行政院國家科學委員會補助專題研究計畫成果報告

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計畫類別：■**個別型計畫**　　□**整合型計畫**
計畫編號：NSC89－2211－E－002－117
執行期間：89 **年** 8 **月** 1 **日至** 90 **年** 7 **月** 31 **日**

計畫主持人：劉佩玲
共同主持人：

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執行單位：國立台灣大學應用力學研究所

中　華　民　國　九十年　十　月三十一日

行政院國家科學委員會專題研究計畫成果報告

結構可靠度再評估之研究（二）

Re-evaluation of Structural Reliability Based on Structural Response

計畫編號：NSC 89-2211-E-002 -117
執行期限：89 **年** 8 **月** 1 **日至** 90 **年** 7 **月** 31 **日**
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一、中文摘要

本計畫的主旨在發展一套監測結構物可靠度的方法，先以有限元素可靠度法計算結構物的初始可靠度。結構物受損後，根據結構反應，以貝氏統計法修正桿件性質分佈。最後，再以修正後的桿件機率分佈重新對結構物作可靠度評估，並利用可靠度法中的敏感度分析定出最佳維修方式。

關鍵詞：可靠度、安全評估

Abstract

The objective of this project is to develop a method to monitor the reliability of structures. The initial reliability of a structure is obtained by the finite element reliability method. After a period of service, nondestructive tests are performed on the structure, and the element properties are reconstructed based on the response of the damaged structure. Then, the reliability of the structure is computed using the updated distributions of element properties. Furthermore, reliability sensitivity analysis is performed to attain the optimal maintenance strategy.

Keywords: reliability, safety assessment.

二、緣由與目的

The nondestructive examination of existing structures has gained increasing interest in recent years. The inspection methods can be divided into two categories: local and global. The local inspection applies nondestructive testing techniques to examine the local properties, defects, degradation, etc. of a structure, while the global inspection applies the system identification methods to construct the global properties of the structure, such as the element rigidities of the structure. The main purpose of such inspection is to evaluate the safety of an existing structure. However, how to apply the inspection results in the safety assessment is an unexplored area.

The reliability analysis is effective in the safety assessment of structures. It can provide a reasonable estimate on the failure probability of a structure if the distributions of the uncertainties in the system are known. However, the distributions of the material properties change when the structure is damaged. Therefore, the distributions should be modified and the reliability of the structure should be re-evaluated when damage occurs.

The Bayesian statistics can be used to update the distribution parameters of a random variable based on new observations. Its applications in engineering can be found in the literature.

This study adopts the Bayesian approach to update the distributions of the structural properties based on the response of the structure. Then, the reliability of the structure is computed using the updated distributions of the properties. Furthermore, sensitivity analysis is performed to decide the follow-up maintenance strategy.

三、可靠度分析

The structural reliability analysis is formulated based on two fundamental assumptions: (1) the state of the structure is defined in the outcome space of a vector of basic random variables, **V**; (2) the structure can be in one of two states, the safe state or the failure state. The state of the structure is determined by the value of a limit-state function $g(\mathbf{v})$, which is formulated such that when $g(\mathbf{v})>0$, the structure is safe, and when $g(\mathbf{v})\le 0$, the structure fails. The failure probability of the structure associated with the specific failure criterion, then, can be obtained by integrating the joint probability density function of **V** in the failure domain. That is

$$P_f=\int_{g(\mathbf{v})\le 0} f_{\mathbf{V}}(\mathbf{v})\,d\mathbf{v}$$

where $f_{\mathbf{V}}(\mathbf{v})$ is the joint probability density function of **V**. It is usually difficult to perform the multifold integral directly. Hence, the first-order reliability method (FORM) is often adopted to estimate the failure probability.

Apparently, the reliability analysis gives accurate results only when the joint distribution $f_{\mathbf{V}}(\mathbf{v})$ is valid. However, $f_{\mathbf{V}}(\mathbf{v})$ changes when the structure is damaged. Such change is reflected in the structural response. Therefore, $f_{\mathbf{V}}(\mathbf{v})$ can be updated based on the response of the damaged structure.

四、貝氏統計

Usually, the statistics of the structural properties are available at the construction stage. Such data may be obtained by experiments, theoretical derivations, and/or engineering judgment. Therefore, some prior information is known about the structural properties.

Suppose structural test is carried out on the target structure. Hence, new information is collected from the test. Apparently, such information should be incorporated to give a new estimate of the structural properties.

The Bayesian approach can incorporate new experimental outcomes to update the distribution of a random variable. Hence, it can be applied to meet our needs.

Let **V** denote the element properties of the target structure with prior distribution $f_{\mathbf{V}}'(\mathbf{v})$, and **t** denote the measured response of the structure, e.g., displacements, strains, etc.. According to the Bayesian formula, the updated probability density of **V** is

$$f_{\mathbf{V}}''(\mathbf{v})=k f_{\mathbf{t}|\mathbf{V}}(\mathbf{t}\mid\mathbf{v})f_{\mathbf{V}}'(\mathbf{v})$$

where $f_{\mathbf{t}|\mathbf{V}}(\mathbf{t}\mid\mathbf{v})$ is the conditional probability density of observing **t** as **V** = **v** $k=\int\int_{-\infty}^{\infty} f_{\mathbf{t}|\mathbf{V}}(\mathbf{t}\mid\mathbf{v})f_{\mathbf{V}}'(\mathbf{v})d\mathbf{v}$ is a normalizing constant.

Take **t** = **u** = measured displacements for example. Suppose the measurement results are independent normal with mean $\mathbf{K}^{-1}\mathbf{F}$ and standard deviations σ_u, where **K** is the stiffness matrix of the structure, and **F** is the load vector. Then, $\mathbf{C}_{\mathbf{u}}=\sigma_u^2\mathbf{I}$ and

$$\begin{aligned}f_{\mathbf{u}|\mathbf{V}}(\mathbf{u}\mid\mathbf{v})&=\mathrm{N}\left(\mathbf{K}^{-1}\mathbf{F},\mathbf{C}_{\mathbf{u}}\right)\\&=\mathrm{N}\left(\mathbf{K}^{-1}\mathbf{F},\sigma^2\mathbf{I}\right)\\&=\frac{1}{(2\pi)^{m/2}\sigma_u^m}\exp\left[-\frac{1}{2\sigma_u^2}\left(\mathbf{u}-\mathbf{K}^{-1}\mathbf{F}\right)^T\left(\mathbf{u}-\mathbf{K}^{-1}\mathbf{F}\right)\right]\end{aligned}$$

In the above expression, $\mathbf{K}^{-1}\mathbf{F}$ is an implicit function of **V**. That makes the Bayesian update difficult to apply. To overcome this problem, $\mathbf{K}^{-1}\mathbf{F}$ is expanded into a Taylor's series. For linear structures, **K** can be expressed as

$$\mathbf{K}(\mathbf{v})=\sum_e \mathbf{K}_e=\sum_e \mathbf{A}_e\nu_e=\mathbf{K}(\mathbf{v}_0)+\sum_e \mathbf{A}_e\Delta\nu_e$$

where $\mathbf{K}_0=\mathbf{K}(\mathbf{v}_0)$, and $\mathbf{v}_0$ is the vector of the original properties. Hence,

$$\mathbf{K}^{-1}(\mathbf{v}) = \mathbf{K}_0^{-1} - \sum_e \mathbf{K}_0^{-1}\mathbf{A}_e\mathbf{K}_0^{-1}\Delta\nu_e + \text{i}$$

$$\mathbf{K}^{-1}\mathbf{F} \cong \mathbf{K}_0^{-1}\mathbf{F} - \sum_e \mathbf{K}_0^{-1}\mathbf{A}_e\mathbf{K}_0^{-1}\mathbf{F}(\nu_e - \nu_{0e})$$

$$= \mathbf{u}_0 - \mathbf{H}\mathbf{v}$$

where

$$\mathbf{H} = \left[\mathbf{K}_0^{-1}\mathbf{A}_1\mathbf{K}_0^{-1}\mathbf{F} \quad \text{i} \quad \mathbf{K}_0^{-1}\mathbf{A}_m\mathbf{K}_0^{-1}\mathbf{F}\right]$$

and $\mathbf{u}_0 = \mathbf{K}_0^{-1}\mathbf{F} + \mathbf{H}\mathbf{v}_0$. Substitute the above equation into the conditional probability, one gets

$$f_{\mathbf{u}|\mathbf{V}}(\mathbf{u} \mid \mathbf{v})$$

$$= k_u \exp\left[-\frac{1}{2\sigma_u^2}(\mathbf{H}\mathbf{v} - \mathbf{u}_0 + \mathbf{u})^T(\mathbf{H}\mathbf{v} - \mathbf{u}_0 + \mathbf{u})\right]$$

$$= N(\text{ì}_u, \mathbf{C}_u)$$

where k_u is a normalization factor, $\text{ì}_u =$ $\sigma_u^{-2}\mathbf{C}_u\mathbf{H}^T(\mathbf{u}_0 - \mathbf{u})$, and $\mathbf{C}_u = \sigma_u^2(\mathbf{H}^T\mathbf{H})^{-1}$.

Suppose the prior distribution of $\mathbf{V}$ is $f'_{\mathbf{V}}(\mathbf{v}) = \mathrm{N}(\text{«}'_{\mathbf{V}}, \mathbf{C}'_{\mathbf{V}})$, where $\text{«}'_{\mathbf{V}}$ is the prior mean vector, and $\mathbf{C}'_{\mathbf{V}}$ is the prior covariance matrix. Then, the posterior distribution of $\mathbf{V}$ is

$$f''_{\mathbf{V}}(\mathbf{v}) = \mathrm{N}(\text{«}''_{\mathbf{V}}, \mathbf{C}''_{\mathbf{V}})$$

where

$$\text{«}''_{\mathbf{V}} = \left(\mathbf{C}'^{-1}_{\mathbf{V}} + \mathbf{C}_u^{-1}\right)^{-1}\left(\mathbf{C}'^{-1}_{\mathbf{V}}\text{«}'_{\mathbf{V}} + \mathbf{C}_u^{-1}\text{ì}_u\right)$$

$$\mathbf{C}''_{\mathbf{V}} = \left(\mathbf{C}'^{-1}_{\mathbf{V}} + \mathbf{C}_u^{-1}\right)^{-1}$$

are the posterior mean vector and covariance matrix of $\mathbf{V}$, respectively.

Now, consider that the element strains **£** rather than the displacement vector is measured. In the context of finite element analysis, $\varepsilon = \mathbf{B}\mathbf{K}^{-1}\mathbf{F}$.

Again, suppose the measurement results are independent normal with mean $\mathbf{B}\mathbf{K}^{-1}\mathbf{F}$ and standard deviations σ_ε. Then

$$f_{\text{å}|\mathbf{V}}(\text{å} \mid \mathbf{v})$$

$$= \mathrm{N}(\mathbf{B}\mathbf{K}^{-1}\mathbf{F}, \sigma_\varepsilon^2\mathbf{I})$$

$$= \frac{\exp\left[-\frac{1}{2\sigma_\varepsilon^2}(\text{å} - \mathbf{B}\mathbf{K}^{-1}\mathbf{F})^T(\text{å} - \mathbf{B}\mathbf{K}^{-1}\mathbf{F})\right]}{(2\pi)^{m/2}\sigma_\varepsilon^m}$$

$$f_{\text{å}|\mathbf{V}}(\text{å} \mid \mathbf{v}) =$$

$$k_\varepsilon \exp\left[-\frac{1}{2\sigma_\varepsilon^2}(\text{å} - \mathbf{B}(\mathbf{u}_0 - \mathbf{H}\mathbf{v}))^T(\text{å} - \mathbf{B}(\mathbf{u}_0 - \mathbf{H}\mathbf{v}))\right]$$

$$= N(\text{ì}_\varepsilon, \mathbf{C}_\varepsilon)$$

where k_ε is a normalization factor, $\text{ì}_\varepsilon =$ $\sigma_\varepsilon^{-2}\mathbf{C}_\varepsilon\mathbf{H}^T(\mathbf{B}\mathbf{u}_0 - \text{å})$, $\mathbf{C}_\varepsilon = \sigma_\varepsilon^2(\mathbf{H}^T\mathbf{B}^T\mathbf{B}\mathbf{H})^{-1}$.

Once the distributions of the structural properties are modified using the results of system identification, reliability analysis can be carried out using the updated distributions.

五、敏感度分析

Let ¯ be a set of distribution parameters, for example, means and standard deviations. The sensitivity of with respect to ¯ is as follows:

$$\frac{\partial}{\partial ¯} = \frac{\mathbf{y}^{*T}}{} \frac{\partial \mathbf{y}^*(\mathbf{v}^*, ¯)}{\partial ¯}\bigg|_{\mathbf{v}^*}$$

where $\mathbf{v}^*$ is the design point in the original space. Applying the chain rule, one gets

$$\frac{\partial P_{f1}}{\partial ¯} = -¤(\quad)\frac{\partial}{\partial ¯}$$

where ¤ is the standard normal probability density function.

The sensitivity measures can be used to compare the influence of various properties on the structural reliability. If the failure probability is very sensitive to the mean of an element property, deterioration of the element would greatly increase the failure probability of the structure. Such element must be repaired immediately once it is damaged. Therefore, the sensitivity measures are useful in the arrangement of maintenance plan.

The sensitivity measures can also be used to estimate the change of failure probability when the structure is damaged:

$$P_{f1}(¯ - \Delta¯) \cong P_{f1}(¯) - \frac{\partial P_{f1}}{\partial ¯}\Delta¯$$

六、與系統識別修正法之比較

The main difference between the proposed evaluation method and the method developed in a previous study [1] is experimental results **t** used in the Bayesian modification. In Ref. [1], **t** represents the element properties obtained from system identification. In this study, **t** represents the measured response. Obviously, the proposed method has the advantage that no identification is required, and thus is more efficient. It also excludes the identification errors in the evaluation process. However, a new update formula needs to be derived for each type of measurement. Hence, the evaluation method based on identification results may be preferred if the identification is readily available.

七、結論

This paper develops a framework to incorporate the response of the damaged structure in the re-evaluation of structural reliability. Several conclusions can be made from this study:

1. The Bayesian method provides a systematic way of incorporating the structural response to update the distributions of the structural properties.
2. The proposed method does not require system identification in the evaluation process.
3. The sensitivity measures can be used to devise the optimal maintenance plan.
4. As a structure is damaged, the sensitivities of the failure probability associated with the damaged elements increase most rapidly.

七、計畫成果自評

本計畫預計完成的工作項目如下：

1. **建立以結構反應直接修正桿件性質的方法。**
2. **發展電腦程式**
3. **以數值算例驗証直接修正法及可靠度再評估。**
4. **研究量測不同反應對直接修正法之影響。**
5. **比較系統識別修正法及直接修正法。**

在本年度內，本研究已達成這五項預定目標，研究成果目前正在撰寫論文，預定投稿 Structural Safety **期刊。**

八、參考文獻

1. P.-L. Liu and Y.-S. Chen, “The Re-Evaluation Of Structural Reliability Based On Identification Results”, *The Chinese Journal of Mechanics*, vol. 15, no. 3, pp. , 1999. (EI, SCI)