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非線性運動系統之類神經網路順滑控制（二）
Neural/Fuzzy Sliding Mode Control of Nonlinear Motion Control Systems (II)

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主持人：林巍聳 e-mail:weisong@cc.ee.ntu.edu.tw 參與人員：陳俊勝 許書凱 鄭佳弘
執行單位：國立台灣大學電機工程學系

中文摘要
本研究發展出結合可適性模糊邏輯控制器與順滑控制器的設計方法，應用於控制多變數非線性系統時，由於具有自學的功能因此不需要知道系統的模型，而自調後件函數更使系統具有强健性質。整個系統分為循順滑面和向順滑面兩個控制結構，循順滑面時的可適性模糊控制器包含子系統互應抵消控制和等效控制兩個主要部份，撞擊控制器則負責向順滑面的控制，全系統的穩定性均通過李亞普諾夫穩定定理的分析驗證，並成功的應用於二連桿機器手臂的控制。
關鍵詞：可適性模糊控制、多變數非線性系統、順滑控制

Abstract A sliding-mode direct adaptive fuzzy controller is developed for the control of a class of uncertain MIMO nonlinear systems. The controller requires no knowledge of the MIMO nonlinear system and facilitates robust properties by fine-tuning the consequent membership functions. By employing the fuzzy descriptions to overcome the interaction among the subsystems, a fuzzy sliding-mode controller is used to approximate the equivalent control in the neighborhood of the switching hyper-plane, and a hitting control is appended to assure the closed-loop stability. The stability is achieved by assuring the trajectories tracking control of the unknown nonlinear plant to satisfy Lyapunov stability theory. The simulation results of a two-link robotic manipulator confirm that the effect of both the fuzzy approximation error and external disturbance on the tracking error can be attenuated efficiently by the proposed method.
Keywords: adaptive fuzzy, sliding mode, MIMO nonlinear system

1. Introduction

Conventional control theory is well suited for application where the control efforts can be generated based on analytical model [1-2]. There are inevitable unmodelled nonlinearities and uncertain disturbance in their constructed model where conventional control strategies cannot be easily derived. The latest studies consider adding some computationally intelligent methods to the sliding mode control (SMC) by automatically tuning the control parameters. Particularly, integrating fuzzy set theory and SMC into fuzzy controller design have acquired superior performance [3-8]. This approach retains the positive property of SMC but alleviates the chattering, and the fuzzy control rules can be determined systematically by the reaching condition of the SMC.

This paper will address the problem of controlling an unknown multi-input multi-output (MIMO) nonlinear affined system. The goal is to develop a direct adaptive MIMO fuzzy controller to overcome the interaction among the subsystems by a decoupling neural network and to facilitate robust properties by fine-tuning the consequent membership functions. Firstly, a sliding mode controller for robust tracking control of multivariable nonlinear systems is developed by assuming that imposed uncertainties are bounded and satisfy matching conditions. The fuzzy logic control is then designed on the basis of the SMC law. A fuzzy sliding mode control (FSMC) is used to approximate the equivalent control in the neighborhood of the switching hyperplane with on-line fuzzy self-tuning parameters subject to parameter variations in the control object. Secondly, the hitting control is appended to assure that the proposed FSMC can result in a closed-loop system that is stable for the trajectories tracking control of a plant with unknown nonlinear dynamics. As a result, we simultaneously guarantee the global stability of the closed-loop system and obtain a suitable equivalent control when the nominal mathematic model is unknown in advance. The simulations using the proposed method by a two-link manipulator subject to uncertainties is performed to demonstrate the properties of the developed FSMC.

2. Problem Formulation

Consider an MIMO nonlinear system govern by

$$y^{(r)} = f(x) + G(x)u + d(x,t) \tag{1}$$

where $y = (y_1, \cdots, y_m)^T$ and $y^{(r)} \equiv (y_1^{(r_1)}, \cdots, y_m^{(r_m)})^T$ denote the output vector and its derivative, respectively, $r = (r_1, \cdots, r_m)$ with $\sum_{i=1}^{m} r_i = n$ is defined as the system relative degree, $u = (u_1, \cdots, u_m)^T$ is the input, $x = (x_1, x_2, \cdots, x_n)^T = (y_1, \cdots, y_1^{(r_1-1)}, y_2, \cdots, y_m^{(r_m-1)})^T$ is the state vector, $f(x) = (f_1(x), \cdots, f_m(x))^T$, $G(x) = [g_1(x), \cdots, g_m(x)]^T$, $g_i(x) = (g_{i1}(x), \cdots, g_{im}(x))^T$ with $g_{ii} > 0$ and $f_i(x)$, $i = 1, \cdots, m$, are unknown functions, and $d(x,t) = (d_1(x,t), \cdots, d_m(x,t))^T$ is the disturbance with the properties of standard smoothness and it is assumed to have upper bound $D = Diag[D_i]$, that is, $|d_i(x,t)| \le D_i$, $i = 1,\cdots,m$.

Let $y_d = (y_{d1}, y_{d2}, \cdots, y_{dm})^T$ represents the known desired trajectory, the control aim is to determine a controller for the composite nonlinear system described by (1) so that the tracking error represented by

$$\underline{e} = [e_1, \cdots, e_m]^T \tag{2}$$

with $e_i = (e_i, \dot{e}_i, \cdots, e_i^{(r_i-1)})^T = (y_{di} - y_i, \cdots, y_{di}^{(r_i-1)} - y_i^{(r_i-1)})^T$, $i = 1,\cdots,m$, will be attenuated to an arbitrarily small residual tracking error set. Define a generalized error vector to represent a switching manifold as follows:

$$s = \begin{pmatrix} e_1^{(r_1-1)} + \alpha_{11} e_1^{(r_1-2)} + \cdots + \alpha_{1,(r_1-1)} e_1 \\ e_2^{(r_2-1)} + \alpha_{21} e_2^{(r_2-2)} + \cdots + \alpha_{2,(r_2-1)} e_2 \\ \vdots \\ e_m^{(r_m-1)} + \alpha_{m1} e_m^{(r_m-2)} + \cdots + \alpha_{m,(r_m-1)} e_m \end{pmatrix} \tag{3}$$

and $\Lambda_i = (\alpha_{i1}, \cdots, \alpha_{ir_i})^T \in R^{r_i}$ be such that all roots of the polynomial

$$h_i(p) = p^{(r_i)} + \alpha_{i1} p^{(r_i-1)} + \cdots + \alpha_{i,(r_i-1)} p + \alpha_{ir_i} \tag{4}$$

are in the open left-half plane, $i = 1,\cdots,m$. The aim of sliding mode control law is to force the system states approach the sliding surface and then move along the sliding surface to the origin. This implies that the system dynamics will track reference trajectory asymptotically.

3. Description of the Fuzzy Logic System

Various fuzzy models and their control have been successfully applied in many fields [9-12]. The basic configuration of the fuzzy logic system comprises four principal components: fuzzifier, fuzzy rule base, fuzzy inference engine and defuzzifier [13]. The fuzzy control rules are the principal factor to determine the performance of a fuzzy controller. The fuzzy system can uniformly approximate nonlinear continuous functions to arbitrary accuracy [14-15]. Thus we will introduce fuzzy systems, which are expressed as a series expansion of fuzzy basis functions.

The fuzzy logic system performs a mapping from $U \subset R^n$ to $V \subset R^m$. Let $U = U_1 \times \cdots \times U_n$ and $V = V_1 \times \cdots \times V_m$ where $U_k \subset R$, $k = 1,2,\cdots,n$ and $V_i \subset R$, $i = 1,2,\cdots,m$. A multivariable system can be controlled by the following N linguistic rules

$R^{(l)}$: IF x_1 is A_1^l and $\cdots$ and x_n is A_n^l

THEN z_1 is B_1^l and $\cdots$ and z_m is B_m^l (5)

where $l = 1,\cdots,N$, $x_k, k = 1,2,\cdots,n$, are the input variables to the fuzzy system, $z_i, i = 1,2,\cdots,m$, are the output variables of fuzzy system, and the antecedent fuzzy sets A_k^l in U_k and the consequent fuzzy sets B_i^l in V_i are linguistic terms characterized by the fuzzy membership functions $\mu_{A_k^l}(x_k)$ and $\mu_{B_i^l}(z_i)$, respectively. The fuzzy logic system with center-average defuzzifier, product inference and singleton fuzzifier is defined as [14]

$$z_i(x) = \frac{\sum_{l=1}^{N} \mu^l(x) \cdot q_i^l}{\sum_{l=1}^{N} \mu^l(x)} \tag{6}$$

where $\mu^l(x) = \Pi_{k=1}^{n} \mu_{A_k^l}(x_k)$ is the matching degree of the lth rule, and q_i^l is the center of the consequent membership function of the lth rule. If q_i^l is chosen as the design parameter, the adaptive fuzzy system can be viewed as the type of neural network [16]. Therefore, (6) can be rewritten as

$$z_i(x) = \phi_i^T \psi(x) \tag{7}$$

where $\phi_i = (q_i^1, \cdots, q_i^N)^T$ is a parameter vector, and $\psi(x) = (\xi_1, \cdots, \xi_N)^T$ is a regressor, and where the fuzzy basis function is defined as [14]

$$\xi_l = \frac{\Pi_{k=1}^{n} \mu_{A_k^l}(x_k)}{\sum_{l'=1}^{N} \left(\Pi_{k=1}^{n} \mu_{A_k^{l'}}(x_k) \right)} \tag{8}$$

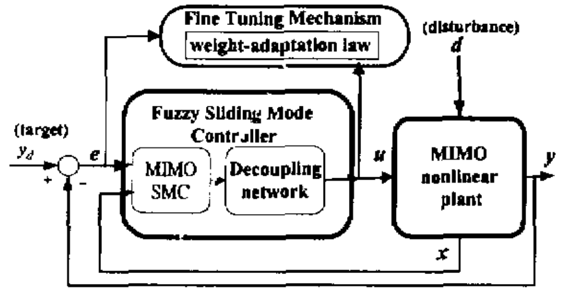

Fig. 1 Configuration of the adaptive FSMC system.

4. Adaptive Fuzzy Sliding Mode Controller

The proposed adaptive fuzzy sliding mode controller is composed of the following three parts: an MIMO SMC, a fine-tuning mechanism on the consequent membership functions of the multi-layer fuzzy system, and a decoupling network shown in Fig. 1 [17]. The multi-layer fuzzy system and the decoupling network are nominal designs based on on-line approximation of the unknown nonlinear functions of the plant. The fine-tuning mechanism is designed to encounter the equivalent uncertainty resulted by the plant uncertainty, the function approximation error, or the external disturbances.

Let u_i^0 be the output of the system's ith MIMO SMC. Then, for the system given in (1), the ith sliding surface is s_i. Hence, this MIMO SMC also has m sliding surfaces to form a switching manifold so that the system exhibits desirable behavior when its trajectories are confined in the sliding surfaces. If the control law is designed such that the sliding mode exists on $s_i = 0$, $i = 1,\cdots,m$, the system error dynamics is dictated by the linear dynamic equation (3). Since (3) satisfies Hurwitz stability criterion from (4), maintaining system states on sliding surface s for all $t > 0$ is equivalent to the tracking problem $y = y_d$, that is, it is required that the system errors converge to zero. Thus, the tracking control problem can be formulated by keeping the error vector in (2) on the sliding surface defined as follows:

$$\dot{s} = y_d^{(r)} - f(x) - G(x)u - d(x,t) + \begin{pmatrix} \sum_{i=1}^{r_1-1} \alpha_{1i} e_1^{(r_1-i)} \\ \sum_{i=1}^{r_2-1} \alpha_{2i} e_2^{(r_2-i)} \\ \vdots \\ \sum_{i=1}^{r_m-1} \alpha_{mi} e_m^{(r_m-i)} \end{pmatrix} \quad (9)$$

In the design of sliding mode controller, an equivalent control is given first such that each state Lyapunov-like condition holds for system stability [18]:

$$\frac{1}{2}\frac{d}{dt}(s_i^2) \le -\eta_i |s_i|,\ \eta_i > 0,\ i = 1,\cdots,m \quad (10)$$

or in sum:

$$\frac{1}{2}\frac{d}{dt}(s^2 s) \le -\sum_{i=1}^{m} \eta_i |s_i|,\ \eta_i > 0, i = 1,\cdots,m \quad (11)$$

Inequality (10) constrains trajectories to point towards the surface $s_i(t)$ such that the distance to sliding surface decreases along all system trajectories, and is referred to as the reaching condition. That is, the states of the system are driven from any initial state to the eventual sliding surface on which sliding mode control takes place.

If the function f, G and d of nonlinear MIMO systems (1) are known and does not take the interconnections among subsystems into consideration, then the control law u^0 can be chosen as follows

$$u^0 = \hat{P}^{-1}(\begin{pmatrix} \sum_{i=1}^{r_1-1} \alpha_{1i} e_1^{(r_1-i)} \\ \sum_{i=1}^{r_2-1} \alpha_{2i} e_2^{(r_2-i)} \\ \vdots \\ \sum_{i=1}^{r_m-1} \alpha_{mi} e_m^{(r_m-i)} \end{pmatrix} - f - d + y_d^{(r)} + K\,\mathrm{sgn}(s)) \quad (12)$$

where $\hat{P} = Diag[g_{ii}]$, $K = Diag[K_i]$ is $m \times m$ positive definite diagonal gain matrix with $K_i > 0$, $\mathrm{sgn}(s) \equiv (\mathrm{sgn}(s_1),\cdots,\mathrm{sgn}(s_m))^T$ and $\mathrm{sgn}(s_i)$ is defined as

$$\mathrm{sgn}(s_i) = \begin{cases} 1 & s_i > 0 \\ 0 & s_i = 0\,, \\ -1 & s_i < 0 \end{cases} \quad i = 1,\cdots,m\,.$$

Therefore the optimal control is

$$u_i^* = g_{ii}^{-1}[\sum_{j=1}^{r_i-1} \alpha_{ij} e_i^{(r_i-j)} - f_i - d_i + y_{di}^{(r_i)} + h_i\,\mathrm{sgn}(s_i)\cdot \eta_i]$$

where $\eta_i > 0$, $h_i = \begin{cases} 1 & ,\text{if } s_i \ne 0 \\ 0 & ,\text{if } s_i = 0 \end{cases}$.

This optimal sliding mode control input u_i^* guarantees the reaching condition of (10).

Since the control of MIMO nonlinear systems directly use the sliding mode control but does not take the interconnections among subsystems into consideration, the interconnections compensating network is needed. Thus the proposed sliding mode controller has a neural part to release the interaction among the subsystems. The output of the controller is combined with u^0 and its modification by decoupling network

$$u(t) = u^0(t) + Mu^0(t) \quad (13)$$

To derive a stable weight adaptation in control matrix, the matrix M be chosen as

$$M = -(I_m + \hat{C}^{-1}\hat{P})^{-1} \quad (14)$$

where I_m denotes a $m \times m$ identity matrix and

$$\hat{C} = \begin{bmatrix} 0 & g_{12} & \cdots & g_{1m} \\ g_{21} & 0 & \cdots & g_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ g_{m1} & g_{m2} & \cdots & 0 \end{bmatrix} \quad (15)$$

Using (12), (13), (14), (15) and the matrix inversion $(A + BCD)^{-1} = A^{-1} - A^{-1}B(DA^{-1}B + C^{-1})^{-1}DA^{-1}$ [19], the formulation of MIMO SMC resolves into

$$u^* = G^{-1}(\begin{bmatrix} \sum_{i=1}^{r_1-1} \alpha_{1i} e_1^{(r_1-i)} \\ \sum_{i=1}^{r_2-1} \alpha_{2i} e_2^{(r_2-i)} \\ \vdots \\ \sum_{i=1}^{r_m-1} \alpha_{mi} e_m^{(r_m-i)} \end{bmatrix} - f - d + y_d^{(r)} + K\,\mathrm{sgn}(s)) \quad (16)$$

where $G = \hat{C} + \hat{P}$. By plugging u^* into (9), we will have $\dot{s} = -K\,\mathrm{sgn}(s)$. Thus, the reaching condition (10) can be easily verified.

In this paper we use direct adaptive fuzzy controller (DAFC), therefore, the parameters of the controller are directly adjusted to reduce some norm of the output error between the plant and the reference model. Due to the existing of fuzzy approximation errors and external disturbances, simply an equivalent control term cannot ensure the stability of the closed-loop system, it is necessary to preserve a hitting control to deal with them. Suppose that the control u due to the DAFC is the summation of a basic fuzzy logic system $\hat{u}(x \mid \underline{\phi})$ and a hitting control $\hat{u}_h$ ($\hat{u}_h = G^{-1}u_h$)

$$u = \hat{u}(x \mid \underline{\phi}) + \hat{u}_h \quad (17)$$

where $\underline{\phi} = [\varphi_1,\cdots,\varphi_m]$, $\hat{u} = (\hat{u}_1,\cdots,\hat{u}_m)^T$ with $\hat{u}_i(x \mid \varphi_i) = \varphi_i^T \cdot \xi_u(x)$, where $\xi_u(x) = (\xi_{u1},\cdots,\xi_{uN})^T$ is a vector of fuzzy bases, $\varphi_i = (\phi_{i1},\cdots,\phi_{iN})$ is the corresponding parameters of fuzzy logic systems, $i = 1,\cdots,m$.

5. Learning Algorithm and Performance Analysis

In direct adaptive fuzzy control, linguistic fuzzy control rules can be directly incorporated into the controllers and the parameters of the controller are directly adjusted to reduce some norm of the output error between the plant and the reference model. As far as the adaptation of the controller parameters are concerned the input applied to one subsystem affecting the other subsystem. Our approach to the solution of such a problem is based on to derive the proper direct adaptive fuzzy control law for the plant model whose structure is represented by exploiting the advantages of the DAFC and the IAFC (indirect adaptive fuzzy controller) into a single controller i.e. both the fuzzy control rules and the fuzzy descriptions can be incorporated into a single controller. Thus the unknown functions $G(x)$ is estimated and the controller is chosen by assuming the estimated parameters being able to representing the true of the plant parameters. This is similar that the IAFSMC (indirect adaptive fuzzy sliding mode control) uses the fuzzy system as approximator for the dynamic systems [17]. In this section, we firstly show how to derive an adaptive law to adjust the controller parameters such that the DAFC can optimally approximate the equivalent control of the FSMC under the situation of unknown functions f and G. Then, we construct the hitting control to guarantee system's stability by the Lyapunov theory so that the ultimately bounded tracking is accomplished.

We now adopt the control $u = \hat{u}(x|\underline{\phi}) + \hat{u}_h$ as (17)

where the hitting control $\hat{u}_h = G^{-1} u_h$, and the fuzzy logic system $\hat{u}_i(x|\varphi_i)$ as (7) is

$$\hat{u}_i(x|\phi_i) = \phi_i^T \cdot \xi_u(x) = \xi_u^{\ T}(x) \cdot \phi_i \tag{18}$$

where $\xi_u(x)$ is vector of fuzzy bases, ϕ_i is the corresponding parameters of fuzzy logic systems, $i = 1,\cdots,m$. Define the parameters $\varphi_i^* \in R^N$ of the best function approximation as

$$\varphi_i^* \equiv \arg \min_{\phi_i \in \Omega_{\phi_i}} [\sup_{x \in \Omega_x} |u_i(x) - \hat{u}_i(x|\varphi_i)|]$$

where Ω_{ϕ_i} is constraint sets for φ_i, $i = 1,\cdots,m$, defined as $\Omega_{\phi_i} = \{\varphi_i : |\varphi_i| \le M_{\phi_i Max}\}$ where $M_{\phi_i Max}$ are specified by the designer. After some straightforward manipulations, the sliding surface equation (9) with the fuzzy control law u in (17) to replace u^* in (16) can be rewritten as

$$\dot{s} = -G \begin{bmatrix} \tilde{\varphi}_1^T \xi_u \\ \tilde{\varphi}_2^T \xi_u \\ \vdots \\ \tilde{\varphi}_m^T \xi_u \end{bmatrix} - K \cdot \mathrm{sgn}(s) - u_h + G[u^* - \hat{u}(x|\underline{\phi}^*)] \tag{19}$$

where $\tilde{\varphi}_i = \varphi_i - \varphi_i^*$ denotes the parameter estimation errors with $\underline{\phi} = [\varphi_1,\cdots,\varphi_m]$. Our design objective involves specifying the control and adaptive laws for φ_i such that the reaching condition (10) is guaranteed.

Theorem 1: Consider nonlinear plant (1) with controller (17), the tracking error allows to use the following adaptive law and hitting control as

$$\dot{\varphi}_i = \rho_i s^T g_i \xi_u \tag{20}$$

$$u_{hi} = \mathrm{sgn}(s_i)[|f_i|_{\max} + \sum_{j=1}^{m} |g_{ij}|_{\max} \cdot |\hat{u}_j| + |y_{di}^{(r_i)}| + |\sum_{j=1}^{r_j-1} \alpha_{ij} e_i^{(r_i-j)}| + D_i] \tag{21}$$

where $i = 1,\cdots,m$. After straightforward manipulation, the time derivative of V is obtained as $\dot{V} = s^T \dot{s} \le 0$.

Proof: Consider the Lyapunov candidate

$$V = V_1 + V_2 + \cdots + V_m = \frac{1}{2}(s^T s + \sum_{i=1}^{m} \frac{1}{\rho_i} \tilde{\varphi}_i^T \tilde{\varphi}_i) \tag{22}$$

where $V_i = \frac{1}{2}(s_i^T s_i + \frac{1}{\rho_i} \tilde{\varphi}_i^T \tilde{\varphi}_i)$, $i = 1,\cdots,m$. By the fact $\dot{\tilde{\varphi}}_i = \dot{\varphi}_i$ and (19), we obtain the derivative of V as

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \cdots + \dot{V}_m \tag{23}$$

where

$$\dot{V} = \sum_{i=1}^{m} \frac{1}{\rho_i} \tilde{\varphi}_i^T (\dot{\tilde{\varphi}}_i - \rho_i s^T g_i \xi_u) - \sum_{i=1}^{m} K_i s_i \,\mathrm{sgn}(s_i) - \sum_{i=1}^{m} s_i u_{hi} + s^T G[u^* - \hat{u}(x|\underline{\phi}^*)]$$

where $g_i(x) = (g_{i1}(x),\cdots,g_{im}(x))^T$, $i = 1,\cdots,m$. If we choose the adaptive law as $\dot{\tilde{\varphi}}_i = \rho_i s^T g_i \xi_u$ and the optimal control (16), then

$$\begin{aligned} \dot{V}_i &= -K_i s_i \,\mathrm{sgn}(s_i) + s_i [\sum_{j=1}^{m} g_{ij} u_j^* - \sum_{j=1}^{m} g_{ij} \hat{u}_j] - s_i u_{hi} \\ &= -K_i s_i \,\mathrm{sgn}(s_i) + s_i [\sum_{j=1}^{r_j-1} \alpha_{ij} e_i^{(r_i-j)} - f_i - d_i + y_{di}^{(r_i)} + K_i \,\mathrm{sgn}(s_i) - \sum_{j=1}^{m} g_{ij} \hat{u}_j] - s_i u_{hi} \\ &\le |s_i| [|\sum_{j=1}^{r_j-1} \alpha_{ij} e_i^{(r_i-j)}| + |f_i| + D_i + |y_{di}^{(r_i)}| + \sum_{j=1}^{m} |g_{ij} \hat{u}_j|] - s_i u_{hi} \end{aligned} \tag{24}$$

We use the fact that u_{hi} has the same sign with s_i, the u_{hi} can be implemented in (21) such that $\dot{V}_i \le 0$, $i = 1,\cdots,m$.

In order to complete the FSMC design, it is necessary to show that the hitting control is enough to force the state trajectory toward the sliding surface as well as to establish asymptotic convergence of the tracking error. Consider the Lyapunov function candidate

$$V_i = \frac{1}{2} s_i^2 \tag{25}$$

Taking the derivative of (25) and using (9), (17), one has

$$\dot{V}_i = s_i (\sum_{j=1}^{r_j-1} \alpha_{ij} e_i^{(r_i-j)} - f_i - \sum_{j=1}^{m} g_{ij} \hat{u}_j + y_{di}^{(r_i)} - d_i) - s_i u_{hi} \tag{26}$$

To ensure (26) being less than zero, the hitting control should be selected as (21). This means that the inequality $\dot{V}_i = s_i \dot{s}_i < 0$ is obtained and the hitting control actually achieves a stable FSMC system.

Conceptually, in sliding mode the equivalent control is used when the state trajectory is near $s_i = 0$, while the hitting control is appended in the case of $s_i \neq 0$ [20]. However, the hitting control will generate a very large control force and causes high-frequency unmodelled dynamics [20]. Therefore, we minimize the hitting control in (21) by a fuzzy function in practical implementation

[21]. Thus, a fuzzy rule base is of the form

If s_i is ZO Then u_i is $u_i = \hat{u}_i$ (27)

If s_i is NZ Then u_i is $u_i = \hat{u}_i + u_{hi}$ (28)

where ZO and NZ denote zero and nonzero fuzzy sets, respectively, and input variable s_i is given in (3). The modified control law of the fuzzy controller for (17) is

$$u_i = \frac{\mu_{ZO}(s_i)\cdot \hat{u}_i + \mu_{NZ}(s_i)[\hat{u}_i + \hat{u}_{hi}]}{\mu_{ZO}(s_i) + \mu_{NZ}(s_i)} \quad (29)$$

where $\mu_{ZO}(s_i)$ and $\mu_{NZ}(s_i)$ is the membership functions of fuzzy sets ZO and NZ, respectively.

6. Simulation Results

We demonstrate the proposed FSMC by the tracking control of a two-link robotic manipulator with 2 degrees of freedom in the rotational angles described by $\boldsymbol{q} = (q_1, q_2)^T$, as shown in. Fig. 2. The dynamic equations describing the motion of the robotic system are of the following form [22]

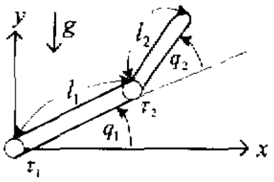

Fig. 2 Model of a two-link robotic manipulator.

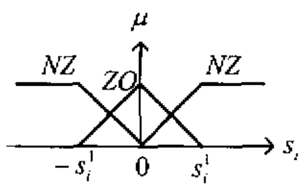

Fig. 3 The fuzzy membership functions of ZO and NZ.

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{h}(\boldsymbol{q},g) = \boldsymbol{\tau} + \boldsymbol{d}(q,\dot{q},t) \quad (30)$$

where $\boldsymbol{\tau}$ is the externally applied torques along the directions of their corresponding generalized coordinates $\boldsymbol{q}$,

$$\boldsymbol{M}(\boldsymbol{q}) = \begin{bmatrix} (m_1+m_2)r_1^2 + m_2 r_2^2 + 2m_2 r_1 r_2 c_2 + J_1 & m_2 r_2^2 + m_2 r_1 r_2 c_2 \\ m_2 r_2^2 + m_2 r_1 r_2 c_2 & m_2 r_2^2 + J_2 \end{bmatrix}$$

$$C(\boldsymbol{q},\dot{\boldsymbol{q}}) = \begin{bmatrix} -m_2 r_1 r_2 s_2 \dot{q}_1(\dot{q}_1 + \dot{q}_2) \\ m_2 r_1 r_2 s_2 \dot{q}_1^2 \end{bmatrix}, \quad \boldsymbol{h}(\boldsymbol{q}) = \begin{bmatrix} ((m_1+m_2)l_1 c_2 + m_2 l_2 c_{12})g \\ (m_2 l_2 c_{12})g \end{bmatrix}$$

with $g = 9.8m/s^2$ is the gravity constant and $\boldsymbol{d}(\boldsymbol{q},\dot{\boldsymbol{q}},t) \in R^2$ is friction forces, and external disturbances. In (30), the nominal parameters m_1, m_2, J_1, J_2, $r_1 = 0.5l_1$, and $r_2 = 0.5l_2$ are the mass, the moment of inertia, the half-length of link 1 and 2, and shorthand notations $c_2 = \cos(q_2)$, $s_2 = \sin(q_2)$, $c_{12} \equiv \cos(q_1 + q_2)$, etc. The combined effects of friction and the external torque disturbance are

$$d_1 = 2.0\sin(\dot{q}_1) + 2.5\sin(\dot{q}_2) + 0.5\sin(t)$$

$$d_2 = 5.0\sin(\dot{q}_1) + 4.0\sin(\dot{q}_2) + 0.4\sin(t)$$

In the control experiments described below, the kinematics and inertial parameters of the arm are chosen as $l_1 = 2.04m$, $l_2 = 1.66m$, $J_1 = J_2 = 4.5kg\cdot m$, $m_1 = 0.60kg$, $m_2 = 7.02kg$, respectively. The trajectories to be followed are described by two decoupled linear systems from (4), the desired coefficients are specified to be $\alpha_{i1} = 2$, $\alpha_{i2} = 1$, $i = 1,2$. The robot is given the following target joint rotations:

$$q_{d1} = (2.5\pi/12)\cdot \sin t$$

$$q_{d2} = (3.75\pi/12)\cdot \cos t$$

with the initial states $q_1(0) = 1.5\text{rad}$, $q_2(0) = -1.2\text{rad}$, $\dot{q}_1(0) = 0\ \text{rad/sec}$ and $\dot{q}_2(0) = 0\ \text{rad/sec}$.

In (20) and (21), the design parameters are given by $\rho_i = 1.2$, $K_i = 1$, $D_i = 5$, $i = 1,2$. The membership functions of states q_1, q_2, $\dot{q}_1$, and $\dot{q}_2$ (represented by generic variable x_i) for the qualitative statements ($N = 5^4 = 625$ regular rule partitions) are defined as

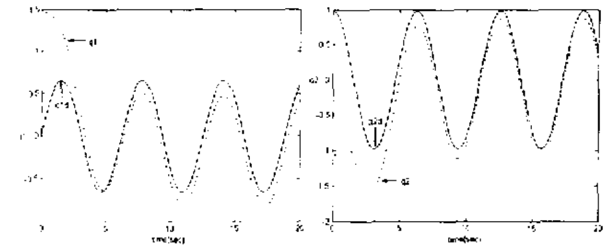

Fig. 4 The tracking curves of q_1, q_{d1} and q_2, q_{d2}.

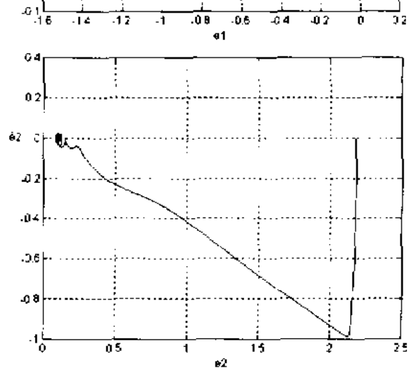

Fig. 5 The state trajectories on the phase plane.

$$\{NB, NS, ZE, PB, PS\}$$

where

$NB: 1/[1+\exp(2.5(x_i + 0.4))]$, $NS: \exp(-0.5(x_i + 0.2)^2)$,

$PB: 1/[1+\exp(-2.5(x_i - 0.4))]$, $PS: \exp(-0.5(x_i - 0.2)^2)$, $ZE: \exp(-0.5x_i^2)$. In (27) and (28), The membership functions of s_i for the fuzzy sets *ZO* and *NZ* are given in triangle function, as shown in Fig. 3. It has a property that, for all s_i, $\mu_{ZO}(s_i)+\mu_{NZ}(s_i)=1$. When holding the condition $|s_i| \geq s_i^1$ with $s_i^1 = 0.3$, it can be seen that the control law is the same as the proposed FSMC. However, the amount of hitting control in region $|s_i| < s_i^1$ is dominated by the grade of the membership function of *NZ*, that is, the hitting control could be attenuated by the grade of *NZ*.

The tracking curves and the state trajectories of the phase plane for $q_1(t)$ and $q_2(t)$ are shown in Fig. 4-5, respectively. The simulation results reveal that the proposed FSMC, encountering the combined effects of friction, parametric uncertainties, unmodeled dynamics and external disturbance, can attenuate the tracking error efficiently. Moreover, without using any *a priori* linguistic information, our adaptive fuzzy sliding mode controller has successfully executed the trajectory following control of the robot system.

7. Conclusion

The goal of this work is the development and implementation of a direct adaptive fuzzy control based SMC for the robust trajectory tracking of MIMO control systems with unknown nonlinear dynamics. This design obtains robustness in the sense that the self-tuning mechanism can automatically adapt the fuzzy controller by using a learning algorithm and the global asymptotic stability of the algorithm is established via Lyapunov stability criterion. The simulation presented in the two-link robotic manipulator control indicates that the proposed approach is capable of achieving a good chattering-free trajectory following performance without the knowledge of plant parameters. Although only the two-link robotic system has been studied in this paper, the proposed control scheme can also be used to address the other class of MIMO nonlinear systems.

9. References

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