0.0082
FE per RA	–	25	265	993	25	265	993
3M	%	1.5	2.7	1.5	93.5	19.7	11.7
4M	%	98.5	97.3	98.5	6.5	80.3	88.3

Table 5 Statistical properties for short column design problem

	F (kN)	M_1 (kN-m)	M_2 (kN-m)	f_y (MPa)	h (m)	b (m)
Mean	2500	250	125	40	μ_h	μ_b
COV	0.2	0.3	0.3	0.1	0/0.05/0.10/0.15	0/0.05/0.10/0.15
Normal case	Normal	Normal	Normal	Normal	Normal	Normal
Non-normal case	Gumbel	Gumbel	Gumbel	Weibull	Lognormal	Lognormal

Table 6 Comparison of the optimization results for short column design problem

	Normal case				Non-normal case			
	0	0.05	0.1	0.15	0	0.05	0.1	0.15
Aoues and Chateauneuf [1]	0.1915	0.2024	0.2372	0.3014	0.2014	0.2088	0.2319	0.2717
PSO-4M–3M–2M[a]	0.1918	0.2031	0.2394	0.3182	0.2133	0.2209	0.2457	0.2871
Relative error[b]	0.2974	0.7110	1.2017	0.9073	0.7338	0.2175	0.3331	0.9724
Ave. runtime[c]	224.16	837.85	824.86	823.39	274.58	945.87	946.12	963.52
FE per RA	3425	15,809	15,809	15,809	3425	15,809	15,809	15,809
3M (%)	2.6[c]				2.9[c]			
4M (%)	97.3[c]				97.1[c]			

[a]Solutions from nine points with tri-variate dimension reduction

[b]Equation (33) is used to calculate the error percentage

[c]COV=0.15

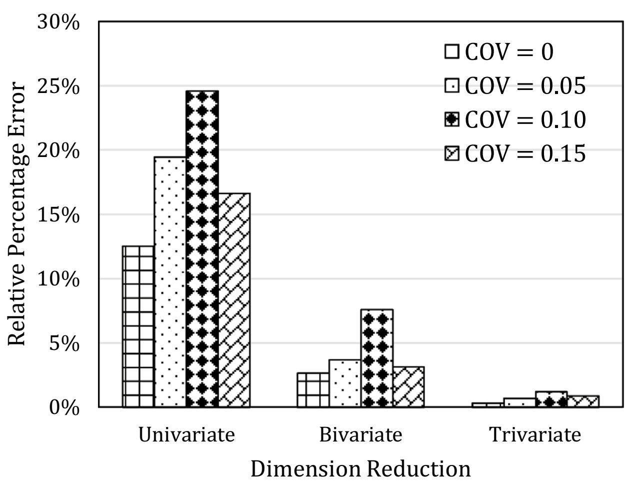

Fig. 4 Relative percentage error of reliability index at optimal point using PSO-4M–3M–2M (normal case)

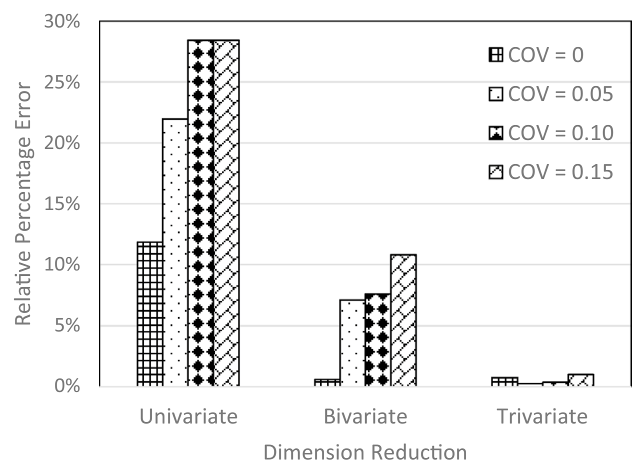

Fig. 5 Relative percentage error of reliability index at optimal point using PSO-4M–3M–2M non-normal case)

to those of literature studies. Furthermore, MCS and Eq. (33) are used to compute the error percentage, ranging from 0.2 to 1.2%. It is seen that the proposed PSO-4M–3M–2M is able to provide a promising solution. The PDF and COV of the random variables only have moderately influence on the error percentage.

The effect of different number of variate used in the dimension-reduction technique is illustrated in Figs. 4 and 5 for normal and non-normal cases, respectively. For both cases, it is clear that increasing number of variate will enhance the solution accuracy. However, the influence of COV on different numbers of variate does not have a clear trend. As shown in Fig. 4, relative error (RE) of COV = 0.05 is about 0.3 times that of COV = 0 in the bi-variate case, and RE of COV = 0.05 is about 0.6 times that of COV = 0 in the univariate case. It seems that the COV of random variable has more influence when the number of variate is smaller. However, Fig. 5 displays an opposite trend, RE of COV = 0.05 is more than four times that of COV = 0 in the bi-variate case, while RE of COV = 0.05 is about two times

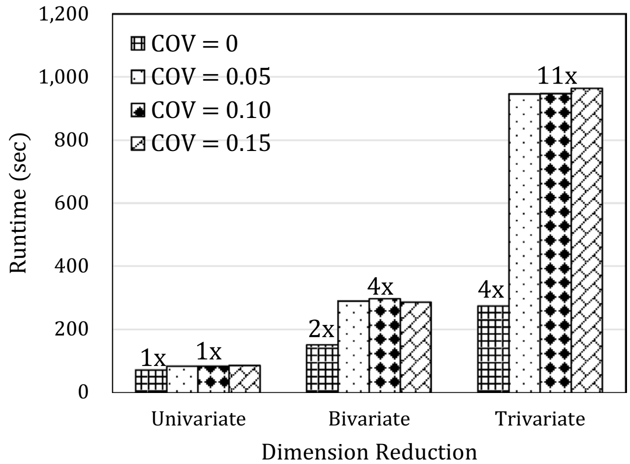

Fig. 6 Runtime for short column design problem using PSO-4M–3M–2M (non-normal case)

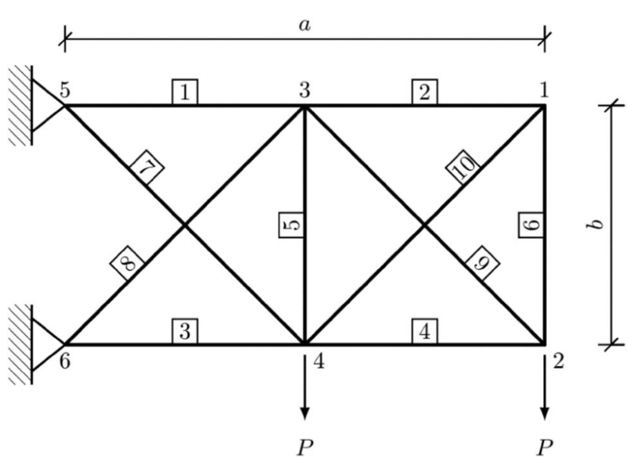

Fig. 7 Illustration of ten-bar truss problem

that of COV = 0 in the univariate case. Figure 6 compares the computational speed for the non-normal case with different COVs and number of variate. It is seen that the cost of increasing number of variate to enhance the accuracy is around 11 times to that of univariate case.

5.3 Ten-bar truss

To examine the applicability of the proposed PSO-4M–3M–2M on a problem of implicit performance, a classical ten-bar truss problem, adapted from Shayanfar et al. [19], is used. The objective of this ten-bar truss problem is to minimize the total weight of a truss structure, as shown in Fig. 7. The mathematical formulation is given in the following equation:

$$\begin{aligned} \text{Min.} w &= \rho \sum_{i=1}^{10} A_i L_i, i = 1, 2, \ldots, 10 \\ \text{s.t. } &\text{Prob}[G(\boldsymbol{d}, \boldsymbol{x}) \le 0] \le \Phi(-\beta^T) \\ &0.10 \le A_i \le 35.0, \end{aligned} \tag{35}$$

where ρ is the material density (0.1 lb/in.^{3}), A_i is the cross area of each truss member, L_i is the length of each truss member, G is the performance function, $\boldsymbol{x}$ is vector of random variable, $\boldsymbol{d} = [A_1, A_2, \ldots, A_{10}]^T$, $G(\boldsymbol{d}, \boldsymbol{x}) = \boldsymbol{u}_2 - 2\,\text{(in.)}\, \boldsymbol{u}_2$ is the vertical displacement at point 2, as shown in Fig. 7, and β^T is the target reliability index (i.e., 3.0). Parameters of a and b indicated in Fig. 7 are 720 and 360 (in.), respectively. As indicated in Eq. (35) and Fig. 7, this ten-bar truss problem has 12 random variables which all follows the normal distribution. The area, A_{1-10}, is both a design variable and a random variable and the design area is the mean value of the random variable, A_i. The external load, P, has a mean of 10^5 lb and the modulus of elasticity, E, has a mean of 10^7 psi. All 12 random variables have a COV that is equal to 0.05.

In addition to examine the suitability of the proposed PSO-4M–3M–2M for a problem with implicit function, effects of univariate, bi-variate and tri-variate are also investigated for the dimension reduction. Each dimension-reduction method uses 7, 9, and 11 points for moment estimation. 20 particles in 150 iterations are used in PSO to find the optimal solution. The inertial parameter is linearly decreasing from 0.9 to 0.6. c_1 and c_2 in Eq. (28) are set to be 2.0.

Table 7 displays the optimal area for each truss member using PSO-4M–3M–2M. It is seen that using more number of variate can effectively reduce the relative error percentages from approximately 5–0.2%. As expected, the tri-variate case delivers the best accuracy. However, the corresponding cost also increases dramatically when the more number of variate is used. Approximately, the cost of using tri-variate is 180 times to that of the univariate case. On the other hand, increasing number of point in moment estimation does not have a significant effect on the accuracy for all three cases. That is, seven-point estimation is able to precisely provide the moments for the calculation of reliability index. Compared to the literature study [19], it is observed that the proposed method provides a slightly better optimal weight and a better reliability measure compared to the previous study. However, due to the inherent randomness in PSO, the optimal weight of the seven-point estimation in the tri-variate case is slightly higher than other solutions obtained in the current study and the weight indicated in the literature study. It is believed that increasing point of the population or iteration number in PSO can solve this problem.

Table 7 Optimal areas for ten-bar truss problem using PSO-4M–3M–2M

No.[a]	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	W^b	β_{MCS}	E^c	T^d	3M/4M (%)
Univariate															
7	35.00	0.10	27.47	19.33	0.10	0.10	3.42	29.15	27.96	0.10	6042.19	2.8401	5.634	74.3	3.7[e]
9	35.00	0.10	29.21	20.20	0.10	0.10	3.79	27.37	27.71	0.10	6051.69	2.8335	5.962	77.2	
11	35.00	0.10	35.00	18.88	0.10	0.10	3.43	27.00	26.44	0.10	6110.82	2.8312	5.964	80.1	
Bi-variate															
7	35.00	0.10	35.00	19.09	0.10	0.10	3.36	27.39	27.22	0.10	6174.96	2.9735	0.896	1002.1	100[e]
9	35.00	0.10	27.57	19.92	0.10	0.10	3.35	29.59	28.49	0.10	6114.10	3.0006	0.045	1174.1	
11	35.00	0.10	35.00	19.02	0.10	0.10	3.43	27.87	26.71	0.10	6174.84	2.9860	0.468	1315.3	
Tri-variate															
7	35.00	0.10	35.00	16.61	0.10	0.10	3.09	24.31	35.00	0.10	6311.04	2.9963	0.120	9460.5	2.3[e]
9	35.00	0.10	35.00	18.90	0.10	0.10	3.44	27.18	27.58	0.10	6180.43	2.9986	0.132	15,203.2	
11	35.00	0.10	27.57	19.75	0.10	0.10	3.39	29.65	28.59	0.10	6118.09	2.9926	0.301	24,240.5	
Shayanfar et al. [19]															
	34.35	0.10	29.68	26.28	0.10	0.10	3.34	28.35	26.14	0.10	6211.30	3.0363	–	–	

[a]No. of point estimate
[b]Weight
[c]Relative error (%)
[d]Average running time (s)
[e]11 point

5.4 2D-mathematical problem

To reveal the applicability of the proposed algorithm in solving an RBDO with system reliability, a literature problem from Liao and Ivan [11], and Aoues and Chateauneuf [1] is investigated. The mathematical formulation of the RBDO problem is described in the following equation:

$$\begin{aligned} &\text{Min.}(\boldsymbol{d}) = (d_1 + d_2 - 8)^2/30 + (d_1 - d_2 - 15)^2/120 \\ &\text{s.t. Prob.}[(g_1 \le 0) \cup (g_2 \le 0) \cup (g_3 \le 0) \cup (g_4 \le 0)] \le 2.275\% \\ &0 \le \boldsymbol{d} \le 10, \end{aligned} \tag{36}$$

where

$$\begin{aligned} g_1(\boldsymbol{X}) &= 1 - ((X_1 - 0.3)^2 X_2)/20 \\ g_2(\boldsymbol{X}) = 1 - (-0.4226X_1 + 0.9063X_2) &+ (0.9063X_1 + 0.4226X_2 - 6)^2 + \\ (0.9063X_1 + 0.4226X_2 - 6)^3 &- 0.6(0.9063X_1 + 0.4226X_2 - 6)^4) \\ g_3(\boldsymbol{X}) &= 1 - 80/(X_1^2 + 8X_2 + 5) \\ g_4(\boldsymbol{X}) &= 1 - (X_1 - X_2 - 2.5)^2/30 - (X_1 + X_2 + 4.5)^2/120 \\ \boldsymbol{X} &\sim N(\boldsymbol{d}, 0.3) \\ \boldsymbol{d} &= [d_1, d_2]. \end{aligned}$$

It is seen that this is a series system reliability problem with nonlinear constraints. The proposed performance function for this problem based on the description of "Methods of moments for system reliability" is defined, as in the following equation:

$$G(\boldsymbol{X}) = \min[g_1, g_2, g_3, g_4]. \tag{37}$$

Table 8 displays the optimization results. It is seen that the error of the proposed algorithm is slightly larger than that of the previous examples. The simplification of system reliability using Eq. (37) and the simplification of reliability index using 4M, 3M, or 2M are the potential sources of the error. As discussed in "The proposed RBDO", within

Table 8 Solutions of the 2D-mathematical problem

	D_1	D_2	Cost	PF_{sys}	PF_{MCS}	% Error	T^b	3M/4M (%)
Liao and Lu [12]	4.401	3.867	1.7461	0.022565	0.022576	0.05	–	
PSO-4M–3M–2M[a]	4.531	3.942	1.7381	0.022742	0.022742	2.63	29.97	34.8

[a]Eleven point of tri-variate approach is used
[b]Average running time (s)

the applicable range, the fourth-moment reliability index, β_{4M}, is expected to provide a more accurate reliability index. However, although this is a general case, it is not always true. It is observed that, in a very few case, β_{3M} provides a better accuracy. Although the proposed RBDO algorithm (PSO-4M–3M–2M) is able to deliver a promising solution for many cases as shown in the numerical examples. More investigation on the moment-based reliability index should be worth for future application.

6 Discussions

RBDO, imposing probabilistic constraints into the optimization procedure, is one of the approaches to find an optimal design considering the effects of nondeterministic information. To verify the satisfaction of probabilistic constraints, reliability analysis must be conducted at each design trial, leading to a double-loop procedure which is often a barrier in application. This study investigates accuracy and efficiency of the moment-based reliability analysis that is incorporated with the ε-constrained PSO for RBDO problems. The proposed algorithm is validated using several RBDO problems, and the performance of accuracy, efficiency, and effects of applicable range is discussed below.

The accuracy of the proposed algorithm is evaluated by comparing the solutions from those of the literatures and MCS. Compared to the literatures, the proposed algorithm is able to find better RBDO solutions for all examples, excepting the solution from Wu et al. [24] in the first example and the solution from Liao and Lu [12] in the fourth example. Compared to the solution of MCS, the relative errors range from 0.008 to 2.63% for four examples if tri-variate method is adopted. The error percentage is often quite small, and the highest error percentage (2.63%) is found from the 2D-mathematical problem, in which system reliability is considered. The simplification of system reliability using Eq. (37) is considered as the potential source of the error. It is seen that the proposed algorithm often delivers a promising accuracy. In addition, a decreasing trend in relative error is observed when a greater number of variates are used for all examples. From the second example, it is found that the PDF and COV of the random variables only have moderately influence on the error percentage. However, increasing number of point in moment estimation does not have significant effect on the accuracy.

Although a greater number of variates can help enhance the solution accuracy, the function evaluation (FE) significantly increases in such approach. Taking the first example for example, the FE needed per reliability analysis is only 25 for univariate method, while it is 265 and 993 for bi-variate and tri-variate cases. In terms of the running time, the cost of using tri-variate is 11 and 180 times to that of univariate case for the short column and the ten-bar truss examples. The ten-bar truss example is used to demonstrate the applicability of the proposed algorithm in a problem with implicit performance functions. That is, finite-element analysis is needed in finding the optimal solution, resulting a longer running time. The running time for the ten-bar truss is 9460 s if the suggested approach (i.e., tri-variate with seven-point estimation) is used, which is not quite efficient, but should be still in an acceptable range.

Two applicable ranges of the 4M method are used for comparison. The empirical range (Eq. 23) is built based on allowable relative error (i.e., 2%) among different approaches. Zhao et al. [32] derived a complete expression of the 4M method with different combinations of skewness and kurtosis, and investigate the monotonicity of each expression, in which a theoretical applicable range was obtained. Tables 9, 10, and 11 display the results of using empirical and theoretical applicable ranges for the cantilever beam, short column, and ten-bar truss problem. For the cantilever beam, it is seen that both equations are able to

Table 9 Comparison of cantilever beam using empirical and theoretical applicable ranges

	w (in.)	t (in.)	Area (in.^{2})	MCS
Case 1[c]				
Equation (23)[a]	2.4502	3.8805	9.52030	0.00135
Equation (24)[b]	2.4497	3.8873	9.52140	0.00135
Case 2[c]				
Equation (23)[a]	2.7021	3.4153	9.2285	0.0013498
Equation (24)[b]	2.7049	3.4119	9.2287	0.0013499

[a]The empirical applicable range is used

[b]The theoretical applicable range is used

[c]Nine-point estimation with tri-variate is used

Table 10 Comparison of short column using empirical and theoretical applicable ranges

	COV = 0	COV = 0.15
Obj. for normal case[c]		
Equation (23)[a]	0.1918	0.3182
Equation (24)[b]	0.1974	0.3183
Difference	2.8%	0.03%
Obj. for non-normal case[c]		
Equation (23)[a]	0.2133	0.2871
Equation (24)[b]	0.2171	0.2871
Difference	1.8%	0%

[a]The empirical applicable range is used

[b]The theoretical applicable range is used

[c]Nine-point estimation with tri-variate is used

Table 11 Comparison of ten-bar truss using empirical and theoretical applicable ranges

Case	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	W^b	Diff.
Tri-variate with 11 points												
A[a]	35.00	0.10	27.57	19.75	0.10	0.10	3.39	29.65	28.59	0.10	6118.09	0.7%
B[b]	35.00	0.10	27.99	20.14	0.10	0.10	3.36	28.89	28.83	0.10	6119.50	

[a]The empirical applicable range is used

[b]The theoretical applicable range is used

find satisfied solutions that is verified by the MCS. Table 10 displays solution differences between two equations, ranging from 0 to 2.8%, indicating that effects of COV and normal/non-normal are insignificant. Similarly, for a problem with implicit performance function such as ten-bar truss problem, using empirical or theoretical applicable ranges does not have considerable change. Observed that the applicable ranges indicated in Eqs. 23 and 24 are similar, outcomes of Tables 9, 10, and 11 are reasonable.

7 Conclusions

Based on the results and discussions found here, the proposed PSO-4M–3M–2M algorithm is able to deliver a promising design and several important observations are described below.

1. The proposed RBDO algorithm is shown to be a stable method, which is not affected by a problem with non-normal random variables or with nonlinear performance functions.
2. In general, the solutions found by PSO-4M–3M–2M, compared to the literatures, are either more optimal or more approaching to the probabilistic constraint for different types of RBDO problems investigated here. This indicates that the moment-based reliability index provides an accurate uncertainty measurement for PSO to find a solution.
3. The accuracy of an RBDO solution is proportional to the number of variate in dimension-reduction method. The drawback of increasing variate number is its computational cost is often higher.
4. Random variable with higher COV generally, but not always decreases the accuracy of the moment-based reliability measurement. Using more variate can dramatically reduce such error.
5. Increasing number of point in moment estimation is not necessary to enhance the accuracy of reliability analysis. Based on the investigated problem, using seven points appears to be adequate for estimating the reliability index.

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