行政院國家科學委員會專題研究計畫 成果報告

非線性內波與梯形斷面透水牆互制作用之研究
研究成果報告(精簡版)

計 畫 類 別 ：個別型
計 畫 編 號 ：NSC 95-2221-E-002-424-
執 行 期 間 ：95 年 08 月 01 日至 96 年 07 月 31 日
執 行 單 位 ：國立臺灣大學工程科學及海洋工程學系暨研究所

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處 理 方 式 ：本計畫可公開查詢

中 華 民 國 96 年 07 月 27 日

行政院國家科學委員會補助專題研究計畫 ■ 成 果 報 告
□ 期中進度報告

非線性內波與梯形斷面透水牆互制作用之研究

計畫類別：■ 個別型計畫　　□ 整合型計畫
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成果報告類型(依經費核定清單規定繳交)：■精簡報告　□完整報告

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中　華　民　國 九 十 六 年 八 月 一 日

報告內容

Abstract

The method for solving analytic solution will make use of Boussinesq and Korteweg de-Vries(KdV) equations to a forcing disturbance moving steadily in a two-layer flow at different boundary conditions. Comparing the result and the relation of the above two equations, we can expand the application to stratified flow. For the purpose of increasing the applied ability of this study, the moving disturbance modeled by different pressure distribution or section area as forcing function under water also be discussed. There is influence of permeability on the distribution of pore pressure in the problem of nonlinear waves in a channel. The general solution is obtained from the application of the theory of Biot. A numerical analysis is also attempted in simulating this flow problem by using numerical analysis scheme. The computed results should be compared to the analytical solution in order to increase the reliability of the result. We also want to setup an experimental frame to get experimental data in this work. After all, it is expected that the efforts of this project would be helpful in the designing of these engineering problems.

Keywords: internal wave, permeability, trapezoidal porous wall

中文摘要

內重力波與具滲透性結構物或邊界的互制作用對海洋工程的發展有極大幫助。藉由研究內重力波並正確估算海中固體邊界受波浪作用之動力變化情形，進而瞭解海域生態、週遭設施的構建基礎之穩定性，再做最安全的設計考量。而其解析模式可藉波動力學理論及固體與流體相互作用的二相理論配合數學解析模式進行分析研究。本文假設流場為具兩種不同密度之雙層流體流場，透水層為具梯形斷面之直立牆，由此針對雙層流中的內重力波求解，並推廣應用於較複雜的情況及邊界條件。本文並應用Biot理論以簡化求解水波作用下多孔彈性底床之孔隙水壓力及有效剪應力，建立一套分析此物理現象的基本機制。

關鍵詞：*內重力波，滲透性、梯形斷面透水牆*

1. Formulation

First of all, the flow field will be divided into three regions shown in Figure 1: permeable wall (P), incident region (I) and transmitted region (T). It exists two-layer flow in each region. Based on the theory derived by Biot (1962), the frame of the wall is assumed to be rigid and the fluid in the wall is incompressible. The continuity equation and momentum equations in region P are

$$\nabla\left(\hat{v}_p\right)_i = 0 (i=1,2) \tag{1}$$

$$\rho_i \frac{\partial\left(\hat{v}_p\right)_i}{\partial \hat{t}} = -\nabla\left(\hat{p}_p\right)_i + \rho_i g - \frac{\mu_i n_0 \hat{w}\hat{d}}{k_s \hat{A}_w}\left(\hat{v}_p\right)_i \qquad (i=1,2) \tag{2}$$

where μ_i indicates the dynamic viscosity, n_0 is the porous ratio, k_s represents the specific permeability, d is the thickness of the wall and $\hat{A}_w$ is the wall's submerged area. And the velocity potential can be assumed as

$$\left(\hat{v}_p\right)_i = -\frac{\rho_i k_s \hat{A}_w}{\mu_i n_0 \hat{d}} \nabla\left(\phi_p\right)_i \qquad (i=1,2) \tag{3}$$

Taking Eq.(3) into Eq.(1) and Eq.(2), it yields

$$\nabla\left(\phi_p\right)_i = 0 (i=1,2) \tag{4}$$

$$\frac{\left(p_p\right)_i}{\rho_i g} + z = \frac{w}{g}\left(\phi_p\right)_i + \frac{\rho_i k_s A_w}{\mu_i n_0 g d}\frac{\partial\left(\phi_p\right)_i}{\partial t} + h_1 \qquad (i=1,2) \tag{5}$$

where h_1 indicates the undisturbed averaged depth of upper layer. As for the region I and T, we also assume fluid in these two regions is incompressible and inviscid. The continuity equation and momentum equations are

$$\nabla^2\left(\phi_I\right)_i = \nabla^2\left(\phi_T\right)_i = 0 \qquad (i=1,2) \tag{6}$$

$$\frac{\left(p_I\right)_i}{\rho_i g} + z = -\frac{1}{g}\frac{\partial\left(\phi_I\right)_i}{\partial t} + h_1 \qquad (i=1,2) \tag{7}$$

$$\frac{\left(p_T\right)_i}{\rho_i g} + z = -\frac{1}{g}\frac{\partial\left(\phi_T\right)_i}{\partial t} + h_1 \qquad (i=1,2) \tag{8}$$

2. Boundary Conditions

To simplify the problem, we assume boundaries at $z = h_1$ and $z = -h_2$ are rigid, therefore

$$\frac{\partial\left(\phi_I\right)_1}{\partial z} = \frac{\partial\left(\phi_T\right)_1}{\partial z} = \frac{\partial\left(\phi_p\right)_1}{\partial z} = 0 \qquad \left(z = h_1\right) \tag{9}$$

$$\frac{\partial\left(\phi_I\right)_2}{\partial z} = \frac{\partial\left(\phi_T\right)_2}{\partial z} = \frac{\partial\left(\phi_p\right)_2}{\partial z} = 0 \qquad \left(z = -h_2\right) \tag{10}$$

The kinematic and dynamic free surface boundary conditions in region I are

$$\frac{\partial(\phi_I)_i}{\partial z}=\frac{\partial(\eta_I)_i}{\partial t} \qquad (i=1,2) \tag{11}$$

$$(\eta_I)_i=-\frac{1}{g}\frac{\partial(\phi_I)_i}{\partial t} \qquad (i=1,2) \tag{12}$$

Eq.(11) and Eq.(12) can be combined as

$$\frac{\partial^2(\phi_I)_i}{\partial t^2}+g\frac{\partial(\phi_I)_i}{\partial z}=0 \qquad (i=1,2) \tag{13}$$

Similarly, in region T, the boundary conditions are

$$(\eta_T)_i=-\frac{1}{g}\frac{\partial(\phi_T)_i}{\partial t} \qquad (i=1,2) \tag{14}$$

$$\frac{\partial^2(\phi_T)_i}{\partial t^2}+g\frac{\partial(\phi_T)_i}{\partial z}=0 \qquad (i=1,2) \tag{15}$$

In the permeable wall, it yields

$$-\frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial(\phi_p)_i}{\partial z}=\frac{\partial(\eta_p)_i}{\partial t} \qquad (i=1,2) \tag{16}$$

$$(\eta_p)_i=\frac{w}{g}(\phi_p)_i+-\frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial(\phi_p)_i}{\partial t} \qquad (i=1,2) \tag{17}$$

Therefore

$$\frac{\partial^2(\phi_p)_i}{\partial t^2}+\frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial(\phi_p)_i}{\partial t}+g\frac{\partial(\phi_p)_i}{\partial z}=0 \qquad (i=1,2) \tag{18}$$

At each interface of each two regions it must satisfy the following conditions

$$-\frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial(\phi_p)_i}{\partial x}=\frac{\partial(\phi_I)_i}{\partial x} \qquad (i=1,2)\left(x=-\frac{d}{2}\right) \tag{19}$$

$$-\frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial(\phi_p)_i}{\partial x}=\frac{\partial(\phi_T)_i}{\partial x} \qquad (i=1,2)\left(x=\frac{d}{2}\right) \tag{20}$$

$$(\phi_p)_i+\frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial(\phi_p)_i}{\partial t}=-\frac{1}{g}\frac{\partial(\phi_I)_i}{\partial t} \qquad (i=1,2)\left(x=-\frac{d}{2}\right) \tag{21}$$

$$(\phi_p)_i+\frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial(\phi_p)_i}{\partial t}=-\frac{1}{g}\frac{\partial(\phi_T)_i}{\partial t} \qquad (i=1,2)\left(x=\frac{d}{2}\right) \tag{22}$$

The Sommerfeld radiation boundary condition also must be satisfied

$$(\phi_I)_i(x\to-\infty)\Rightarrow outgoing \qquad (i=1,2) \tag{23}$$

$$(\phi_T)_i(x\to\infty)\Rightarrow outgoing \qquad (i=1,2) \tag{24}$$

3. Boundary Value Problem

We assume the velocity potential is periodic, $\phi=\exp(iwt)$, and define the porous Reynold number as $R=\frac{\rho k k_s A_w}{\mu_i n_0 d}$. Taking both the velocity potential and the Reynold number into the above equations, it yields

In region I

$$\nabla^2(\phi_I)_1 = \nabla^2(\phi_I)_2 = 0 \qquad \left(-\infty < x < -\tfrac{d}{2}, -h_2 < z < h_1\right) \tag{25}$$

$$\frac{\partial(\phi_I)_1}{\partial z} = 0 \qquad \left(-\infty < x < -\frac{d}{2}, z = h_1\right) \tag{26}$$

$$\frac{\partial(\phi_I)_2}{\partial z} = 0 \qquad \left(-\infty < x < -\frac{d}{2}, z = -h_2\right) \tag{27}$$

$$(\phi_I)_i - \frac{g}{w^2}\frac{\partial(\phi_p)_i}{\partial z} = 0 \qquad \left(-\infty < x < -\frac{d}{2}, z = 0\right) \tag{28}$$

$$\frac{\partial(\phi_I)_i}{\partial x} = -n_0 R \frac{\partial(\phi_p)_i}{\partial x} \qquad \left(x = -\frac{d}{2}, -h_2 < z < h_1\right) \tag{29}$$

$$(\phi_I)_i(x \to -\infty) \Rightarrow outgoing \qquad (i = 1,2) \tag{30}$$

In region T

$$\nabla^2(\phi_T)_1 = \nabla^2(\phi_T)_2 = 0 \qquad \left(\frac{d}{2} < x < \infty, -h_2 < z < h_1\right) \tag{31}$$

$$\frac{\partial(\phi_T)_1}{\partial z} = 0 \qquad \left(\frac{d}{2} < x < \infty, z = h_1\right) \tag{32}$$

$$\frac{\partial(\phi_T)_2}{\partial z} = 0 \qquad \left(\frac{d}{2} < x < \infty, z = -h_2\right) \tag{33}$$

$$(\phi_T)_i - \frac{g}{w^2}\frac{\partial(\phi_T)_i}{\partial z} = 0 \qquad \left(\frac{d}{2} < x < \infty, z = 0\right) \tag{34}$$

$$\frac{\partial(\phi_T)_i}{\partial x} = -n_0 R \frac{\partial(\phi_p)_i}{\partial x} \qquad \left(x = \frac{d}{2}, -h_2 < z < h_1\right) \tag{35}$$

$$(\phi_T)_i(x \to -\infty) \Rightarrow outgoing \qquad (i = 1,2) \tag{36}$$

In region P

$$\nabla^2(\phi_p)_1 = \nabla^2(\phi_p)_2 = 0 \qquad \left(-\frac{d}{2} < x < \frac{d}{2}, -h_2 < z < h_1\right) \tag{37}$$

$$\frac{\partial(\phi_p)_1}{\partial z} = 0 \qquad \left(-\frac{d}{2} < x < \frac{d}{2}, z = h_1\right) \tag{38}$$

$$\frac{\partial(\phi_p)_2}{\partial z} = 0 \qquad \left(-\frac{d}{2} < x < \frac{d}{2}, z = -h_2\right) \tag{39}$$

$$(\phi_p)_i - \frac{g}{w^2}\frac{\partial(\phi_p)_i}{\partial z} = 0 \qquad \left(-\frac{d}{2} < x < \frac{d}{2}, z = 0\right) \tag{40}$$

$$(\phi_p)_i + i\left[Rw(\phi_p)_i + \frac{w}{g}(\phi_I)_i\right] = 0 \qquad \left(x = -\frac{d}{2} - h_2 \le z \le h_1\right) \tag{41}$$

$$(\phi_p)_i + i\left[Rw(\phi_p)_i + \frac{w}{g}(\phi_T)_i\right] = 0 \qquad \left(x = \frac{d}{2} - h_2 \le z \le h_1\right) \tag{42}$$

4. Analysis

Most of natural porous media should satisfy the following relation

$$n_0 R = \frac{\rho k_s A_w}{\mu_i d} \sim \left(10^{-8} \sim 1\right) \tag{43}$$

We assume $R << 1$, therefore

$$(v_I)_i \sim O[(\phi_I)_i] \tag{44}$$

$$(v_T)_i \sim O[(\phi_T)_i] \tag{45}$$

$$\left(v_p\right)_i \sim O\left[R\left(\phi_p\right)_i\right] \tag{46}$$

Now $(\phi_I)_i$, $(\phi_T)_i$ and $\left(\phi_p\right)_i$ can be expanded (Huang & Chwang, 1990)

$$(\phi_I)_i = (\phi_I)_i^{(0)} + (\phi_I)_i^{(00)} + R(\phi_I)_i^{(1)} + O\left(R^2\right) \tag{47}$$

$$(\phi_T)_i = R(\phi_T)_i^{(1)} + O\left(R^2\right) \tag{48}$$

$$\left(\phi_p\right)_i = \left(\phi_p\right)_i^{(1)} + R\left(\phi_p\right)_i^{(2)} + O\left(R^2\right) \tag{49}$$

where $(\phi_I)_i^{(0)}$ indicates the incident condition. We take Eq.(47) to Eq.(49) into all governing equations and assume the incident wave to be

$$\eta_I = a_4 \exp\left[-ik_0\left(x + \frac{d}{2}\right)\right]\exp(iwt) \qquad (z = 0) \tag{50}$$

Therefore

$$(\phi_I)_1^{(0)} = a_1^{(0)} \exp\left[-ik_0\left(x + \frac{d}{2}\right)\right]\cosh[k_o(z - h_1)] \tag{51}$$

$$(\phi_I)_2^{(0)} = a_2^{(0)} \exp\left[-ik_0\left(x + \frac{d}{2}\right)\right]\cosh[k_o(z + h_2)] \tag{52}$$

$$(\phi_I)_1^{(00)} = a_1^{(0)} \exp\left[ik_0\left(x + \frac{d}{2}\right)\right]\cosh[k_o(z - h_1)] \tag{53}$$

$$(\phi_I)_2^{(00)} = a_2^{(0)} \exp\left[ik_0\left(x + \frac{d}{2}\right)\right]\cosh[k_o(z + h_2)] \tag{54}$$

Taking Eq.(51) to Eq.(54) into Eq.(12), it yields

$$a_1^{(0)} = \frac{ia_4 g}{w\cosh(-k_0 h_1)} = \frac{iaw}{k\cosh(-k_0 h_1)}\frac{\coth(kh_2) + r_\rho \coth(kh_1)}{1 - r_\rho} \tag{55}$$

$$a_2^{(0)} = \frac{ia_4 g}{w\cosh(k_0 h_2)} = \frac{iaw}{k\cosh(k_0 h_2)}\frac{\coth(kh_2) + r_\rho \coth(kh_1)}{1 - r_\rho} \tag{56}$$

From governing equations and boundary conditions, all velocity potentials are

$$\left(\phi_p\right)_1^{(1)} = 8\pi a_4 \sum_{n=1}^{\infty} \left(\prod_{n0} 1\right) (-1)^{n-1} \frac{\sinh\left[(\alpha_n)_1 \left(x - \frac{d}{2}\right)\right]}{\sinh\left[(\alpha_n)_1 d\right]} \cos\left[(\alpha_n)_1 (z - h_1)\right] \tag{57}$$

$$\left(\phi_p\right)_2^{(1)} = 8\pi a_4 \sum_{n=1}^{\infty} \left(\prod_{n0} 2\right) (-1)^{n-1} \frac{\sinh\left[(\alpha_n)_2 \left(x - \frac{d}{2}\right)\right]}{\sinh\left[(\alpha_2)_1 d\right]} \cos\left[(\alpha_n)_2 (z + h_2)\right] \tag{58}$$

$$\begin{aligned} \left(\phi_T\right)_1^{(1)} = \left(a_T\right)_1^{(1)} \exp\left[ik_0 \left(x - \frac{d}{2}\right)\right] \cosh\left[k_0 (z - h_1)\right] \\ + \sum_{j=1}^{\infty} \left(b_T\right)_{1j}^{(1)} \exp\left[k_j \left(x - \frac{d}{2}\right)\right] \cosh\left[k_j (z - h_1)\right] \end{aligned} \tag{59}$$

$$\begin{aligned} \left(\phi_T\right)_2^{(1)} = \left(a_T\right)_2^{(1)} \exp\left[ik_0 \left(x - \frac{d}{2}\right)\right] \cosh\left[k_0 (z - h_1)\right] \\ + \sum_{j=1}^{\infty} \left(b_T\right)_{2j}^{(1)} \exp\left[k_j \left(x - \frac{d}{2}\right)\right] \cosh\left[k_j (z + h_2)\right] \end{aligned} \tag{60}$$

$$\begin{aligned} \left(\phi_I\right)_1^{(1)} = \left(a_I\right)_1^{(1)} \exp\left[-ik_0 \left(x - \frac{d}{2}\right)\right] \cosh\left[k_0 (z - h_1)\right] \\ + \sum_{j=1}^{\infty} \left(b_I\right)_{1j}^{(1)} \exp\left[-k_j \left(x - \frac{d}{2}\right)\right] \cosh\left[k_j (z - h_1)\right] \end{aligned} \tag{61}$$

$$\begin{aligned} \left(\phi_I\right)_2^{(1)} = \left(a_I\right)_2^{(1)} \exp\left[-ik_0 \left(x - \frac{d}{2}\right)\right] \cosh\left[k_0 (z + h_2)\right] \\ + \sum_{j=1}^{\infty} \left(b_I\right)_{2j}^{(1)} \exp\left[-k_j \left(x - \frac{d}{2}\right)\right] \cosh\left[k_j (z - h_1)\right] \end{aligned} \tag{62}$$

5. Results and Discussion

Inserting Eq.(57) and Eq.(59) into Eq.(3), we get the velocity inside the wall

$$\frac{\left(v_p\right)_1^{(1)}}{a_4 w} = 4\pi^2 C_{w1} R \sum_{n=1}^{\infty} (-1)^{n-1} \left(\prod_{n0} 1\right) \frac{\cosh\left[(\alpha_n)_1 \left(x + \frac{d}{2}\right)\right]}{\sinh\left[(\alpha_n)_1 d\right]} \cos\left[(\alpha_n)_1 (z - h_1)\right] \cos wt \tag{63}$$

$$\frac{\left(v_p\right)_2^{(1)}}{a_4 w} = 4\pi^2 C_{w2} R \sum_{n=1}^{\infty} (-1)^{n-1} \left(\prod_{n0} 2\right) \frac{\cosh\left[(\alpha_n)_2 \left(x + \frac{d}{2}\right)\right]}{\sinh\left[(\alpha_n)_2 d\right]} \cos\left[(\alpha_n)_2 (z + h_2)\right] \cos wt \tag{64}$$

with

$$C_{w1} = \frac{g}{w^2 h_1}, \quad C_{w2} = \frac{g}{w^2 h_2} \tag{65}$$

The pressure inside the wall is

$$\frac{\left(p_p\right)_1^{(1)}}{\rho g a_4}=\sum_{n=1}^{\infty}\frac{8\pi a(-1)^{n-1}}{2n-1}\left(\prod_{n0}1\right)\frac{\sinh\left[(\alpha_n)_1\left(x+\frac{d}{2}\right)\right]}{\sinh\left[(\alpha_n)_1 d\right]}\cos\left[(\alpha_n)_1(z-h_1)\right]\cos wt \tag{66}$$

$$\frac{\left(p_p\right)_2^{(1)}}{\rho g a_4}=\sum_{n=1}^{\infty}\frac{8\pi a(-1)^{n-1}}{2n-1}\left(\prod_{n0}2\right)\frac{\sinh\left[(\alpha_n)_2\left(x+\frac{d}{2}\right)\right]}{\sinh\left[(\alpha_n)_2 d\right]}\cos\left[(\alpha_n)_2(z-h_1)\right]\cos wt \tag{67}$$

The corresponding free surface profile is

$$\begin{aligned}\frac{(\eta_T)_1^{(1)}}{a_4}&=8\pi^3 R n_0 C_{w1}\{\frac{\sinh(k_0h_1)}{w^2h_1+g\sinh^2(k_0h_1)}\sum_{n=1}^{\infty}\left(\prod_{n0}1\right)\left(\prod_{n0}1\right)\frac{2n-1}{\sinh\left[(\alpha_n)_1 d\right]}\\&\cdot\cos\left[wt+k_0\left(x-\frac{d}{2}\right)\right]\cosh\left[k_0(z-h_1)\right]\\&+\sum_{j=1}^{\infty}(b_T)_{1j}^{(1)}\frac{\sinh(k_jh_1)}{w^2h_1+g\sinh^2(k_jh_1)}\sum_{n=1}^{\infty}\left(\prod_{n0}1\right)\left(\prod_{nj}1\right)\frac{2n-1}{\sinh\left[(\alpha_n)_1 d\right]}\\&\cdot\exp\left[k_j\left(x-\frac{d}{2}\right)\right]\cosh(k_jh_1)\sin wt\}\end{aligned} \tag{68}$$

$$\begin{aligned}\frac{(\eta_T)_2^{(1)}}{a_4}&=8\pi^3 R n_0 C_{w2}\{\frac{\sinh(k_0h_2)}{w^2h_2+g\sinh^2(k_0h_2)}\sum_{n=1}^{\infty}\left(\prod_{n0}2\right)\left(\prod_{n0}2\right)\frac{2n-1}{\sinh\left[(\alpha_n)_2 d\right]}\\&\cdot\cos\left[wt+k_0\left(x-\frac{d}{2}\right)\right]\cosh\left[k_0(z+h_2)\right]\\&+\sum_{j=1}^{\infty}(b_T)_{2j}^{(1)}\frac{\sinh(k_jh_2)}{w^2h_2+g\sinh^2(k_jh_2)}\sum_{n=1}^{\infty}\left(\prod_{n0}2\right)\left(\prod_{nj}2\right)\frac{2n-1}{\sinh\left[(\alpha_n)_2 d\right]}\\&\cdot\exp\left[k_j\left(x-\frac{d}{2}\right)\right]\cosh(k_jh_2)\sin wt\}\end{aligned} \tag{69}$$

When the porous Reynold number is small, the seapage phenomenon will occur at the interface of region P and I or T, i.e. $\eta_p-\eta_I\neq 0$ and $\eta_p-\eta_T\neq 0$. From Eq.(68) and Eq.(69), there is a local disturbance due to the singularity at the interface. Because the porous Reynold number R is the ratio of inertial force to viscous force, the inertial effect may be neglected while R is very small. Then the permeability and the thickness of the wall will mainly affect the transmitted wave. The transmitted wave will be small while the value of R is small as well. But, however, if the thickness of the porous wall is greater than the water depth, the wall can be deemed impermeable. The permeable coefficient K_T defined the ratio of amplitude of transmitted wave to that of incident wave is as

$$K_T=8\pi^3 R n_0 C_{w2}\left[\frac{\sinh(k_0h_2)}{w^2h_2+g\sinh^2(k_0h_2)}\right]\sum_{n=1}^{\infty}\left(\prod_{n0}2\right)\left(\prod_{n0}2\right)\frac{2n-1}{\sinh\left[(\alpha_n)_2 d\right]}\cosh(k_0h_2) \tag{70}$$

The relations of K_T versus R, C_{w2} and $\frac{d}{2h_2}$ are displayed in Figure 2 to Figure 5.

Though some results are obtained, this problem is still worthy of studying in many scopes. How to simulate the practical more exactly is the next goal we hope to achieve.

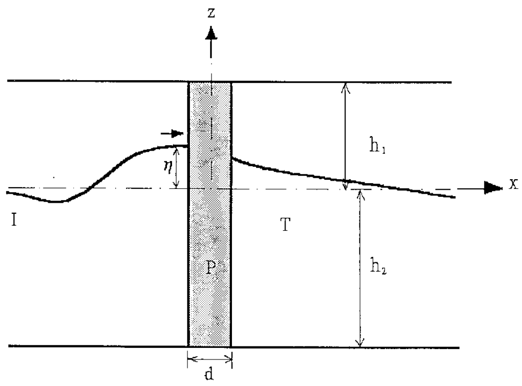

Fig.1 Analytical domain

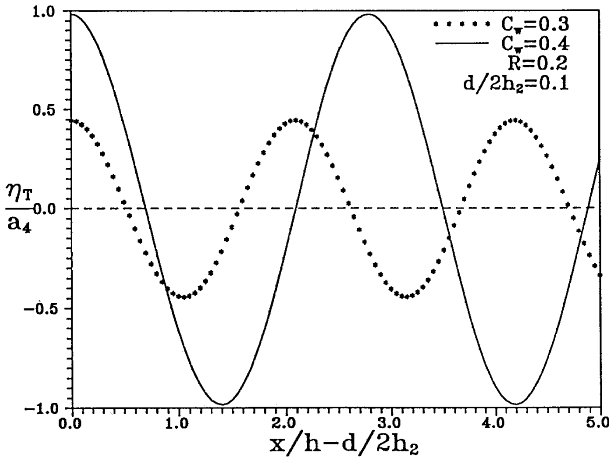

Fig.2 Amplitude of transmitted wave

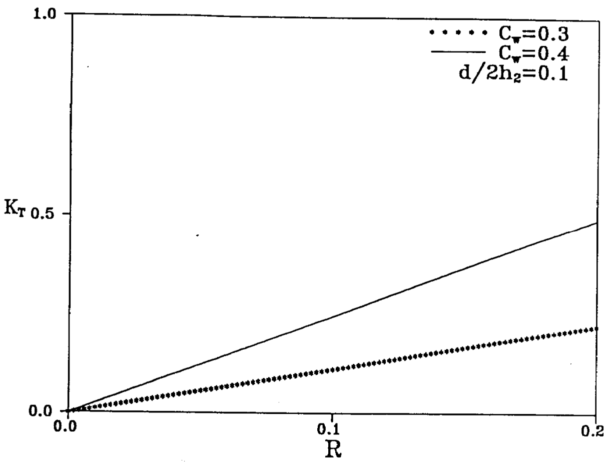

Fig.3 The relation between K_T and R

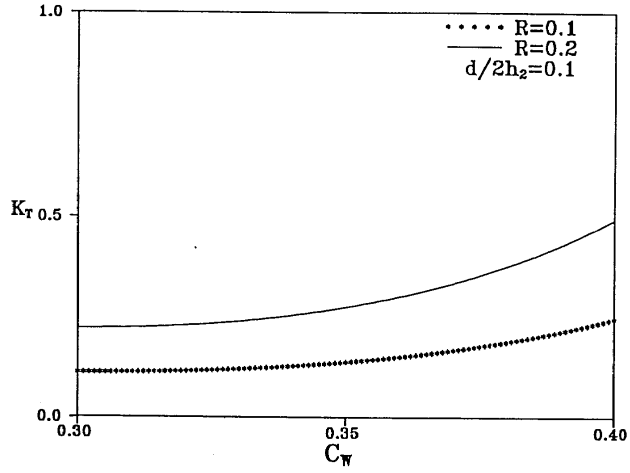

Fig.4 The relation between K_T and C_w

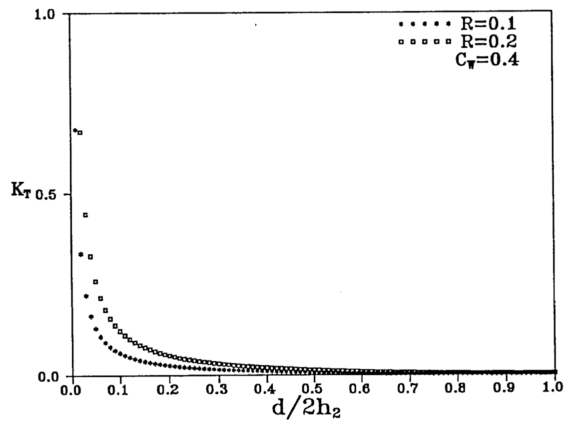

Fig.5 The relation between K_T and $d/2h_2$

6. References

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計畫成果自評：

綜合以上的結論可知，以理論分析方式求解層變流域內重力波通過透水邊界問題，再應用於不同起始及邊界條件的問題上，需先確認問題的複雜性及解析方法的可行性，尤其為簡化問題所給定的假設條件，務必符合真實的流場或海況，希望對研究領域的提昇有所助益。至於往後的研究方向可以朝更高階、非線性動力方面來深究，並佐以實驗和數值方法，使流場系統複雜且隨機的行為可以被充分瞭解，而最終的目的還是在於確實且清楚的瞭解整體的物理現象及複雜行為。

自評成果為：本計畫之研究成果高度符合計畫書中所規劃的研究內容。