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A New Robust Model Reference Control for a Class of Multivariable Unknown Plants

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ABSTRACT

Motivated by the recent works [2] [14], a new robust model reference control (MRC) scheme for a class of multivariable unknown plants is presented in this paper. The controller is devised using the concept of variable structure design which prevails in the robust control context. Such a new scheme solves the model reference adaptive control (MRAC) problem for a multivariable plant of interest subject to exactly the same conditions but with better performance. It is shown that the global stability of the overall system is achieved and the tracking errors will converges to a residual set whose size can be directly related to the size of unmodeled dynamics and output disturbances explicitly. Furthermore, in the absence of unmodeled dynamics and output disturbances, the tracking error can be driven to zero in finite time.

1. Introduction :

With the advance in designing robust adaptive controllers for single–input single–output (SISO) uncertain dynamical systems, a multi–input multi–output (MIMO) model reference adaptive control (MRAC) scheme has been established [1] [2]. Since the parametrization issue was solved, the research problems will be to focus on the design of the controller and the adaptation law for robust MRAC of MIMO plants. The object is achieved in the SISO case, although the control performance are different from one to the other, by using the concept of persistency of excitation, e.g. [3] [4] [5] or by modifying the adaptation law, e.g. [6] [7] [8] [9] [10]. Also, a general framework is proposed in [11] to analyze a wide class of robust adaptive laws. The methods described in [11] are further transformed into work in dealing with the robust MRAC of MIMO plants.

Recently, some researches are interested in the controller design with variable structure concept for either SISO plants [12] [13] [14] [15] or MIMO plants [16]. In this paper, we propose a new model reference control (MRC) scheme using variable structure design for a class of MIMO plants. This is an outgrowth of [14] and [15] which use the concept of variable structure adaptation and give an improvement in transient response and convergence property. A modified version by fixing the control parameters at some constant values is used to control a linear fast time–varying unknown plant [17]. Motivated by the researches above, a combined technique is developed for the MIMO model reference control. It can be shown that the well behaved transient performance still holds with all closed loop signals remaining uniformly bounded. Also note that the complexity in a standard MIMO MRAC design is reduced since only a simple control law is used.

The paper is organized as follows: in section *2*, we give a detailed problem formulation and the control structure for an MIMO MRC scheme. The resulting error model and the robust controller for a class of plants are presented in section *3*. Section *4* gives a simulation to demonstrate the effect of the robust controller. Finally, a conclusion is made in section *5*.

2. Problem Formulation and Controller Design :

In this paper, we use the concept of modified left interactor (MLI) matrix and modified right interactor (MRI) matrix [2] for the parametrization of a MIMO plant $P_0(s)$.

2.1 System Description :

Consider an MIMO linear time–invariant plant with N inputs and N outputs described by the following transfer matrix

$$P_u(s) = P_0(s)[I + \mu\Delta P_1(s)] + \mu\Delta P_2(s)$$
$$P_0(s) = Z_p(s)R_p^{-1}(s) \qquad (2.1)$$

where $P_0(s)$ represents the nominal plant transfer matrix and $\mu\Delta P_1(s)$ $\mu\Delta P_2(s)$ are multiplicative and additive unmodeled dynamics respectively with some $\mu \geq 0$. The control objective is to design a control law $u_p(t)$ such that the output $y_p(t)$ of the plant tracks the output $y_m(t)$ of a linear time–invariant reference model, i.e. $y_m = W_m(s)\ ref$, where $W_m(s)$ is a stable strictly proper rational matrix and $ref\ (t)$ is a uniformly bounded reference input signal vector. To make the problem more tractable, several assumptions on the plant and the unmodeled dynamics are made in the following:

(A1) The MLI (MRI) matrix $\xi_\ell^m(s)$ ($\xi_r^m(s)$) of $P_0(s)$ is known.

(A2) An upper bound ν on the observability index of $f_\ell^{-1}(s)\xi_\ell^m(s)P_0(s)$ (resp. $f_r^{-1}(s)P_0(s)\xi_r^m(s)$) is known.

(A3) The matrix K_ℓ (resp. K_r) such that $K_{\ell p}^m K_\ell$

$= \Gamma_\ell > 0$ (resp. $K^m_{rp} K_r = \Gamma_r > 0$) is positive definite is known.

(A4) $P_0(s)$ is nonsingular and has stable zeros.

(A5) The unmodeled dynamics $\Delta P_1(s)$ and $\Delta P_2(s)$ are stable proper and strictly proper transfer matrices respectively. Furthermore, there exists $\gamma > 0$ such that $\|\Delta P_1(s)\|_\infty$ and $\|P_0^{-1}(s)\Delta P_2(s)\|_\infty \le \gamma$, where $\|H(s)\|_\infty = \bar{\sigma}(H(jw))$ and $\bar{\sigma}(.)$ denotes the largest singular value of the argument matrix.

Remark 2.1 [2] : The MIMO linear time–invariant plant $y_P(s) = P_0(s)u_P(s)$ can be represented as

$$y_P(s) = f_\ell(s)(\xi^m_\ell(s))^{-1}x(s)$$
$$x(s) = P_\ell(s)u_P(s) \tag{2.2}$$

or

$$y_P(s) = P_r(s)v(s)$$
$$u_P(s) = f_r^{-1}(s)\xi^m_r(s)v(s) \tag{2.3}$$

where $P_\ell(s) = f_\ell^{-1}(s)\xi^m_\ell(s)P_0(s)$ (resp. $P_r(s) = f_r^{-1}(s) P_0(s)\xi^m_r(s)$) is an N x N transfer matrix whose MLI (resp. MRI) matrix is $f_\ell(s)I$ (resp. $f_r(s)I$) ; $f_\ell(s)$ (resp. $f_r(s)$) is an arbitrary Hurwitz polynomial of degree d_ℓ (resp. d_r) and d_ℓ (resp. d_r) $\geq$ the maximum degree of the elements of $\xi^m_\ell(s)$ (resp. $\xi^m_r(s)$). □□□

Remark 2.2 : In this paper, the plant $P_0(s)$ will in general be assumed to have a diagonal MLI or MRI matrix which can be specified with only the knowledge of relative degree of each matrix entry. □□□

Remark 2.3 : The assumptions (A1) – (A4) are equivalent to the relative degree, upper bound for the order of transfer function, the sign of high frequency gain and minimum phase assumptions in model reference control scheme for SISO plant. □□□

Furthermore, the plant is assumed to be operated subject to bounded output disturbances $\zeta_0 \in R^{n\times 1}$ (which are usually modeled in a real system as measurement noise), i.e. $\hat{y}_P = y_P + \zeta_0$.

2.2. Controller Structure Design :

By the way of parametrization described in (2.2) and (2.3), we can use the standard MRC structure for MIMO plants in a way similar to that in an SISO case. In this section, the MRC structure based on MRI matrix formulation is discussed and that based on MLI matrix can be obtained similarly as in as [2]. We now express the system equation in the following form:

$$\hat{y}_P(s) = G_r(s)v(s) + \zeta_0(s)$$
$$u_P(s) = f_r^{-1}(s)\xi^m_r(s)v(s) \tag{2.4}$$

where

$$G_r(s) = P_r(s)[I + \mu\Delta P_{1r}(s)] + \mu\Delta P_{2r}(s)$$
$$P_r(s) = f_r^{-1}(s)P_0(s)\xi^m_r(s) = Z_{pr}(s)R^{-1}_{pr}(s)$$
$$\Delta P_{1r}(s) = (\xi^m_r(s))^{-1}\Delta P_1(s)\xi^m_r(s)$$
$$\Delta P_{2r}(s) = f_r^{-1}(s)\Delta P_2(s)\xi^m_r(s) \tag{2.5}$$

and $Z_{pr}(s)$ and $R_{pr}(s)$ are right coprime polynomial matrices of dimension N × N and $R_{pr}(s)$ is column proper. The standard MRC structure will be used and the control input u_P for the plant is given as:

$$v = C_r(s,\theta_1)N_r^{-1}(s)v + D_r(s,\theta_2,\theta_3)N_r^{-1}(s)\hat{y}_P + K_0\, ref + \nu_P$$
$$u_P = f_r^{-1}(s)\xi^m_r(s)v \tag{2.6}$$

where

$$C_r(s,\theta_1) = \theta_{11}s^{\nu-2} + \dots\dots + \theta_{1,\nu-1}$$
$$D_r(s,\theta_2,\theta_3) = \theta_{21}s^{\nu-2} + \dots\dots + \theta_{2,\nu-1} + \theta_3 N_r(s)$$
$$\theta_1 = [\theta_{11},\dots\dots, \theta_{1,\nu-1}]^T$$
$$\theta_2 = [\theta_{21},\dots\dots, \theta_{2,\nu-1}]^T \tag{2.7}$$

and $N_r(s) = diag\{n_r(s)\}$; $n_r(s)$ is an arbitrary monic stable polynomial of degree ν–1 and K_0 is a constant matrix. The design of the control input is similar to that in [2] but with a difference in the additional term ν_P to be specified later. Under this structure and with the condition of $\mu = 0$, $\zeta_0 = 0$, and $\nu_P = 0$, it can be easily shown that there exist constant matrices θ_1^*, θ_2^*, θ_3^* and K_0^* such that the closed loop transfer function matrix matches the reference model $W_m(s)$. It can be easily observed that a suitable choice for the reference model is simply $f_r^{-1}(s)I$ whose strictly positive real (*SPR*) property can be easily checked from the order of the polynomial $f_r(s)$. For an analysis on error models and Lyapunov design in the next section, a definition of the concept of " generalized relative degree " for an MIMO plant is naturally given as follows:

Definition 2.1 :

Consider an MIMO linear time–invariant plant $P_0(s)$ to be parametrized as either (2.2) or (2.3) so that a simple MLI matrix $f_\ell(s)I$ or MRI matrix $f_r(s)I$ is obtained. Then the plant $P_0(s)$ is defined as a system of generalized relative degree n if $f_\ell(s)$ or $f_r(s)$ is an n–th order polynomial. □□□

3. Robust MRC Design For MIMO Plants with Generalized Relative Degree One :

In this section, the MRI matrix $f_r(s)I$ is considered, where $f_r(s)$ is a first order Hurwitz polynomial, such that the reference model $f_r^{-1}(s)I$ is *SPR*.

3.1 Error Model :

It can be easily verified that the state–space representation of (2.4) (2.5) can be given with a minimal realization (A_P, B_P, C_P) of the transfer matrix $P_r(s)$ as follows:

$$\dot{X}_P = A_P X_P + B_P v + B_P \mu\zeta_i \,;\, X_P \in R^n$$
$$\hat{y}_P = C_P X_P + \zeta_0 \tag{3.1}$$

where $\mu\zeta_i$ represents the effect of unmodeled dynamics satisfying:

$$\dot{X}_\zeta = A_\zeta X_\zeta + B_\zeta v$$
$$\zeta_i = C_\zeta X_\zeta + D_\zeta v \qquad (3.2)$$

with $(A_\zeta, B_\zeta, C_\zeta, D_\zeta)$ being a minimal realization of the transfer matrix $\Delta P_{1r}(s) + P_0^{-1}(s)\Delta P_{2r}(s)$ which is proper stable from assumptions (A4) and (A5) so that A_ζ is Hurwitz. Also define the signal vector w and $\hat{w}$ with dimension $(2n \times \nu)$ as :

$$\hat{w} = \begin{bmatrix} ref \\ w_1 \\ w_2 \\ \hat{y}_P \end{bmatrix} = \begin{bmatrix} ref \\ w_1 \\ w_2 \\ y_P \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \zeta_0 \end{bmatrix} = w + \hat{\zeta}_0 \qquad (3.3)$$

where

$$w_1 = S(s)N^{-1}(s)v = (sI - \Lambda)^{-1}Bv$$
$$w_2 = S(s)N^{-1}(s)\hat{y}_P = (sI - \Lambda)^{-1}B\hat{y}_P \qquad (3.4)$$

with $S(s) = [I, sI, \dots, s^{\nu-2}I]^T$, and $det(sI - \Lambda) = n_r(s)$ so that the control input u_P can be rewritten as:

$$v = \theta^T \hat{w} + \nu_P \ ; \ u_P = f_r^{-1}(s)\xi_r^m(s)\, v \qquad (3.5)$$

with $\theta = [K_0, \theta_1^T, \theta_2^T, \theta_3]^T$. Thus, a state space representation of the plant loop is given by:

$$\frac{d}{dt}\begin{bmatrix} x_P \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} A_P & 0 & 0 \\ 0 & \Lambda & 0 \\ BC_P & 0 & \Lambda \end{bmatrix}\begin{bmatrix} x_P \\ w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} B_P \\ B \\ 0 \end{bmatrix}(\theta^T \hat{w} + \nu_P) + \begin{bmatrix} B_P \\ 0 \\ 0 \end{bmatrix}\mu\zeta_i + \begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix}\zeta_0$$

$$y_P = [C_P, \ 0, \ 0]\begin{bmatrix} x_P \\ w_1 \\ w_2 \end{bmatrix} \qquad (3.6)$$

By the " matching condition " described in last section, the reference model $f_r^{-1}(s)I$ can be realized by a nonminimal state-space (A_c, B_c, C_c) as:

$$\frac{d}{dt}\begin{bmatrix} x_m \\ w_{1m} \\ w_{2m} \end{bmatrix} = \begin{bmatrix} A_P + B_P\theta_1^* C_P & B_P\theta_1^{*T} & B_P\theta_2^{*T} \\ B\theta_3^* C_P & \Lambda + B\theta_1^{*T} & B\theta_2^{*T} \\ BC_P & 0 & \Lambda \end{bmatrix}\begin{bmatrix} x_m \\ w_{1m} \\ w_{2m} \end{bmatrix} + \begin{bmatrix} B_P \\ B \\ 0 \end{bmatrix} K_0^* \, ref$$

$$y_m = [C_P, \ 0, \ 0]\begin{bmatrix} x_m \\ w_{1m} \\ w_{2m} \end{bmatrix} \qquad (3.7)$$

Define the state error vector e as:

$$e = \begin{bmatrix} x_P \\ w_1 \\ w_2 \end{bmatrix} - \begin{bmatrix} x_m \\ w_{1m} \\ w_{2m} \end{bmatrix} \qquad (3.8)$$

so that the error model can be derived from the subtraction of (3.7) from (3.6) as:

$$\dot{e} = A_c e + B_c(\phi^T w + \nu_P) + B_{c1}\mu\zeta_i + B_{c2}(\theta)\zeta_0$$
$$e_0 = y_P - y_m = C_c e \qquad (3.9)$$

where $B_{c1} = [B_P^T, 0, 0]^T$, $B_{c2}(\theta) = [\theta_3 B_P^T, \theta_3 B^T, B^T]^T$, and $\phi = \theta - \theta^*$ is the parameter error vector.

3.2 Robust Controller Design :

A new MRC scheme using variable structure design concept is given in this section. This method is an outgrowth from the one proposed by Fu [14], [15] for SISO MRAC scheme with unknown plant of relative degree one or two whose transient response is drastically improved. But, because of the fact that the control parameters converge to zero at a rapid rate by using variable structure adaptation, a modification which fixes the control parameters θ at a zero vector (or zero matrix as MIMO plants are considered) at the very beginning has been used for the control of linear fast time–varying unknown plant successfully [17]. Furthermore, the complexity due to a lot of computation for adaptation is reduced when the control parameters are fixed. Hence we will focus on the choice of ν_P in (2.6) so that the overall system is guaranteed to be globally stable and the tracking error is ensured to converge to a residual set whose size is a simple function of μ and of the bound on ζ_0 when they are small. The following theorem gives the robust property with global stability and bounded tracking performance.

Theorem 3.1 :

Consider the error system (3.9) satisfying assumption (A1) – (A5) with ν_P being specified as :

$$\nu_P = -K_r sign\,(\hat{e}_0)(\beta_0 \,|w| + \beta_1) \qquad (3.10)$$

where $\hat{e}_0 = e_0 + \zeta_0$, $sign(\hat{e}_0) = [sign(\hat{e}_{01}), sign(\hat{e}_{02}), \dots, sign(\hat{e}_{0n})]^T$ and $|.|$ denotes the two morn of a vector or induced two norm of a matrix. Let $\theta(t) = 0 \ \forall \ t \geq 0$, then there exists $\mu^* > 0$ such that for all $\mu \in (0,\mu^*]$ all signals inside the closed loop system are uniformly bounded and the tracking error will converge to a residual set whose size can be written as an explicit class K function of μ and ρ where $\rho \geq |\zeta_0|$ is the upper bound for output disturbances. □□□

Proof:

If the control parameter θ is set to be zero, then $\phi = -\theta^*$ and the error system will be:

$$\dot{e} = A_c e + B_c(-\theta^{*T} w + \nu_P) + B_{c1}\mu\zeta_i + B_{c2}\zeta_0$$
$$e_0 = C_c e \qquad (3.11)$$

where $B_{c2} = [0, 0, B^T]^T$. Furthermore, the input signal v simply becomes ν_P. Construct a Lyapunov function:

$$V(e, x_\zeta) = \frac{1}{2} e^T P_e e + \frac{1}{2}\mu x_\zeta^T P_\zeta x_\zeta \qquad (3.12)$$

where $P_e = P_e^T > 0$ satisfies

$$P_eA_c + A_c^TP_e = -2Q_e \;;\; P_eB_cK_r = C_c^T \quad (3.13)$$

due to the *SPR* property of the transfer matrix $C_c(sI - A_c)^{-1}B_cK_r$ for some $Q_e > 0$ and $P_\zeta > 0$ satisfying

$$P_\zeta A_\zeta + A_\zeta^TP_\zeta = -2Q_\zeta \quad (3.14)$$

for some $Q_\zeta > 0$. Then the time derivative of V along the trajectories of (3.2) and (3.11) subject to ν_P (3.10) can be computed as :

$$\dot V = -e^TQ_ee + (\hat e_0 - \zeta_0)^TK_r^{-1}(-\theta^{*T}w + \nu_P)$$
$$+ \mu e^TP_eB_{c1}\zeta_i + e^TP_eB_{c2}\zeta_0 - \mu x_\zeta^TQ_\zeta x_\zeta + \mu x_\zeta^TP_\zeta B_\zeta\nu_P \quad (3.15)$$

Let q_{e1}, q_{e2}, $q_{\zeta1}$, $q_{\zeta2}$ be strictly positive such that :

$$q_{e1}I \le Q_e \le q_{e2}I\,,\; q_{\zeta1}I \le Q_\zeta \le q_{\zeta2}I \quad (3.16)$$

Then

$$\dot V \le -q_{e1}|e|^2 - \mu q_{\zeta1}|x_\zeta|^2 + \hat e_0^T K_r^{-1}\Big(-\theta^{*T}\hat w + \theta^{*T}\hat\zeta_0 + \nu_P\Big) - \zeta_0^T K_r^{-1}\Big(-\theta^{*T}w + \nu_P\Big) + \mu|e||P_eB_{c1}|\Big(|C_\zeta||x_\zeta| + |D_\zeta||\nu_P|\Big) + |e||P_eB_{c2}||\zeta_0| + \mu|x_\zeta||P_\zeta B_\zeta||\nu_P| \quad (3.17)$$

Since $\hat e_0^T\ sign(\hat e_0) = |\hat e_0|_1$ where $|.|_1$ means the one norm of a vector and

$$|\nu_P| = |-K_r\, sign(\hat e_0)(\beta_0|\hat w| + \beta_1)| \le k_1|\hat w| + k_2 \quad (3.18)$$

for some k_1, $k_2 > 0$, we can find that

$$\dot V \le -q_{e1}|e|^2 - \mu q_{\zeta1}|x_\zeta|^2 - |\hat e_0|_1\Big(\beta_0|\hat w| + \beta_1 - |K_r^{-1}||\theta^*||\hat w| - |K_r^{-1}||\theta^*||\zeta_0|\Big) + |\zeta_0||K_r^{-1}|\Big(k_1|\hat w| + k_2 + |\theta^*||w|\Big) + \mu k_3|e||x_\zeta| + \mu k_4|e|\Big(k_1|\hat w| + k_2\Big) + \mu k_5|x_\zeta|\Big(k_1|\hat w| + k_2\Big) + k_6|e||\zeta_0| \quad (3.19)$$

where $k_3 = |P_eB_{c1}||C_\zeta|$, $k_4 = |P_eB_{c1}||D_\zeta|$, $k_5 = |P_\zeta B_\zeta|$ and $k_6 = |P_eB_{c2}|$. If we choose the control parameters

$$\beta_0 > \beta_0^* = |K_r^{-1}||\theta^*|$$
$$\beta_1 > \beta_1^* = \rho|K_r^{-1}||\theta^*| \quad (3.20)$$

then $\dot V$ will be subject to the following inequality:

$$\dot V \le -q_{e1}|e|^2 - \mu q_{\zeta1}|x_\zeta|^2 + |e|\Big(\mu k_7|\hat w| + \mu k_8 + k_6\rho\Big) + |x_\zeta|\Big(\mu k_9|\hat w| + \mu k_{10}\Big) + \mu k_3|e||x_\zeta| + \rho\Big(k_{11}|\hat w| + k_{12}|w| + k_{13}\Big) \quad (3.21)$$

for some suitable $k_7 \sim k_{13} > 0$. Also it can be easily seen that the relation between w and e is

$$w = w_m + Ge \quad (3.22)$$

where

$$w_m = [ref^T,\ w_{m1}^T,\ w_{m2}^T,\ y_m^T]^T$$

and

$$G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ C_P & 0 & 0 \end{bmatrix} \quad (3.23)$$

which implies that

$$|\hat w| \le |w_m| + |G||e| + |\zeta_0| \le k_{14} + k_{15}|e| + \rho \quad (3.24)$$

for some constants k_{14} and $k_{15} > 0$. Hence,

$$\dot V \le -\Big(q_{e1} - k_{16}\mu - k_{19}\mu^{2/3}\Big)|e|^2 - \mu\Big(q_{\zeta1} - k_{17}\mu^{1/3}\Big)|x_\zeta|^2 + \Big(\mu k_{18} + k_6\rho\Big)|e| + \mu k_{19}|x_\zeta| + k_{20}\rho \quad (3.25)$$

with suitable constants $k_{16} \sim k_{20}$, where we use the fact that

$$\mu ab \le \tfrac{1}{2}(\mu^{4/3}a^2 + \mu^{2/3}b^2) \quad (3.26)$$

Hence, there exists $\mu^* > 0$ such that

$$\tfrac{1}{2}q_{e1} > (k_{16}\mu^* + k_{17}\mu^{*2/3})$$
$$\tfrac{1}{2}q_{\zeta1} > k_{17}\mu^{*1/3} \quad (3.27)$$

and thus

$$\dot V \le -\tfrac{1}{2}q_{e1}|e|^2 - \tfrac{1}{2}\mu q_{\zeta1}|x_\zeta|^2 + \max(\mu,\rho)k_{21}|e| + \mu k_{19}|x_\zeta| + k_{20}\rho \quad (3.28)$$

for some $k_{21} > 0$. We now define the vector X as

$$X = (e^T,\ \mu^{1/2}x_\zeta^T)^T \quad (3.29)$$

so that we can see that there exist constants $\alpha_1 \sim \alpha_4 > 0$ such that

$$\alpha_1|X|^2 \le V \le \alpha_2|X|^2$$
$$\dot V \le -\alpha_3|X|^2 + \alpha_4|X| + k_{20}\rho \quad (3.30)$$

where

$$\alpha_1 = min\,(\tfrac{1}{2}p_{e1},\ \tfrac{1}{2}p_{\zeta1})$$
$$\alpha_2 = man\,(\tfrac{1}{2}p_{e2},\ \tfrac{1}{2}p_{\zeta2}) \quad (3.31)$$

with

$$p_{e1}I \le P_e \le p_{e2}I \text{ and } p_{\zeta1}I \le P_\zeta \le p_{\zeta2}I \quad (3.32)$$

and

$$\alpha_3 = min\,(\tfrac{1}{2}q_{e1},\ \tfrac{1}{2}q_{\zeta1})$$
$$\alpha_4 = max\,(\mu^{1/2}k_{19},\ max\,(\mu,\rho)k_{21}) \quad (3.33)$$

Consequently, the ultimate boundedness property [18] [19] of the overall system can be concluded now for all $\mu < \mu^*$. Furthermore, the ultimate bound of e and, hence, e_0 can be shown to be a class K function of the following form as a result of [18]

$$g_0\sqrt{\frac{\alpha_2}{\alpha_1}}\left(\frac{\alpha_4}{2\alpha_3} + \sqrt{\frac{\alpha_4{}^2}{4\alpha_3} + k_{20}\rho}\right) \quad (3.34)$$

for some $g_0 > 0$. This completes our proof. ΔΔΔ

<u>*Corollary 3.2*</u> :

Consider the system as described in (2.1) but in the absence of unmodeled dynamics and output dis– turbances. Then the controller (3.10) will drive the output error to zero in finite time with all closed loop signals remaining uniformly bounded. □□□

<u>*Proof*</u>:

In the absence of unmodeled dynamics and output disturbance, i.e. ,$\mu = 0$ and $\zeta_0 = 0$, the error model described in (3.11) will be modified as the following form:

$$\dot{e} = A_c e + B_c(-\theta^{*T} w + \nu_P) \;;\; e_0 = C_c e \qquad (3.35)$$

Hence, by a simple Lyapunov function $V(e) = e^T P e$, $P = P^T > 0$, it can be easily shown by a similar procedure of the proof for Theorem *3.1* that $\dot{V}(e) \le -m_1 V(e)$ for some positive constant m_1 which concludes that the state error e will converge to zero at least exponentially fast. Furthermore,

$$\begin{aligned} & e_0^T \Gamma_r^{-1} \dot{e}_0 \\ &= e_0^T \Gamma_r^{-1}(C_c A_c e + C_c B_c(-\theta^{*T} w + \nu_P)) \\ &= e_0^T \Gamma_r^{-1} C_c A_c e + e_0^T \Gamma_r^{-1} K_{rp}^m K_r\Big(-sign(e_0)(\beta_0 |w| + \beta_1) \\ &\quad - K_r^{-1}\theta^{*T} w\Big) \\ &\le |e_0|_1\Big(m_2|e| - (\beta_0|w| + \beta_1 - |K_r^{-1}||\theta^*||w|)\Big) \qquad (3.36) \end{aligned}$$

for some positive constant m_2. Since $|e|$ approaches zero at least exponentially fast and (3.20) is assumed, there exists a finite $T > 0$ such that

$$e_0^T \Gamma_r^{-1} \dot{e}_0 \le -m_3|e_0|_1 \qquad (3.37)$$

for all $t > T$ and for some $m_3 > 0$, which implies that the switching surface $e_0 = 0$ will be reached in finite time [14]. ΔΔΔ

4. Simulation :

A 2 × 2 transfer matrix $P_0(s)$ is given for computer simulation.

$$P_0(s) = \begin{bmatrix} \frac{3}{s-1} & \frac{1}{s+3} \\ \frac{1}{(s+2)^2} & \frac{1}{s-3} \end{bmatrix} \qquad (4.1)$$

It can be easily observed that the modified right interactor matrix $\xi_r^m(s)$ can be chosen as $(s+2)I$ which is diagonal and the high frequency gain matrix

$$K_{rp}^m = K_{rp} = \lim_{s\to\infty} P_0(s)\xi_r^m(s) = \begin{bmatrix} 3 & 1 \\ 0 & 1 \end{bmatrix} \qquad (4.2)$$

is nonsingular and positive definite. Hence, we will choose the reference model as $(s+2)^{-1}I$ and the matrix $K_r = I$ for our control design such that ν_P is

$$\nu_P = -\begin{bmatrix} sign(e_{01}) \\ sign(e_{02}) \end{bmatrix} (\beta_0|w| + \beta_1) \qquad (4.3)$$

Also note that the observability index for the plant $P_0(s)$ is 3. In the following simulations, the initial conditions for $P_0(s)$ is assumed to be 3 and 2.5 for diagonal elements and in the task of tracking, the reference input $ref_1(t) = 2$ and $ref_2(t) = 2sin(t)$ are applied.

4.1 Ideal case :

In the absence of unmodeled dynamics and output disturbance, the control parameters β_0 and β_1 are chosen as 20 and 0 respectively. Fig.1 and Fig.2 are the tracking performance for e_{01} and e_{02} whose finite time convergence property is observed!

4.2 Unmodeled dynamic and output disturbance :

Consider the multiplicative and additive unmodeled dynamics with $\mu = 0.001$

$$\Delta P_1(s) = \begin{bmatrix} \frac{s+1}{s+4} & 0 \\ 0 & \frac{s+1}{s+4} \end{bmatrix} \qquad (4.4)$$

$$\Delta P_2(s) = \begin{bmatrix} \frac{1}{s+2} & \frac{1}{s+5} \\ 0 & \frac{1}{s+10} \end{bmatrix} \qquad (4.5)$$

and output disturbance

$$\zeta_0 = \begin{bmatrix} 0.1\cos(t) \\ 0.1\sin(t) \end{bmatrix} \qquad (4.6)$$

The control parameters are now chosen as $\beta_0 = 20$ and $\beta_1 = 10$ respectively. From Fig.3 and Fig.4 we can see that the output tracking performance is still acceptable for the existence of unmodeled dynamics and disturbance.

5. Conclusion :

In this paper, a new model reference control (MRC) scheme for a class of MIMO systems was presented. With moderate unmodeled dynamics and bounded output disturbances, the controller, which combines the characteristics of variable structure design [14] [15] and null adaptation process [17], not only stabilizes the overall systems but also drives the tracking error to a residual set whose size can be directly related to an explicit function of μ and ρ. It is noteworthy, however, that such an MRC scheme solves the same problem as the MRAC schemes usually do but the former remarkably simplifies the complex computation conventionally required by the latter.

In the absence of unmodeled dynamics and output disturbances, the output error will be driven to zero in finite time (modulo some chattering afterwards). As indicated in [14] [15], this convergence property is a remedy to the undesirable slow convergence usually appear in traditional MRAC schemes. From the simulation results, the drastic improvement for convergence performance is observed.

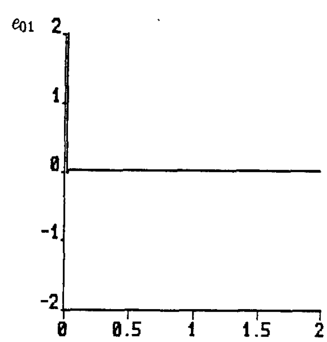

Fig.1

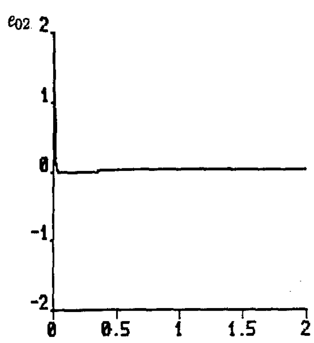

Fig. 2

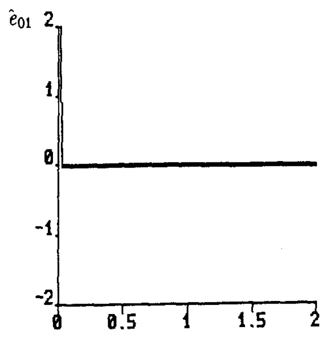

Fig. 3

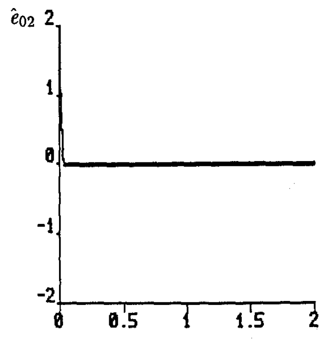

Fig. 4

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