Nonlinear and Switching Control for the HVAC System

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Abstract

In this paper, we integrate nonlinear control and supervisory control theory to design a controller of a heating, ventilating and air conditioning (HVAC) system. First we design the nonlinear controllers by Lyapunov theory and backstepping. Then we create the discrete abstraction of this system and thus derive the discrete event system model from the continuous plant. Finally we design the supervisory controller of the discrete event model and thus complete the design of a hybrid control system. With the switching logic controlled by the supervisor and the good performance of the nonlinear controller, we can achieve many specifications with this systematic design.

Key Words: HVAC system, nonlinear control, switching system.

I. Introduction

HVAC systems for buildings are major consumers of electrical energy through the world [1]. Improving the energy efficiency of HVAC system while maintaining the comfort is the primary goal in the development of controller design. With increasing complexity of modern HVAC systems, how to control and optimize the operation with guaranteed performance, stability and reliability becomes a challenging issue since the air conditioning process is highly non-linear; the interaction between the temperature and humidity control loops is significant and the constraints imposed by non-ideal actuator behavior are considerable. To control such systems efficiently and effectively in the presence of dynamic interaction and random disturbance so as to conserve energy while maintaining the desired thermal comfort level requires more than a conventional methodology. A good controller for the air-handling units (AHUs) is extremely desirable for human comfort and energy saving. Classical HVAC control techniques such as the ON/OFF controllers (thermostats) and the proportional-integral-derivative (PID) controllers are still very popular because of their low cost [2]. Some advanced control method such as adaptive control and predicative control are also studied in literatures [3], [5], [4]. In [6], the actuators' dynamics is considered and the feedback linearization approach is adopted to design the controller. However in the long run, these controllers are expensive since they operate at a very low-energy efficiency.

Hybrid systems theory [7] which combine the continuous-variable dynamical systems(CVDS) and discrete event systems(DES) and the interactions of them is a promising approach to improve the control of HVAC systems. CVDS, as the name, are usually modelled by differential equations and theoretically measured by norms. DES are systems characterized with event-driven and discrete states that use different performance measurement due to discrete dynamics, and therefore the space of interest are often not metric [8]. Because of the different nature of CVDS and DES, we can not directly use conventional design methodologies for hybrid systems. Control of hybrid systems usually involves with multi-layer controller architecture which has better performance then the single controller by the effect of switching [9]. For the multi-layer controller, the logical control in the upper layer is used to decide the switching rules. The servomechanism control in the lower layer is used to satisfy the performance specification or constraints of the CVDS. In this paper we design a satisfactory nonlinear controller in the lower layer for the heating and cooling subsystems and a supervisor in the upper layer for decision making.

From discrete abstraction, we can extract a DES model from the hybrid system with some properties preserved [10], [11]. In the DES model, the occurrence of discrete events cause the transitions between discrete states (e.g., HeatingOn-CoolingOff, HeatingOff-CoolingOff, ... etc.). The supervisory controller oversees the event sequence and control the on/off behaviors of system. When the system operation stays in one mode, the continuous state trajectory (temperature) with the corresponding differential equation will be controlled by the nonlinear controller. The process completes the control of hybrid systems. Briefly, DES and CVDS may affect each other and causing the system highly complex and difficult to be controlled. We discuss the HVAC system, both the DES and CVDS, and then design the supervisor and nonlinear controller for it.

The rest of this paper is organized as follows: In section 2, we give some preliminaries about the hybrid system model and some concepts of supervisory control. In section 3 we introduce the model of a HVAC system and apply Lyapunov method with the backsteeping technique to design the nonlinear controller for the heating and cooling subsystems. In section 4, discrete abstraction of the continuous plant is performed and the supervisor is designed. Thus we complete the design of a hybrid control system. Finally, section 5 concludes the paper.

II. PRELIMINARIES

In this section, we introduce the structure of hybrid system models and fundamental concepts of supervisory control theory.

A. Hybrid System Model

Hybrid control systems consist of three parts [11]: the "plant" contains all the continuous dynamics, including the continuous controller. The "supervisor" is a discrete event system which controls the logic. The supervisor is usually described as a finite automaton and designed by the DES theory. The "interface" plays a key role in hybrid systems, it connects the continuous and discrete dynamics, like a A/D, D/A converter, see Fig. 1. Supervisory control of hybrid systems is based on the discrete event system theory. *Discrete abstraction* is used to approximate the continuous plant into a DES model and thus the techniques of supervisory control can be applied. It is a important step in this approach. Researches of discrete abstraction are active in the field of hybrid systems.

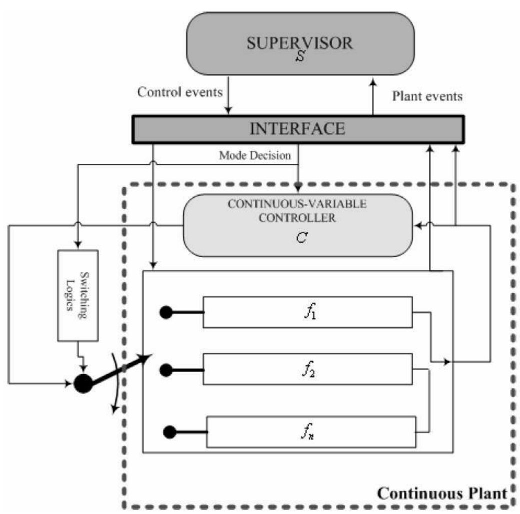

Fig. 1. A switching control system model with the supervisor.

B. Ramadge-Woham Supervisory Control Theory

The Ramadge-Woham supervisory control theory provides a mathematical framework for the design of discrete event systems. Here we follow the definitions and notations in [12] and give a brief introduction to the theory. A discrete event system can be described by an *automaton*:

$$G = \{X, \Sigma, \delta, x_0, X_m\}$$

where X is the set of discrete states, Σ is the set of events (or alphabets), $\delta : X \times \Sigma \to X$ is the state transition function , x_0 is the initial state, and X_m is the set of the *marked* states. Let Σ^* denote the set of all finite strings from the event set Σ, e.g., $u = \sigma_1\sigma_2 \in \Sigma^*$ where $\sigma_1, \sigma_2 \in \Sigma$. A *language* is the set of all sample paths that demonstrate the possible behaviors of G. That is

$$L(G) = \{t \in \Sigma^* | \delta(x_0, t)!\}$$

where $\delta(x_0,t)!$ means that there exists a state $x' \in X$ such that $\delta(x_0,t) = x'$. Let $\Gamma(x)$ be the feasible events at state x, i.e., events that could happen at state x. Now we can start to discuss the control of DES.

The main concept of supervisory control is to restrict the "illegal" behaviors that we do not want. We care about whether the states start from the initial states can be transited to the marked states by available event sequences. Generally the specification of the DES is defined by a language K, and what the supervisor need to do is to make the language $L(S/G)$ belongs to the specification, that is, $L(S/G) \subseteq K$, where S/G means that G is supervised by the supervisor S. We can regard the supervisor $S : L(G) \to 2^{\Sigma}$ as a function that maps a event sequence of $L(G)$ to the feasible event set. For any event sequence t that G generated so far, the feasible event set function of controlled DES S/G is

$$\Gamma_{S/G}\left(\delta(x_0,t)\right) = S(t) \cap \Gamma\left(\delta(x_0,t)\right)$$

The procedure of supervisory control of a discrete event system is summarized as follows:

1) According to the problem we concerned, label the possible "illegal" states in set X. The illegal states mean the states that violate our requirement, e.g., precedence constraints or dead locks.
2) Let H be the automaton obtained from removing the illegal states from G.
3) Let the system specification $K = L(H)$ and design the supervisor to make the system behave well.

III. PLANT OF THE HVAC SYSTEM

As we mentioned above, the "plant" represents all the continuous dynamics of the hybrid control system, including the nonlinear controllers which are the candidates of switchings.

A. HVAC system

The air in the room is assumed to have a uniform temperature distribution and the heat loss between components is neglected. The system operates as shown in Fig. 2. The outdoor air enters the system at temperature $T_0(t)$ and volumetric flow rate $f_0(t)$. Air with temperature T_0 and flow rate $f(t)$ passes through the heat exchanger where an amount of heat is exchanged with the air. Since we have the assumption of perfect mixing, the air temperature within and exiting the heat exchanger is $T_2(t)$, which represents the supply air temperature. After being cooling or heating in the heat exchanger, the air at temperature T_2 passes into the thermal space with the help of fan and the air temperature in the thermal space is $T_3(t)$. The heat load in the room is included as Q_O. Air leaving the thermal space is drawn through the fan and some portion of it excluded from the system whereas the reminder is recirculated to mix with the fresh air from outdoor.

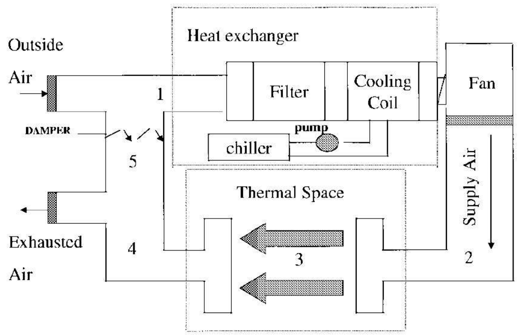

Fig. 2. Model of the HVAC system [4]

The schematic layout of a HVAC system is illustrated in Fig. 3, where a group of components working together to *move* heat to somewhere (heating), and to *remove* heat from somewhere it is not wanted (cooling) and to *put* it there as it is un-subjectable (ventilating)[13]. The HVAC system is complex and there are some researches working on it with supervisory control, e.g., [14].

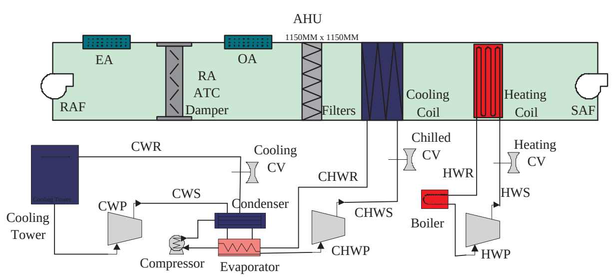

Fig. 3. HVAC schematic layout.

For the system operates with k different modes, we model it as the equation of this form:

$$\dot{x} = \Sigma_k f_k(x, u_k, \Theta_k)$$

where Θ_k indicates which mode is operating now. Θ_k is a time-varying variable which has binary value 0 or 1 and is determined by the supervisor. Here 0 and 1 means OFF and ON respectively. Consider the single-zone thermal system in Fig. 3 The following thermal dynamic equations [15] are derived from the principle of conservation of energy

$$\begin{aligned} \rho c_p V_{he}\frac{dT_2}{dt} &= f\rho c_p(T_1 - T_2) + \Theta_H \eta_H C(T_2, T_3, \Theta_H) \\ &\quad -\Theta_C \eta_C C(T_2, T_3, \Theta_C) \\ \rho c_p V_{ts}\frac{dT_3}{dt} &= f\rho c_p(T_2 - T_3) + Q_L \end{aligned} \tag{1}$$

where T_3 is the air temperature in the building, T_2 is the resulting temperature of heat exchanger and T_o is the temperature outside. $T_1 = T_3 + \frac{f_o}{f}(T_o - T_3)$ is the mixed temperature obtained from T_o and T_3, where $\frac{f_o}{f}$ means the system-to-fresh-air volumetric flow-rate ratio. Θ_H and Θ_C indicate the heating and cooling operation mode. η_H and η_C are heating and cooling constants, respectively. $C(T_2, T_3, \Theta_H)$ and $C(T_2, T_3, \Theta_C)$ are our controller used for heating and cooling. System variables and parameters are described in Table I and the parameter values within are adopted from [16].

TABLE I
HVAC SYSTEM VARIABLES AND PARAMETERS

ρ	air density	$1.19\ Kg/m^3$
c_p	constant pressure specific heat of air	$1005\ \ J/Kg^\circ C$
V_{he}	effective heat exchanger volume	$1.719\ m^3$
V_{ts}	effective thermal space volume	$1655.115\ m^3$
f_o	volumetric ventilation airflow rate	$2\ m^3/s$
f	volumetric circulate airflow rate	$8.0231\ m^3/s$
t	time	sec
T_i	temperature at location i	$^\circ C$
Q_L	thermal load	$Watt$

B. Nonlinear Controller Design For The Plant

Let the continuous states $x = [x_1, x_2]^T = [T_2, T_3]^T$ and make a substitution of T_1 to (1), then we have

$$\begin{aligned} \dot{x}_1 &= \tfrac{1}{V_{he}}\Big[(T_o - x_2) f_0 + (x_2 - x_1) f \\ &\qquad + \tfrac{\Theta_H \eta_H C(x_1, x_2, \Theta_H) - \Theta_C \eta_C C(x_1, x_2, \Theta_C)}{\rho c_p}\Big] \\ \dot{x}_2 &= \tfrac{1}{V_{ts}}\left[(x_1 - x_2) f + \tfrac{Q_L}{\rho c_p}\right] \end{aligned} \tag{2}$$

The objective of the nonlinear controller is to regulate the state $x_2 = T_3$ to a set point $x_2^* = T_3^*$. In the following procedure we will use the concept of backstepping control to complete the task. Define the error $e_2 = x_2 - x_2^*$, then

$$\dot{e}_2 = \dot{x}_2 = \frac{1}{V_{ts}}\left[(x_1 - x_2) f + \frac{Q_L}{\rho c_p}\right] \tag{3}$$

and (2) can be rewritten as

$$\begin{aligned} \dot{x}_1 &= f(x_1, e_2) + \frac{1}{V_{he}\rho c_p}(\Theta_H \eta_H C(x_1, e_2 + x_2^*, \Theta_H) \\ &\quad -\Theta_C \eta_C C(x_1, e_2 + x_2^*, \Theta_C)) \qquad (4) \\ \dot{e}_2 &= \dot{x}_2 = h(x_1, e_2) \qquad (5) \end{aligned}$$

where

$$\begin{aligned} f(x_1, e_2) &= \frac{-f}{V_{he}} x_1 + \frac{f - f_o}{V_{he}} e_2 + \frac{T_o f_o}{V_{he}} + \frac{f - f_o}{V_{he}} x_2^* \\ h(x_1, e_2) &= \frac{f}{V_{ts}} x_1 - \frac{f}{V_{ts}} e_2 - \frac{f}{V_{ts}} x_2^* + \frac{Q_L}{V_{ts}\rho c_p} \end{aligned}$$

Suppose that e_2 can be stabilized by a smooth state feedback control when $x_1 = \phi(e_2)$. If there is a Lyapunov function $V_1(e_2)$ that satisfies the inequality

$$\dot{V}_1 = \frac{\partial V_1}{\partial e_2}[h(\phi(e_2), e_2)] \le -W(e_2) \tag{6}$$

where $W(e_2)$ is always positive, then e_2 will converges to zero and this means $x_2 \to x_2^*$. Let $z = x_1 - \phi(e_2)$ as the error of x_1, then

$$\begin{aligned} \dot{z} &= \dot{x}_1 - \dot{\phi}(e_2) \\ \dot{e}_2 &= h(z + \phi(e_2), e_2) \end{aligned} \tag{7}$$

From above equations we will stabilize e_2 by control z. Now define a Lyapunov function $V_2(x_1,e_2) = V_1(e_2) + \frac{1}{2}z^2$ and let $\dot{z} = v$, then

$$\begin{aligned}\dot{V}_2 &= \frac{\partial V_1(e_2)}{\partial e_2}\dot{e}_2 + z\dot{z} \\ &= \frac{\partial V_1(e_2)}{\partial e_2}[h(z+\phi(e_2),e_2)] + zv\end{aligned}$$

Choosing $v = -\frac{\partial V_1(e_2)}{\partial e_2}(\frac{\partial h}{\partial e_2}) - kz$ with a positive constant k, then

$$\dot{V}_2(x_1,e_2) \le W(e_2) - kz^2 < 0$$

By Lyapunov analysis we know that x_1 and e_2 are asymptotically stable. Since $v = \dot{x}_1 - \dot{\phi}(e_2)$ and $z = x_1 - \phi(e_2)$, from (4) and apply $v = \frac{\partial V_1(e_2)}{\partial e_2}(\frac{\partial h}{\partial e_2}) - kz$ we have

$$\begin{aligned}\dot{x}_1 &= v + \dot{\phi}(e_2) = \left[-\frac{\partial V_1(e_2)}{\partial e_2}(\frac{\partial h}{\partial x_1}) - k(x_1 - \phi(e_2))\right] \\ &\quad + \frac{\partial \phi}{\partial e_2}h(x_1,e_2) \\ &= f(x_1,e_2) + \frac{1}{V_{he}\rho c_p}(\Theta_H \eta_H C(x_1,e_2+x_2^*,\Theta_H) \\ &\quad -\Theta_C \eta_C C(x_1,e_2+x_2^*,\Theta_C))\end{aligned}$$

and thus yields the controller

$$\begin{aligned}C(x_1,x_2,\Theta_H) &= \frac{V_{he}\rho c_p}{\eta_H}\Big(-\frac{\partial V_1(e_2)}{\partial e_2}(\frac{\partial h}{\partial x_1}) \\ &\quad -k(x_1-\phi(e_2)) + \frac{\partial \phi}{\partial e_2}h(x_1,e_2) - f(x_1,e_2)\Big)\end{aligned}$$

when $\Theta_H = 1,\ \Theta_C = 0$, and

$$\begin{aligned}C(x_1,x_2,\Theta_C) &= \frac{V_{he}\rho c_p}{\eta_C}\Big(-\frac{\partial V_1(e_2)}{\partial e_2}(\frac{\partial h}{\partial x_1}) \\ &\quad -k(x_1-\phi(e_2)) + \frac{\partial \phi}{\partial e_2}h(x_1,e_2) - f(x_1,e_2)\Big)\end{aligned}$$

when $\Theta_H = 0,\ \Theta_C = 1$.

C. Simulations of the Nonlinear Controller

Using the system parameter values in Table I and suppose the reference set point of $T_3^* = 21.66°C$ and the outdoor temperature $T_o = 29.44°C$, then we have

$$\begin{aligned}V_{he}\rho c_p &= -4.864 \times 10^{-4} \\ f(x_1,e_2) &= -4.667x_1 + 3.503(e_2+x_2^*) + 110.145 \\ h(x_1,e_2) &= 4.847 \times 10^{-3}x_1 - 4.847 \times 10^{-3}(e_2+x_2^*) \\ &\quad -0.148\end{aligned}$$

Note that $e_2 = x_2 - x_2^* = T_3 - T_3^*$. By the analysis in previous subsection, we choose $V_1(e_2) = \frac{1}{2}(e_2+x_2^*)^2$ and

$$\phi(e_2) = \frac{-k_2(e_2+x_2^*) + 4.847 \times 10^{-3}(e_2+x_2^*) - 0.148}{4.847 \times 10^{-3}}$$

with a positive constant k_2, then $\dot{V}_1(e_2) = -k_2(e_2 + x_2^*)^2 = -k_2x_2^2 < 0$. Hence the controller $C(x,\Theta) = C(x_1,x_2,\Theta_H)\Theta_H + C(x_1,x_2,\Theta_C)\Theta_C$ is derived from previous section. Note that the value of Θ_H and Θ_C will be determined by the supervisor which will be discussed in next section.

Fig. 4 shows the cooling performance of the nonlinear controller with $\Theta_C = 1$, $T_3^* = 21.66°C$, $T_o = 29.44°C$ and the thermal load $Q_L = 84970W$. Fig. 5 shows the temperature tracking to $25°C$ and $35°C$ consecutively with $T_o = 30°C$ and the thermal load $Q_L = 8000 \times (T_o - T_3)W$. From the figures we can see that the performance is satisfactory with respect to the responding time and the transient response.

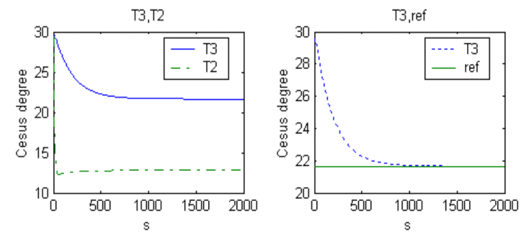

Fig. 4. Cooling performance.

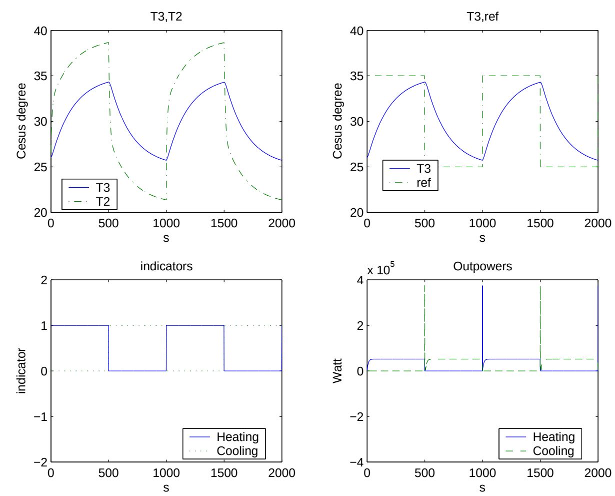

Fig. 5. Temperature tracking.

Since the number of switchings are always finite in a finite time (*nonzeno*) and each subsystem is asymptotically stable, it can be shown that the Lyapunov like function is decreasing at every instants when the mode switches in and hence by the analysis of multiple Lyapunov function [17], we know that the overall system is stable under switchings.

IV. Discrete Abstraction and Supervisory Control of the HVAC System

Once we designed a set of nonlinear controllers for the continuous plant, the problem is how to determine the switching mechanism or the control modes. This is the main objective of the supervisor. To design supervisory controller for the hybrid systems, we have to approximate our continuous plant into a discrete event model and thus the Ramadge-Woham framework can be applied.

A. Discrete Abstractoin

Discrete abstraction is the technique that used to reduce a continuous dynamic system into a discrete event system while preserving the important dynamics we care about in the original continuous system. The direct way to extract the DES model from the continuous plant is partitioning the state space by several hypersurfaces [11]. In our interest of the HVAC system, we would like to keep the temperature $x_2 \equiv T_3$ in some fixed range. So we divide the continuous state space into three parts

$$x_2 < 20,\ 20 < x_2 < 25, x_2 > 25$$

This can be done by defining the smooth functions

$$\begin{aligned} h_1(x) &= x_2 - 20 \\ h_2(x) &= 25 - x_2 \end{aligned}$$

We collect the continuous states in each state space partition as the DES plant states $\tilde{p}_1, \tilde{p}_2$ and $\tilde{p}_3$, respectively. To define the abstracted DES model, we introduce the transition relation of it. When the continuous state cross form one discrete state $\tilde{p}_i$ to another state $\tilde{p}_j$, a *plant event* $\tilde{x}_{ij}$ is generated. Thus when a plant event occurs, it means there might be a state transition (see Fig. 6). The transited DES plant state is effected by the previous DES state and the the control signal Θ, hence the whole DES plant model could be characterized as an automaton $G = (\tilde{P}, \tilde{X}, \Theta, \psi, \lambda)$, where $\tilde{P}$ is the set of DES states, $\tilde{X}$ is the set of plant events, Θ is the set of control signals which is given by the supervisor, $\psi : \tilde{P} \times \Theta \rightarrow 2^{\tilde{P}}$ is the transition relation of DES states and plant events, and $\lambda : \tilde{P} \times \tilde{P} \rightarrow 2^{\tilde{X}}$ is the transition relation of DES states and control signals. The relation can be summarized as follows

$$\tilde{p}[n-1] \times \Theta \xrightarrow{\psi} \tilde{p}[n]$$

$$\tilde{p}[n-1] \times \tilde{p}[n] \xrightarrow{\lambda} \tilde{x}[n]$$

Note the causality of the transitions between each components.

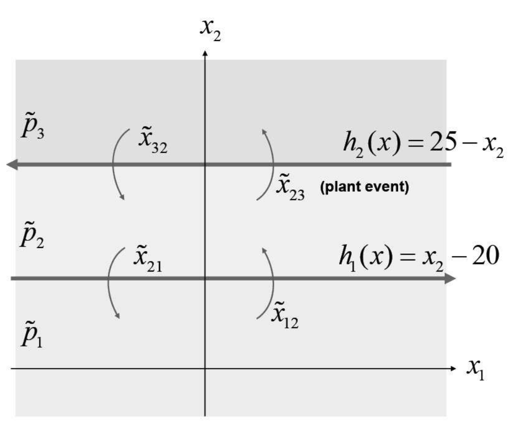

Fig. 6. Partition of the continuous plant state space.

For the setup of heating-cooling control mode, the system will identify the initial DES plant state. If the initial state is $\tilde{p}_1$, Θ_C will be zero for all the times (i.e., cold days, heating mode on). Conversely, if the initial state is $\tilde{p}_3$, Θ_H will be zero.

How to judge the occurrence of the plant events is an important part of discrete abstraction. The popular method to determine whether the continuous state cross the hypersurface h_i is by performing the gradient analysis at the boundaries of adjacent partitions on the continuous state space [11].The concept can be interpreted by Fig. 7. Suppose the DES plant states $\tilde{p}_b$ and $\tilde{p}_c$ are adjacent, if the trajectory cross the hypersurface $h_i(x) = 0$ from $\tilde{p}_b$ to $\tilde{p}_c$, then the following conditions must be satisfied: when $\tilde{p}_b$ is in the place of $h_i(x) < 0$ and $h(\xi) = 0$, then $\nabla_x h(\xi) \cdot f(\xi, C) > 0$. And if $\tilde{p}_b$ is in the place of $h_i(x) > 0$, then $\nabla_x h(\xi) \cdot f(\xi, C) < 0$. This is clear from Fig. 7. After defining the partitions and

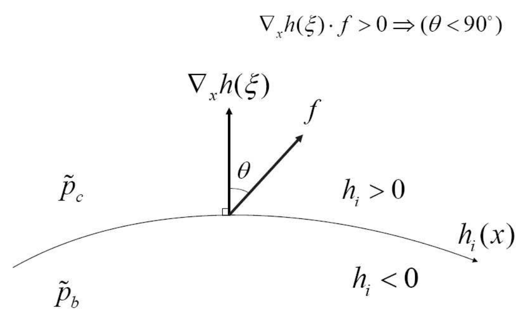

Fig. 7. The gradient analysis on the hypersurface.

transition relations, the DES model of the continuous plant is derived and the DES model automaton is shown as Fig. 8.

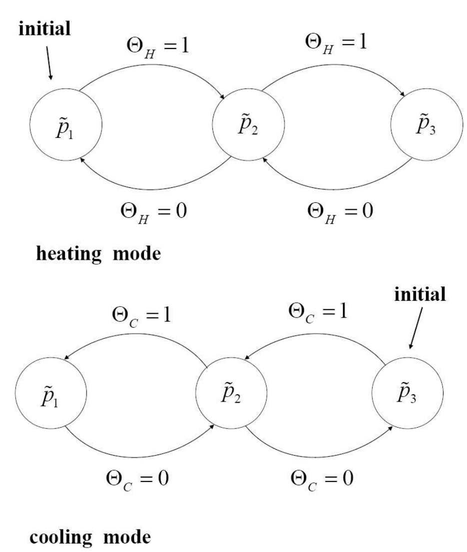

Fig. 8. DES plant model of the HVAC system.

B. Supervisory Controller

The supervisory controller S oversees the HVAC system and determines the value of Θ for the nonlinear controller $C(x, \Theta)$. We define the supervisor $S = (\tilde{S}, \tilde{X}, \Theta, \delta, \phi)$, where

$\tilde{S}$ is the supervisory controller states and the transition flow is

$$(\tilde{x}[n] \times \tilde{s}[n-1]) \xrightarrow{\delta} \tilde{s}[n] \xrightarrow{\phi} \Theta$$

Fig. 9 shows the the state transition diagram of the supervisor and to meet the specification $20 \sim 25$, we define

$$\begin{aligned}
\phi(\tilde{s}_1) &= \Theta_H = 1 \Longleftrightarrow \text{(heating mode on)} \\
\phi(\tilde{s}_2) &= \Theta_H = 0 \Longleftrightarrow \text{(heating mode off)} \\
\phi(\tilde{s}_3) &= \Theta_C = 1 \Longleftrightarrow \text{(cooling mode on)} \\
\phi(\tilde{s}_4) &= \Theta_C = 0 \Longleftrightarrow \text{(cooling mode off)}
\end{aligned}$$

With the DES model and the supervisor, the nonlinear controller designed in section 3 will be supervised and the temperature will always keep in 20°C to 25°C. We can save energies with this design because we can turn off the control in some situation which is specified by supervisor (e.g., $\tilde{s}_2$ and $\tilde{s}_4$). Of course, we can only use the nonlinear controller to regulate the desired temperature, this depends on the demand. Control of the whole system, including the continuous plant and the discrete logic, is thus completed.

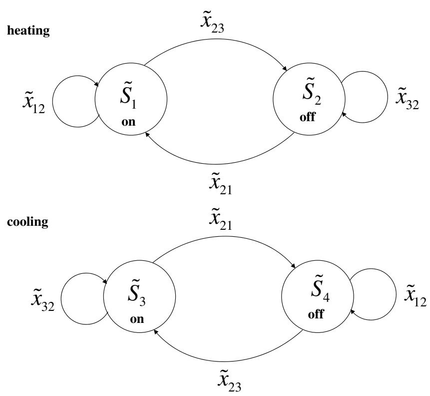

Fig. 9. Supervisor for the HVAC system.

V. CONCLUSIONS

In this paper, we discuss the HVAC system with both the continuous and discrete dynamics and complete a systematical design for this hybrid control system. First we design the nonlinear controller for the heating and cooling subsystems inside HVAC system. Then we extract a discrete event system model from the continuous plant by discrete abstraction. A supervisor for the DES plant is then designed and hence both the continuous and discrete phases of the hybrid system are controlled. A practical HVAC system contains many complex behaviors that we do not describe in this paper. There are still many problems to be considered such as the interactions between components other then the heating and cooling exchangers, and the the discrete abstraction of them . We will consider more working components of this system and include analysis of the nondeterminism of discrete abstraction in the future work.

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