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Technical Briefs

Noncircular Turning of Workpieces With Sharp Corners

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This note deals with noncircular machining of workpieces with sharp corners. Tool positioning for directly tracking a profile with sharp corners requires large actuator inputs at the sharp corners, which may easily go beyond the actuator saturation limit and cause serious dimensional errors. To solve this problem, a method for generating the nonsmooth profiles from multiple smooth profiles, which are followed by the tool, is given. This method utilizes the following nature of turning operations: the cutting tool practically passes one point in the axial direction of a workpiece several times and the machined shape is attained when the depth of cut is maximum.

1 Introduction

The implementation of noncircular machining, which refers to turning of workpieces to noncircular cross-sections by directly actuating the cutting tool in the direction normal to the rotating workpiece, has become practical due to advances in digital computing hardware and digital control theory [1, 2, 3]. In noncircular machining, the achievable shape complexity is constrained by the acutator bandwidth and saturation limit, and the dimensional accuracy is mainly dependent on the performance of the digital tracking control algorithm which lets the cutting tool follow the prescribed profiles. When profiles with sharp corners are generated by noncircular machining, significant dimensional errors appear at those corners because of the inability of the actuator to generate nonsmooth motions.

Instead of letting the tool directly track the profiles with sharp corners, this note gives a method to synthesize nonsmooth profiles from multiple smooth profiles based on the following nature of turning operations: tool feed per revolution in the axial direction of the workpiece is usually very small. Practically, it can be assumed that the cutting tool passes one point in the axial direction of a workpiece several times and the machined shape is attained when the depth of cut is maximum or equivalently when the radial tool position is minimum. This implies that the tool may not engage in cutting in some portions of tool path; therefore, a nonsmooth profile can be generated from smooth tool paths which comprise engaging and nonengaging cutting tool paths. This idea is formalized in Section 2 and experimentally verified in Section 3.

2 Synthesizing Sharp Corners by Smooth Profiles

Consider the cylindrical coordinate system (r, θ, z) for turning operations where the z axis represents the spindle axis, θ represents the angular position of the tool with respect to the workpiece, and r represents the radial position of the cutting point (see Fig. 1). Suppose the tool passes a certain axial position z_o in m workpiece revolutions and denote the tool path of the ith workpiece revolution by $r(\theta, z_o)=f_i(\theta)$ where $0 \leq \theta < 2\pi$, $i=1, 2, \ldots, m$ and z_o is dropped from $f_i(\theta)$ for the simplicity of presentation. The final machined shape is

$$r=f(\theta)=\min_{1\leq i\leq m} f_i(\theta) \quad (1)$$

$f(\theta)$ is continuous but its derivative is not necessarily so. Sharp corner appears at θ where the derivative is discontinuous. The definition of sharp corner is given below.

Definition: A shape $r=f(\theta)$ is said to have a sharp corner at θ^* if $df(\theta)/d\theta$ is discontinuous at θ^*, i.e., $df(\theta^*_-)/d\theta \neq df(\theta^*_+)/d\theta$, where θ^*_- and θ^*_+ are the left-hand and right-hand neighbor of θ^*, respectively.

Our goal is to utilize equation (1) to synthesize a profile, $r=f(\theta)$, with sharp corners by a set of profiles, $r=f_i$, $i=1, 2, \ldots, m$, without sharp corners. However, it is not possible to synthesize every kind of sharp corners by this method. Synthesizability is defined below.

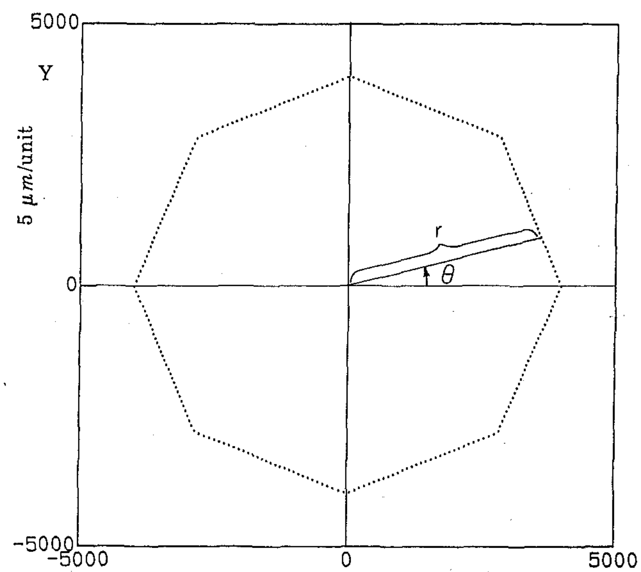

Fig. 1 Cross section of an equilateral octagon

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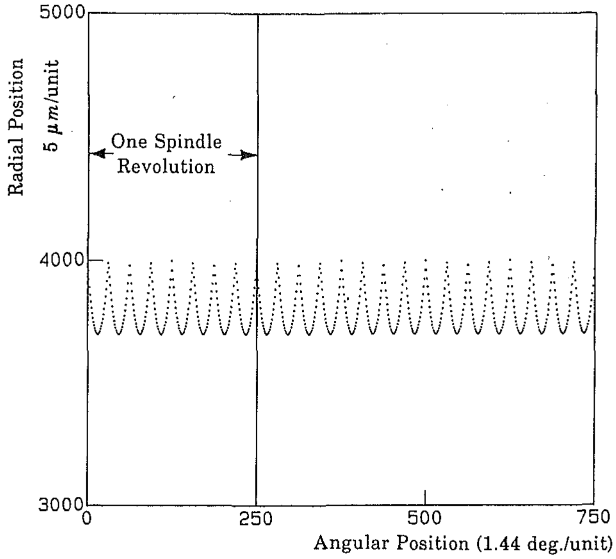

Fig. 2 Profile of an octagon in the r-θ plane

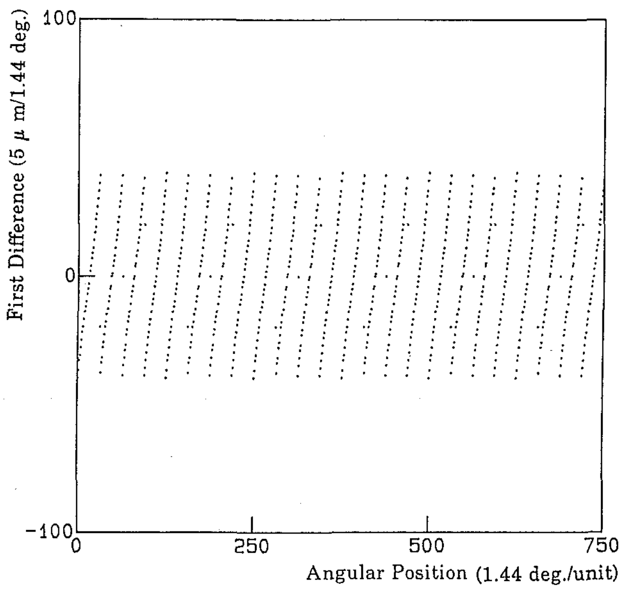

Fig. 3 First difference of the nonsmooth profile in Fig. 2

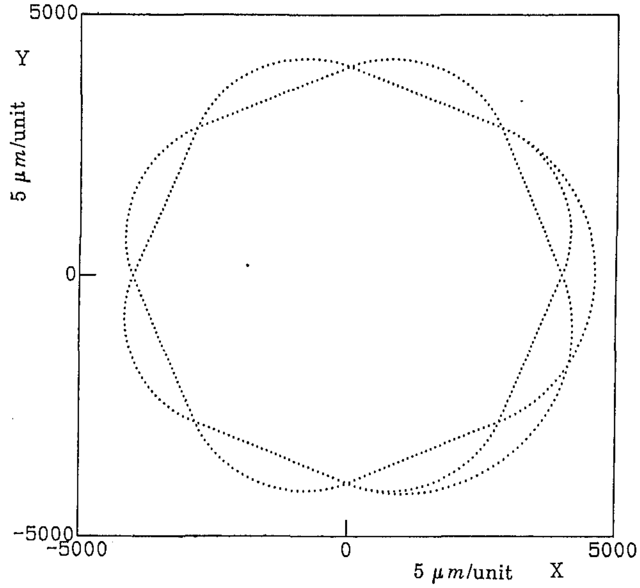

Fig. 4 Synthesizing profiles for an octagon in the x-y plane

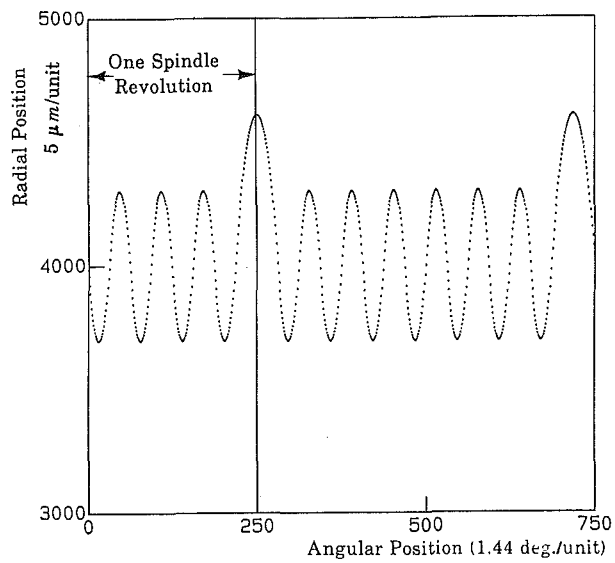

Fig. 5 Synthesizing profiles for an octagon in the r-θ plane

Definition: A shape $r=f(\theta)$ with a sharp corner at θ^* is said to be synthesizable at θ^* if it can be generated according to equation (1) by a set of shapes $f_i(\theta)$ which are continuously differentiable at θ^*. A shape with sharp corners is synthesizable if every sharp corner is synthesizable.

The necessary and sufficient condition for a synthesizable sharp corner is given in the following theorem.

Theorem 1: A shape $r=f(\theta)$ with a sharp corner at θ^* is synthesizable if and only if $df(\theta^*_-)/d\theta > df(\theta^*_+)/d\theta$ [3].

Corollary (polygonal shape): Every convex polygon is synthesizable.

In constructing a smooth tool path for machining a sharp corner, the two edges around the sharp corner should not engage in cutting consecutively. They must be cut in different workpiece rotations and connected smoothly by a noncutting tool path. A systematic method of constructing the synthesizing profiles for any N-sided polygon is given in the following theorem.

Theorem 2: Given an N-polygon or a cross-sectional shape with N sharp corners, number every sides from 0 to $N-1$. Then all N sides can be covered in m workpiece revolutions in the order $mod\,(im, N)$, $i=0, 1, \ldots$ where $m>1$ is co-prime with N and $mod(i\,m, N)$ is the modular of $i\,m$ divided by N.

The proof of the above theorem is not given since it is self-evident. As an example, consider an octagon. The eight sides of the octagon are numbered 0, 1, 2, . . . , 7. By Theorem 2, m can be 3, 5, . . . which are co-prime with 8. Take $m=3$ for instance, the eight sides are covered in three revolutions in the sequence $0\rightarrow3\rightarrow6\rightarrow1\rightarrow4\rightarrow7\rightarrow2\rightarrow5$. Since no adjacent sides are cut consecutively, a smooth tool path can be constructed by *inserting noncutting curves which smoothly connect the cutting curves.*

This theorem can be utilized in determining the cutting sequence for an N-sided polygon. For minimizing the cutting time, the smallest m is preferred. In most cases, the smallest m is either 2 or 3. For odd sided polygons, we can always choose $m=2$. Obvious exceptions are $N=6$ and 12 where 5 is the smallest number co-prime with N.

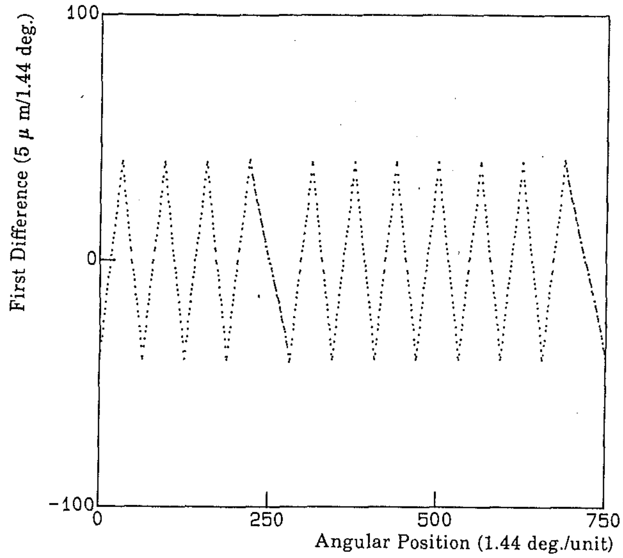

Fig. 6 First difference of the smooth profile in Fig. 5

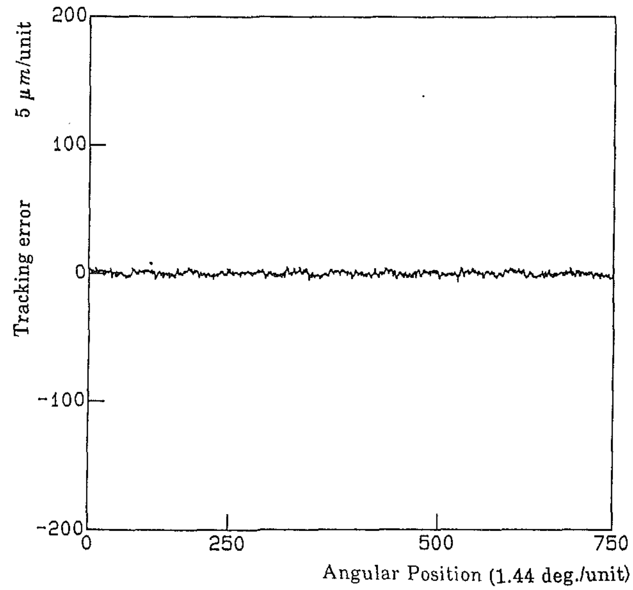

Fig. 8 Experimental result: tracking of the smooth profile

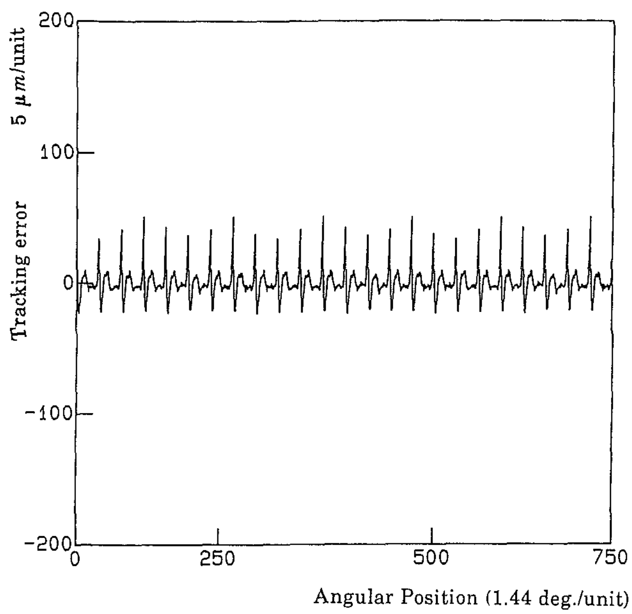

Fig. 7 Experimental result: tracking of the nonsmooth profile

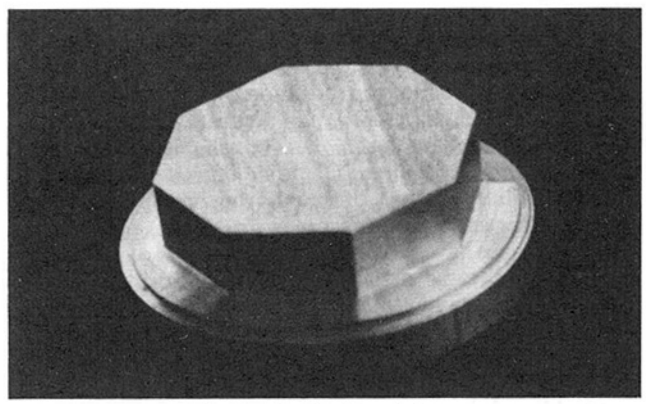

Fig. 9 Machined workpiece

3 Experimental Results

An equilateral octagon is considered in the experiment. The octagonal profile is represented by 250 points uniformly distributed over one rotation as shown in the X-Y plane in Fig. 1 and in the r-θ plane in Fig. 2. The tool velocity shown in Fig. 3, which is the first difference of the radius in Fig. 2, is discontinuous at those eight corners.

As aforementioned, the octagonal shape considered here can be synthesized by a smooth tool path in three workpiece revolutions. An example of such smooth profiles is shown in the X-Y plane in Fig. 4 and in the r-θ plane in Fig. 5. The velocity shown in Fig. 6, which is the first difference of the radius in Fig. 5, is now continuous at those sharp corners.

Experiments were conducted on a computer controlled electro-hydraulic servo system described in detail in [2, 3]. In order to let the tool accurately traverse the specified profile, a digital repetitive control algorithm which has been proven to have superior tracking performance [2, 3, 4] is applied. At 600 rpm spindle speed, which corresponds to 2.5 KHz digital control sampling rate, Figs. 7 and 8 show the experimental results of tracking the nonsmooth and smooth profiles specified by Figs. 2 and 5, respectively. The tracking error in Fig. 8 is much smaller than that in Fig. 7 and is as small as the system noise level. Figure 9 is a photograph of the machined workpiece.

4 Conclusion

In the noncircular machining operation, direct tracking of nonsmooth profiles results in significant dimensional errors due to the actuator bandwidth and saturation limitations. Using smooth profiles to synthesize nonsmooth profiles has proven useful in reducing such errors. This method greatly increases the class of shapes that can be generated by the noncircular machining operation.

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