Joint Weighted Least Squares Estimation of Frequency and Timing Offset for OFDM Systems over Fading Channels

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***Abstract*— This paper presents an algorithm for joint estimation of carrier frequency offset and timing frequency offset in orthogonal frequency division multiplexing (OFDM) systems. A weighted least squares algorithm based on a pilot-aided scheme is proposed. The algorithm operates near the Cramér-Rao bound in the variance of estimation errors. Simulations of several estimation algorithms in a multipath fading channel show that this algorithm provides the most accurate estimation in both the carrier frequency offset and the timing frequency offset.**

I. Introduction

The OFDM modulation technique offers an attractive solution to high-rate data access for its robustness against frequency-selective multipath fading and its simple equalization scheme. Moreover, it is also very efficient in spectrum utilization since the spectra of adjacent subcarriers overlap. OFDM has therefore been adopted in several standards such as DVB-T, VDSL, and IEEE 802.11a. However, it is also well known that OFDM systems are very sensitive to synchronization errors, which can cause *inter-carrier interference* (ICI) and degrade system performance.

All the OFDM standards mentioned above have dedicated pilot subcarriers to facilitate the synchronization tasks in the receivers. Various pilot-aided carrier frequency offset estimation and timing frequency offset estimation algorithms have been proposed [1], [2], [3], [4], [5], [6], [7]. Most of them utilized the detected phase of the received frequency-domain complex data in the pilot subcarriers. The phase shift in the received complex data due to carrier frequency offset is identical in all subcarriers of an OFDM symbol if ICI is neglected. Classen[1], Kapoor[2] and Moose[3] have taken advantage of this fact to build their algorithms. The phases of the received pilot-subcarrier data are extracted and the phase difference between two consecutive symbols in a pilot subcarrier is computed in [1] and [2]. The phase differences are then averaged among all pilot subcarriers or over several OFDM symbols to estimate the carrier frequency offset. On the other hand, a maximum likelihood method is adopted in [3] for the carrier frequency offset estimation. Also the complex data of the pilot subcarriers, instead of their phases, are averaged. However, these algorithms may produce biased estimation when there exists timing frequency offset, which occurs frequently in most communication systems.

Timing frequency offset, unlike carrier frequency offset, causes phase shifts that are proportional to the subcarrier index as well as the offset itself. A very popular class of schemes estimate the timing offset by computing a slope from the plot of measured pilot subcarrier phases versus pilot subcarrier index [4], [5], [6], [7]. In [4], the slope is obtained by averaging over phase differences between pairs of adjacent pilot subcarriers. On the other hand, both the phase and magnitude of the pilot subcarriers in a single OFDM symbol are used in the slope calculation in [5]. In [6] and [7] linear regression is adopted in the estimation of the slope. All of these four algorithms examine only the phases of the pilot subcarriers in one symbol, which are influenced by not only the timing offset but also frequency selective fading. Therefore, the estimated timing frequency offset can be severely degraded. One way to get around this problem is to first take the phase difference of the same pilot subcarrier in two adjacent OFDM symbols so that the frequency selective fading, being same in the two symbols, will be canceled.

Sliskovic[8] proposed to jointly estimate the carrier frequency offset and the timing frequency offset. In addition, a weight is assigned to each pilot subcarrier during averaging since the subcarriers suffer from different levels of fading and thus may have different SNR. The timing frequency offset is estimated by first computing the phase difference between a pair of adjacent pilot

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subcarriers, computing the difference of that amount in two OFDM symbols, and then weighted averaging. The carrier frequency offset can be estimated by first calculating the pilot-subcarrier phase difference between two OFDM symbols, removing the quantity contributed by the previously estimated timing frequency offset, and finally weighted averaging. Thus in this algorithm the accuracy of carrier frequency offset estimation depends on not only the noise contained in the measurements but also the accuracy of timing frequency offset estimation.

In this paper, a weighted least squares algorithm, also based on the pilot-aided scheme, is proposed. Both AWGN and multipath fading channels are considered and the optimal weight assignment that achieves the Cramér-Rao bound is presented.

The rest of the paper is organized as follows. The OFDM signals with both carrier and timing synchronization errors are analyzed in Section II. The proposed weighted least squares algorithm is presented in Section III. In Section IV, the simulation results that demonstrate the potential of the proposed method comparing to others are shown. Finally, a conclusion is given in Section V.

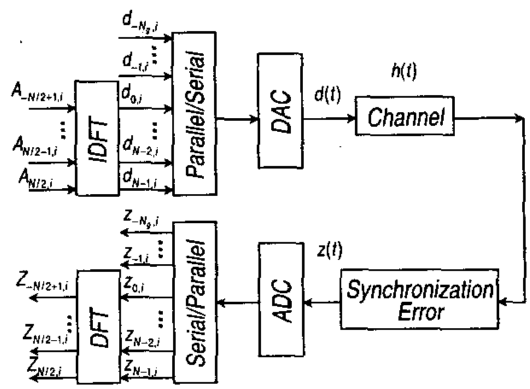

Fig. 1. System model of a OFDM communication system with N subcarriers.

II. OFDM SIGNALS WITH SYNCHRONIZATION ERRORS

An OFDM baseband signal is given by modulating N complex data ($A_{k,i}$) using the *inverse discrete Fourier transform* (IDFT) on N subcarriers as shown in Fig. 1. The subcarriers spacing is $1/T$, which is the inverse of the symbol duration. The n-th time-domain sample of the i-th symbol can be expressed as

$$d_{n,i} = \frac{1}{N}\sum_{k=-N/2+1}^{N/2} A_{k,i}e^{j2\pi nk/N} \quad n=-N_g,\cdots,N-1. \tag{1}$$

Note that in order to combat *inter-symbol interference* (ISI), a cyclic prefix of N_g samples is inserted at the beginning of a symbol.

Assume that the received signal is influenced by a multipath fading channel with a channel impulse response

$$h(t) = \sum_r h_r(t)\cdot\delta(t-\tau_r(t)). \tag{2}$$

The amplitude and delay of the r-th path are denoted by $h_r(t)$ and $\tau_r(t)$. Consequently after convolving with the channel impulse response, the received signal takes the form of

$$z(t) = \sum_r h_r(t)\cdot d(t-\tau_r(t)) + n(t), \tag{3}$$

In addition to multipath channel fading, oscillator mismatch and Doppler effect inflict the received signal with carrier frequency offset and timing frequency offset. If some carrier frequency offset Δf and timing frequency offset $\delta\cdot T_s$ exist, where T_s is the sampling interval in the transmitter, the m-th received sample of the l-th symbol (counting from the end of the cyclic prefix) is given by

$$z_{m,l} = z(t)\cdot e^{j2\pi\Delta ft}|_{t=lN_T(1+\delta)T_s+N_g(1+\delta)T_s+m(1+\delta)T_s}, \quad m=0,1,\cdots,N-1, \tag{4}$$

where N_T is equal to $N+N_g$. The receiver then drops the cyclic prefix and passes the N samples to an N-point *discrete Fourier transform* (DFT) block. After DFT, the complex data on the k-th subcarrier consists of three components: signal term $S_{k,l}$, interference term $I_{k,l}$ and noise term $V_{k,l}$.

$$Z_{k,l} = S_{k,l} + I_{k,l} + V_{k,l}. \tag{5}$$

If H_k represents the channel complex response on subcarrier k, then the signal term can be expressed as

$$S_{k,l} = A_{k,l}\cdot H_k\cdot e^{j2\pi\frac{1}{N}(lN_T+N_g)\phi_{kk}} \cdot e^{j\pi\phi_{kk}(1-\frac{1}{N})}\cdot s(\pi\phi_{kk}), \tag{6}$$

where

$$\phi_{pk} = (1+\delta)\cdot(\epsilon+p) - k \tag{7}$$

and

$$s(\pi\phi_{pk}) = \frac{sin(\pi\phi_{pk})}{Nsin(\frac{\pi\phi_{pk}}{N})}. \tag{8}$$

Note that $\epsilon = \Delta f\cdot T$ is the normalized frequency offset. The two parameters ϕ_{kk} and $s(\pi\phi_{kk})$ represent the distortion of the kth-subcarrier data in phase and magnitude, respectively. Moreover, the synchronization error destroys the orthogonality between subcarriers and the ICI term, $I_{k,l}$, is induced and it is given by.

$$I_{k,l} = \sum_{p=-N/2+1, p\neq k}^{N/2} A_{p,l} \cdot H_p \cdot e^{j2\pi \frac{1}{N}(lN_T+N_g)\phi_{pp}} \cdot e^{j\pi\phi_{pk}(1-\frac{1}{N})} \cdot s(\pi\phi_{pk}) \quad (9)$$

Note that in the case with small δ and ϵ, $s(\pi\phi_{kk})$ is close to 1 and $s(\pi\phi_{pk})$ is near zero, and the ICI term can be ignored. Consider the case when only the carrier frequency offset exists, $S_{k,l}$ will be rotated by (in addition to H_k)

$$2\pi\frac{\epsilon}{N}(lN_T + N_g) + \pi\epsilon(1 - \frac{1}{N}). \quad (10)$$

Note that this phase is independent of k and is identical in every subcarrier. On the contrary, in the case when only some timing frequency offset δ exists, $S_{k,l}$ will be rotated by (in addition to H_k)

$$2\pi\frac{\delta k}{N}(lN_T + N_g) + \pi\delta k(1 - \frac{1}{N}), \quad (11)$$

which is proportional to the subcarrier index k as well as the symbol index l.

III. Joint Estimation Algorithms

In the AWGN channel, $H_k = 1$ for all k and $S_{k,l}$ is distorted in phase and amplitude caused only by synchronization errors. This effect is clearly seen in Fig. 2, which illustrates the phases of subcarrier data in two consecutive OFDM symbols when they are distorted by noise, carrier frequency offset, and timing frequency offset. The carrier frequency offset is 0.05 subcarrier spacing and the timing frequency offset is 100 p.p.m. The received data contain ICI and noise, therefore the extracted phases deviate from the two ideal straight lines. From the extracted subcarrier data phases, the *least-squares* (LS) method can estimate the two straight lines, from which the carrier frequency offset and the timing frequency offset can be derived. In the multipath fading channel, the complex channel response H_k distorts the received data in magnitude and in phase. Moreover, signals on the deeply-faded subcarriers have low SNR, while those on the subcarriers with little fading have high SNR. Obviously, the data phases of the subcarriers with higher SNR are much more reliable than those with lower SNR in the estimation process.

We believe that the concept of weighting the data in each subcarrier is necessary because data of deeply-faded subcarriers should be assigned smaller weights to minimize their adverse effect on estimation accuracy. In addition, estimation based on the phase difference across two OFDM symbols has the advantage of removing the common channel phase. Finally, linear regression provides better estimation since it can find simultaneously the best slope and intercept in terms of least squared error. In light of all the above considerations, we propose a weighted least squares (WLS) algorithm for joint estimation of the carrier frequency offset and timing frequency offset.

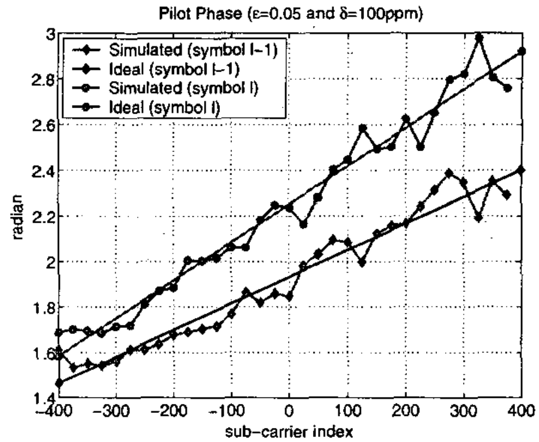

Fig. 2. Phases of subcarrier data in two OFDM symbols with $\epsilon = 0.05$, $\delta = 100$ppm, and Gaussian noise.

Let $\mathbf{y} = [y_0\ y_1\ \cdots\ y_{J-1}]^T$ represents J pilot subcarrier phase differences between two consecutive OFDM symbols and $y_j = \psi(Z_{x_j,l}Z^*_{x_j,l-1})$, where $\psi()$ is the phase of its argument and x_j is the j-th pilot subcarrier index. Also let $\mathbf{b} = [m, d]^T$ be an estimation of the slope m and the intercept d of the optimal straight line in the y_j versus pilot subcrrier index plot, then $\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{n}$, where

$$\mathbf{X} = \begin{bmatrix} x_0 & x_1 & \cdots & x_{J-1} \\ 1 & 1 & 1 & 1 \end{bmatrix}^T. \quad (12)$$

The phase noise component $\mathbf{n} = [e_0\ e_1\ \cdots\ e_{J-1}]^T$ represents the errors in phase contributed by ICI and AWGN and $e_j = y_j - \psi(S_{x_j,l}S^*_{x_j,l-1})$. The Gauss-Markov solution for minimum-variance least-squares estimation, which achieves the Cramér-Rao bound [9], is given by

$$\hat{\mathbf{b}} = (\mathbf{X^T W X})^{-1}\mathbf{X^T W y}, \quad (13)$$

where $\mathbf{W} = E\{\mathbf{n}\mathbf{n}^\mathbf{T}\}^{-1}$.

For simplicity, assume that the residual synchronization error is so small that ICI can be neglected. The off-diagonal terms of $E\{\mathbf{n}\mathbf{n}^{\mathrm{T}}\}$ are contributed by ICI and can be ignored, hence the matrix inverse operation can be avoided and the j-th diagonal component of $\mathbf{W}$, denoted as w_j, is given by

$$w_j = \sigma_{e_j}^{-2}, \tag{14}$$

where $\sigma_{e_j}^2$ is the variance of e_j.

Assume that the channel response is almost static in the duration of two symbols, then y_j is approximated by.

$$\begin{aligned} y_j &= \psi(Z_{x_j,l} \cdot Z^*_{x_j,l-1}) \\ &\approx \psi(S_{x_j,l} \cdot S^*_{x_j,l-1} + S_{x_j,l} \cdot V^*_{x_j,l-1} \\ &\quad + V_{x_j,l} \cdot S^*_{x_j,l-1} + V_{x_j,l} \cdot V^*_{x_j,l-1}) \\ &= \psi(E_s|H_{x_j}|^2 e^{j2\pi \frac{N_T}{N} \phi_{x_j,x_j}} s^2(\pi\phi_{x_j,x_j}) \\ &\quad + S_{x_j,l} \cdot V^*_{x_j,l-1} + V_{x_j,l} \cdot S^*_{x_j,l-1} \\ &\quad + V_{x_j,l} \cdot V^*_{x_j,l-1}) \end{aligned} \tag{15}$$

The first term in the right-hand side of the above equation is the desired output component with a phase related to the two synchronization errors ϵ and δ in ϕ_{x_j,x_j} and a power equal to the squared received signal power. In high-SNR cases, the noise component in y_j depends mainly on the two signal-noise product terms, whose power scales with $E_s N_o |H_{x_j}|^2$. Thus, assuming high SNR, w_j can be approximated by [10]

$$w_j \cong \frac{E_s \cdot |H_{x_j}|^2}{N_o}. \tag{16}$$

In the AWGN channel, w_j is proportional to the inverse of the Guassian noise power in the j-th pilot subcarrier. On the other hand, in the multipath channel w_j scales with $|H_{x_j}|^2$ and inversely with the Guassian noise power. In this case, the carrier frequency offset ϵ as well as the timing frequency offset δ can be derived from (13) and we have

$$\epsilon = \frac{(\sum_{j=0}^{J-1} w_j x_j^2)(\sum_{j=0}^{J-1} w_j y_j) - (\sum_{j=0}^{J-1} w_j x_j)(\sum_{j=0}^{J-1} w_j y_j x_j)}{(2\pi \frac{N_T}{N}) \left[(\sum_{j=0}^{J-1} w_j)(\sum_{j=0}^{J-1} w_j x_j^2) - \left(\sum_{j=0}^{J-1} w_j x_j \right)^2 \right]} \tag{17}$$

and

$$\delta = \frac{(\sum_{j=0}^{J-1} w_j)(\sum_{j=0}^{J-1} w_j y_j x_j) - (\sum_{j=0}^{J-1} w_j x_j)(\sum_{j=0}^{J-1} w_j y_j)}{(2\pi \frac{N_T}{N}) \left[(\sum_{j=0}^{J-1} w_j)(\sum_{j=0}^{J-1} w_j x_j^2) - \left(\sum_{j=0}^{J-1} w_j x_j \right)^2 \right]}. \tag{18}$$

IV. Comparison of Simulated Performance

Assume that the carrier frequency offset and the timing frequency offset both exist, which is the typical case in communication systems. The *root-mean-square* (RMS) errors in the estimated offsets using different algorithms over a multipath fading channel are shown in Fig. 3 and Fig. 4. A carrier frequency error $\epsilon = 0.05$ and a timing frequency error $\delta = 20$ p.p.m. are introduced in the simulation. As can be seen from the two figures, the proposed WLS algorithm achieves, in most cases, the minimum RMS error in both carrier frequency offset estimation and timing frequency offset estimation. Note that even though a small-ICI assumption is made in deriving the WLS joint estimation algorithm, the residual synchronization error is so small in the tracking mode that the ICI can indeed be neglected.

V. Conclusion

In this paper, we proposed a joint estimation algorithm that can estimates both the carrier frequency offset and the timing frequency offset. With a weighted least squares technique, the proposed algorithm indeed generates offset estimates with minimum RMS errors. Therefore, we believe that this algorithm can greatly enhance the performance of OFDM-based communication receivers by reducing residual synchronization error and thus suppressing ICI.

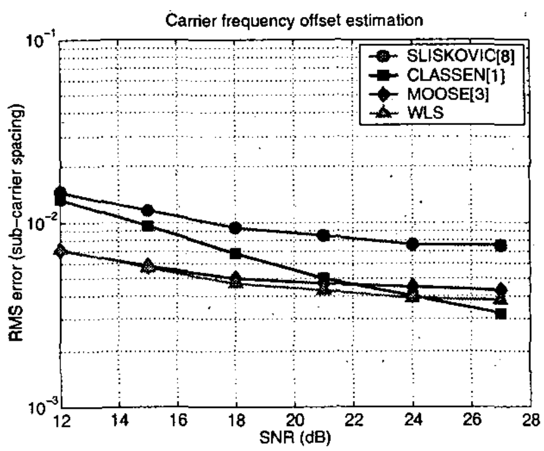

Fig. 3. Simulated carrier frequency estimation errors by different algorithms .

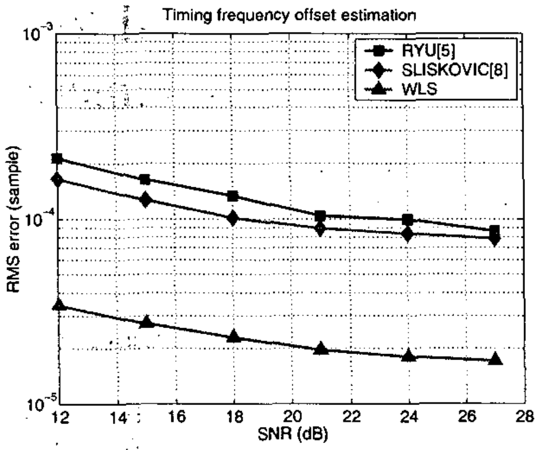

Fig. 4. Simulated timing frequency estimation errors by different algorithms.

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