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Intermediate Product R&D, Firm Size, and Firm Growth

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Abstract

If we identify the number of firm-specific intermediate inputs as the size of a firm, then we can regard the growth of a firm as the process of accumulating new firm-specific intermediate inputs through its R&D. In this paper, we attempt to build a firm growth model with R&D in intermediate input innovation. Through R&D in intermediate input innovation, a firm can reduce its production costs indefinitely. Assuming that the firm is a monopoly in its product market facing a constant elasticity demand function, there is an optimal process of R&D in intermediate input innovation that maximizes the present value of the firm. In this paper, we investigate the optimal growth path of the firm under different values of demand elasticity, elasticity of substitution among intermediate inputs, and market interest rate in a deterministic environment. We study how a firm's growth rate is related to its size. Then we extend the model to the case with uncertainty.

Keywords: Intermediate Product Innovation, R&D, Firm size, Firm Growth

1 Introduction

How an individual firm grows has long been an interesting issue in the literature of the theory of the firm. Early in 1931, Gibrat proposed the famous Gibrat's Law of proportionate effect: Growth rates are independent of firm size. Later on, Hart and Prais (1956), Pasigian and Hymer(1962), Mansfield (1962), Evans (1987), and others attempt to test Gibrat's Law using industrial data. Baumol (1962) was the first to try to build a Model to explain the growth of a firm. Jovanovic (1982) considers learning effects and concludes that new firms grow fast than old firms. Cabral (1995) uses sunk costs to explain the negative relationship between firm sizes and growth rates. In this paper, we want to set up an infinite horizon dynamic optimization model in which a firm is engaging in R&D activities over time to maximize its present value or the expected utility of its present value. The growth of the firm is determined by its R&D process. We obtain a relationship between the growth rate of a firm and its firm size. Thus, we provide an R&D model to explain the growth of a firm.

Our basic idea is that, through R&D in intermediate input innovation, a firm can become more efficient and hence reduce its production costs. Also, depending on the market condition and the degree of substitution among inputs, there is an incentive for the firm to continue the process of R&D in intermediate input innovation indefinitely. Furthermore, if we identify the number of firm-specific intermediate inputs as the size of a firm, then we can regard the growth of a firm as the process of accumulating new firm-specific intermediate inputs through its R&D. Based upon these considerations, we setup a firm growth model with R&D in intermediate input innovation. Assuming that the firm is a monopoly in its product market facing a constant elasticity demand function, there is an optimal process of R&D in intermediate input innovation that maximizes the present value or the expected utility of the present value of the firm.[1]

[1] See Sun (1999) and Han (2000) for early versions of the model.

In this paper, we characterize the optimal R&D path of the firm under different values of demand elasticity, elasticity of substitution among intermediate inputs, and market interest rate in both deterministic and stochastic environments. We find that, for a given degree of substitution among inputs, there is a critical value of the demand elasticity such that the optimal R&D process is a constant growth path when the demand elasticity equals the critical value. In such case, the firm is growing at a constant rate, independent of the firm size. If the demand elasticity is higher than the critical value, the optimal growth rate is decreasing with the firm size and therefore a small firm will growth faster than a large firm.

Chou (2001) formulates a static model of R&D on intermediate inputs and derives the optimal (equilibrium) ratio of a firm's R&D expenditures to its total sales for different market structures. In view that R&D is a dynamic issue and that a static model is insufficient to capture the intertemporal incentives behind R&D decisions, this paper explicitly consider the dynamic optimization issues of a firm.

In the next section, we formulate the deterministic dynamic R&D model and derive the optimal growth path. In section 3 we extend the model to the case of a risk neutral monopoly facing a controlled diffusion R&D process. Section 4 consider the case when the monopolist is a risk averter. Section 5 conclude.

2 A dynamic R&D model of the growth of a firm

Consider an infinitely lived monopoly firm. At each time t, the inverse market demand function is given by

$$P_t = Aq_t^{-\theta}, \qquad A > 0, 0 < \theta < 1, \tag{1}$$

where P_t (q_t) is the price set (quantity sold) by the monopoly at time t.

We assume that at time t the firm has a Dixit-Stiglitz-Ethier type CES production function with differentiated production inputs:[2]

$$q_t = \left[\int_0^{N_t} (x(v))^\alpha dv\right]^{1/\alpha}, \quad 0 < \alpha < 1. \tag{2}$$

where N_t is the number of firm-specific intermediate inputs existing at time t, each $v \in [0, N_t]$ represents an intermediate input, and $x(v)$ is the quantity of v-input employed in the production. To simplify the problem, we assume that each unit of v-input costs one dollar to the monopolist.

2.1 Cost minimization, cost function, and profit at time t

By symmetry, $x(v)$ is a constant for all $v \in [0, N_t]$, $x(v) = x_t$. Therefore,

$$q_t = N_t^{\frac{1}{\alpha}} x_t \tag{3}$$

and the production cost function of the firm is

$$PC(q_t) = N_t^{1-\frac{1}{\alpha}} q_t. \tag{4}$$

The static profit maximization problem and the solution at time t are

$$\max_{q_t} A g_t^{1-\theta} - N_t^{1-\frac{1}{\alpha}} q_t \quad \Rightarrow \quad q_t = \left[\frac{(1-\theta)A}{N_t^{1-\frac{1}{\alpha}}}\right]^{\frac{1}{\theta}},$$

The total revenue and profits are given by

$$TR = (1-\theta)^{\frac{1}{\theta}-1} A^{\frac{1}{\theta}} N_t^{(\frac{1}{\theta}-1)(\frac{1}{\alpha}-1)}, \quad \pi_t = \theta(1-\theta)^{\frac{1}{\theta}-1} A^{\frac{1}{\theta}} N_t^{(\frac{1}{\theta}-1)(\frac{1}{\alpha}-1)}. \tag{5}$$

[2]The production function can be generalized to

$$q = f\left(\left[\int_0^{N_t} (x(v))^\alpha dv\right]^{\frac{1}{\alpha}}, K_t\right)$$

where $f(\cdot, K_t)$ is a homothetic function and K_t represents other production factors.

2.2 Innovation and intertemporal profit maximization

Assume that the monopoly does intermediate input R&D to increase the number of inputs N_t. The cost of R&D at time t depends on the intensity of its innovation activities represented by the time derivative $\dot{N}_t$:

$$RC(\dot{N}_t) = F\dot{N}_t^2, \qquad F > 0. \tag{6}$$

Assuming a constant discount rate r, the present value of the monopoly profit stream is[3]

$$\Pi = \int_0^\infty e^{-rt}\left[TR(q_t) - PC(q_t) - RC(\dot{N}_t)\right] dt = \int_0^\infty e^{-rt}\left[\pi_t - F\dot{N}_t^2\right] dt.$$
$$= F\int_0^\infty e^{rt}\left[kN^{2+\epsilon} - \dot{N}_t^2\right] dt, \qquad k \equiv \frac{\theta}{F(1-\theta)}[A(1-\theta)]^{1/\theta}, \quad \epsilon \equiv \frac{1-\alpha-\theta}{\alpha\theta} - 1. \tag{7}$$

Assumption 3: $\epsilon \leq 0$ and $\frac{r^2}{4} > \frac{kN_0^\epsilon(2+\epsilon)(1+\epsilon)}{2}$

If assumption 3 is not satisfied, the integral divergent and the maximum value is ∞.[4] For the dynamic problem to have a closed form solution, we make the following approximation

$$N^{2+\epsilon} \approx \frac{N_0^\epsilon(2+\epsilon)(1+\epsilon)}{2}N^2 - (2+\epsilon)\epsilon N_0^{1+\epsilon}N + \frac{\epsilon(1+\epsilon)}{2}N_0^{2+\epsilon}. \tag{8}$$

That is, we approximate the static profit function around $N = N_0$ by quadratic Taylor's expansion. Ignoring the constant term and the factor F, the problem becomes

$$\max \int_0^\infty e^{-rt}\left\{k\left[\frac{N_0^\epsilon(2+\epsilon)(1+\epsilon)}{2}N^2 - (2+\epsilon)\epsilon N_0^{1+\epsilon}N\right] - \dot{N}_t^2\right\} dt. \tag{9}$$

Given the initial size of the firm N_0, the optimal R&D process can be derived using calculus of variation method. The Euler equation is

$$\ddot{N} - r\dot{N} + \frac{kN_0^\epsilon(2+\epsilon)(1+\epsilon)}{2}N = \frac{k(2+\epsilon)\epsilon N_0^{1+\epsilon}}{2}. \tag{10}$$

[3] r can be interpreted as the sum of discount rate and death rate of the market. See Dixit and Pindyck (1994) Ch 4.

[4] We can verify this by considering the case of constant growth of N_t.

The solution is given by

$$N(t) = \frac{N_0}{1+\epsilon}\left(\epsilon + e^{\lambda t}\right), \quad \text{where } \lambda \equiv \frac{r}{2} - \sqrt{\frac{r^2}{4} - \frac{kN_0^{\epsilon}(2+\epsilon)(1+\epsilon)}{2}}. \tag{11}$$

In the above, the transversality condition is used.

At $t = 0$, the R&D expenditure is

$$F\dot{N}_0^2 = \frac{F\lambda^2}{(1+\epsilon)^2} N_0^2. \tag{12}$$

The total sales (total revenue) are

$$\theta^{-1} F k N_0^{2+\epsilon}. \tag{13}$$

Therefore, the instantaneous R&D expenditure to output sales ratio at $t = 0$ is

$$\rho \equiv \frac{RC}{TR} = \frac{F\lambda^2}{\theta^{-1}Fk(1+\epsilon)^2 N_0^{\epsilon}} = \frac{F(\lambda/N_0^{\frac{\epsilon}{2}})^2}{(1+\epsilon)^2 A^{\frac{1}{\theta}}(1-\theta)^{\frac{1}{\theta}-1}}. \tag{14}$$

Since

$$\frac{\lambda}{N_0^{\frac{\epsilon}{2}}} = \frac{r}{2N_0^{\frac{\epsilon}{2}}} - \sqrt{\left(\frac{r}{2N_0^{\frac{\epsilon}{2}}}\right)^2 - \frac{k(2+\epsilon)(1+\epsilon)}{2}}$$

is a decreasing function of $\frac{r}{2N_0^{\frac{\epsilon}{2}}}$, it follows that $\partial\rho/\partial N_0 < 0$ when $\epsilon < 0$. Thus, our model shows that large firms will have a lower R&D expenditure ratio than small firms for $\epsilon < 0$.

3 A stochastic dynamic model of R&D

In this section we incorporate uncertainty into the dynamic model of the last section. Now we regard the R&D process as a controlled diffusion process and apply stochastic dynamic programming method to derive the optimal R&D behavior of a monopoly firm.

3.1 Disturbed innovation process with obsolescence

To formulate the model in terms of dynamic programming, we first distinguish between the time derivative dN/dt and the intensity of R&D activities, $u(t)$, which is now the control variable. We assume that the N_t follows a controlled diffusion process given by

$$dN = [u(t) - \delta N(t)]dt + \sigma N(t)dZ(t), \tag{15}$$

where $dZ(t)$ is the instantaneous increment of a standard random walk process (Brownian motion) $Z(t)$ and σ represents the magnitude of the disturbance. The term $-\delta N(t)$ implies that some of the existing inputs become obsolescent at every instant of time and δ is the obsolete rate. Given a time path of $u(t)$ or a control rule $u(t) = f(N(t))$, equation (15) is a stochastic differential equation (SDE). Given the initial value $N(0)$, the solution of the SDE will be a diffusion process describing the probability law of the time path of $N(t)$.

3.2 The dynamic optimization of a risk neutral monopoly

In this subsection we assume that the monopolist is risk neutral and tries to maximize the expected present value of its profit stream. The problem becomes

$$\max_{u(t), 0 \le t < \infty} E\left[\int_0^\infty e^{-rt}\left\{k\left[\frac{N_0^\epsilon(2+\epsilon)(1+\epsilon)}{2}n^2 - (2+\epsilon)\epsilon N_0^{1+\epsilon}n\right] - u(t)^2\right\}dt\right] \tag{16}$$

subject to

$$dn = [u(t) - \delta n(t)]dt + \sigma n(t)dZ(t), \quad n(0) = N_0.$$

In the above, we use n as the state variable so that we can distinguish between the initial value and state variable in the dynamic programming setup. Define the stationary value function $V(N)$ as

$$V(N) = \max E\left[\int_0^\infty e^{-rt}\left\{k\left[\frac{N_0^\epsilon(2+\epsilon)(1+\epsilon)}{2}n^2 - (2+\epsilon)\epsilon N_0^{1+\epsilon}n\right] - u^2\right\}dt\right],$$

$$dn = [u(t) - \delta n(t)]dt + \sigma n(t)dZ(t), \quad n(0) = N.$$

$V(N)$ satisfies the Hamilton-Jacobi-Bellman (HJB) equation:

$$rV(N) = \max_{u \geq 0} \frac{k(2+\epsilon)}{2} \left[N_0^\epsilon(1+\epsilon)N^2 - 2\epsilon N_0^{1+\epsilon}N\right] - u^2 + V'(N)(u - \delta N) + \frac{1}{2}\sigma^2 N^2 V''(N) \tag{17}$$

The first order condition of the above maximization with respect to u is

$$u = \frac{V'(N)}{2}.$$

Using the FOC to eliminate u from the HJB equation, we obtain the differential equation (DE) for $V(N)$ as follows:

$$rV = k\left[\frac{N_0^\epsilon(2+\epsilon)(1+\epsilon)}{2}N^2 - (2+\epsilon)\epsilon N_0^{1+\epsilon}N\right] + \frac{(V')^2}{4} - \delta N V' + \frac{1}{2}\sigma^2 N^2 V''. \tag{18}$$

Using the undetermined coefficient method, the solution of the DE is

$$V(N) = \lambda_0 + \lambda_1 N + \lambda_2 N^2, \tag{19}$$

where

$$\lambda_2 \equiv \frac{r + 2\delta - \sigma^2}{2} - \sqrt{\frac{(r + 2\delta - \sigma^2)^2}{4} - \frac{kN_0^\epsilon(2+\epsilon)(1+\epsilon)}{2}},$$

$$\lambda_1 = \frac{-k(2+\epsilon)\epsilon N_0^{1+\epsilon}}{r + \delta - \lambda_2} = \frac{-2k(2+\epsilon)\epsilon N_0^{1+\epsilon}}{r + \sigma^2 + \sqrt{(r + 2\delta - \sigma^2)^2 - 2kN_0^\epsilon(2+\epsilon)(1+\epsilon)}}, \qquad \lambda_0 = \frac{\lambda_1^2}{4r},$$

and

$$u = \lambda_2 N + \frac{\lambda_1}{2}.$$

At $t = 0$, the (gross) R&D expenditure is

$$F(u(0))^2 = F(\lambda_2 N_0 + \frac{\lambda_1}{2})^2. \tag{20}$$

The R&D to sales ratio is

$$\rho = \frac{F(\lambda_2 / N_0^{\frac{\epsilon}{2}})^2}{(1+\epsilon)^2 A^{\frac{1}{\theta}}(1-\theta)^{\frac{1}{\theta}-1}}\left[1 - \frac{\epsilon(\delta - \sigma^2)}{r + \delta - \lambda_2}\right]^2. \tag{21}$$

To compensate for the obsolescence of existing inputs, the firm will increase ρ. Also, when R&D uncertainty increases, λ_2 will increase and hence ρ will be higher.

For the special case when $\epsilon = 0$, $\lambda_0 = \lambda_1 = 0$ and the decision rule is

$$u(t) = \lambda_2 N(t) = \left[\frac{r + 2\delta - \sigma^2}{2} - \sqrt{\frac{(r + 2\delta - \sigma^2)^2}{4} - k}\right].$$

4 The case of a mean–standard deviation utility

In the case of a risk neutral monopolist, the expected utility function is represented by a linear functional of the stochastic profit stream, i.e., the mean of the present value of the stream. When the monopolist is a risk averter, its expected utility function has to be replaced by a nonlinear functional of the stochastic profit stream. In this subsection we simply assume that the monopolist maximizes a function of the mean and standard deviation of the present value of the stochastic profit steam. [5] Therefore, the problem of the firm becomes

$$\max E(\Pi) - \gamma\sqrt{V(\Pi)}, \quad \Pi \equiv \int_0^\infty e^{-rt}\left\{k\left[\frac{N_0^\epsilon(2+\epsilon)(1+\epsilon)}{2}N^2 - (2+\epsilon)\epsilon N_0^{1+\epsilon}N\right] - u(t)^2\right\}dt. \tag{22}$$

To simplify the presentation, we consider the case when $\epsilon = 0$. The objective function of the maximization is therefore

$$E[\Pi] - \gamma\sqrt{V[\Pi]} = E\left[\int_0^\infty e^{-rt}\left[kN^2 - u(t)^2\right]dt\right] - \gamma\sqrt{V\left[\int_0^\infty e^{-rt}\left[kN^2 - u(t)^2\right]dt\right]}.$$

In the case of risk neutral monopoly, the optimal strategy is a simple decision rule $u(t) = \lambda N(t)$ when $\epsilon = 0$. Therefore, we assume that the monopoly uses such a decision rule. The monopolist will choose a λ to maximize its mean-standard deviation

[5] The probability distribution of Π as a function of λ is not easy to obtain. Therefore, an expected utility maximization problem

$$\max_\lambda E[U(\Pi(\lambda))]$$

is difficult for other utility functions, say, $U(\Pi) = \Pi^\beta/\beta$.

utility. A direct computation shows that

$$E[\Pi]=\frac{(k-\lambda^2)N_0^2}{r-2\lambda-\sigma^2},\qquad V[\Pi]=\frac{(k-\lambda^2)^2\sigma^2N_0^4}{(r-2\lambda-\sigma^2)^2(r-2\lambda-3\sigma^2)}.$$

$E[\Pi]=\infty$ when $\frac{r-\sigma^2}{2}<\lambda<\sqrt{k}$ and $V[\Pi]=\infty$ when $\frac{r-3\sigma^2}{2}<\lambda<\sqrt{k}$. The optimization does not make sense if $E[\Pi]=V[\Pi]=\infty$. Therefore we make the following assumption:

Assumption 4: $0<\sqrt{k}<\frac{r-\sigma^2}{2}$ and $0<r-3\sigma^2$.

Since the optimization will exclude the case $\frac{r-3\sigma^2}{2}<\lambda<\sqrt{k}$, we do not need $\sqrt{k}<\frac{r-3\sigma^2}{2}$. However, if $r-3\sigma^2<0$, $V[\Pi]=\infty$ for all $\lambda\geq 0$ and the optimization does not make sense either.

The mean-standard deviation utility as a function of λ is

$$h(\lambda)\equiv\frac{(k-\lambda^2)N_0^2}{r-2\lambda-\sigma^2}\left[1-\frac{\gamma\sigma}{(r-2\lambda-3\sigma^2)^{1/2}}\right].\tag{23}$$

The monopolist will choose a $\lambda\in[0,\min\{\sqrt{k},\frac{r-(3+\gamma^2)\sigma^2}{2}\}]$ to maximize $h(\lambda)$.

By Weierstrauss theorem, an optimal λ^* always exists. However, the closed form of λ^* is not available. Some comparative statics are not difficult to obtain. By direct computation, we have

$$\frac{\partial^2 h}{\partial k\,\partial\lambda}>0,\qquad\frac{\partial^2 h}{\partial\gamma\,\partial\lambda}<0.$$

Therefore,

$$\frac{\partial\lambda^*}{\partial k}>0,\qquad\frac{\partial\lambda^*}{\partial\gamma}<0.$$

The signs of other comparative statics are ambiguous.

5 Conclusion

In this paper we derived the optimal R&D process for a monopoly firm. When $\epsilon=\frac{1-\alpha-\theta}{\alpha\theta}-1=0$, the R&D process is characterized by a constant growth path. When $\epsilon<0$ ($\epsilon>0$), the process exhibits decelerating growth rate (accelerating growth rate).

Thus, the paper provides a criterion to resolve the issue of whether large firms grow faster than small firms. If $\epsilon < 0$, ($\epsilon > 0$) then the growth rate of a firm is decreasing (increasing) and hence a small firm will grow faster (slower) than a large firm. Since ϵ is a function of the demand elasticity and elasticity among firm-specific inputs, our criterion is empirically testable. Therefore, our model provide a testable hypothesis for future empirical research.

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