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ORDER AND PRODUCTION SCHEDULING/RESCHEDULING FOR FLOW SHOPS*

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Abstract

This paper presents an integrated framework for order and production schedulings of a discrete–part, make–to–order flow shop. We first formulate the integrated problem and then decompose it by applying Lagrangian relaxation into subproblems of order scheduling (OS), production scheduling (PS) and overtime capacity allocation (OCA), which match operational functions in a manufacturing organization, and form a dual problem to optimize the Lagrange multipliers for relaxation, which are interpreted as marginal costs. The solution algorithm has a structure that clearly captures the interrelationship among subproblems and provides a framework for integration. Exploiting such a framework, we also develop effective rescheduling algorithms for handling new order insertion and capacity loss.

1. Introduction

In the practice of production control, scheduling the manufacturing of customer orders is broken into order scheduling (OS) and production scheduling (PS) according to the time scales of decisions involved [VBW89]. OS determines the schedule of filling orders by estimating the capacity of the factory, has a time scale in weeks and is in the higher level of decision hierarchy. On the other hand, PS is in the lower level and is used to decide the hourly or daily production schedule for the factory so that order delivery promises set by OS are met. Current practice of OS uses simple Master Production Scheduling (MPS) to estimate the coarse

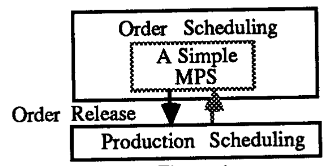

Figure 1

production capacity of the factory [VBW89]. The promised deliveries thus generated may either pose ill–structure production requirements to PS or just cannot be met. When there is a significant capacity loss or material shortage in the shop floor, there is lack of an effective method to adjust both OS and PS accordingly and predict the potential impacts to customers. These situations may not only result in customers' dissatisfaction but also create production chaos at the shop floor, such as highly unbalanced resource utilization and special efforts to rush late orders through. Such deficiencies are due to the fact that the interactions between OS and PS, especially the dynamic aspect, are not well accounted for in the current OS practice by just using simple MPS to model PS.

In this paper we exploit recent advancements in scheduling theory and algorithms [CLH91, CAB90, LHM90, CoH88, Gra82] to propose a better integrated framework for OS and PS and develop its associated solution methodology. We first formulate both OS and PS as one optimization problem in Section 2, where given a set of customer orders and production constraints, we want to find a manufacturing schedule to fill customer orders just–in–time and minimize production costs. The optimization problem is decomposed in Section 3 by Lagrangian relaxation of resource and demand constraints into three groups of subproblems : order scheduling (OS), production scheduling (PS), and overtime capacity allocation (OCA). A dual problem is formed to optimize the Lagrange multipliers, which

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have an economic interpretation as marginal costs. Section 4 briefly describes a two-level solution algorithm which we developed in [ChH91] for the above optimization problems. We then investigate the interrelationship between decisions of order and production schedulings from the view point of mathematical decomposition and coordination among the three groups of subproblems and the dual problem. In Section 5, we exploit the economic interpretation of Lagrange multipliers and the structure of our algorithm to develop fast rescheduling algorithms for handling new order insertion and unexpected capacity loss. Numerical results for rescheduling is given in Section 6. We conclude in Section 7.

2. An Integrated Problem Formulation

Consider that a set of orders have been placed to a make-to-order flow shop without set-up effects, each demanding a few different types of products with a due date. The production of each type of products involves a sequence of operations by different machining facilities or labors. Different types of products may have operations that require the same manufacturing resource. Assembly and disassembly operations are not considered in this paper. Let us first define the following notations and variables.

<u>Input Variables and Notations:</u>

T : the time horizon of scheduling;
T_1 : the length of a time cycle;
t : time cycle index;
N : total number of cycles in T;
I : total number of product types;
i : product type index, $i = 1,\dots,I$;
J : total number of machine groups;
j : machine group index, $j = 1,\dots,J$;
N_j : the number of identical machines in machine group j;
O_{jt} : the maximum overtime capacity of machine group j at cycle t;
t_{ij} : the processing time of a type-i product by a machine in group j;
L_{ijt} : the input buffer level of type-i products at machine group j+1;
h_{it} : per unit type-i product holding cost during cycle t;
o_{jt} : per unit overtime cost of machine group j during cycle t;
K : total number of orders;
d_k : the due date of order k;
D_{ki} : the desired number of type-i products in order k;
r_{it} : the maximum available quantity of the raw material for type-i products at cycle t;

<u>Decision Variables :</u>

P_{ijt} : number of type-i products processed at machine group j during cycle t;
F_{jt} : number of overtime units of machine : group j committed during cycle t;
R_{it} : number of type-i products released for manufacturing at the beginning of cycle t;
X_{kt} : order delivery variable, where $X_{kt} = 1$ if order k is to be delivered at the end of the t-th cycle; $X_{kt} = 0$ otherwise.

To convey our main idea but without loss of generality, we assume that all types of products go through all the J machine groups in the ascending group index and that buffers are infinite. The processing requirements for each machine group may vary among different types of products. Considering the level of details in modelling for order scheduling purpose, we select a cycle time $T_1 >> t_{ij}$ for all i's and j's, and assume that a batch of P_{ijt} units of type-i parts scheduled for processing by machine group j during cycle t can be finished during the cycle and are moved together. Production flows in the line of production must satisfy the following set of constraints.

<u>Production Flow Balance Equations</u>

$$L_{i0(t+1)} = L_{i0t} - P_{i1t} + R_{it}, \qquad (2\text{-}1.a)$$

$$L_{ij(t+1)} = L_{ijt} - P_{i(j+1)t} + P_{ijt},\ j=1,\dots,J-1, \qquad (2\text{-}1.b)$$

$$L_{iJ(t+1)} = L_{iJt} + P_{iJt}, \qquad (2\text{-}1.c)$$

$t=1,\dots,N$, $i=1,\dots,I$, with L_{ij1}, $j=0,1,\dots,J$ given.

<u>Material Availability Constraints</u>

$$0 \le R_{it} \le r_{it}, \qquad (2\text{-}2.a)$$

$$0 \le P_{i1t} \le L_{i0t} + R_{it}, \qquad (2\text{-}2.b)$$

$$0 \le P_{ijt} \le L_{i(j-1)t},\ j=2,\dots,J,\ \forall\, i,t; \qquad (2\text{-}2.c)$$

<u>Capacity Constraints</u>

$$\sum_{i=1}^{I} t_{ij} P_{ijt} \le N_j T_1 + F_{jt} T_1, \qquad (2\text{-}3)$$

$$0 \leq F_{jt} \leq O_{jt}, \forall j,t; \qquad (2\text{–}4)$$

End Product Availability Constraints

$$\sum_{t=1}^{n} P_{iJt} - \sum_{t=1}^{n} \sum_{k=1}^{K} D_{ki} X_{kt} \geq 0, \quad i=1,..,I, \quad t=1,..,N. \qquad (2\text{–}5)$$

Single Delivery Constraints

$$\sum_{t=1}^{N} X_{kt} = 1 \quad , \forall k . \qquad (2\text{–}6)$$

Our objective of scheduling is to determine the delivery dates of orders and their respective production schedules that meet all the system constraints and minimize the production costs due to holding inventory, use of overtime capacities and earliness and overdue penalty. The integrated order and production scheduling problem is then formulated as

$$(\text{IP}) \quad \min_{X,R,P,F} \sum_{i=1}^{I} \sum_{t=1}^{N} \{[\sum_{n=1}^{t} R_{in} - \sum_{n=1}^{t} \sum_{k=1}^{K} D_{ki} X_{kn}] h_{it}\} + \sum_{j=1}^{J} \sum_{t=1}^{N} F_{jt} o_{jt} + \sum_{k=1}^{K} V_k(\sum_{t=1}^{N} t X_{kt} - d_k)$$

subject to constraints (2.1–2.6) and integrality of decision variables,

where the V_k function penalizes both early and late deliveries [OLC90],[ChH91].

3. Lagrangian Relaxation and Scheduling Subproblems

3.1 Decomposition by Lagrangian Relaxation

In problem (IP), we observe that the coupling among different types of products is due to their competition for production capacities (inequality (2.3)). The coupling among different orders arises from their competition for end products (inequality (2.5)). Applying Lagrangian relaxation [Lue84] to these two coupling constraints, we form a dual problem of (IP) with $\underline{\lambda}$ and $\underline{\pi}$ as the Lagrange multipliers for relaxing (2.3) and (2.5) respectively :

$$(\text{D}) \quad \max_{\underline{\lambda}, \underline{\pi} \geq \underline{0}} [\, L(\underline{\lambda},\underline{\pi}) \equiv \sum_{k=1}^{K} L_{1k}(\underline{\lambda}) + \sum_{i=1}^{I} L_{2i}(\underline{\lambda},\underline{\pi}) + \sum_{j=1}^{J} L_{3j}(\underline{\pi}) - \sum_{j=1}^{J} \sum_{t=1}^{N} \pi_{jt} N_j T_1 \,],$$

where

Order Scheduling subproblem,

OS_k, $k = 1,2,\dots,K$

$$L_{1k}(\underline{\lambda}) \equiv \min_{X_{kt}} V_k(\sum_{t=1}^{N} tX_{kt} - d_k) + \sum_{i=1}^{I} \sum_{t=1}^{N} (\lambda_{it} - h_{it}) \sum_{n=1}^{t} D_{ki} X_{kn}$$

subject to constraint (2.6) for order k ;

Production Scheduling subproblem,

PS_i, $i = 1,2,\dots,I$

$$L_{2i}(\underline{\lambda},\underline{\pi}) \equiv \min_{R_{it},\, P_{ijt}} \sum_{j=1}^{J} \sum_{t=1}^{N} b_{ijt} P_{ijt} + \sum_{t=1}^{N} \sum_{n=t}^{N} h_{in} R_{it}$$

subject to the flow balance equation (2.1) and the material availability constraint (2.2) for type–i product, where

$$b_{ijt} \equiv \begin{cases} \pi_{jt} t_{ij} , & j=1,..,J-1 ; \\ \pi_{Jt} t_{iJ} - \sum_{n=t}^{N} \lambda_{in} , & j=J. \end{cases}$$

Overtime Capacity Allocation subproblem,

OCA_j, $j = 1,2,\dots,J$

$$L_{3j}(\underline{\pi}) \equiv \min_{F_{jt}} \sum_{t=1}^{N} (o_{tj} - \pi_{jt} T_1) F_{jt}$$

subject to constraint (2–4).

3.2 Economic Interpretations

In our relaxation procedure above, the Lagrange multiplier π_{jt} can be interpreted as the marginal cost (or shadow price) of using a group j machine for one cycle while λ_{it} can be interpreted as the marginal cost of acquiring a type–i product for order delivery during the cycle. Now consider the objective function of OS subproblem. As a unit of type–i product delivered at t has to be acquired during [t,N], the total acquiring cost is $\sum_{n=t}^{N} \lambda_{in}$. On the other hand, the holding cost that needs not be incurred due to this delivery is $\sum_{n=t}^{N} h_{in}$. The total cost for acquiring all the needed products to deliver order k at cycle t is then $\sum_{i=1}^{I} \sum_{n=t}^{N} (\lambda_{in} - h_{in}) D_{ki}$. We now clearly see that the delivery decision for order k is merely making a tradeoff between the earliness/overdue penalty and the cost for acquiring necessary end products.

In the PS subproblem, the production costs are mostly due to machine utilization and in–process inventory. Once a type–i product is finished at cycle t, it can possibly be acquired for delivery and therefore has a value of $\sum_{n=t}^{N} \lambda_{in}$. So its net cost at cycle t is the machine utilization cost minus the completion value. PS thus determines the minimum cost production and loading. We can similarly interpret OCA.

4. Solution Methodology and Integration

4.1 Algorithm Development

Our methodology for finding a near–optimal solution of problem IP consists of three parts as follows :

(1) Algorithms for Subproblems

An OS subproblem is solved by direct enumeration while an OCA subproblem is solved by using complementary slackness. Solving a PS subproblem is essentially solving a minimum cost linear network flow problem. Interested readers may refer to [ChH91] for more details.

(2) A Subgradient Algorithm for the Dual Problem

Let { X, R, P, F } be the optimal solution to subproblems for the given Lagrange multipliers $\underline{\lambda}$ and $\underline{\pi}$. We define the subgradient of the dual function as

$$g^1_{it} \equiv \frac{\partial}{\partial \lambda_{it}} L(\underline{\lambda},\underline{\pi})|_{\underline{\lambda},\underline{\pi}} = -[\sum_{n=1}^{t} P_{iJn} - \sum_{n=1}^{t}\sum_{k=1}^{K} D_{ki} X_{kn}], \qquad (4\text{–}1.a)$$

$$g^2_{jt} \equiv \frac{\partial}{\partial \pi_{jt}} L(\underline{\lambda},\underline{\pi})|_{\underline{\lambda},\underline{\pi}} = \sum_{i=1}^{I} t_{ij} P_{ijt} - N_j T_1 - F_{jt} T_1, \qquad (4\text{–}1.b)$$

$i = 1,\ldots,I,\ j = 1,\ldots,J$, and $t = 1,\ldots,N$.

The subgradient method proposed by Polyak [Pol69] is adopted to update $\underline{\lambda}$ and $\underline{\pi}$. Iterative application of algorithms in (1) and (2) may converge to an optimal dual solution $\{ \underline{\lambda}^*, \underline{\pi}^*, X^*, R^*, P^*, F^* \}$.

(3) A Heuristic for Finding A Good Feasible Solution

The solution $\{ X^*, R^*, P^*, F^* \}$ obtained under relaxation may violate capacity constraint (2–3) and/or the end–product availability constraint (2–5). Our heuristic scheme uses unallocated overtime capacity and a "pull–then–push" procedure for the production flow networks to resolve capacity violations in a philosophy of minimum change. The end–products produced under the adjusted schedule are then allocated to orders by the Earliest Due Date first rule and order delivery dates are determined accordingly.

4.2 Implications to Integration

The iterative algorithmic structure above captures the interactions among OS, PS and OCA and provides a framework for integrating order and production schedulings. In addition to the economic interpretations in subsection 3.2, we observe in (4–1.a) that the end–product cost profile is affected by the difference between the accumulated quantities of end–products and the quantities for delivering orders. If the difference is positive at cycle t, there are more end–products than needed. So the price of acquiring such an end–product, say λ_{it}, should be lowered, which in turn makes cost at the last (J–th) machine, b_{iJt}, increase. As a result, P_{iJt} is very likely to decrease in order to minimize the production cost. This effect then propagates through the flow balance equation to the whole production schedule. Since the demands for production resources are now varied, the marginal cost profile $\underline{\pi}$ of machine utilization is also varied according to (4–1.b), which in turn affects the allocation of overtime capacity. Actually, the above reasoning process can be initiated from the perspective of either OS, PS or OCA to see the interactions among each other. Furthermore, the economic interpretations of multipliers may be used for costing the scheduling processes.

5. Rescheduling

In this Section we explore how our integrated order and production scheduling algorithm can be exploited for effective rescheduling in response to the uncertainties of order insertion and capacity loss.

5.1 Order Insertion

When a customer intends to place a new order, the customer usually specifies a preferred window of delivery. In inserting a new order, the manufacturer generally evaluates the following two factors : (1) the production cost incurred and (2) its impact to the smoothness of the originally scheduled production.

Let $S_N \equiv \{ X_N, R_N, P_N, F_N, \lambda_N, \pi_N \}$ be an existing production schedule obtained by our nominal algorithm. A new order arrives at period n and the desired delivery window is $[N_b, N_e]$, where $N_b \geq n$. Let $P_I(n,d)$ be the rescheduling problem of inserting the new order, by minimizing the increase of production cost, into the existing production schedule with the order due date set at $d \in [N_b, N_e]$. We solve $P_I(n,d)$ in a similarly iterative procedure of the nominal algorithm by using S_N as a starting point.

As the dimension of Lagrange multipliers of the dual problem does not increase with the insertion of a new order as long as it does not require new types of products, we initialize the multipliers $\{ \lambda_I, \pi_I \}$ for $P_I(n,d)$ by $\{ \lambda_N, \pi_N \}$. In solving the subproblems of the dual of $P_I(n,d)$ to maintain the smoothness of the original production, let $[n, N_f]$, where $N_f \geq n$, be a user defined window within which the part of original schedule $\{ X_N, R_N, P_N, F_N \}$ is desired to be fixed. Schedule after N_f can be arbitrarily modified. Available nominal and overtime capacities in $[n, N_f]$ are calculated accordingly. The three types of subproblems in the dual of $P_I(n,d)$ are solved the same as they are in the nominal algorithm by adding the new order into demands. The same iterative procedure and the feasibility adjustment heuristic as the nominal algorithm are then applied to obtain a production schedule with the new order inserted. Let C(d) be the resultant production cost. We finally solve $P_I(n,d)$ for all $d \in [N_b, N_e]$ and find $d^* = \arg\min_d C(d)$ as the promised delivery cycle for the new order.

5.2 Capacity Loss

As a capacity loss occurs due to machine breakdown or labor shortage, the original schedule may become infeasible because of excessive productions scheduled for the available capacity. We extend the feasibility adjustment heuristic to solve this problem by

(1) fixing the part of the original schedule that has been executed up to n,
(2) removing the excessive production flows from their respective material flow network in a way of minimum increase cost and
(3) reroute, in a way of minimum increase in cost, the removed flows into their individual flow network from n and on with arc capacities properly adjusted.

6. Numerical Results of Rescheduling

We have preliminarily demonstrated in [ChH91] that the nominal algorithm is near–optimal (with duality gap less than 7%) and computationally efficient (190 minute for a problem with I=4, K=20, T=50 and J=9). In this Section we construct two scenarios to examine the effectiveness of our rescheduling scheme. The flow shop mainly produces four different brands of products every season. There are nine machine groups in the shop. Data about this shop is listed in the Tables shown below.

Production Process of the Four Brands of Products

part type	processing time/machine								
	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9
1	1	3	1	7	1	3	4	x	4
2	1	3	1	9	1	3	x	5	4
3	1	3	1	7	3	4	4	x	4
4	1	3	1	x	x	3	x	5	4

where x denotes that the product need not be processed on the machine group

Machine Capacity

machine group	1	2	3	4	5	6	7	8	9
normal capacity	14	40	40	40	40	40	6	5	40
overtime capacity	14	20	20	20	20	20	6	5	20

Contents and Due Dates of Orders

order \ type	1	2	3	4	due date	order \ type	1	2	3	4	due date
1	90	64	0	0	7	11	0	0	0	192	22
2	0	0	192	128	10	12	384	0	0	0	26
3	198	0	0	0	12	13	0	520	0	0	30
4	0	120	0	0	12	14	384	0	0	0	30
5	0	0	192	128	14	15	400	0	0	0	35
6	192	0	0	0	16	16	368	0	0	0	38
7	0	136	0	0	16	17	0	512	0	0	38
8	192	0	0	0	18	18	0	0	300	0	42
9	0	120	0	0	17	19	0	0	468	0	46
10	0	384	0	0	22	20	192	0	0	512	46

Other parameters are chosen as : time horizon T = 50 cycles, cycle T1=32 hrs, holding cost coefficient

h_{it}=0.04, ∀ i,t, overtime cost coefficient o_{jt}=10, ∀ j,t, and overdue penalty $V_k(x)=\begin{cases} 100\ x, & \text{if } x \geq 0\ ; \\ -10\ x, & \text{otherwise}\ ; \end{cases}$

Example 1 (*New Order Insertion*)

Suppose a near–optimal schedule has been obtained for the above baseline example. Results of applying the rescheduling algorithm to evaluate the costs of the four candidate delivery dates are listed as follows.

d	19	20	21	22
total cost	7649.67	7691.90	7102.9	7222.48
CPU time	1569	1552	1530	1478

It can be seen from the above Table that d = 21 with cost = 7102.96 is the best cycle for delivering the new order. It takes about 25 minutes to finish the rescheduling for each optimal delivery date, which may not be very satisfactory for practical applications. However, the CPU time can be improved by fine tuning of the initial stepsize and the total number of iterations of the subgradient algorithm and the stopping criteria to within 10 min or less.

Example 2 (*Rescheduling Due To Machine Failure*)

Suppose at the beginning of cycle 17 machine group 8 breaks down for one cycle. Since group 8 happens to be the bottleneck in the baseline example, this scenario represents the occurrence of a significant capacity loss. It takes 1563.99 sec for our algorithm to reschedule and the resultant cost is 7559.0399 with duality gap as 13.838%. Among all the orders, deliveries of orders 18 and 19 have to be delayed for one cycle due to this capacity loss. The order manager may want to inform the corresponding customers about it.

7. Conclusions

We have formulated an integrated framework for order and production scheduling/rescheduling and developed the corresponding solution algorithms. Algorithmic modules of our nominal scheduling algorithm match the operational functions in a manufacturing organization and the mathematical relationships among them provide us with insights for integrating these functions. Numerical experimentations shows that our rescheduling schemes for handling both order insertion and capacity loss converge to near–optimal solutions.

References

[CAB90] L. Cavalho, R. Almeida, C. Bispo, J. Sentiero," An Integrated Environment for Planning and Scheduling in Flow Shop Manufacturing Plants ", *Proceeding of Rensselaer's 2nd International Conference on Computer Integrated Manufacturing*, New York, May 21–23, 1990, pp. 55–63

[ChH91] S.C. Chang and F.S. Hsieh," Integrated Order and Production Scheduling ", *Proceedings of the Fourth International IFIP Conference on Computer Applications in Production and Engineering : Integration Aspects*, CAPE'91 Bordeaux, France, 10–12 Septemper 1991 North–Holland, pp. 621–628

[CLH91] S.C. Chang, D.Y. Liao, F.S. Hsieh," Scheduling Flexible Flow Shops Of No Setup Cost By A Lagrangian Relaxation And Network Flow Approach ", *Proceedings of the 1991 IEEE International Conference on Decision and Control,* Sacramento, California, April 1991, pp.1054–1059

[CoH88] R. Conterno, Y. C. Ho," Order Scheduling Problem in Manufacturing Systems ", *Int. J. Prod.* Res. Vol. 26, no.9, 1988 pp.1487–1510

[Gra82] S.C. Graves," Using Lagrangian Techniques To Solve Hierarchical Production Planning Problems ", *Management Science* Vol. 28, No. 3, March 1982, pp.841–852

[LHM90] P. B. Luh, D. J. Hoitomt, E. Max and K. R. Pattipati," Schedule Generation and Reconfiguration for Parallel Machines ", *IEEE Transaction on Robotics and Automation*, Vol.6, no. 6, Dec., 1990, pp. 687–696.

[Lue84] Luenberger, D. G., *Linear and Nonlinear Programming*, Addison Wesley, 1984

[OLC90] T.A. Owens, P.B. Luh, S.K. Cochran, P. Weisser, " A Hybrid Optimization and Knowledge based Approach to Job Shop scheduling ", *Proceeding of Rensselaer's 2nd International Conference on Computer Integrated Manufacturing*, New York, May 21–23, 1990, pp. 32–38.

[Pol69] Polyak, B.T., " Minimization of Unsmooth Functionals ", USSR Computational Math. and Math. Physics, Vol. 9, 1969, pp. 14–29.

[VBW89] T.E. Vollmann, W.L. Berry, D.C. Whybark, *Manufacturing Planning and Control Systems, 2nd ed.*, The Dow Jones–Irwin/APICS Series in Production Management, Homewood, IL 1989