CHINESE JOURNAL OF PHYSICS VOL. 21 NO. 1, 2 SPRING/SUMMER, 1983

Variation of Wave Vectors in Mean Free Path Theory in a Nucleus

DER-RUENN Su (蘇德潤)

Physics Department, National Taiwan University, Taipei 107

(Received 23 December, 1982)

As the imaginary part of the wave vector is considered to vary over a statistical width mainly 2Δ, the mean free path λ of a nucleon is theoretically calculated to fit the empirical values perfectly. Further Δ/λ is shown to have a universal value $\sqrt{2}-1$.

RECENTLY, the mean free path of a particle in the nuclear medium has been intensively studied. The necessity of a further theoretical investigation is stated in Ref. 1. In this note, I shall show that the present theory predicts a mean free path fit the experimental results[2] entirely. The points of view of the present theory are (i) as soon as the wave vector is considered to be complex in mean free path considerations, it varies over a "width" for its imaginary part, even prossibly its real part too. (ii) I believe that the Bernstein polynomial approximation[3] for a function is surely better than usual Taylor series approximation. (iii) The Green's function method is applicable in the many-body theory of nuclear matter. The numerical results are taken from the review article of Jeukenne, Lejeune and Mahaux[4] and mathematical results are compared with the experimental results of Nadasen *et al.*[2]

Similar to the treatment in Ref. 1, the energy of a nucleon in a nuclear medium is described in the Green's function method by the dispersion relation

$$E=k^2/2m+U(k,\ E)+iW(k,\ E). \tag{1}$$

As this equation is solved for real E, k is necessarily complex

$$k=k_R+ik_I \tag{2}$$

In case of existence of a "width" 2A of k_I due to the statistical nature of the mean free path λ, we have around the central average

$$k_I=1/2\lambda \tag{3}$$

the imaginary part of k actually varied in the region

$$k_I=\left[\frac{1}{2(\lambda+\Delta)},\ \frac{1}{2(\lambda-\Delta)}\right]\equiv k_i\left[1-\frac{\Delta}{\lambda},\ 1+\frac{\Delta}{\lambda}\right] \tag{4}$$

where we define the quantity A and central value

$$k_i=\frac{\lambda}{2(\lambda^2-\Delta^2)} \tag{5}$$

(1) J. W. Negele and K. Yazaki, Phys. Rev. Lett. 47, 71 (1981).

(2) A. Nadasen, P. Schwandt, P.P. Singh, W. W. Jacobs, A. D. Bacher, P.T. Debevec, M.D. Kaitchuck and J.T. Meek, Phy. Rev. **C23**, 1023 (1981).

(3) G. G. Lorentz, "Bernstein Polynomials". (Univ. Toronto Press, Toronto, 1953). For convenience, cf. any Theory and Approximations of Functions text.

(4) J. P. Jeukenne, A. Lejeune and C. Mahaux, Phys. Rept. 25, 83 (1876).

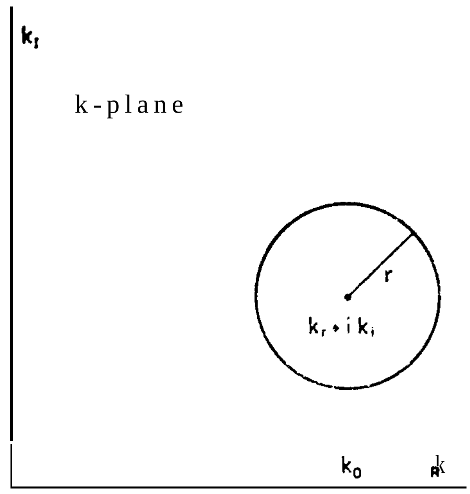

Fig. i. Schematic graph for the complex &-plane. The width 2Δ of the k_I schemes to cover k values inside a circle of radius $r=k_i\Delta/\lambda$. Actually, k_i and r are first order small quantities.

and the "radius" of variation

$$r=k_i\Delta/\lambda \tag{6}$$

I extend this variation to include the possible real part of k, k_R and draw a k-plane in Fig. 1. Hereafter I shall consider only a complex k inside the shown circle centered at k_r+ik_i with a radius r. *Now* the energy E in (1) is expanded into the Bernstein polynomial of degree 2 as

$$E(k)=E_-(1-Z)+E_+Z \tag{7}$$

where the variable Z and the coefficients E_- and E_+ are

$$Z\equiv[k-k_r-i(k_i-r)]/2r \qquad E_-\equiv E[k_r+i(k_i-r)] \qquad E_+\equiv E[2r+k_r+i(k_i-r)] \tag{8}$$

This results explicitly

$$\begin{aligned} E(k)=&\{[k_r+i(k_i-r)]^2/2m+U[k_r+i(k_i-r),E_-]+iW[k_r+i(k_i-r),E_-]\}(1-Z)\\ &+\{[2r+k_r+i(k_i-r)]^2/2m+U[2r+k_r+i(k_i-r),E_+]\\ &\quad+iW[2r+k_r+i(k_i-r),E_+]\}Z \end{aligned} \tag{9}$$

By assuming the approximations

$$\{U[2r+k_r+i(k_i-r),E_+]-U[k_r+i(k_i-r),E_-]\}/2r\approx\frac{\partial U}{\partial k}(k_r+ik_i)\equiv U'$$

$$\{W[2r+k_r+i(k_i-r),E_+]-W[k_r+i(k_i-r),E_-]\}/2r\approx\frac{\partial W}{\partial k}(k_r+ik_i)\equiv W' \tag{10}$$

We get the results that the zeroth order equation of (9) is

$$k_r^2/2m+U[k_r+i(k_i-r),E]=E \tag{11}$$

and $W(k, E)$ is a first order quantity. In comparison with the first approximation, eq. (5) in Ref. 1,

$$E\approx k_0^2/2m+U(k_0, E)$$

(11) implies up to $0(r^0)$

$$k_r=k_0=\{2m[E-U[k_r+i(k_i-r),E]]\}^{1/2} \tag{12}$$

The first order equation of (9) is

$$k_r(k_i-r)/m+W[k_r+i(k_i-r),E]-i[k-k_r-i(k_i-r)]U' \\ -ik_r[k-k_r-i(k_i-r)]/m=0 \tag{13}$$

Along the line $k=k_r+ik_I$ and averaging $k_I=k_i$ we get the averaged result

$$k_i=-W\{k_r/m+U'\Delta/\lambda\}^{-1} \tag{14}$$

For A=O, this result confirms with that of the previous phonomenological local theory. For $\Delta=\lambda$, this result is the same one as that in the nonlacal theory given in Ref. 1 for k,.

Consequently the mean free path is presently obtained from (5) as

$$\lambda=\frac{1}{2k_i[1-(\Delta/\lambda)^2]}=-\frac{k_r}{2mW}\left\{1+\frac{m\Delta}{k_r\lambda}U'\right\}\Big/\left[1-\left(\frac{\Delta}{\lambda}\right)^2\right] \tag{15}$$

With the estimations in Ref. 1, mU'/k_r is approximately $2/3$ at low energies. If we identify 1 in (15) above with the correct nonlocal expression λ in eq. (13) in Ref. 1, we get

$$\Delta/\lambda\approx0.46 \tag{16}$$

It is improper to ignore A in comparison with 1.

A different approach to the mean free path from the above argument comes from the energy width argument. The energy difference for two different values of k, or equivalently two different values of Z, Z_1 and Z_2, is

$$E_2-E_1=\{2r[r+k_r+i(k_i-r)]/m+U[2r+k_r+i(k_i-r),E_+]-U[k_r+i(k_i-r),E_-] \\ +i[W[2r+k_r+i(k_i-r),E_+]-W[k_r+i(k_i-r),E_-]]\}(Z_2-Z_1) \tag{17}$$

from (9) above. Again along $k=k_r+ik_I$, up to O(r), the maximum value of E_2-E_1 or the energy width is

$$-i\Gamma/2=2ir\{k_r/m+U'\} \tag{18}$$

The quasiparticle lifetime Γ implies the mean free path

$$\lambda=v/\Gamma$$

where the group velocity v is derived from (11) as

$$v\equiv\frac{dE}{dk}=k_r/m+U' \tag{19}$$

From (18) and (19) we get

$$\lambda=-(4r)^{-1}=-(4k_i\Delta/\lambda)^{-1} \tag{20}$$

Substituting (14) for k_i we get

$$\lambda=-\frac{k_r}{2mW}\left\{1+\frac{m\Delta}{k_r\lambda}U'\right\}\Big/2\frac{\Delta}{\lambda} \tag{21}$$

Usually the expressions for λ in *(15)* and (21) are considered to be the same. This leads the equation for Δ/λ as

$$(\Delta/\lambda)^2+2\Delta/\lambda-1=0 \tag{22}$$

Solving this equation we get

$$\Delta/\lambda=\sqrt{2}-1\approx0.41 \tag{23}$$

This value is very close to the one given in (16) which is the estimation from many-body theory. Actually Δ/λ given in (23) should be better theoretically because it is derived from the identification or equivalency of the mean free path definitions defined from k_I and lifetime Γ respectively. Further (22) is independent of all other quantities apart from Δ/λ itself. It must have some universal properties. To compare Δ/λ given in (23) further with the many-body theoretical

Table 1. The values of Δ/λ calculated from (15) using the results given in Ref. 4 for $k_F = 1.35 \mathrm{fm}^{-1}$

k/k_F	*0.5*	*0.8*	*0.9*	*1.1*	1.2	1.4	1.5	1.8	2.0
Δ/λ	0.491	0.487	0.485	0.479	0.475	0.467	0.461	0.440	0.418

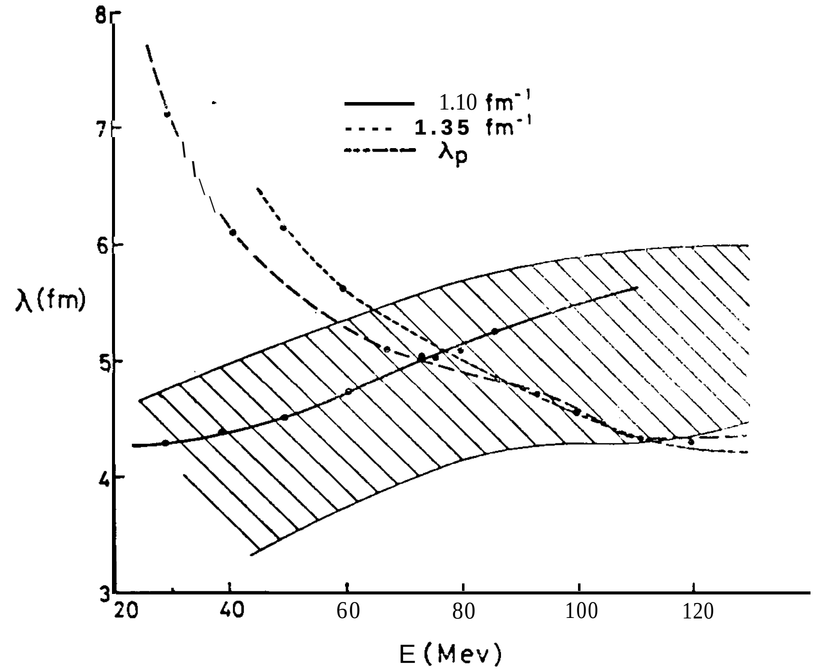

Fig. 2. The values of the mean free path λ calculated from the equivalent equations (15) and (21). Here k_F is chosen to be $1.10 \mathrm{fm}^{-1}$ (full curve) and $1.35 \mathrm{fm}^{-1}$ (long dash curve), Δ/λ to be $\sqrt{2}-1$ and W and U' from Ref. 4. The results here are compared with the previous results of Ref. 1 (λ_p-curve as short dash curve). All results here are compared with the experimental results of Ref. 2 (shaded band)

Numeral Table for Fig. 2

$k_F(\mathrm{fm}^{-1})$	E(MeV)	29.2	39.2	50.0	60.8	16.2	85.4	
1.10	λ(fm)	4.29	4.36	4.54	4.78	5.04	5.25	
1.35	E(MeV)	29.6	42.3	67.3	73.3	93.5	110.4	130.0
	λ(fm)	7.10	6.11	5.11	5.06	4.75	4.37	4.27

estimations, I recheck the results given in Ref. 4 and tabulated them in Table 1. There k_F is chosen to be 1.35 fm^{-1}. It is seen the universal value given in (23) nearly coincided with the estimation from Ref. 4 at $k=2k_F$ using the same process as the derivation of (16).

To derive λ, I use $\Delta/\lambda=\sqrt{2}-1$ which is equivalent to use both (15) and (21). The results are shown in Fig. 2. For the values of W, I accept from Fig. 23 in Ref. 4 and for those of mU'/k, from Figs. 18 and 19 in Ref. 4. The experimental detected range (shaded band) from Ref. 2 is compared. It is noted that the curve for $k_F=1.10 fm^{-1}$ is entirely laid within the experimental detected range. Even for $k_F=1.35 fm^{-1}$, the present theory results better than that given in Ref. 1 (λ_p-curve in Fig. 2). Incidently, it is clear that the values of k_F influence the results very much,