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Exponentially Stable Nonlinear Field-Oriented PI-Controller for Speed Regulation of Induction Motor

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Abstract

A simple control law for speed regulation of induction motor is proposed in this paper. This controller is designed based on the concept of the field orientation and the property of linear PI-controller, and is shown to be robust with respect to variation of system parameters and load structure. In fact, the local exponential stability of the system around any desired speed can be proved by a simple iterative criterion derived in this paper. Numerical simulations are also given to show the effectiveness of the presented control law.

Nomenclature

i_{qs} (i_{ds})	:	q-(d-)*axis stator current*
V_{qs} (V_{ds})	:	q-(d-)*axis input stator voltage*
λ_{or} (λ_{dr})	:	q-(d-)*axis stator flux*
ω_r	:	*rotor speed*
T_e	:	*electromagnetic torque*
T_L	:	*mechanical load torque*
$R_s(R_r)$	:	*ststor (rotor) resistance*
$L_s(L_r)$	:	*stator (rotor) inductance*
L_m	:	*mutual inductance*
P	:	*number of poles*
J_m	:	*inertia of the rotor*

$$
\begin{aligned}
D &= (L_sL_r - L_m^2) \\
\tilde{a}_4 &= (R_r/L_r) \\
\tilde{a}_3 &= L_m\tilde{a}_4 \\
\tilde{b} &= L_m/D \\
\tilde{a}_1 &= (R_sL_r)/D + \tilde{b}\tilde{a}_3 \\
\tilde{a}_2 &= \tilde{b}\tilde{a}_4 \\
\tilde{c} &= \frac{1}{\sigma L_s} \\
K_t &= \frac{3PL_m}{4L_r} \\
a_1 &= L_rR_s/D \\
a_2 &= L_mR_s/D \\
a_3 &= L_mR_r/D \\
a_4 &= L_sR_r/D \\
a_5 &= K_t/(J_m + J_L) \\
f(x_5) &= f_L(x_5)/(J_m + J_L)
\end{aligned}
$$

1 Introduction

Pioneered by Blaschke [2], field orientation is the most widely used method for servo control of induction motor, just the same as the PID controller for linear systems. In most of the existing nonlinear controllers for induction motors [4][7], the rotor flux is made to converge asymptotically to a fixed constant, which is to prevent the saturation of the magnetic circuit. But, the flux saturation remains due to the resultant flux in the stator and the rotor [5], and hence the control schemes still can hardly achieve their origional goal satisfactorily.

The nonlinear adaptive control schemes are also applied when some imprecisely known parameters are concerned [6][7], such as the rotor resistance, changing with temperature variation, and the coefficients of the load characteristics. But since the control law will normally depend on the type of load structure, the form of the adaptive schemes will have to be changed when the load type is changed.

Given the above observation, here we try to devise a simple controller for the speed regulation problem of induction motor driving a more general class of load. The flux tracking is not considered here. With the robustness of general PI-control, the unknown parameters in the system as mensioned previously need not be estimated. Because the transformation between the actual input voltages and the d-q-axis voltages is stationary, which can be done easily by some appropriate analog circuit design, and because the computation time needed in the control process is relatively little, the proposed control scheme has a higher promise to be successfully implemented with more ease. So far to the best of the auther's knowledge, this is the first paper which analyzes the stability propertly of the field-oriented linear PI-controller, though it is already widly used in industry applications.

2 Field-Oriented PI-Controller

(A) Dynamical model of induction motor:

As being well-known, the dynamical model of an induction motor can be simplified by a d-q-axis coordinate transformation to some rotation-reference frame [5]. But for implementation feasibility, the stationary-reference frame is more popularly used. Thus, here we adopt the following d-q-axis coordinat-transformed dynamical equations of an induction motor [4]:

$$
\begin{aligned}
\dot{i}_{qs} &= -\tilde{a}_1 i_{qs} + \tilde{a}_2\lambda_{qr} - \tilde{b}\omega_r\lambda_{dr} + \tilde{c}V_{qs} \\
\dot{i}_{ds} &= -\tilde{a}_1 i_{ds} + \tilde{b}\omega_r\lambda_{qr} + \tilde{a}_2\lambda_{dr} + \tilde{c}V_{ds} \\
\dot{\lambda}_{qr} &= \tilde{a}_3 i_{qs} - \tilde{a}_4\lambda_{qr} + \omega_r\lambda_{dr} \\
\dot{\lambda}_{dr} &= \tilde{a}_3 i_{ds} - \omega_r\lambda_{qr} - \tilde{a}_4\lambda_{dr} \\
J_m\dot{\omega}_r &= T_e - T_L \\
T_e &= K_t(\lambda_{dr} i_{qs} - \lambda_{qr} i_{ds}), \qquad (1)
\end{aligned}
$$

where the states and the parameters are defined as shown in the nomenclature.

(B) Analysis of mechanical load:

In many mechanical system, the load torque is a strictly increasaing function of the speed ω if the inertial part does not exist or were to be neglected. For example, the load torque of the fan-load system can be modeled as " $b_0 + b_1\omega_r + b_2 sign(\omega_r)\omega_r^2$ " which is a strictly increasing function of ω_r. For this reason, throughout the paper we will consider the class of mechanical loads which satisfies the following assumption:

(A1) $T_L = J_L\dot{\omega}_r + f_L(\omega_r)$ with $\frac{df_L(\omega_r)}{d\omega_r} > 0$, $\forall\omega_r \in R$, and $f_L(0) = 0$, where J_L is the mechanical inertia of the payload.

(C) Field-oriented control:

The basic concept of a field-oriented control of an induction motor at steady state is to find the input voltages V_{ds}, V_{qs} such that $[\lambda_{qr}\ \ \lambda_{dr}]^T[V_{ds}\ \ V_{qr}]$ is maximum under the constraint that $V_{qs}^2 + V_{ds}^2$ is a fixed constant, where λ_{qr} (or λ_{dr}) is the q- (or d-)axis rotor flux[1][2]. Applying such a control concept to the input voltages design of an induction motor, an intuitive realization of the field-oriented control law is the following:

$$
\begin{aligned}
\tilde{c}V_{qs} &= \frac{\lambda_{dr}}{\sqrt{\lambda_{qr}^2 + \lambda_{dr}^2}}V \\
\tilde{c}V_{ds} &= \frac{-\lambda_{qr}}{\sqrt{\lambda_{qr}^2 + \lambda_{dr}^2}}V
\end{aligned}
$$

which clearly satisfies the voltage constraint " $(V_{qs}^2 + V_{ds}^2) = (V/\tilde{c})^2$ " at any time instant. Of course, at present V is no longer a constant. Instead, it offers one D.O.F. (degree of freedom) control to the system, but normally it will converge to a constant when the system approaches to the steady state.

On the other hand, we would like to show that, given a desired speed ω_d, there exists a proper constant V such that the steady state of the system exactly achieves the purpose of speed regulation, i.e., $\omega_r = \omega_d$. To this end, we further simplify the dynamics shown in (1) by introducing a nonlinear state transformation as follows:

$$
\begin{aligned}
x_1 &= h^2(i_{qs}^2 + i_{ds}^2) + \tilde{b}^2h^2(\lambda_{qr}^2 + \lambda_{dr}^2) + 2\tilde{b}h^2(i_{qs}\lambda_{qr} + i_{ds}\lambda_{dr}) \\
x_2 &= \lambda_{qr}^2 + \lambda_{dr}^2 \\
x_3 &= \tilde{b}h(\lambda_{qr}^2 + \lambda_{dr}^2) + (i_{qs}\lambda_{qr} + i_{ds}\lambda_{dr}) \\
x_4 &= h(i_{qs}\lambda_{dr} - i_{ds}\lambda_{qr}) \\
x_5 &= \omega_r,
\end{aligned}
$$

where $h = D/L_r$, under which the dynamical equations shown in (1) can be transformed to the following ones:

$$
\begin{aligned}
\dot{x}_1 &= -2a_1x_1 + 2a_2x_3 + \frac{2x_4}{\sqrt{x_2}}V \\
\dot{x}_2 &= -2a_4x_2 + 2a_3x_3 \\
\dot{x}_3 &= a_3x_1 + a_2x_2 - (a_1 + a_4)x_3 + x_5x_4 \\
\dot{x}_4 &= -x_5x_3 - (a_1 + a_4)x_4 + \sqrt{x_2}V \\
\dot{x}_5 &= a_5x_4 - f(x_5), \qquad (2)
\end{aligned}
$$

where the parameters a_1, a_2, a_3, a_4, a_5 and the function $f(x_5)$ are defined in the nomenclature. Thus, it can be verified that the system (2) has $[x_{1s}\ x_{2s}\ x_{3s}\ x_{4s}\ x_{5s}]$ as its unique equilibrium point, where

$$
\begin{aligned}
x_{1s} &= (\frac{m}{a_3})^2(a_4^2 + z^2) \\
x_{2s} &= m^2 \\
x_{3s} &= \frac{a_4}{a_3}m^2 \\
x_{4s} &= \frac{z}{a_3}m^2 \\
x_{5s} &= \omega_d, \qquad (3)
\end{aligned}
$$

provided

$$
\begin{aligned}
V &= (\frac{m}{a_3})[a_4\omega_d + (a_1 + a_4)z] \\
d &= a_1a_4 - a_2a_3 \\
z^2 &+ \omega_d z = d \\
m &= \sqrt{\frac{a_3 f(\omega_d)}{a_5 z}}. \qquad (4)
\end{aligned}
$$

In order to make the conditions in (3) plausible, additional assumption is needed as shown in the following:

(A2) $a_1 < 3a_4$.

To justify this assumption, we can use a linear current feedback to reduce the stator resistance R_s, and hence to reduce the parmeters a_1 and a_2. The details about the reason why the stator resistance can be reduced by a linear current feedback can be found in reference [5].

(D) PI-controller for speed regulation:

Controller design based on the system structure often involves complicated algorithms which may likely lead to high implementation cost. This fact thus motivates the need of designing a simple PI-controller that can be applicable to a vast class of system structure. Now, we consider the load which is modeled as in assumption (A1), and let the amplitude of the input voltage V be designed as:

$$\begin{aligned}\dot{x}_6 &= k_i(\omega_d - x_5)\\ V &= x_6 + k_p(\omega_d - x_5),\end{aligned} \quad (5)$$

By combining systems (2) and (5), we can readily conclude that $x_s = [x_{1s}, x_{2s}, x_{3s}, x_{4s}, x_{5s}, x_{6s}]^T$, where x_{is}'s, $i = 1, .., 5$, are defined in (3) and

$$x_{6s} = (\frac{m}{a_3})[a_4\omega_d + (a_1 + a_4)z], \quad (6)$$

is actually a unique equilibrium point of such an augmented system. In order to analyze the stability of the property of this eqlilbrium point, we will require some mathematical preliminaries which will be shown in the next Section. In fact, later in Section 4, we will show that under the following assumption:

(A3) $\quad 0.8L_sL_r \leq L_m^2,$

the equilibrium point x_s is exponentially stable under proper design of the gains k_i and k_p.

Remark: The air gap between the stator and the rotor is generally made sufficiently small so as to increase the efficiency of energy conversion between the electrical part and the mechanical part, and to reduce the reluctance (mainly caused by the air gap) significantly, which, in turn, implies that the ratio $\frac{L_m^2}{L_sL_r}$ will approach unity. As a consequence, the above assumption is quite reasonable.

3 Mathematical Preliminary

In this section, we will develop a simple criterion to check whether a matrix A_e is Hurwitz or not, where the matrix A_e is extended by a given Hurwitz matrix, two vector, and a scalar under a strictly positive real constraint. This result will be frequently used throughout the paper.

Proposition 1 *Given a matrix $A \in R^{n\times n}$, two vectors $b \in R^{n\times 1}$, $c \in R^{1\times n}$, and a scalar $d \in R$, if the matrix A is Hurwitz and $Re\{-d-c(j\omega I-A)^{-1}b\} > 0$ for all $\omega \in R$, then the extended matrix $A_e = \begin{bmatrix} A & b \\ c & d \end{bmatrix}$ is Hurwitz.*

By some simple calculations, we can obtain the following corollary whose proof is omitted here.

Proposition 2 *Given polynomials $P(s)$, $Q_1(s)$, and $Q_2(s)$. If both $Re\{\frac{Q_1(j\omega)}{P(j\omega)}\}$ and $Re\{\frac{Q_2(j\omega)}{P(j\omega)}\}$ are larger than zero for all $\omega \in R$, then we have $Re\{\frac{P(j\omega)}{Q_1(j\omega)+cQ_2(j\omega)}\} > 0$ for all $\omega \in R$ and any $c > 0$.*

4 Stability Analysis

Referring to the closed-loop ststem (2) and (5), which is rewritten as:

$$\begin{aligned}\dot{x}_1 &= -2a_1x_1 + 2a_2x_3 + \frac{2x_4}{\sqrt{x_2}}V\\ \dot{x}_2 &= -2a_4x_2 + 2a_3x_3\\ \dot{x}_3 &= a_3x_1 + a_2x_2 - (a_1 + a_4)x_3 + x_5x_4\\ \dot{x}_4 &= -x_5x_3 - (a_1 + a_4)x_4 + \sqrt{x_2}V\\ \dot{x}_5 &= a_5x_4 - f(x_5)\\ \dot{x}_6 &= k_i(\omega_d - x_5)\\ V &= x_6 + k_p(\omega_d - x_5),\end{aligned} \quad (7)$$

with the equilibrium point x_s as defined above, for any given $\omega_d \in R$, we are going to prove that the equilibrium point x_s is unique and is exponentially stable. The following theorem will give a complete statement of the conditions under which the aforementioned properties hold.

Theorem 1 *For arbitrary desired speed $\omega_d \in R$, suppose that the assumptions* **(A1)**, **(A2)**, *and* **(A3)** *are satified. If the controller (5) satisfies $\frac{16d}{5a_4} < \frac{k_i}{k_p} < \frac{a_1+a_4}{2}$, then x_s is the unique exponentially stable equilibrium point of the system (7).*

Proof:

Since $x_2 = \lambda_{qr}^2 + \lambda_{dr}^2$ and $0.8L_sL_r \leq L_m^2$, we always have $d > 0.2a_1a_4$ and $m \geq 0$. Without lose of generality, we assume $\omega_d \geq 0$.

(1) Proof of uniqueness

For any given desired speed ω_d, the equation $m = \sqrt{\frac{a_3 f(\omega_d)}{a_5 z}}$ and the assumption on f, i.e., $f(0) = 0$ and $\frac{df(\omega_d)}{d\omega_d} > 0$, imply that $\omega_d z \geq 0$ and $z = (-\omega_d + \sqrt{\omega_d^2 + 4d})/2$. Then the equilibrium point x_s is unique if x_{6s} is a one-to-one function of ω_d.

Define $g_1(\omega_d) = \frac{\omega_d}{\sqrt{z}} + (1 + \frac{a_1}{a_4})\sqrt{z}$, then we have the following relations:

$$\begin{aligned}x_{6s} &= (\frac{a_4}{a_3})(\sqrt{\frac{f(\omega_d)}{a_5}})[\frac{\omega_d}{\sqrt{z}} + (1 + \frac{a_1}{a_4})\sqrt{z}]\\ &= (\frac{a_4}{a_3})(\sqrt{\frac{f(\omega_d)}{a_5}})g_1(\omega_d),\\ z' &= \frac{dz}{d\omega_d} = \frac{-z}{\sqrt{\omega_d^2 + 4d}},\\ g_1'(\omega_d) &= \frac{dg_1(\omega_d)}{d\omega_d}\\ &= \frac{1}{\sqrt{z}} - \frac{\omega_d z'}{2z^{3/2}} + (1 + \frac{a_1}{a_4})(\frac{z'}{2\sqrt{z}})\\ &= (\frac{1}{2\sqrt{z}})[2 + \frac{\omega_d}{\sqrt{\omega_d^2 + 4d}} - (1 + \frac{a_1}{a_4})(\frac{z}{\omega_d^2 + 4d})].\end{aligned}$$

Since $a_1 < 3a_4$ from **(A2)**, we have

$$\begin{aligned}g_1'(\omega_d) &\geq (\frac{1}{\sqrt{z}})(1 - \frac{2z}{\sqrt{\omega_d^2 + 4d}})\\ &= \frac{\omega_d}{z(\omega_d^2 + 4d)} > 0.\end{aligned}$$

Because $g_1'(\omega_d) > 0$ for all $\omega_d > 0$, $g_1(\omega_d)$ is a strictly increasing function of ω_d. Given the strictly increasing property of $g_1(\omega_d)$ and $f(\omega_d)$ with respect to ω_d, we can infer that x_{6s} is also a strictly increasing function of ω_d.

As a result, x_{6s} is a one-to-one function of ω_d and hence x_s is uniquely determined by ω_d.

(2) Proof of exponential stability:

Let J be the Jacobian matrix of the overall system at the equilibrium point x_s for an arbitrary desired speed ω_d, then we define $J' = LJL^{-1}$ where $L = \begin{bmatrix} I_{4\times 4} & 0_{4\times 2} \\ 0_{2\times 2} & \begin{matrix} 1 & 0 \\ -k_p & 1 \end{matrix} \end{bmatrix}$. Now, define A_n to be the n'th principle minor of the matrix J', then we want to prove that J' is Hurwitz through following steps based on the results of Proposition 1 and 2.

Step 1: A_2 is Hurwitz for any given ω_d.

This result can be checked easily.

Step 2: A_3 is Hurwitz for any given ω_d.

Rewrite the matrix A_3 as $\begin{bmatrix} A_2 & b_2 \\ c_2 & -(a_1 + a_4) \end{bmatrix}$, then we have the following inequality by assumption (A1) (A2) and (A3):

$$Rea_1 + a_4 - c_2(j\omega I - A_2)^{-1}b_2 > 0 \quad \forall\ \omega \in R.$$

By virtue of Proposition 1, we can conclude that the matrix A_3 is Hurwitz for all $\omega_d \in R$.

Step 3: By the same procedures, we can find that J' and A_n are Hurwitz for n=4, 5.

By Lyapunov's direct theorem, the equilibrium point x_s is locally exponentially stable for any desired speed $\omega_d \in R$. The proof is hence completed. Q.E.D.

5 Computer Simulation

In this section, the performance of the presented controller, as it is applied to an induction motor, will be demonstrated by a numberof simulation examples. The characteristics of the motor are listed below:

$R_s = 3.745$, $R_r = 3.583$, $L_s = 0.1633$, $L_r = 0.1633$, $L_m = 0.15467$, $P = 6$, $J_m = 0.0284$, 6 *poles*, 220V AC, $1Hp$.

The mechanical load torque is chosen to be $T_L(\omega) = J_L\dot{\omega} + b_0 + b_1\omega + b_2 sign(\omega)\omega^2 + T_d$ whose parameters are listed as follows:

$J_L = 0.1$, $b_0 = 7$, $b_1 = 0.061$, $b_2 = 0.005$, $T_d = 12$ at the time interal [0.4 0.6].

For practical implementation, the amplitude of the input voltage is limited. In our case, the upper bound of the input voltage is 300 V. The desired speed ω_d is 40 (rad/sec) (about 382 (rpm)) and the control gain k_p is selected as $k_p = \frac{3.4 \times d}{a_4}$. As shown in Fig. 1, if k_i is designed to be 0.04, it will induce significant stator currents and rotor fluxes which may cause the magnetic circuit to be saturated and hence, greater power rating of the driver is required. Moreover, the disturbance rejection is much worse. Hence, we modify the control gain according to the criterion in the conventional PI-control approach, in which case the gain k_i is designed as 0.25. It is shown in Fig. 2 that under such gain selection the disturbance can be rejected perfectly, although the stator currents and rotor fluxes are still large. Hence, we propose a strategy to reduce the stator currents and rotor fluxes. This is similar to the so-called "soft-start" problem in industrial applications. Since the large transient currents are caused by the large initial speed error, we construct a modified speed command ω_{ds} by virture of Theorem 1 as follows:

Step 1: Select a $\Delta\omega > 0$,

Step 2: Find the integer p that $p\Delta\omega < \omega_d \le (p+1)\Delta\omega$,

Step 3: At any time, let $m(t)$ be the integer such that $(m(t)-1)\Delta\omega < \omega_r(t) \le (m(t)\Delta\omega - 0.3)$,

Step 4: Choose the modified speed command ω_{ds} as:

$$\omega_{ds} = \begin{cases} \Delta\omega, & if\ m(t) < p \\ \omega_d, & if\ m(t) = p \\ (m(t)+1)\Delta\omega, & if\ m(t) > p \end{cases}$$

After replanning this new speed commend ω_{ds} with $\Delta\omega = 2$, the stator currents can be reduced significant, but at the expense of larger rising time, as shown in Fig. 3.

6 Conclusion

In this paper, we have presented a field-oriented PI-controller for an induction motor, which can drive wide class of loads. Though it is difficult to prove the gloabl stability of the overall system through Lyapunov analysis, we have proposed a simple iterative algorithm to prove that the system has a locally exponentially stable equilibrium point at any desired speed. Moreover, the control law is independent of the rotor resistence (a parameter which normally varies with the temperature) and it only uses the speed of rotor and the orientation of rotor flux as feedback, which made it more feasible.

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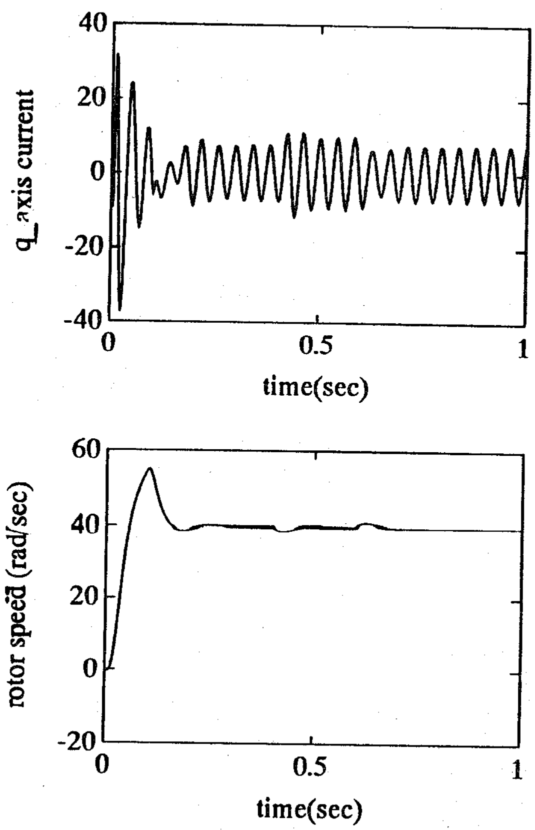

Fig. 1

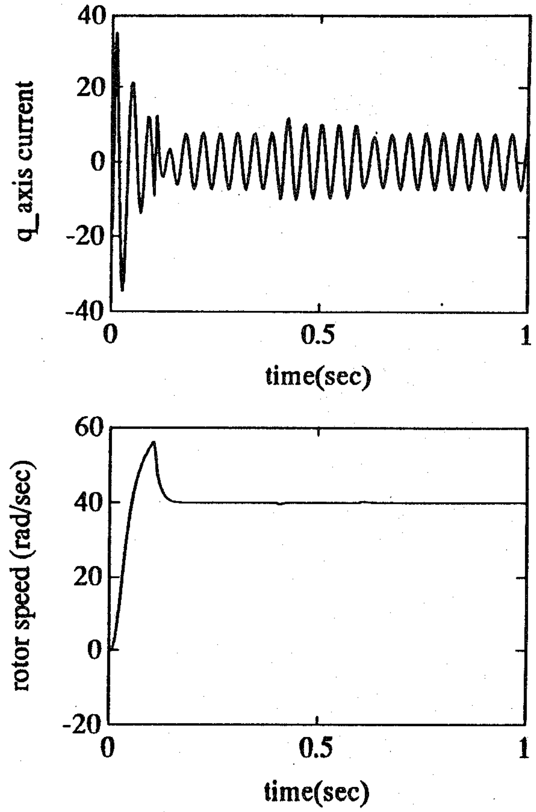

Fig. 2

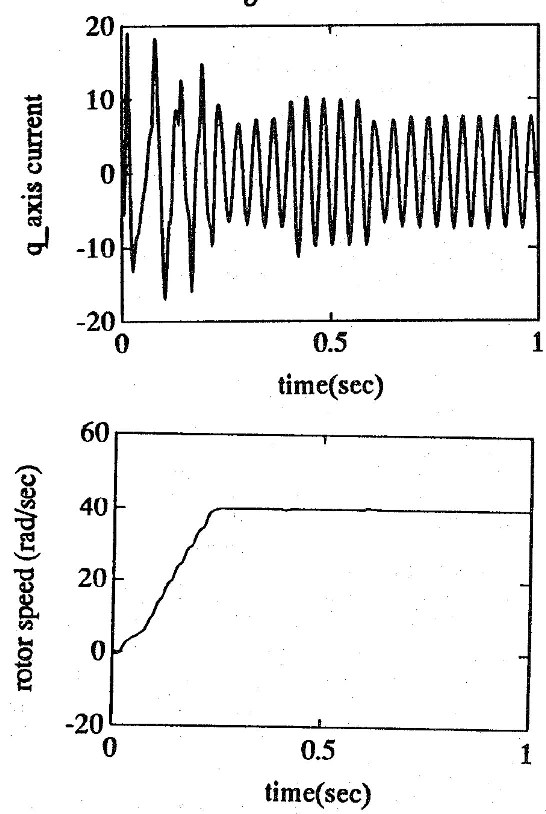

Fig. 3