

Fast Optimization of FIR Decision-Feedback Equalizers for Wireless Data Communications

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Abstract- Decision-feedback equalization (DFE) is an effective means for mitigating the effects of inter-symbol interference (ISI). In general, the computation of DFE filter coefficients is computationally intensive. Although several fast methods have been proposed in the literature, these methods either give approximate solutions to the DFE optimization problem or are "optimal" only under certain assumptions.

This paper describes a new fast algorithm for FIR DFE optimization that eliminates the limitations of the previously methods while obtaining the exact "optimal" solutions. Simulation results show that by using the new algorithm, substantial bit-error-rate improvement of 2-3 dB can be obtained without significantly increasing complexity.

I. INTRODUCTION

High-speed wireless data communication systems encounter delay-spread multipath fading propagation. The realization of reliable transmission hence requires measures to combat the performance degradation due to inter-symbol interference (ISI). An effective method for mitigating the effects of ISI is decision-feedback equalization[1,2]. A decision-feedback equalizer (DFE) consists of a feedforward filter (FFF) and a feedback filter (FBF). In practice, the FFF and FBF are linear finite-impulse-response (FIR) filters. The FFF and FBF coefficients are computed, based on the minimum mean-square error (MMSE) criterion, from estimates of the channel pulse response (CPR), defined as symbol-spaced samples of the convolution of the transmitter filter, channel impulse response, and receiver filter, and correlation matrix of the noise at the input of the DFE. Direct computation of the optimal MMSE DFE filter coefficients can be found in various previous studies[2-4]. In general, the computation of these coefficients requires the solution to a set of N_f complex linear equations, which in turn requires the inversion of an $N_f \times N_f$ complex matrix – a computationally intensive operation requiring $O(N_f^3)$ complex multiplications. Computationally efficient methods for DFE optimization are therefore desirable for low-power applications such as wireless packet data communications. A fast algorithm for FIR DFE filter optimization based on the Fast Fourier Transform (FFT) has previously been proposed[5]. Although computationally efficient, this method does not give the exact MMSE solutions. An alternative approach, based on the fast Cholesky decomposition[6], gives the exact MMSE solutions under certain assumptions. However, significant performance degradation can result when these assumptions do not hold, thus limiting the applicability of this approach. As will be shown by simulation, the approach in [6] incurs significant bit-error-rate (BER) degradation in wireless communication systems

that employ tight non-Nyquist receiver filtering for adjacent channel interference considerations.

This paper describes a novel algorithm for computing the exact MMSE FFF and FBF coefficients without the limitations of the approach proposed in [6]. This algorithm is a novel application of the generalized Schur recursions[7]. As will be shown later, under general conditions the BER performance of this algorithm significantly outperforms the approach in [6] while slightly increasing the computational complexity.

The system model is described in Section 2. In Section 3, we describe the new fast algorithm in detail. Computer simulation results and complexity analysis are presented in Section 4 while conclusions are given in Section 5.

II. SYSTEM MODEL

A baseband-equivalent model for a communication system using linear modulation is shown in Fig. 1. In the transmitter, the modulator maps information bits onto information symbols in the signal constellation. The transmitter filter then applies linear pulse-shaping to generate a modulated signal, whose baseband-equivalent representation is given by

$$u(t) = \sum_k x_k g(t - kT) \quad (1)$$

where the complex random variable x_k is the k -th information symbol, $g(\bullet)$ is the impulse response of the transmitter filter, and T is the symbol-period.

The modulated signal $u(t)$ is transmitted through a communication channel that is modeled as a multipath-fading channel corrupted by complex additive white Gaussian noise (AWGN). The baseband-equivalent output of the channel is given by

$$r(t) = \sum_{m=1}^M a_m \exp(j\phi_m) u(t - \tau_m) + \eta(t) \quad (2)$$

where M is the number of paths in the multipath channel, a_m , ϕ_m , and τ_m are the gain, phase, and delay of the m -th multipath component, respectively, $\eta(\bullet)$ is the complex AWGN, and j is the square-root of -1 .

At the receiver, the output of the channel is first filtered using a linear time-invariant receiver filter with impulse response $f(\bullet)$ and sampled at symbol rate to obtain the discrete-time received signal, given by

$$y_k = y(kT + t_0) \quad (3)$$

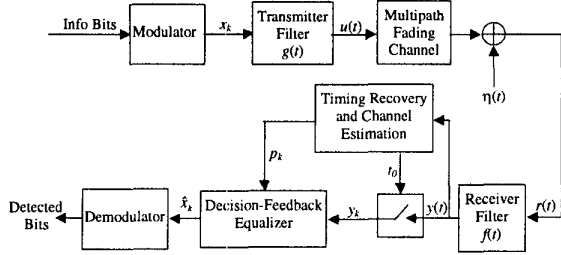


Fig. 1. The baseband-equivalent model of a communication system.

where $y(\bullet)$ is the output of the receiver filter, and t_0 is an appropriate sampling instant determined by a suitable timing recovery algorithm. The discrete-time received signal y_k is then passed through the DFE. The decisions of the DFE $\hat{x}_k$ are then passed to the demodulator to obtain the detected bits.

As shown in Fig. 2, a DFE consists of a feedforward filter (FFF), a feedback filter (FBF), an adder, and a decision device. The FFF and FBF are linear time-invariant FIR filters. The FFF reduces the precursor ISI (ISI from future symbols) in the received signal while the FBF uses past decisions to cancel the post-cursor ISI (ISI from past symbols). Mathematically, the output of the DFE at time k can be expressed as

$$z_k \equiv \sum_{j=0}^{N_f-1} w_j^* y_{k+\Delta-j} + \sum_{j=1}^{N_b} b_j^* \hat{x}_{k-j} \quad (4)$$

where N_f and N_b are respectively the number of FFF and FBF taps, and w_j and b_j are respectively the FFF and FBF coefficients, y_k is the input of the DFE at time k , and $\hat{x}_k$ is the decision at time k . In (4), the “decision delay” Δ is a non-negative integer. The FFF and FBF coefficients are computed, based on the minimum mean-square error (MMSE) criterion, from estimates of the channel pulse response (CPR) and correlation matrix of the noise at the input of the DFE. The CPR is defined as symbol-spaced samples of the convolution of the transmitter filter, channel impulse response, and receiver filter. Mathematically, the CPR is given by

$$p_k = \sum_{m=1}^M a_m \exp(j\phi_m) p(t - \tau_m) \Big|_{t=kT+t_0} \quad (5)$$

where

$$p(t) = (g \otimes f)(t). \quad (6)$$

In (6), “ $\otimes$ ” denotes linear convolution. In practice, the CPR has finite duration and must be estimated using a channel estimation algorithm. In this paper, the estimated CPR is given by p_k , $k=0,1,\dots,v$, where v is a nonnegative integer representing the memory of the channel. The FFF coefficient vector $\mathbf{w}$ and FBF coefficient vector $\mathbf{b}$ are given by [4]

$$\mathbf{R}_p \mathbf{w} = \mathbf{p} \quad (7)$$

and

$$\mathbf{b} = -\mathbf{H}^* \mathbf{w} \quad (8)$$

where “ $*$ ” denotes Hermitian transposition, $\mathbf{R}_p$ is an $N_f \times N_f$ complex positive-definite matrix given by

$$\mathbf{R}_p = \mathbf{P}\mathbf{P}^* - \mathbf{H}\mathbf{H}^* + \mathbf{R}_N, \quad (9)$$

$\mathbf{P}$ is an $N_f \times (N_f+v)$ complex matrix expressed as

$$\mathbf{P} = \begin{bmatrix} p_0 & p_1 & \cdots & p_v & 0 & \cdots & 0 \\ 0 & p_0 & p_1 & \cdots & p_v & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & p_0 & p_1 & \cdots & p_v \end{bmatrix}, \quad (10)$$

$\mathbf{H}$ is an $N_f \times N_b$ complex matrix given by

$$\mathbf{H} = \mathbf{P} \begin{bmatrix} \mathbf{0}_{(\Delta+1), N_b} \\ \mathbf{I}_{N_b} \\ \mathbf{0}_{(N_f+v-N_b-\Delta+1), N_b} \end{bmatrix}, \quad (11)$$

$\mathbf{p}$ is an $N_f \times 1$ column vector given by

$$\mathbf{p} = \mathbf{P} \mathbf{e}_\Delta, \quad (12)$$

and $\mathbf{R}_N$ is the $N_f \times N_f$ complex correlation matrix of the noise samples at the input of the DFE. The elements of $\mathbf{R}_N$ are given by

$$R_{i,j} = E[n_{k-i} n_{k-j}^*], \quad i, j = 0, \dots, (N_f - 1) \quad (13)$$

where n_k is the discrete-time complex noise at the input of the DFE. In (11), $\mathbf{0}_{a,b}$ denotes a $a \times b$ block of zeroes, and $\mathbf{I}_a$ represents an identity matrix of dimension $a \times a$. In (12), $\mathbf{e}_\Delta$ is a column vector whose Δ -th element is equal to 1 and all other elements are equal to zero. Note that solving (7) requires the inversion of the $N_f \times N_f$ complex matrix $\mathbf{R}_p$ – a computationally intensive operation that requires $O(N_f^2(N_f+v))$ complex multiplications. Furthermore, forming the matrix $\mathbf{R}_p$ as in (9) requires $O(N_f^3)$ complex multiplications. Fast and efficient methods for computing the DFE filter coefficients are therefore desirable for low-power applications such as wireless communications.

III. FAST ALGORITHMS FOR DFE FILTER OPTIMIZATION

Low-complexity algorithms for computing FIR DFE filter coefficients have previously been studied. An FFT-based algorithm [5] having a complexity of $O(N_f \log_2 N_f)$ only provides an approximation to the optimal MMSE solutions given in (7) and (8). On the other hand, an approach based on fast Cholesky decomposition [6] has also been proposed. This approach yields the optimal (MMSE) solution assuming that

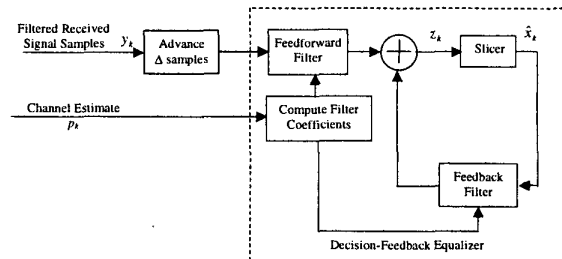


Fig. 2. A decision-feedback equalizer.

1) the discrete-time noise at the input of the DFE is white, 2) $N_b = v$, and 3) $\Delta = N_f - 1$. In this approach, an $N_f \times 2$ complex matrix is used to initialize the fast Cholesky decomposition, and sequences of planar rotations or Householder reflections are subsequently used for generating the DFE filter coefficients[6]. Unfortunately, the applicability of this approach is limited to systems in which the aforementioned assumptions approximately hold true. In particular, for wireless communication systems that are interference-limited, or that apply tight, non-Nyquist receiver filtering (due to, e.g., adjacent channel interference considerations), the white-noise assumption (assumption 1) is typically untrue. In this situation, applying this approach can lead to significant performance degradation, as will be shown later by simulation. Furthermore, the number of feedback taps is fixed (due to assumption 2), making this approach inflexible because it is often desirable to reduce the number of feedback taps to less than v in order to minimize the effects of feedback error propagation. Despite these limitations, the approach proposed in [6] is nevertheless simple and elegant. In this paper, this approach will be used as a baseline for comparison, and will be referred to simply as the "previous approach".

In order to eliminate the limitations of the previous approach, we describe a novel fast algorithm for DFE optimization based on the generalized Schur recursions[7]. In this algorithm, an $N_f \times N_f$ complex lower triangular matrix $\mathbf{L}$ with positive real diagonal elements is computed recursively from an $N_f \times 4$ initial generator matrix $\mathbf{G}^{(0)}$ using sequences of planar and hyperbolic rotations[8], such that^a

$$\mathbf{R}_p = \mathbf{L}\mathbf{L}^* \quad (14)$$

The *exact* MMSE FFF coefficients are next obtained by substituting (14) into (7) and using forward eliminations and backward substitutions[9]. Finally, the *exact* MMSE FBF coefficients are obtained from (8). Since the derivations for the algorithm are very tedious and lengthy, the algorithm is simply stated without proof.

Let $\mathbf{l}_k$, $k = 0, \dots, (N_f - 1)$ denote the columns of the lower triangular matrix $\mathbf{L}$. Furthermore, without loss of generality, it is assumed that

$$\sum_{j=0}^v |p_j|^2 + R_{00} = 1. \quad (15)$$

The proposed new algorithm for $N_f \geq v$, $0 \leq N_b \leq v$, and $\Delta = N_f - 1$ is as follows^b:

Step 1. Let $\mathbf{G}^{(0)}$ be an $N_f \times 4$ complex matrix with elements $G_{ii}^{(0)}$ and having the form shown in Fig. 3. The four columns of $\mathbf{G}^{(0)}$ are given by^c

$$G_{i0}^{(0)} \equiv \sum_{j=i}^v p_j p_{j-i}^* + R_{i0}, \quad i = 0, \dots, (N_f - 1); \quad (16)$$

^a $\mathbf{L}$ is the lower-triangular Cholesky factor for $\mathbf{R}_p$.

^b The method can be easily extended to the arbitrary case. However, for a given system, these assumptions on the DFE parameters generally yield the best performance.

^c Note that by definition, $G_{00}^{(0)} \equiv 1$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ x & 0 & 0 & x \\ x & 0 & 0 & x \\ x & 0 & x & x \\ x & 0 & x & x \\ x & 0 & x & x \\ x & x & x & x \\ x & x & x & x \\ x & x & x & x \\ x & x & x & x \end{bmatrix} \begin{array}{l} \leftarrow \text{Row } 0 \\ \\ \leftarrow \text{Row } (N_f - v) \\ \\ \leftarrow \text{Row } (N_f + N_b - v) \\ \\ \leftarrow \text{Row } (N_f - 1) \end{array}$$

Fig. 3. The initial generator matrix $\mathbf{G}^{(0)}$.

$$G_{i1}^{(0)} \equiv \begin{cases} 0 & i = 0, \dots, (N_f + N_b - v - 1) \\ p_{N_f + N_b - i}, & i = (N_f + N_b - v), \dots, (N_f - 1) \end{cases}; \quad (17)$$

$$G_{i2}^{(0)} \equiv \begin{cases} 0 & i = 0, \dots, (N_f - v - 1) \\ p_{N_f - i} & i = (N_f - v), \dots, (N_f - 1) \end{cases}; \quad (18)$$

$$G_{i3}^{(0)} \equiv \begin{cases} 0 & i = 0 \\ G_{i0}^{(0)} & i = 1, \dots, (N_f - 1) \end{cases}. \quad (19)$$

$$\mathbf{M} \leftarrow \mathbf{G}^{(0)};$$

$$\mathbf{l}_0 \leftarrow \mathbf{m}_0 \text{ (the first column of } \mathbf{M}\text{)}.$$

Step 2. For $k=1, \dots, (N_f - v - 1)$ repeat the following: Obtain $\mathbf{G}^{(k)}$ by down-shifting the first column of $\mathbf{M}$ by one element while keeping the rest unchanged^d (see Fig. 4);

$$\mathbf{M} \leftarrow \mathbf{G}^{(k)} \Omega \text{ (hyperbolic rotation), where}$$

$$\Omega \equiv \frac{1}{\sqrt{|G_{k0}^{(k)}|^2 - |G_{k3}^{(k)}|^2}} \begin{bmatrix} G_{k0}^{(k)} & 0 & 0 & -G_{k3}^{(k)} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -G_{k3}^{(k)*} & 0 & 0 & G_{k0}^{(k)} \end{bmatrix}; \quad (20)$$

$$\text{and } \mathbf{l}_k \leftarrow \mathbf{m}_0 \text{ (the first column of } \mathbf{M}\text{)}.$$

Step 3. For $k=(N_f - v), \dots, (N_f - v + N_b - 1)$, repeat the following: obtain $\mathbf{G}^{(k)}$ by down-shifting the first column of $\mathbf{M}$ by one element while keeping the rest unchanged;

$$\mathbf{M} \leftarrow \mathbf{G}^{(k)} \Omega \text{ (hyperbolic rotation), where}$$

$$\Omega \equiv \frac{1}{\sqrt{|G_{k0}^{(k)}|^2 - |G_{k3}^{(k)}|^2}} \begin{bmatrix} G_{k0}^{(k)} & 0 & 0 & -G_{k3}^{(k)} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -G_{k3}^{(k)*} & 0 & 0 & G_{k0}^{(k)} \end{bmatrix}; \quad (21)$$

$$\mathbf{M} \leftarrow \mathbf{M} \Omega \text{ (hyperbolic rotation), where}$$

$$\Omega \equiv \frac{1}{\sqrt{|M_{k0}|^2 - |M_{k2}|^2}} \begin{bmatrix} M_{k0} & 0 & -M_{k2} & 0 \\ 0 & 1 & 0 & 0 \\ -M_{k2}^* & 0 & M_{k0} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad (22)$$

^d Note that, for all k , the first k rows of $\mathbf{G}^{(k)}$ are all zeros.

and $\mathbf{l}_k \leftarrow \mathbf{m}_0$ (the first column of $\mathbf{M}$).

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \Lambda & X & X & X \\ O & X & X & X \\ X & X & X & X \\ \nabla & X & X & X \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & X & X & X \\ \Lambda & X & X & X \\ O & X & X & X \\ X & X & X & X \end{bmatrix}$$

Fig. 4. Down-shifting the first column.

Step 4. For $k=(N_f-v+N_b), \dots, (N_f-1)$, repeat the following: obtain $\mathbf{G}^{(k)}$ by down-shifting the first column of $\mathbf{M}$ by one element;

$\mathbf{M} \leftarrow \mathbf{G}^{(k)} \Omega$ (hyperbolic rotation), where

$$\Omega \equiv \frac{1}{\sqrt{|G_{k,0}^{(k)}|^2 - |G_{k,3}^{(k)}|^2}} \begin{bmatrix} G_{k,0}^{(k)} & 0 & 0 & -G_{k,3}^{(k)} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -G_{k,3}^{(k)*} & 0 & 0 & G_{k,0}^{(k)} \end{bmatrix}; \quad (23)$$

$\mathbf{M} \leftarrow \mathbf{M} \Omega$ (hyperbolic rotation), where

$$\Omega \equiv \frac{1}{\sqrt{|M_{k,0}|^2 - |M_{k,2}|^2}} \begin{bmatrix} M_{k,0} & 0 & -M_{k,2} & 0 \\ 0 & 1 & 0 & 0 \\ -M_{k,2}^* & 0 & M_{k,0} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad (24)$$

$\mathbf{M} \leftarrow \mathbf{M} \Omega$ (planar rotation), where

$$\Omega \equiv \frac{1}{\sqrt{|M_{k,0}|^2 + |M_{k,1}|^2}} \begin{bmatrix} M_{k,0} & -M_{k,1} & 0 & 0 \\ M_{k,1}^* & M_{k,0} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad (25)$$

and $\mathbf{l}_k \leftarrow \mathbf{m}_0$ (the first column of $\mathbf{M}$).

Step 5. For $i = 0, \dots, (N_f-1)$, compute

$$v_i \equiv \frac{1}{L_{ii}} \left(p_{N_f-i-1} - \sum_{j=0}^{i-1} L_{ij} v_j \right). \quad (\text{Forward elimination})$$

Step 6. For $i = (N_f-1), (N_f-2), \dots, 0$, compute

$$w_i^* \equiv \frac{1}{L_{ii}} \left(v_i^* - \sum_{j=i+1}^{N_f-1} L_{ji} w_j^* \right). \quad (\text{Backward substitution})$$

Step 7. For $i = 1, \dots, N_b$, compute $b_i^* = -\sum_{j=i}^v p_j w_{i-j+N_f-1}^*$.

The flow chart summarizing the steps of the new algorithm is shown in Fig. 5. It should be noted that the matrix $\mathbf{R}_p$ is never computed in the new algorithm.

IV. SIMULATION RESULTS AND COMPLEXITY ANALYSIS

The performance of the DFE is evaluated using computer simulation for a wireless communication system. The modulation and burst formats follow the specifications of

Enhanced Data Rates for GSM Evolution (EDGE)[10]. In particular, the modulation scheme is $3\pi/8$ -shifted 8-level phase shift keying (8-PSK) with "linearized GMSK" pulse-shaping[11], and the symbol rate is approximately 270.833 ksymbols/second. The channel is modeled as the GSM Typical Urban (TU) channel corrupted by AWGN. The channel is assumed to be stationary within each burst, and uncorrelated from burst to burst. In the receiver, a tight non-Nyquist filter is used as the receiver filter $f(\bullet)$. Symbol-timing recovery and channel estimation are next performed based on the midamble training sequence embedded in each burst. The DFE filter coefficients are computed from the CPR estimate using floating-point implementations of both the previous approach proposed in [6] and the new DFE optimization algorithm described here. It is assumed that there is no frequency offset between the transmitter and the receiver. The number of FFF coefficients is the same for both optimization algorithms. The average channel bit error rates (raw bit error rates) are shown as functions of E_b/N_0 in Fig. 6 for both approaches, where E_b is the energy per bit and $N_0/2$ is the power spectral density of the AWGN.

It can be seen from Fig. 6 that, for the case simulated, the proposed algorithm outperforms the previous approach by 2-3 dB. This is because, as mentioned earlier, the previous approach implicitly assumes that the noise at the input of the DFE is uncorrelated from sample to sample. When a tight, non-Nyquist receiver filter is used, the DFE input noise is colored (i.e., correlated from sample to sample). Using the previous approach, therefore, results in a *mismatched* DFE[12]. The proposed algorithm, on the other hand, makes no assumption on the second-order statistics of the DFE input noise samples. Therefore the algorithm yields significantly better performance than the previous approach when tight receiver filtering is used.

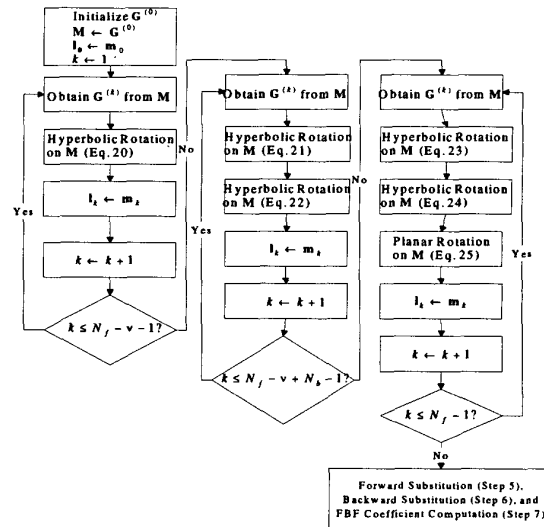


Fig. 5. Flow chart of new DFE optimization algorithm.

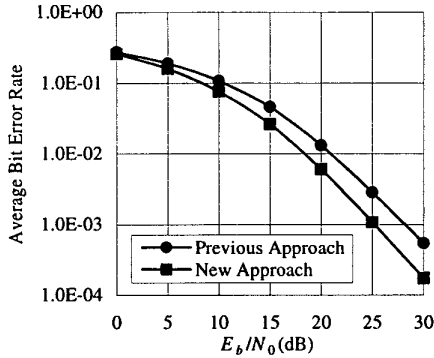


Fig. 6. Bit error rate comparison for the previous method and methods. The modulation scheme is $3\pi/8$ -shifted 8-PSK with linearized GMSK pulse shaping.

Detailed complexity analysis for implementation on a Motorola fixed-point digital signal processor (DSP) shows that the number of operations (clock cycles) for computing the DFE FFF and FBF coefficients using the “previous” approach is approximately

$$C_{\text{Previous}} \approx 30vN_f + 163N_f + 8v - 14v^2. \quad (26)$$

Note that N_b does not appear in (26) because the previous approach implicitly assumes that $N_b = v$. On the other hand, the approximate complexity for implementing a direct computation and the new algorithm are, respectively,

$$C_{\text{Direct}} \approx \frac{2}{3}N_f^3 + 30N_f^2 + 95N_f + 4v^2 \quad (27)$$

and

$$C_{\text{new}} \approx 14N_f^2 + 16vN_f + 205N_f - 14v^2 - 150v. \quad (28)$$

Note that (27) and (28) are obtained by assuming $N_b = v$. Unlike the previous approach, it is possible to set N_b to be smaller than v for the new and direct computation methods, therefore (27) and (28) actually represent complexity upper bounds. Complexity estimates in (26), (27), and (28) are plotted as functions of the number of channel taps v in Fig. 7. It is assumed that the $N_f = 2v$ because this choice of N_f typically yields good BER performance for a channel with memory v . It can be seen that for large v , the complexity of the new algorithm is slightly (approximately 10-25%) higher than the previous approach, and is significantly lower than the direct computation method.

V. CONCLUSION

A new computationally efficient algorithm for DFE filter optimization is described in this paper. This algorithm is a novel application of the generalized Schur recursions. The exact MMSE DFE coefficients are recursively computed using an initial 4-column generator matrix and sequences of hyperbolic and planar rotations. Although not shown in this paper, the new algorithm can be generalized to fractionally-spaced DFE's [1] and DFE's with receiver diversity [4].

Simulation results show that the algorithm described in pa-

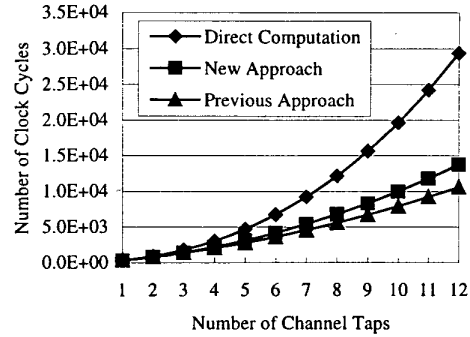


Fig. 7. The computational complexity for the direct computation approach, new algorithm, and previous approach as functions of the number of channel taps v .

per significantly outperforms a previous method [6] in terms of bit error rate when the receiver employs tight non-Nyquist receiver filtering due to, e.g., adjacent channel considerations in wireless communications. Furthermore, this algorithm is also more flexible than the previous approach. Detailed analysis for a fixed-point digital signal processor shows that the complexity for implementing the algorithm is only slightly higher (approximately 10-25%) than the approach proposed in [6], and is significantly lower than the direct computation method. The proposed new algorithm is, therefore, suitable for high-speed, low-power applications such as wireless packet data communications.

REFERENCES

- [1] S. U. H. Qureshi, “Adaptive Equalization,” *IEEE Proceedings*, Vol. 73, pp. 1349-1387, September 1985.
- [2] C. Belfiore and J. Park, Jr., “Decision Feedback Equalization,” *IEEE Proceedings*, Vol. 67, No. 8, pp. 1143-1156, August 1979.
- [3] N. Al-Dhahir and J. M. Cioffi, “MMSE Decision-Feedback Equalizers: Finite-Length Results,” *IEEE Trans. on Inf. Th.*, Vol. 41, No. 4, pp. 961-975, July 1995.
- [4] Y. Lee and D. C. Cox, “Adaptive DFE with Regularization for Indoor Wireless Data Communications,” 1997 *IEEE GlobeCom*, pp. 47-51.
- [5] I. Lee and J. M. Cioffi, “A Fast Computation Algorithm for the Decision Feedback Equalizer,” *IEEE Trans. on Comm.*, Vol. 43, No. 11, pp. 2742-2749, November 1995.
- [6] N. Al-Dhahir and J. M. Cioffi, “Fast Computation of Channel-Estimate Based Equalizers in Packet Data Transmission,” *IEEE Trans. on Signal Proc.*, Vol. 43, No. 11, pp. 2462-73, Nov., 1995.
- [7] H. Lev-Ari and T. Kailath, “Triangular Factorization of Structured Hermitian Matrices,” *Operator Theory: Advances and Applications*, Vol. 18, pp. 301-326.
- [8] G. H. Golub and C. F. Van Loan, *Matrix Computations*, 3rd Ed., The Johns Hopkins University Press, 1996, pp. 611-613.
- [9] —, pp. 88-90.
- [10] A. Furuskar *et al.*, “EDGE: Enhanced Data Rates for GSM and TDMA/136 Evolution,” *IEEE Personal Communications*, Vol. 6, No. 3, pp. 56-66, June 1999.
- [11] U. Mengali and M. Morelli, “Decomposition of M -ary CPM Signals into PAM Waveforms,” *IEEE Trans. on Inf. Th.*, Vol. 41, No. 5, pp. 1265-1275, September 1995.
- [12] N. Al-Dhahir and J. M. Cioffi, “Mismatched Finite Complexity MMSE Decision Feedback Equalizers,” *IEEE Trans. on Signal Proc.*, Vol. 45, No. 4, pp. 935-944, April 1997.