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PID tuning rules for SOPDT systems: Review and some new results

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Abstract

PID controllers are widely used in industries and so many tuning rules have been proposed over the past 50 years that users are often lost in the jungle of tuning formulas. Moreover, unlike PI control, different control laws and structures of implementation further complicate the use of the PID controller. In this work, five different tuning rules are taken for study to control second-order plus dead time systems with wide ranges of damping coefficients and dead time to time constant ratios (D/τ). Four of them are based on IMC design with different types of approximations on dead time and the other on desired closed-loop specifications (i.e., specified forward transfer function). The method of handling dead time in the IMC type of design is important especially for systems with large D/τ ratios. A systematic approach was followed to evaluate the performance of controllers. The regions of applicability of suitable tuning rules are highlighted and recommendations are also given. It turns out that IMC designed with the Maclaurin series expansion type PID is a better choice for both set point and load changes for systems with D/τ greater than 1. For systems with D/τ less than 1, the desired closed-loop specification approach is favored. © 2004 ISA—The Instrumentation, Systems, and Automation Society.

Keywords: PID control; Controller tuning; Maclaurin series; Internal model control; Second-order plus dead time process

1. Introduction

In spite of innovations in predictive and advanced control techniques, most of the chemical industries until today use PID loops. In process control applications more than 95% of the controllers are of PID type. The maintenance and operation of PID controllers are easy and also they are robust in nature. It has been mentioned that more than 98% of the control loops in pulp and paper industries are controlled by PI loops. A PI controller, generally recommended for first-order plus dead time (FOPDT) dynamics, has two tuning parameters (controller proportional gain Kc and integral time constant τ_I). The ideal continuous time domain PI controller has the following structure:

$$K = \mathrm{PI} = K_c\left(1 + \frac{1}{\tau_i s}\right).$$

There are many tuning formulas available for PI controllers in the literature. Ziegler and Nichols [1], Astrom and Hagglund [2], Cohen and Coon [3], and Tyreus and Luyben [4] proposed tuning methods based on the process reaction curve. The Tyreus and Luyben tuning rule, based on frequency domain ultimate values, performs better for processes with a low D/τ ratio. Rivera *et al.* [5] and Zhuang and Atherton [6] discussed tuning based on performance minimization criteria. Smith and Corripio [7] presented tuning rules us-

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ing the direct synthesis design method. The model based tuning rule, namely, IMC [8], is explained in the literature.

Due to the high frequency gain of the derivative term, the closed-loop performance of processes (with a large D/τ ratio) with PID controllers may not give significant achievement over the same with PI. But some observers have found that the PID controllers perform better in making the response faster than PI and have been reported to be superior for FOPDT processes with large dead time to time constant ratios (Luyben [9]). These controllers are sufficient for processes where the dominant dynamics are of the second-order type. Chen and Seborg [10] and Lee *et al.* [11] have presented PID tuning rules for SOPDT systems. Huang *et al.* [12] presented inversed based design methods with a modified PID controller for different kinds of model structures. In general, there are four PID structures available in the literature. The use of the ideal PID controller, with the following structure, is limited due to its sensitivity with noisy signals (we call it PID0):

$$K=\mathrm{PID0}=K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right).$$

The two most common types of PID controller with the following series and parallel structures are widely used in the industry. The series PID takes the form of

$$K=\mathrm{PID1}=K_c\left(1+\frac{1}{\tau_I s}\right)\left(\frac{\tau_D s+1}{\alpha\tau_D s+1}\right).$$

This is termed as PID1 and the parallel PID has the following structure:

$$K=\mathrm{PID2}=K_c\left(1+\frac{1}{\tau_I s}+\frac{\tau_D s}{\alpha\tau_D s+1}\right).$$

It is denoted as PID2. The above-mentioned PID controllers have three tuning parameters, K_C, τ_I, and τ_D. Another type of PID controller includes the filter to the ideal PID. That is,

$$K=\mathrm{PID3}=K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\left(\frac{1}{\tau_f s+1}\right).$$

Thus this PID3 structure has four tuning parameters.

In general, there are two different ways to derive PID tuning parameters: frequency domain and time domain techniques. Both techniques can use parametric and nonparametric models. In this work we use tuning rules involving parametric models. A desired closed-loop trajectory is specified in the direct synthesis approach. The accuracy of the tuning rules depends on the accuracy of process-parameter identification methods. In the literature, the number of PID tuning rules for SOPDT processes are very few compared to the same for FOPDT systems. Some of the tuning rules work better for set-point changes and some of them are better for load disturbance. Chen and Seborg [10] used Taylor's series expansion of time delay terms and presented tuning rules for FOPDT as well as SOPDT processes using the direct synthesis method for both set-point as well as load changes.

Second-order plus dead time processes are rich in dynamics as they include underdamped, critically damped, and overdamped systems. As in the overdamped systems $\tau_{P1}\gg\tau_{P2}$ the corresponding results can be extended to FOPDT also. Hence we choose here the family of SOPDT for study. For the second-order plus dead time (SOPDT) process, very few tuning rules are available. Tuning rules are synthesized from ''ultimate cycle data,'' ''direct synthesis,'' or ''robust controller'' criteria. Which tuning method to select and what derivative algorithm to use for a SOPDT system are still not very clear. Some of the tuning methods are appropriate for low dead time to time constant ratio (D/τ) while others perform better in high D/τ values. Again, there exist different controller tuning procedures for underdamped, critically damped, or overdamped SOPDT systems. Hence, to give a clarification in the confusing picture of choosing correct PID controller for a SOPDT process to achieve better performance, we consider different process models with different D/τ ratio and damping coefficient (ξ) values. The tuning methods discussed in this paper are IMC-PID with filter, IMC-Chien [13], IMC-Maclaurin [11], Honeywell PID [14], and PID with desired closed-loop response trajectory (we call it closed-loop specified PID, in short CS-PID). These controllers are of PID1/PID2/PID3 structures and hence can be practically implemented.

In deriving PID controller parameters, the pure time delay is generally approximated as Padé series (zero- or first-order approximation). The effect of approximating the dead time is realized in de-

Table 1
Tuning rules investigated.

Sl. No.	Tuning rule	Applicable models	Methods used for approximation of dead time	Type of PID used
1	IMC-PID	FOPDT	Padé series	PID-3
2	IMC-Chien	SOPDT	Taylor series	PID-2
3	IMC-Maclaurin	SOPDT	Maclaurin series expansion	PID-2
4	Honeywell	SOPDT		PID-1
5	Closed-loop Spec. PID	SOPDT		PID-1 & 3

teriorating performance. Hence, in this paper, one of the existing tuning rules (IMC-Maclaurin) is used where the time delay term has been expanded by infinite exponential series without truncation and the controller is approximated as a Maclaurin series. This provides a tuning rule with faster response with less overshoot and which is robust. At the end, the applicability of proper tuning methods for SOPDT process models is suggested.

2. PID Controller

2.1. Process studied

We consider SOPDT processes with D/τ values ranging from 0.01 to 10 (seven different D/τ values) and damping coefficient ξ, ranging from 0.2 to 5.0 (nine different ξ values). Hence the process has the following structure:

$$G_P(s)=\frac{K_P e^{-Ds}}{\tau^2 s^2+2\xi\tau s+1}, \quad (1)$$

where K_P is the process open-loop gain, τ is the process time constant, D is the dead time, and ξ is damping coefficient. Thus we have 63 different process models with K_P and τ as unity.

2.2. Tuning methods

We consider five different tuning rules for the PID controller here. Table 1 summarizes the different tuning rules adopted in this study. The time-delay component in the closed-loop equation of IMC-PID is approximated using first-order Padé or modified Padé's approximations. IMC-Chien uses Taylor series approximation for the dead time component. The use of approximating methods deteriorates exact values of the integral time constant (τ_I) and derivative time constant (τ_D). Hence this problem can be avoided by expanding time-delay component in an infinite exponential series without truncating the successive terms and by approximating the PID controller in the form of a Maclaurin series.

2.2.1. IMC tuning

With Padé approximation, Rivera *et al.* [5] proposed a PID design of IMC strategy which needs selection of the tuning parameter λ that is almost equivalent to the closed-loop time constant. Morari and Zafiriou [8] proposed the value of λ as a function of dead time (D) and time constant (τ) for FOPDT. It is possible to approximate a

Table 2
Modeling—approximated FOPDT $[G_m(s)=K_m e^{-D_m s}/(\tau_m s+1)]$ process from SOPDT process.

Parameters	Critically damped	Overdamped	Underdamped
Gain	$K_m=K_p$	$K_m=K_p$	$K_m=K_p$
Time constant	$\tau_m=1.641\ \tau$	$\tau_m=[0.828+0.812(\tau_{p2}/\tau_{p1})+0.172e^{-6.9\tau_{p2}/\tau_{p1}}]\tau_{p1}$	$\tau_m=2\xi\tau$
Dead time	$D_m=0.505\ \tau+D$	$D_m=\frac{1.116\tau_{p2}\tau_{p1}}{\tau_{p1}+1.208\tau_{p2}}+D$	$D_m=\frac{\tau}{2\xi}+D$

Table 3
Tuning—IMC-PI and IMC-PID control algorithms for FOPDT process.

Controller	K_c	τ_I	τ_D
PI	$\frac{2\tau_m+D_m}{2K_P\lambda}$	$\tau_m+0.5D_m$	
PID	$\frac{2\tau_m+D_m}{2K_P(\lambda+D_m)}$	$\tau_m+0.5D_m$	$\frac{\tau_m D_m}{2\tau_m+D_m}$
Filter lag	$\tau_f=\frac{\lambda D_m}{2(\lambda+D_m)}$		
	where $\lambda=\max(0.25D_m, 0.2\tau_m)$ for PID controller		

SOPDT system to a FOPDT process where the approximated process parameters are given in Table 2. Table 3 shows the algorithms to calculate PI or PID3 controller parameters.

2.2.2. IMC-Chien PID tuning

Chien [13] presented a robust PID controller structure for SOPDT processes with the following parameters:

$$K_C=\frac{2\xi\tau}{K_P(\lambda+D)},\quad \tau_I=2\xi\tau,\quad \tau_D=\frac{\tau}{2\xi}. \tag{2}$$

For the overdamped SOPDT process the above parameters become

$$K_C=\frac{\tau_{P1}+\tau_{P2}}{K_P(\lambda+D)},\quad \tau_I=\tau_{P1}+\tau_{P2},$$

$$\tau_D=\frac{\tau_{P1}\tau_{P2}}{\tau_{P1}+\tau_{P2}} \tag{3}$$

with $\lambda=\max(0.25D, 0.2\tau)$.

2.2.3. IMC-Mac PID tuning

Maclaurin's PID controller is based on the result of Lee *et al.* [11]. Morari and Zafiriou [8] proposed the IMC controller to be

$$G_C(s)=\frac{1}{(\lambda s+1)^n G_{P-}(s)}, \tag{4}$$

where $G_{P-}(s)$ is the minimum phase part of the process model, λ is the tuning parameter, and n is chosen such that $G_C(s)$ becomes realizable or proper. $G_C(s)$ can be expanded in Maclaurin's series as

$$G_C(s)=\frac{f(s)}{s}$$

or

Table 4
IMC-Maclaurin settings for FOPDT & SOPDT process.

Process	K_c	τ_I	τ_D
FOPDT	$\frac{\tau_I}{K_P(\lambda+D)}$	$\tau+\frac{D^2}{2(\lambda+D)}$	$\frac{D^2}{2(\lambda+D)}\left(1-\frac{D}{3\tau_I}\right)$
SOPDT Underdamped	$\frac{\tau_I}{K_P(2\lambda+D)}$	$2\xi\tau-\frac{2\lambda^2-D^2}{2(2\lambda+D)}$	$\tau_I-2\xi\tau+\frac{\tau^2-\frac{D^3}{6(2\lambda+D)}}{\tau_I}$
SOPDT Critically damped	$\frac{\tau_I}{K_P(2\lambda+D)}$	$2\tau-\frac{2\lambda^2-D^2}{2(2\lambda+D)}$	$\tau_I-2\tau+\frac{\tau^2-\frac{D^3}{6(2\lambda+D)}}{\tau_I}$
SOPDT Overdamped	$\frac{\tau_I}{K_P(2\lambda+D)}$	$(\tau_{P1}+\tau_{P2})-\frac{2\lambda^2-D^2}{2(2\lambda+D)}$	$\tau_I-(\tau_{P1}+\tau_{P2})+\frac{\tau_{P1}\tau_{P2}-\frac{D^3}{6(2\lambda+D)}}{\tau_I}$
Tuning parameter	$\lambda=\max(0.25D, 0.2\tau)$		

$$G_C(s)=\frac{1}{s}\left(f(0)+f'(0)s+\frac{f''(0)}{2!}s^2+\cdots\right). \tag{5}$$

The coefficients of s^0, s^1, and s^2 in the right-hand side of the above equation (considering only the first three terms) can be equated to a PID controller equation, from which one can get PID control parameters Kc, τ_I, and τ_D as

$$K_C=f'(0),\quad \tau_I=\frac{K_C}{f(0)},\quad \tau_D=\frac{f''(0)}{2K_C}. \tag{6}$$

The PID controller parameters and tuning parameter (λ) are presented in Table 4. The algorithm is implemented on the PID2 structure.

2.2.4. Honeywell PID tuning

Astrom *et al.* [14] proposed a tuning rule for overdamped SOPDT processes which has an industrial PID structure similar to PID-1. The controller parameters are given as follows. For an overdamped process

$$K_C=\frac{3}{K_P\left(1+\frac{3D}{\tau_{P1}+\tau_{P2}}\right)},\quad \tau_I=\tau_{P1}+\tau_{P2},$$

$$\tau_D=\frac{\tau_{P1}\tau_{P2}}{\tau_{P1}+\tau_{P2}}. \tag{7}$$

A rearrangement on K_c gives

$$K_C=\frac{\tau_{P1}+\tau_{P2}}{K_P\left(\frac{\tau_{P1}+\tau_{P2}}{3}+D\right)}. \tag{7a}$$

This expression is similar to the IMC-Chien tuning except that the filter time constant is fixed to $(\tau_{p1}+\tau_{p2})/3$. In the present work, for an underdamped process, $\tau_{p1}+\tau_{p2}=2\xi\tau$ and $\tau_{p1}\tau_{p2}=\tau^2$ are substituted in the above formula for K_C, τ_I, and τ_D. This tuning rule will be called the Honeywell tuning rule here after.

2.2.5. Closed-loop specified PID

The approach of Huang *et al.* [12], is extended by taking the forward loop transfer function as

$$G_PG_C=\frac{e^{-Ds}}{2Ds(\alpha\tau_Ds+1)}, \tag{8}$$

$$\text{Let}\quad G_P=\frac{K_Pe^{-Ds}}{\tau^2s^2+2\xi\tau s+1}. \tag{9}$$

Then, G_c can be written as

$$G_C(s)=\frac{\tau^2s^2+2\xi\tau s+1}{2K_PDs(\alpha\tau_Ds+1)}. \tag{10}$$

Because G_c has a structure similar to PID-3, we obtain the PID controller parameters as

$$K_C=\frac{\xi\tau}{K_PD},\quad \tau_I=2\xi\tau,\quad \tau_D=\frac{\tau}{2\xi},\quad \tau_f=\alpha\tau_D. \tag{11}$$

In the case of overdamped processes, we have the process transfer function as

$$G_P=\frac{K_Pe^{-Ds}}{(\tau_{P1}s+1)(\tau_{P2}s+1)},\qquad \tau_{P1}>\tau_{P2}.$$

Then

$$G_c(s)=\frac{\tau_{P1}}{2K_PD}\left(1+\frac{1}{\tau_{P1}s}\right)\left(\frac{\tau_{P2}s+1}{\alpha\tau_Ds+1}\right).$$

This above equation is similar to the practical PID controller with PID1 form (Smith and Corripio, [7]). By comparing, we can get the controller settings as

$$K_C=\frac{\tau_{P1}}{2K_PD},\quad \tau_I=\tau_{P1},\quad \tau_D=\tau_{P2}. \tag{12}$$

For critically damped systems we have

$$G_P=\frac{K_Pe^{-Ds}}{(\tau s+1)^2} \tag{13}$$

and the controller parameters (PID1 structure) become

$$K_C=\frac{\tau}{2K_PD},\quad \tau_I=\tau,\quad \tau_D=\tau. \tag{14}$$

2.3. Discussion

If we see all of the above tuning rules, we find that the first three (IMC-PID, IMC-Chien, and IMC-Mac) depend on a tuning parameter λ. Honeywell PID and CSPID have no such parameter but Eqs. (3) and (7a) are of almost similar structures except for $\lambda=(2\xi\tau)/3$. In the case of IMC-

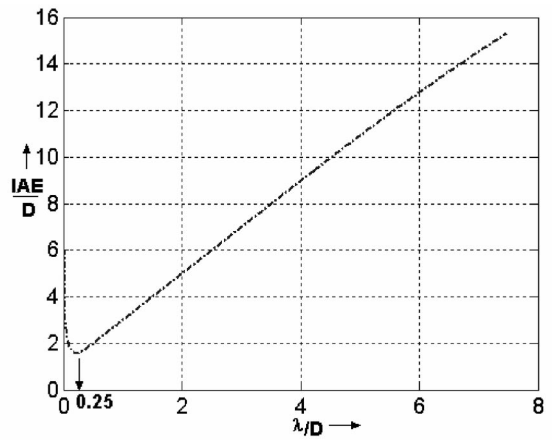

Fig. 1. Variation of IAE vs λ/D for a SOPDT process $1.0e^{-3S}/(s+1)^2$ with the IMC-Maclaurin-PID tuning rule.

Mac PID, a potential problem may arise as there is a possibility of τ_D becoming negative due to some values of λ.

3. Results

3.1. Closed-loop control

Several simulation examples with different SOPDT process models are used to show the performances of PID controllers. Closed-loop responses for set-point and load changes are obtained. First, we discuss the results with the set-point change. In the entire simulation, α was taken as 0.1. All the above five tuning methods were used to calculate PID controller parameters. IMC-

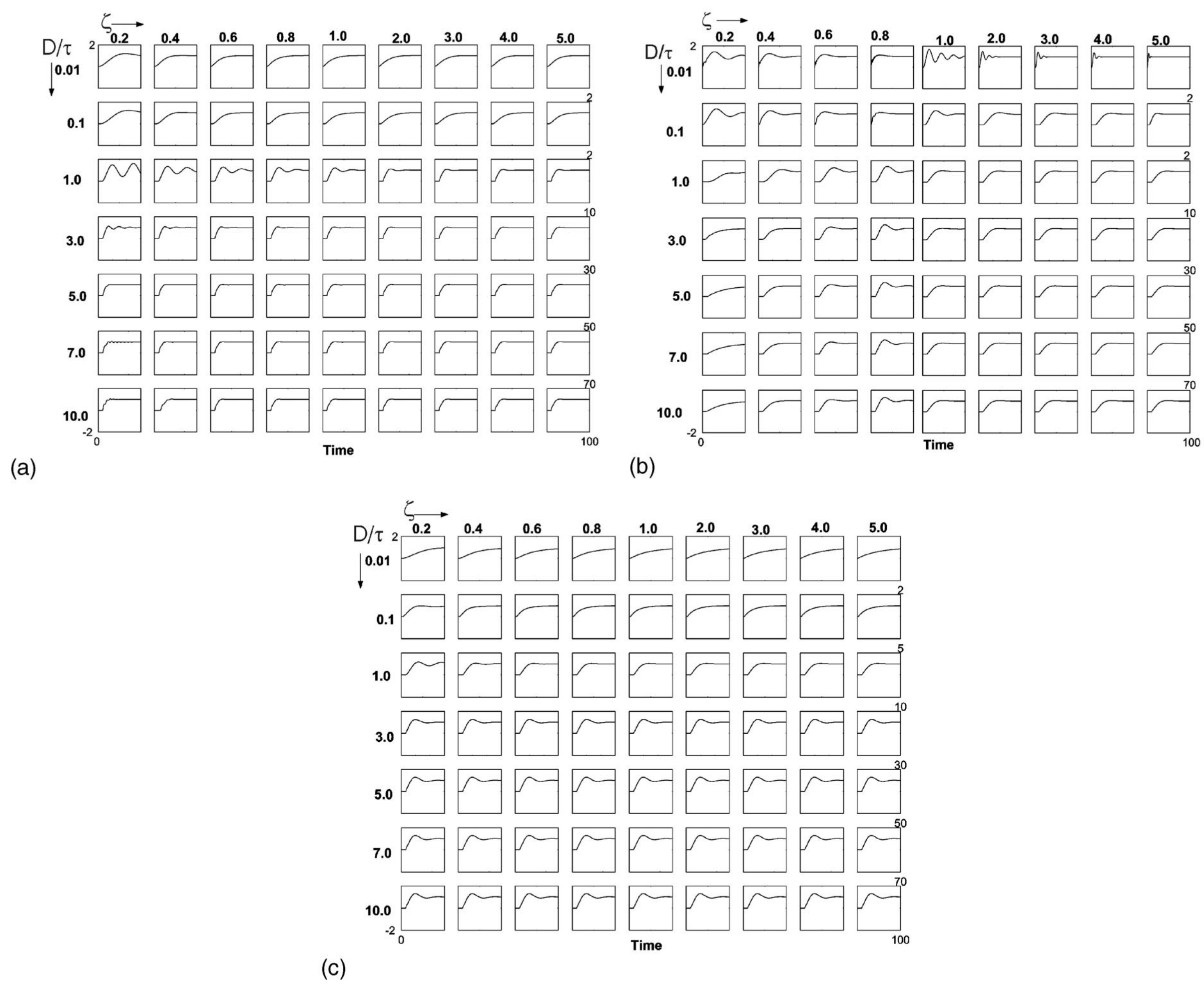

Fig. 2. (a) Set-point responses for the SOPDT models using IMC-Maclaurin PID settings (time in units of D). (b) Set-point responses for the SOPDT models using CS-PID settings (time in units of D). (c) Set-point responses for the SOPDT models using IMC-Chien PID settings (time in units of D).

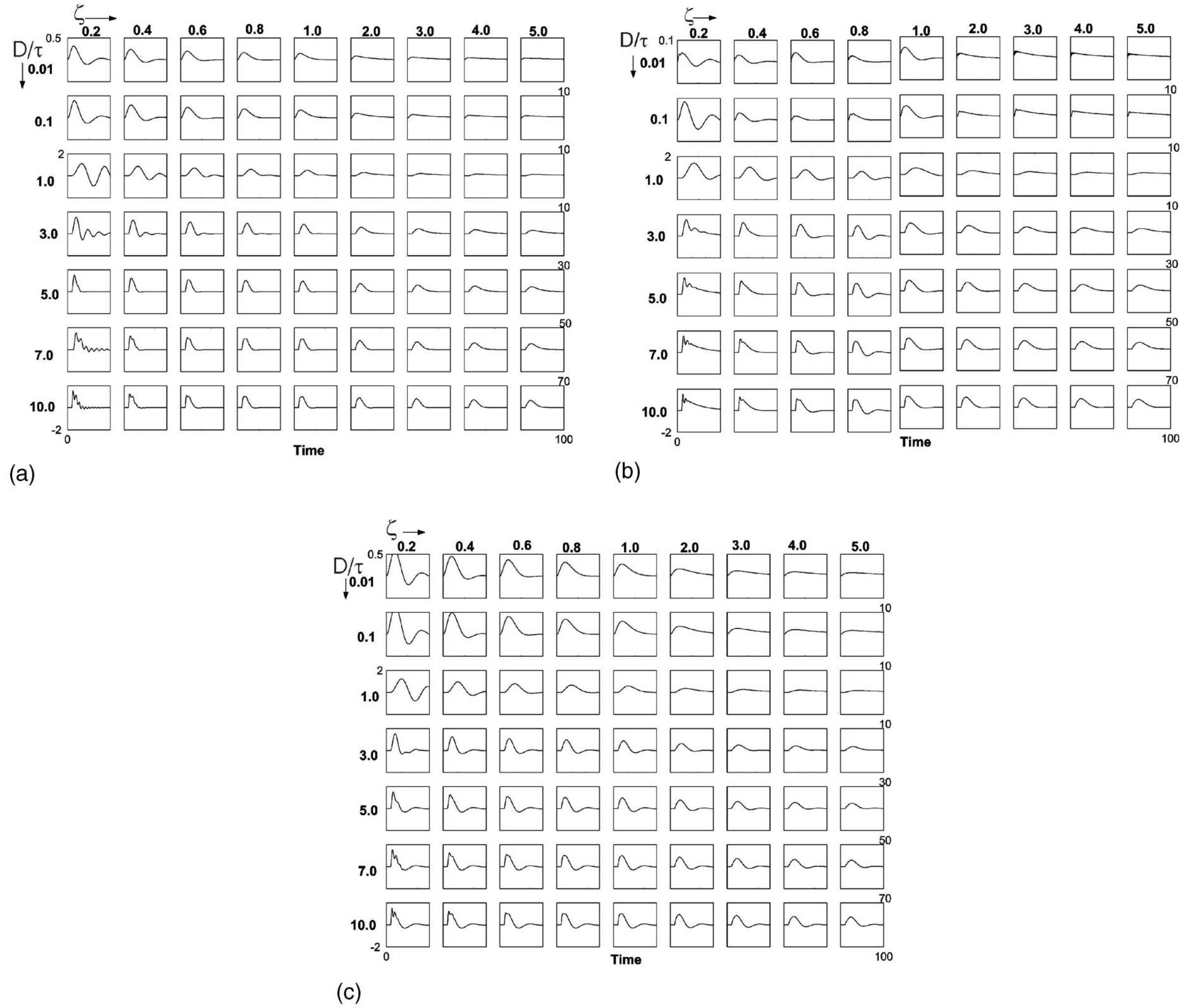

Fig. 3. (a) Load responses for the SOPDT models using IMC-Maclaurin PID settings (time in units of D). (b) Load responses for the SOPDT models using CS-PID settings (time in units of D). (c) Load responses for the SOPDT models using IMC-Chien PID settings (time in units of D).

PID, IMC-Chien, and Maclaurin-PID need to calculate the tuning parameter λ. Fig. 1 shows the optimum value of tuning parameter λ for IMC-Maclaurin PID. IAE becomes minimum at $\lambda=0.25D$. Each of the process models was controlled by different tuning rules for set-point changes. The closed-loop responses with IMC-Maclaurin for all the process models (with different D/τ and ξ) studied are shown in Fig. 2(a). Similar responses for set-point changes obtained with CS-PID and IMC-Chien tuning rules are shown in Figs. 2(b) and 2(c), respectively. For each model, response (y axis: value range $=0-2$: set point is at 1) is plotted against time (x axis: value range $=0-10D$). In these figures, the ξ value slowly starts from 0.2 in the left and increases to 5.0 horizontally in the extreme right. Similarly, the D/τ value starts from 0.01 in the top and increases to 10 in the bottom. Similarly, closed-loop responses under load changes with IMC-Mac, CS-PID, and IMC-Chien tuning rules are shown in Figs. 3(a), 3(b), and 3(c), respectively.

The integral of absolute error (IAE) values were computed in each case. Table 5 shows the normalized IAE values for set-point change. The damping factor of a process increases along the column while the D/τ ratio increases row wise. The last

Table 5
Normalized IAE values ($=\mathrm{IAE}/\mathrm{IAE}_{\min}$) for set-point changes. U is unstable.

D/τ	$\xi=0.2$	0.4	0.6	0.8	1.0	2.0	3.0	4.0	5.0	CONT
0.01	3.5565	3.6049	2.5872	2.2544	3.1764	11.077	20.4169	32.0564	45.2873	IMC-Pid
	1.5102	2.4474	2.4291	2.3840	3.6985	11.613	16.1406	19.6576	22.5033	IMCChn
	1.0000	1.0000	1.0000	1.0000	1.5569	4.8092	6.5911	7.9864	9.1403	IMCMac
	2.0564	1.2628	1.1897	1.2867	2.4771	15.4448	32.115	52.0817	74.4699	Honwel
	3.4023	3.0602	2.0958	1.6174	1.0000	1.0000	1.0000	1.0000	1.0000	CS-PID
0.1	2.9622	3.0115	2.3668	2.1099	2.2043	3.9899	5.5603	7.3038	9.1015	IMC-Pid
	1.3090	2.0506	2.2022	2.1812	2.4851	4.1183	4.4551	4.6177	4.7126	IMCChn
	1.0000	**1.0000**	**1.0000**	**1.0000**	1.1490	1.8997	2.0385	2.1041	2.1439	IMCMac
	U	1.5326	1.3445	1.3061	1.7314	5.3656	8.5036	11.6114	14.705	Honwel
	3.2762	2.8168	2.0839	1.6160	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	CS-PID
1	1.3705	1.7965	1.6568	1.5401	1.5262	1.6414	1.7319	1.8576	2.0152	IMC-Pid
	1.0000	**1.0000**	**1.0000**	**1.0000**	1.0392	1.2514	1.3237	1.3562	1.3724	IMCChn
	9.3349	1.6528	1.2325	1.0573	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	IMCMac
	U	U	4.4848	1.5085	1.2083	1.3548	1.8326	2.2939	2.7428	Honwel
	1.3729	1.4362	1.4629	1.4732	1.1264	1.2765	1.3398	1.3681	1.3818	CS-PID
3	1.4713	1.7080	1.7827	1.8103	1.8203	1.7810	1.7492	1.7141	1.6977	IMC-Pid
	1.1392	1.3080	1.3643	1.3905	1.4054	1.4264	1.4306	1.4328	1.4339	IMCChn
	1.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	IMCMac
	173.81	3.2384	1.6859	1.5389	1.4899	1.3785	1.3479	1.3676	1.4248	Honwel
	2.6069	1.4930	1.3944	1.6194	1.4032	1.3987	1.3997	1.4004	1.4008	CS-PID
5	1.7912	1.8101	1.8092	1.8039	1.7976	1.7652	1.7546	1.7462	1.7210	IMC-Pid
	1.5183	1.5153	1.5069	1.4991	1.4931	1.4838	1.4832	1.4840	1.4851	IMCChn
	1.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	IMCMac
	2.0537	1.3851	1.4620	1.5025	1.5131	1.4549	1.3939	1.3612	1.3509	Honwel
	3.2862	1.6511	1.4165	1.5767	1.4220	1.3988	1.3964	1.3963	1.3969	CS-PID
7	1.6644	1.7714	1.7694	1.7653	1.7627	1.7442	1.7390	1.7368	1.7357	IMC-Pid
	1.4194	1.5008	1.4948	1.4898	1.4861	1.4782	1.4775	1.4785	1.4799	IMCChn
	1.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	IMCMac
	1.4680	1.3821	1.4852	1.5274	1.5428	1.5046	1.4440	1.3994	1.3711	Honwel
	3.0054	1.6330	1.3890	1.5342	1.4108	1.3935	1.3915	1.3918	1.3927	CS-PID
10	1.6710	1.7223	1.7262	1.7252	1.7234	1.7093	1.7048	1.7035	1.7033	IMC-Pid
	1.4411	1.4790	1.4792	1.4768	1.4743	1.4658	1.4632	1.4634	1.4643	IMCChn
	1.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	IMCMac
	1.2243	1.4513	1.5315	1.5620	1.5722	1.5399	1.4892	1.4466	1.4123	Honwel
	2.9054	1.6173	1.3713	1.5017	1.4034	1.3887	1.3854	1.3851	1.3858	IMC-Pid

column indicates five tuning rules against each row. The elements in the table indicate the corresponding normalized-IAE values. For each process model, five IAE values are obtained with five different tuning rules [with the same tuning parameter $\lambda=\max(0.25D,0.2\tau)$] wherever applicable, in the present work the value of D differs but $\tau=1$]. A minimum IAE is sorted out of these five IAE values. Normalized IAE values are calculated by dividing the actual IAE by the minimum IAE for a particular process. The sum (for each tuning rule) of all these normalized-IAE values [also Fig. 4(a)] reveals that IMC-Maclaurin PID and CS-PID tuning rules perform better in overall SOPDT process models compared to IMC-Chien, IMC-PID, and Honeywell PID. Similar exercises, as mentioned above, are performed for the case of load changes (see Table 6). Though the results are similar to that of the set-point change case, the CS-PID tuning rule not only performs better in the region with $D/\tau\leqslant 1$ but also dominates overall, followed by IMC-Mac and IMC-Chien.

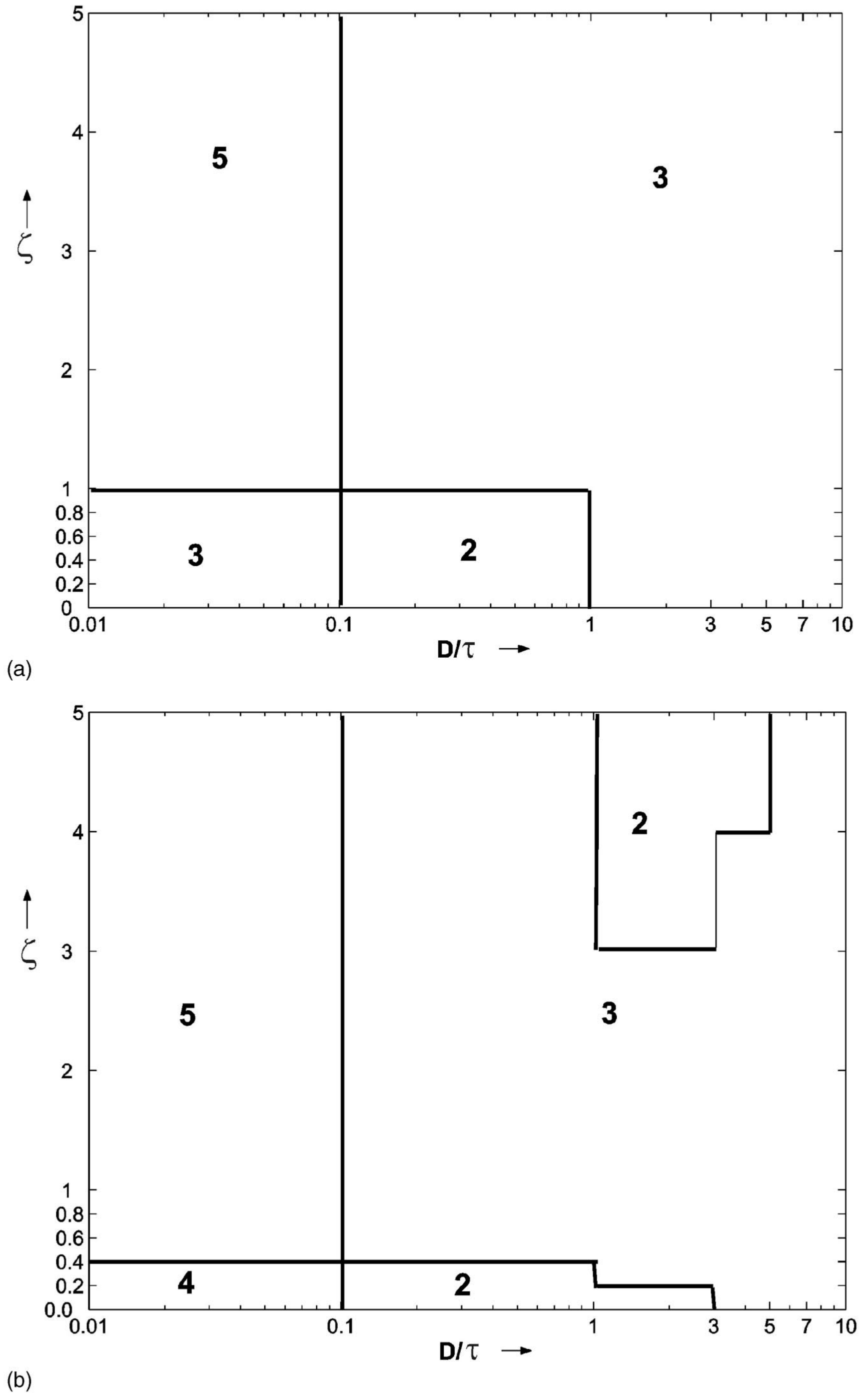

Fig. 4. (a) Demarcation of region for application of different tuning rules (3 IMC-Maclaurin PID; 2: IMC-Chien; 5: Closed-loop spec. CS-PID): set-point case. (b) Approximate demarcation of region for application of different tuning rules (3: IMC-Maclaurin PID; 2: IMC-Chien; 4: Honeywell tuning rule; 5: Closed-loop spec. CS-PID): load-disturbance case.

According to the performance of the controller, the tuning rules are ranked and are shown in Table 7 (set-point case) and Table 8 (load-disturbance case). All 63 process models are divided into some zones like the underdamped zone ($\xi<1$), the overdamped zone ($\xi\geqslant1$), or the low dead time to time constant ratio zone ($D/\tau\leqslant1$), and the high dead time to time constant ratio zone ($D/\tau<1$).

Table 6
Normalized IAE values ($=\mathrm{IAE}/\mathrm{IAE}_{\min}$) for load changes. U is unstable.

D/τ	$\xi=0.2$	0.4	0.6	0.8	1.0	2.0	3.0	4.0	5.0	CONT
0.01	12.6329	12.6635	14.8812	26.3178	16.910	37.490	58.335	78.835	99.1500	IMC-Pid
	13.0565	30.9763	53.8168	78.9302	50.500	50.500	50.500	50.500	50.5000	IMCChn
	6.1947	15.6137	24.4950	32.6899	20.500	20.500	20.500	20.500	20.5000	IMCMac
	1.0000	6.5569	20.2970	42.1163	33.8350	67.165	100.500	133.83	167.155	Honwel
	1.1605	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	CS-PID
0.1	2.0032	1.9744	2.0287	3.3553	2.1410	4.1990	6.2835	8.3335	10.3645	IMC-Pid
	1.9292	3.8825	5.8497	8.6043	5.5000	5.5000	5.5000	5.5000	5.4995	IMCChn
	1.0000	2.1429	2.9560	3.9922	2.5000	2.5000	2.5000	2.5000	2.5000	IMCMac
	U	**1.0000**	2.4716	4.9131	3.8335	7.1665	10.5000	13.833	17.1655	Honwel
	1.5953	1.1628	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	CS-PID
1	1.2001	1.1980	1.3532	1.2821	1.1098	1.1598	1.4378	1.7111	1.9820	IMC-Pid
	1.4719	**1.0000**	1.2360	1.3364	1.3259	1.3333	1.3333	1.3333	1.3334	IMCChn
	8.4914	1.0802	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	IMCMac
	U	U	3.2481	1.3063	1.1099	1.5555	2.0000	2.4445	2.8889	Honwel
	1.0000	1.0836	1.2706	1.1343	1.3430	1.3334	1.3333	1.3333	1.3334	CS-PID
3	1.0824	1.4104	1.6190	1.6756	1.6420	1.2307	1.1374	1.1417	1.2433	IMC-Pid
	1.0000	1.1830	1.3409	1.3856	1.3610	1.0387	**1.0000**	**1.0000**	**1.0000**	IMCChn
	1.2096	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	1.1246	1.1250	1.1250	IMCMac
	U	3.4158	1.6896	1.4862	1.3746	1.0398	1.2496	1.4166	1.5833	Honwel
	1.9818	1.2469	1.2906	1.4844	1.4447	1.3349	1.4995	1.4999	1.5000	CS-PID
5	1.8029	1.7865	1.7641	1.7425	1.7161	1.4894	1.2594	1.1760	1.1890	IMC-Pid
	1.5735	1.5359	1.5101	1.4871	1.4615	1.2611	1.0646	**1.0000**	**1.0000**	IMCChn
	1.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	1.0783	1.1809	IMCMac
	2.3368	1.4571	1.4782	1.4825	1.4598	1.2291	1.0657	1.1043	1.3121	Honwel
	3.2741	1.6328	1.3933	1.5194	1.4312	1.3934	1.3398	1.4378	1.5745	CS-PID
7	1.4500	1.7514	1.7500	1.7340	1.7186	1.6066	1.4295	1.2691	1.1684	IMC-Pid
	1.2550	1.5046	1.4990	1.4837	1.4688	1.3706	1.2159	1.0803	**1.0000**	IMCChn
	1.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	1.0271	IMCMac
	1.8271	1.3913	1.4922	1.5170	1.5131	1.3710	1.1962	1.0812	1.0530	Honwel
	2.5921	1.6190	1.3815	1.5061	1.4156	1.4185	1.3798	1.3431	1.3694	CS-PID
10	1.5481	1.6904	1.6897	1.6816	1.6726	1.6143	1.5318	1.4147	1.3044	IMC-Pid
	1.3532	1.4749	1.4721	1.4640	1.4554	1.4036	1.3269	1.2218	1.1252	IMCChn
	1.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	IMCMac
	1.1959	1.4501	1.5211	1.5441	1.5460	1.4559	1.3338	1.2119	1.1225	Honwel
	2.6557	1.6131	1.3672	1.4779	1.4041	1.4075	1.4109	1.3845	1.3580	CS-PID

For a particular zone (for example, underdamped zone $\xi<1$), the average IAE is calculated by dividing the sum of all normalized IAE's by the number of process models under this zone.

These average IAE's thus obtained are displayed in Table 7 (set-point case) and Table 8 (load-disturbance case). This shows the region of applicability of different tuning rules in SOPDT process models. In the case of set-point change, mainly, IMC-Maclaurin and IMC-Chien work better in the underdamped region while CS-PID and IMC-Mac tuning can be recommended for overdamped processes. CS-PID and IMC-Mac tuning can be recommended for SOPDT process models with low D/τ values while processes with high D/τ need IMC-Maclaurin or IMC-Chien tuning. From Fig. 4(a), one can observe the region of suitability for different tuning rules. IMC-Maclaurin PID covers most of the significant region in this figure. IMC-Chien tuning can be recommended for underdamped SOPDT process models with moderate D/τ values while CS-PID works better

Table 7
Ranking of tuning rules (set-point changes). Average IAE=(total cumulative normalized IAE)/(number of process).

Tuning rule	Normalized (IAE) for				Average IAE	Ranking				
	$\xi<1$	$\xi\geqslant 1$	$D/\tau\leqslant 1$	$D/\tau>1$		$\xi<1$	$\xi\geqslant 1$	$D/\tau\leqslant 1$	$D/\tau>1$	Over all
IMC-PID	2.0221	5.2502	8.4649	1.7392	3.8155	4	4	4	4	4
IMC-Chien	1.5549	3.7039	5.7552	1.4531	2.7488	2	3	3	2	3
IMC-Maclaurin	1.3313	1.8405	3.1284	1.0	1.6142	1	2	2	1	2
Honeywell	33.255 +4U	7.3355	45.7512 +3U	6.3088 +U	18.8554 +4U	5	5	5	5	5
Closed-loop Spec. PID	1.9898	1.2696	2.0098	1.6095	1.5897	3	1	1	3	1

for overdamped SOPDT systems with low D/τ values. Underdamped systems with high D/τ values can be better controlled by IMC-Maclaurin PID. The proposed CS-PID tuning rule is suitable for overdamped processes with $D/\tau=0.1$ and 1.0.

In the case of load disturbance [Fig. 4(b)], it has been found that CS-PID and IMC-Mac PID tuning rules work better in the entire region of SOPDT models compared to IMC-Chien PID.

The IMC-Mac PID tuning rule can be applicable to underdamped or overdamped regions with moderate and higher D/τ ratio for the load change case. The IMC-Chien tuning rule can be used for either highly underdamped or FOPDT types of processes with moderate D/τ. In Fig. 4, the regions of IMC-Chien and Honeywell PID are somewhat approximate.

3.2. Robustness

The closed-loop log modulus with a PID-2 controller using the IMC-Maclaurin tuning rule is calculated and is shown in Fig. 5. Damping coefficients are in the X axis, D/τ values are in the Y axis, and the corresponding log modulus in dB are plotted in the Z axis. The closed-loop log modulus with IMC-Mac PID (Fig. 5) cuts the 2-dB plane and it shows a maximum up to 8 dB for processes with low D/τ and with low damping coefficients. The vertical distance between these two planes (the MAC-PID plane and the 2-dB plane) represents the measure of actual robustness of the corresponding controller.

3.3. PID vs PI

The closed-loop performance of a process can generally be improved by the use of PID controller over PI controller. The minimum value of λ is found from the graph of IAE/D vs λ/D for a particular process using IMC-Mac tuning rule. The optimum tuning parameter (λ_{opt}) values for PID as well as PI are found for each of the SOPDT process models. With these λ_{opt} the ratio

Table 8
Ranking of tuning rules (load changes). Average IAE=(total cumulative normalized IAE)/(number of process).

Tuning rule	Normalized (IAE) for				Average IAE	Ranking				
	$\xi<1$	$\xi\geqslant 1$	$D/\tau\leqslant 1$	$D/\tau>1$		$\xi<1$	$\xi\geqslant 1$	$D/\tau\leqslant 1$	$D/\tau>1$	Over all
IMC-PID	3.8239	10.211	19.5396	1.5037	7.3725	2	4	3	3	3
IMC-Chien	8.0219	8.8718	23.2737	1.2883	8.4941	4	3	4	2	4
IMC-Maclaurin	4.1738	4.0189	10.5074	1.0242	4.0877	3	2	2	1	2
Honeywell	59.428 +4U	16.877	98.6375 +3U	5.0921 +U	33.7889 +4U	5	5	5	5	5
Closed-loop Spec. PID	1.4801	1.2864	1.4325	1.5662	1.3725	1	1	1	4	1

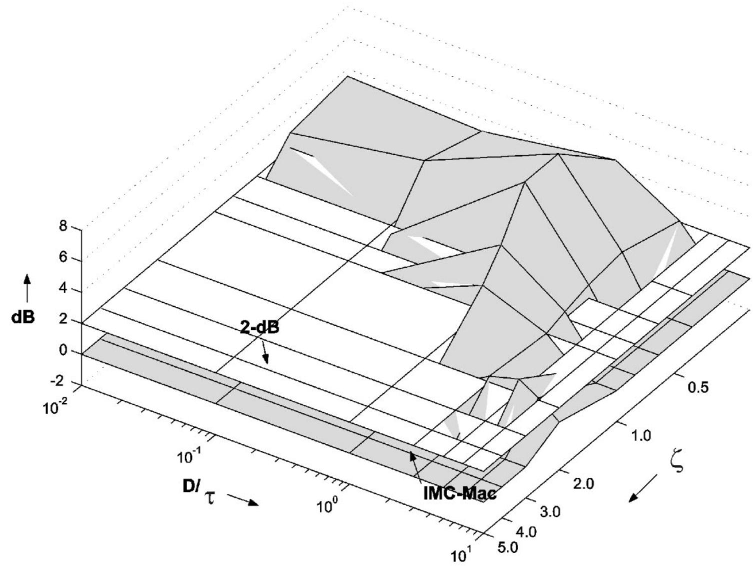

Fig. 5. Closed-loop log modulus of SOPDT processes with IMC-Maclaurin PID tuning rule (compared with 2-dB plane).

between IAE_{PID} and IAE_{PI} are found. Thus-obtained IAE ratios are shown in Table 9. It can be seen from the table that the margin of improvement of PID controller decreases with the increase of D/τ. The results are consistent with all different damping coefficients as shown in Table 9. The processes in the last column resemble FOPDT systems where we find an improvement of $\sim 30\%$ in IAE with PID controller over PI.

4. Conclusion

In this study, five existing PID-controller tuning rules are studied along with a new tuning rule based on closed-loop trajectory specification of $G_P G_C = e^{-Ds}/2Ds(\alpha \tau_D s + 1)$ for finding the appropriate tuning rule for SOPDT systems. Four tuning rules are based on the IMC design where the controller (G_c) is found from the process model (G_P). Three of them are based on the Taylor's series expansions for the dead time and IMC-Maclaurin is based on the Maclaurin series approximation on the whole controller. These tuning rules are used to tune controller parameters of an industrial PID controller implemented in a closed-loop structure with SOPDT models. Performance study reveals the following:

(i) For a process with high D/τ ratio $(D/\tau > 1)$: use *IMC-Maclaurin* settings for both set-point and load changes.

Table 9
Values of ratio of IAE_{PID}/IAE_{PI} for IMC-Mac tuning rule.

ξ D/τ	0.2	0.4	0.6	0.8	1.0	2.0	3.0	4.0	5.0
0.01	0.0438	0.0796	0.1107	0.1461	0.1934	0.3921	0.5225	0.6118	0.6743
0.1	0.1014	0.1717	0.2253	0.2750	0.3244	0.5070	0.6114	0.6774	0.7225
1.0	0.6544	0.9016	0.7948	0.7491	0.7155	0.6859	0.6835	0.6866	0.6905
3.0	0.9547	0.8501	0.8063	0.7787	0.7587	0.7140	0.6940	0.6837	0.6778
5.0	0.8603	0.8473	0.8187	0.7997	0.7861	0.7393	0.7144	0.6990	0.6885
7.0	0.9636	0.9144	0.8522	0.8193	0.8029	0.7576	0.7313	0.7139	0.7017
10.0	1.0284	0.9671	0.9125	0.8677	0.8363	0.7779	0.7516	0.7335	0.7199

(ii) For a process with low D/τ ratio ($D/\tau \leqslant 1$): use *CS-PID* tuning for both set-point and load changes.
(iii) For overall SOPDT process models when considering both set-point and load changes, use *CS-PID* settings.

The comparison between PI and PID controllers is also investigated. The results show that much improved performance can be achieved using a PID controller (over a PI one) for systems with smaller D/τ ratio (Table 9).

Acknowledgment

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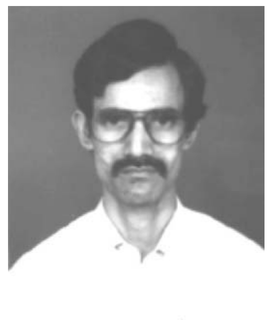

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