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Bank Runs and Interest Rates

Shikuan Chen and Yi-Cheng Kao*

The term structure of interest rates and the phenomenon of bank runs are widely but separately studied in the literature. In this paper, we introduce a simple term structure of interest rates in a bank runs model. By allowing depositors to use a symmetric mixed strategy, the probability of bank runs can be derived. Our simulation shows that a low short-term real interest rate or a high long-term real interest rate would raise the probability of bank runs. This result is consistent with recent empirical findings.

Keywords: probability of bank runs, term structure of interest rates, symmetric mixed strategy
JEL classification: E43, E44, G21

1 Introduction

The term structure of interest rates has significant predictive power for the real economic activities. For example, Estrella and Mishkin (1997) find that the yield curves have significant predictive power for the real economic activity and inflation in the United States and Europe. On the other hand, the term structure of interest rates is also relevant to the banking system, particularly in the extreme episodes of bank runs. When many bank runs

*Department of International Business, National Taiwan University and Department of Business Administration, Chung Yuan Christian University. Correspondence: Yi-Cheng Kao, e-mail: kao@cycu.edu.tw. We are grateful for comments and suggestions by Ming-Jen Chang, Yuanchen Chang, Nan-Kuang Chen, Yehning Chen, Chung-Ming Kuan, Jer-Yuh Wan, and participants at the 2008 NTU International Business Conference, the 2009 Annual Meeting of Taiwan Economic Association, and the Workshop on Econometric and Financial Studies after Crisis, 2010, Academia Sinica. Finally, we want to thank two anonymous reviewers of the journal for their useful suggestions and careful recommendations.

occur simultaneously, it is banking crises and will result in bank failures. For example, Hagen and Ho (2007) find that low short-term real interest rates would raise the likelihood of banking crises. In addition, Arena (2008) shows that, in the crises, failing banks have higher interest rates of long maturities than surviving banks. However, previous models containing the term structure on bank runs or banking crises, such as Waldo (1985), Allen and Gale (1998), and Chang and Velasco (2000), have no direct explanation for those empirical findings.

In this paper, we extend the banking model of Diamond and Dybvig (1983) to investigate the random nature of bank runs with aforementioned empirical results on interest rates. Two points are emphasized here. First, both a short-term and a long-term investment are available and hence a simple term structure of real interest rates could be constructed. Second, depositors use a symmetric mixed strategy for the game of withdrawing their funds. In Diamond and Dybvig (1983), there are multiple equilibria that bank runs or no bank runs are possible. Following Diamond and Dybvig (1983), the bank runs are typically modeled as an equilibrium triggered by exogenous sunspots; see, e.g., Cooper and Ross (1998), Peck and Shell (2003), Ennis and Keister (2006), Martin (2006), Andolfattoa, Nosalb, and Wallace (2007), Ennis and Keister (2009), Gu (2011), and Azrieli and Peck (2012), among others. In those models, depositors use pure strategies to withdraw their funds and the uncertainty appears in the timing of their demand for consumption. However, if depositors use a mixed strategy such that their withdrawing actions are governed by the equilibrium distribution, then we have a third equilibrium, called the symmetric mixed-strategy Nash equilibrium. We apply this equilibrium concept to model the withdrawing behavior of depositors.

Palacios-Huerta (2003) shows that, in the real world, players do use the mixed strategy when the payoff is relevant in the professional game. Evidences of using mixed strategies have also been found by Walker and Wooders (2001), Chiappori, Levitt, and Groseclose (2002), and Hsu, Huang, and Tang (2007), among others. As a bank consists of dominant depositors, one depositor's consideration for others' actions could be modeled by the mixed-strategy Nash equilibrium. To describe such circumstances, depositors are assumed to use a symmetric mixed strategy in our model. That is, in the equilibrium, depositors who need the funds later still have the same probability of withdrawing their funds early. In this situation, we can calculate the probability that the number of early withdrawing depositors exceeds the expectation from the bank. This probability depends on the payoff structure and hence the term structure of interest rates is relevant. As a result, the uncertainty for depositors appears not only in the timing of consumption, but also in the expectation of others' randomly withdrawing actions.

Given an explicit utility function, we simulate the probability with different cases of the term structure. Our simulation shows that the probability is a decreasing function of the short-term rates but an increasing function of the long-term rates. The explanations are as follows. First, a low short-term real interest rate means that the bank has fewer resources to satisfy the expected and unexpected early withdrawing demand. In such case, a typical patient depositor would take a greater probability of early withdrawing to ensure that other patient depositors are indifference between withdrawing early and late in the mixed-strategy Nash equilibrium. Second, a high long-term real interest rate implies a high potential loss for depositors if the bank eventually cannot provide the deposit funds.

Although the symmetric mixed-strategy Nash equilibrium is emphasized, it should be recognized that the model still has multiple equilibria. Our point is that when a bank consists of dominant depositors, they are likely to use the mixed strategy as the players in Palacios-Huerta (2003), and therefore the interaction of them could be modeled by such equilibrium. Regarding the multiple equilibria problem, Goldstein and Pauzner (2005) extend the Diamond and Dybvig (1983) model to show the unique equilibrium by the global game approach of Carlsson and Damme (1993) and Morris and Shin (1998). However, in Goldstein and Pauzner (2005), the bank would be more vulnerable if the deposit contract provides a higher short-term repayment rate, which may support early empirical work of Demirgüc-Kunt and Detragiache (1998) but is contrary to the recent finding of Hagen and Ho (2007).

According to criterions such as the cost of rescue operations, prolonged bank holidays, and deposit freezes, Demirgüc-Kunt and Detragiache (1998) define the events of banking crises over 1981–1994 and suggest that banks are more vulnerable with high real interest rates. However, such event-based method tends to identify banking crises with sample selection bias and it is hard to date the policy intervention exactly. Hence event-based studies have different results on identifying banking crises (see Hagen and Ho (2007)). To overcome such identification problems, Hagen and Ho (2007) develop a new index based on money market pressure called the IMP and then take extreme values of the IMP as objective signals to identify banking crises. Hagen and Ho (2007) further apply the IMP method to investigate the banking crises among 47 countries over 1980–2001 and instead find that low short-term real interest rates would raise the likelihood of banking crises, which can be explained in our model.

Moreover, regarding interest rates, the differences between Goldstein and Pauzner (2005) and our model are as follows. In our model, the bank provides an optimal contract to all depositors according to the returns of short-term and long-term investments, while in Goldstein and Pauzner (2005) the bank determines the short-term rate regardless of the return of short-term investment. Hence, our model characterizes the term structure of interest rates as a market constraint to banks. In fact, the short-term interest rates used in Hagen and Ho (2007) is the money market interest rates, which cannot be determined by a single bank in any competitive market. Since our model consists of depositors and a representative bank in a competitive environment, we interpret the result as a theoretical support to the findings of Hagen and Ho (2007). That is, the short-term real interest rate, as a market price to banks, is crucial in the determination of bank runs and banking crises.

In practice, the short-term interest rates are mainly determined by the monetary authorities in most countries. When the short-term interest rate is very low, it should also be considered the possible adverse effect on raising the probability of bank runs. This consideration is particularly important when the inflation pressure is high, because the short-term real rate would decrease in such situation. For example, Laeven and Valencia (2008) and Laeven and Valencia (2012) find that bank runs are common feature of banking crises and inflation rates tend to be high prior the 42 crises they investigated. In this aspect, our study provides a complementary explanation that a lower real short-term rate may make the banking system more vulnerable.

The rest of the paper is organized as follows. Section 2 presents the model and constructs the probability of bank runs. Section 3 simulates the model and computes the probability with different parameters. Section 4 discusses the implications and the relation between our result and other studies. Section 5 concludes.

2 The Model

The formulation of banking here is similar to Diamond and Dybvig (1983) that banks provide risk sharing for depositors. However, regarding the investing opportunities, we explicitly distinguish the short-term and the long-term investments, and also include the liquidation cost introduced by Cooper and Ross (1998). As a result, a simple term structure of real interest rates and its relation to the probability of bank runs can be constructed in the mixed-strategy Nash equilibrium.

2.1 The Environment

There are N agents living for three periods, $t = 0, 1, 2$. At $t = 0$, all agents decide whether to deposit their one unit of endowment into the bank. There are two types of agents: impatient and patient. An impatient agent derives utility only from consumption at $t = 1$ and a patient one derives utility only from consumption at $t = 2$. Nevertheless, at $t = 0$, all agents are uncertain about their types and only know a probability $\pi \in (0, 1)$ that he or she will be an impatient one. Therefore, at $t = 1$, πN agents learn that they are impatient while others learn that they are patient. An agent's type is private information and none of others knows the type. For simplicity, π is assumed to be constant and all agents know the exact value of π at $t = 0$. Let c_1 and c_2 be the consumption at $t = 1$ and $t = 2$, respectively. The expected utility of agent at $t = 0$ is

$$\pi u\left(c_1\right) + (1 - \pi) u\left(c_2\right), \tag{1}$$

where $u(\cdot)$ is the utility function satisfying $u'(c) > 0 > u''(c)$ and $u(0) = 0$.

There are two possible investing technologies for agents in the economy. The first one is the short-term investment with the rate of return of r over one period. We call r the short-term real interest rate. This short-term investment is available at $t = 0$ or at $t = 1$. The second one is the long-term investment with the rate of return of R^2 over two periods. Clearly, the long-term investment is available only at $t = 0$ and the total return is given at $t = 2$. For comparison, the long-term real interest rate for one period is denoted by R, the square root of R^2. Hence, r and R construct the term structure of real interest rates in the model. We further impose a reasonable assumption that $1 \leq r < R$. Moreover, if the long-term investment is interrupted at $t = 1$, it only yields the rate of return of $(1 - \tau)$, where τ is the liquidation cost introduced by Cooper and Ross (1998) and satisfies $0 < \tau < 1$.

In the economy, the banking industry is perfectly competitive so that each bank earns zero profit in the equilibrium. For simplicity, we only assume a representative bank that maximizes agents' expected utility when it designs the deposit contract in the competitive market. As a result, all agents are willing to accept the deposit contract provided by the bank at $t = 0$. Let I be the long-term investment, the optimal contract is solved from the following problem:

$$\begin{aligned} \max_{I} \quad & \pi u(c_1) + (1-\pi)u(c_2) \\ \text{s.t.} \quad & N\pi c_1 = N(1-I)r, \\ & N(1-\pi)c_2 = NIR^2. \end{aligned} \tag{2}$$

Suppose that I^* is the optimal long-term investment in equation (2); the optimal deposit contract offered by the bank is then $\{c_1^*, c_2^*\} = \{(1 - I^*)r/\pi, I^*R^2/(1-\pi)\}$. Therefore, depositors who go to the bank at $t = 1$ can withdraw c_1^* and others who go to the bank at $t = 2$ can withdraw c_2^*. Since the first order condition of equation (2) is $u'(c_1^*) = (R^2/r)u'(c_2^*)$, we then have $c_1^* < c_2^*$ by the assumptions of $1 \leq r < R$ and $u''(c) < 0$.

Given the optimal deposit contract, a question at $t = 0$ is why agents do not invest the endowments by themselves. Specifically, each of them can choose an optimal I to solve the following self-investment problem:

$$\begin{aligned} \max_{I} \quad & \pi u(c_1) + (1-\pi)u(c_2) \\ \text{s.t.} \quad & c_1 = (1-I)r + I(1-\tau), \\ & c_2 = (1-I)r^2 + IR^2. \end{aligned} \tag{3}$$

Let I' be the optimal long-term investment in (3). The optimal consumption set is then $\{c_1', c_2'\} = \{(1 - I')r + I'(1 - \tau), (1 - I')r^2 + I'R^2\}$. Notice that all agents have learned their type at $t = 1$, so each of them will choose to consume c_1' or c_2' depending on the type. An impatient agent will interrupt the long-term investment with liquidation cost and then consume c_1' at $t = 1$, because she or he does not need the consumption at $t = 2$. On the contrary, a patient agent does not consume the yields of short-term investment at $t = 1$ and will reinvest them over one period again for the final consumption at $t = 2$. Since the liquidation cost τ appears in the constraint of (3), the expected utility over $\{c_1^*, c_2^*\}$ is strictly greater than that over $\{c_1', c_2'\}$. We illustrate such domination of the deposit contract in Fig-

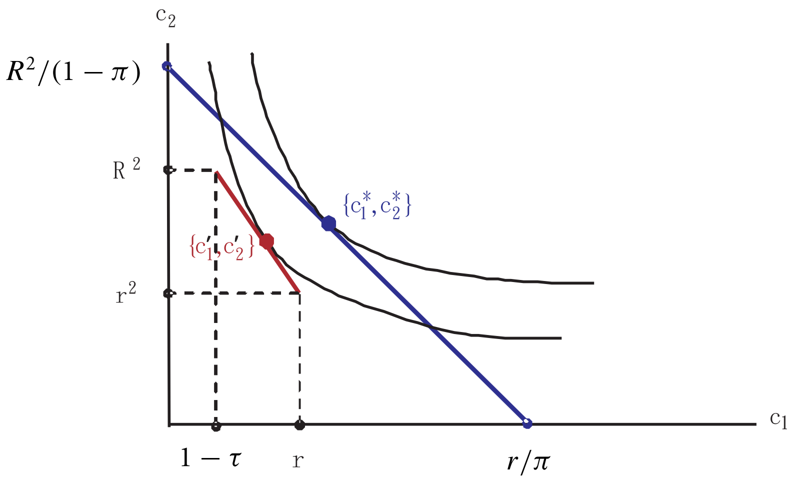

Figure 1: The Deposit Contract and the Self-Investment Allocation

ure 1. As Figure 1 shows, all agents will accept the deposit contract $\{c_1^*, c_2^*\}$ provided by the bank at $t = 0$.

Because the optimal deposit contract is $\{c_1^*, c_2^*\} = \{(1-I^*)r/\pi, I^*R^2/(1-\pi)\}$, it raises a question that $c_1^* = (1-I^*)r/\pi$ may be small than 1 if $\pi > (1-I^*)$. In this case, all agents will still accept the deposit contract $\{c_1^*, c_2^*\}$ offered by the bank, because at $t = 0$ agents are uncertain about their type and they will invest some long-term investment in the self-investment problem. However, if the long-term investment is interrupted at $t = 1$, it only yields the rate of return of $(1-\tau) < 1$. Thus, the introduction of liquidation cost induces agents to accept the deposit contract even though c_1^* may be small than 1. In general, Appendix A shows that the expected utility of $\{c_1^*, c_2^*\}$ is strictly greater than that of $\{c_1', c_2'\}$.

In the optimal deposit contract, if the long-term investment is interrupted at $t = 1$, the bank will also have not enough resources to satisfy the c_2^* withdrawing because of such liquidation losses. This consideration is examined in the following subsection.

2.2 The Probability of Bank Runs

As in Diamond and Dybvig (1983), there are two pure-strategy equilibria in the economy. For impatient depositors, at $t = 1$, they realize the demand for consumption immediately, so they will withdraw c_1^* for sure. For patient depositors, if they believe other patient depositors do not withdraw c_1^* early, the best response is to withdraw c_2^* at $t = 2$. This constitutes a Nash equilibrium. In this Nash equilibrium, the bank runs doses not occur. In contrast, if every patient depositor believes that other patient depositors will withdraw c_1^* at $t = 1$, the best response now is running to withdraw c_1^* early. This is because depositors know that the bank must interrupt the long-term investment and will have not enough resources to satisfy the c_2^* withdrawing. In this case, the bank runs is occurring as the other Nash equilibrium.

The equilibrium of bank runs described above implies that there exist some exogenous mechanisms that coordinate patient depositors to early withdraw their funds at $t = 1$. In fact, if depositors instead use the mixed strategies such that their withdrawing actions are random, then there also exists a third equilibrium called the mixed-strategy Nash equilibrium in the model. In some cases, it would be better to model the interaction of depositors by applying the concept of mixed strategies. For instance, when the bank consists of some dominant depositors, one's consideration for others' actions could be properly characterized by the probability of bank runs in the mixed-strategy Nash equilibrium. In order to construct such probability, we now allow patient depositors to use a symmetric mixed strategy such that each of them has the probability p of withdrawing c_1^* at $t = 1$ and has the probability $(1 - p)$ of withdrawing c_2^* at $t = 2$.

In the mixed-strategy Nash equilibrium, the dominant strategy for all impatient depositors is still to withdraw c_1^* because they need the funds to consume at $t = 1$. For patient depositors, in the Nash equilibrium, each of them has the probability p^* to early withdraw such that other patient depositors are indifference between withdrawing c_1^* at $t = 1$ and withdrawing c_2^* at $t = 2$. Mathematically, p^* is solved from

$$\begin{aligned} u\left(rc_1^*\right) = {} & u\left(c_2^*\right)(1-p)^{N(1-\pi)-1} \\ & + \sum_{m=1}^{N(1-\pi)-1} u(c(m))n(m)p^m(1-p)^{N(1-\pi)-1-m}, \\ \text{where } c(m) = {} & \max\left\{\frac{NI^*(1-\tau)-mc_1^*}{N(1-\pi)-m},\ 0\right\} \quad \text{and} \\ n(m) = {} & \frac{(N(1-\pi)-1)!}{m!(N(1-\pi)-1-m)!}. \end{aligned} \tag{4}$$

Note that c_1^* and c_2^* are functions of r and R in general and therefore p^* should depend on the term structure of real interest rates. The left-hand side of equation (4) is simply that a typical patient depositor withdraws c_1^* at $t = 1$ and reinvests the funds into the short-term investment, and finally consumes rc_1^* at $t = 2$. In other words, $u(rc_1^*)$ is the utility for a patient depositor who withdraws c_1^* early.

The right-hand side of equation (4) is the expected utility of a typical patient depositor who wants to withdraw c_2^* at $t = 2$, given that other patient depositors use a mixed-strategy with p of withdrawing c_1^* at $t = 1$. This expectation consists of two parts. The first part is the outcome that all other patient depositors also withdraw c_2^* at $t = 2$ and this outcome occurs with the probability of $(1 - p)^{N(1-\pi)-1}$. The second part are expectation for other outcomes that m patient depositors have withdrawn c_1^* at $t = 1$. In this case, the final amount of consumption for this patient depositor is $c(m)$, a decreasing function of m, and it could be zero if m is too large. Specifically, given $m \in [1, N(1 - \pi) - 1]$, the patient depositor obtain $(NI^*(1 - \tau) - mc_1^*)/(N(1 - \pi) - m)$ if the value is positive. However, if $(NI^*(1 - \tau) - mc_1^*)/(N(1 - \pi) - m) \leq 0$, then the patient depositor will obtain nothing. For an event with a positive m, the occurring probability is the product of $p^m(1-p)^{N(1-\pi)-1-m}$ and $n(m)$, where $n(m)$ is defined as the combinations of m among $N(1 - \pi) - 1$. Summarizing all the possibilities of m gives the expected utility as the second part in the right-hand side of equation (4).

As a result, the mixed-strategy Nash equilibrium is that all impatient depositors withdraw c_1^* for sure and all patient depositors withdraw c_1^* with the probability p^*. Thus, with the probability p^*, each patient depositor will run to the bank and withdraw their funds early. In this paper, we define the probability of bank runs as more than expected number of depositors to withdraw c_1^*, i.e., $1-[(1-p^*)^{N(1-\pi)}]$. This probability is strictly increasing in p^*. Therefore, we can analyze the probability of bank runs through p^*. For simplicity, we sometimes call p^* the probability of bank runs directly. Note that p^* depends on the optimal solutions of c_1^*, c_2^*, and I^*. In order to investigate the relation between p^* and the term structure of interest rates, we further assume explicit utility functions for depositors and then simulate the p^* in the next section.

3 Simulation

In this section, we simulate the probability of bank runs by specifying the utility function and parameters. In particular, the ranges of interest rates are

Table 1: Descriptive Statistics of Interest Rates in the U.S.

	Mean	Median	Std. dev.	Minimum	Maximum
Effective federal funds rate	5.64%	5.22%	3.30%	0.08%	18.65%
10-year Treasury note yield	6.51%	6.05%	2.63%	2.43%	14.62%

Note: The interest rates are quarterly rates obtained by averaging monthly rates from the FRED database. The sample period is from 1954:Q3 to 2013:Q1

limited by the model assumption of $1 \leq r < R$ and the U.S. data of the effective federal funds rate and yield rate of 10-year Treasury note are used to represent nominal short-term and long-term rates. These rates are quarterly interest rates obtained by averaging monthly rates from the FRED database. The sample period is from 1954:Q3 to 2013:Q1. Table 1 summarizes some descriptive statistics of the data. Notice that those rates are nominal ones and the highest rate is 18.65% of the effective federal funds rate in 1980 when the inflation rate is 13.51%, and hence the real rate is about 5.14% in this case. Therefore, the ranges of real interest rates in the following simulation are limited by $1 \leq r < R \leq 1.06$.

We further assume that depositors have the same utility function as follows:

$$u(c) = \begin{cases} 1 + \ln c, & \text{if } c \geq 1, \\ c, & \text{if } c < 1. \end{cases} \tag{5}$$

Substituting the utility function of equation (5) into equation (2) yields the optimal solution as $c_1^* = r$, $c_2^* = R^2$, and $I^* = 1 - \pi$. Notice that we assume $u(c) = c$ for $c < 1$ since $1 + \ln c$ is not defined when $c < 1$. However, the values of optimal solution, $c_1^* = r$ and $c_2^* = R^2$, are not smaller than one because of $1 \leq r < R$. Thus, the expected utility of agent will be $\pi(1 + \ln r) + (1 - \pi)(1 + \ln R^2)$ at the optimal solution, so that the deposit contract still serves as a role of risk sharing in the model.

According to the solution, the probability of bank runs is determined by the following equation:

$$\begin{aligned} 1 + \ln r^2 = & \left(1 + \ln R^2\right)(1-p)^{N(1-\pi)-1} \\ & + \sum_{m=1}^{N(1-\pi)-1} u\left(\max\left\{\frac{N(1-\pi)(1-\tau) - mr}{N(1-\pi) - m}, 0\right\}\right) \times \\ & n(m) p^m (1-p)^{N(1-\pi)-1-m}. \end{aligned} \tag{6}$$

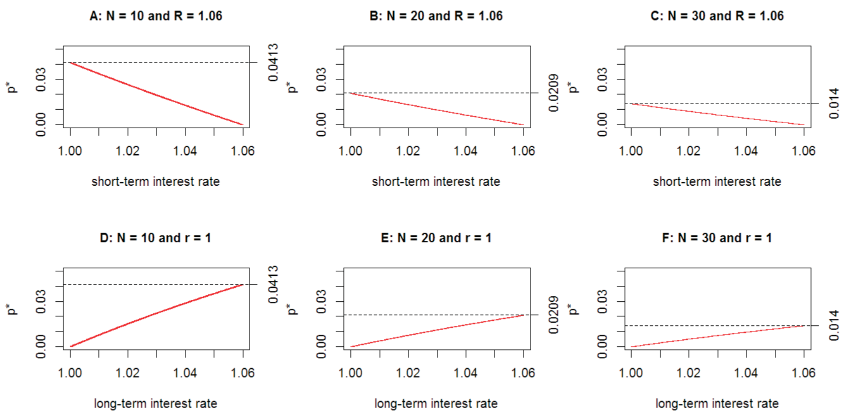

Figure 2: Simulations of the Probability of Bank Runs

Equation (6) is derived from substituting $c_1^* = r$, $c_2^* = R^2$, and $I^* = 1 - \pi$ into equation (4). Because we have p to the power of $N(1-\pi) - 1$, there is no closed form solution in (6). We therefore simulate the p^* implied by equation (6) under different parameters and show the result in Figure 2. Moreover, we also show the existence and uniqueness of the equilibrium p^* in Appendix B.

The basic case we simulate is according to the average short-term and long-term real interest rates from the FRED database, which are $r = 1.01512$ and $R = 1.02515$, with parameters of $\pi = 0.5$, $\tau = 0.5$, and $N = 20$. The simulated p^* is 0.00249 in the basic case. Moreover, given $\pi = 0.5$ and $\tau = 0.5$, we simulate the p^* implied by equation (6) under different interest rates and N and then show the result in Figure 2. Clearly, the probability of bank runs is a decreasing function of short-term real interest rates but an increasing function of long-term real interest rates. These results are reasonable for several reasons. First, a low short-term real interest rate means that the bank has fewer resources to satisfy the expected and unexpected early withdrawing demand. In such case, a typical patient depositor would take a greater probability of early withdrawing to ensure that other patient depositors are indifference between withdrawing early and late in the mixed-strategy Nash equilibrium. Second, a high long-term real interest rate implies a high potential loss for depositors if the bank cannot provide c_2^* eventually. Thus, patient depositors would also take a higher probability

Table 2: The probability of Bank Runs under Various Parameters

	$R = 1.06$			$r = 1.00$		
	$r = 1.01$	$r = 1.03$	$r = 1.05$	$R = 1.01$	$R = 1.03$	$R = 1.06$
$\pi = 0.10$	0.00747	0.00498	0.00249	0.00000	0.00491	0.00737
$\pi = 0.50$	0.01683	0.01010	0.00336	0.00000	0.00733	0.01467
$\pi = 0.90$	0.08472	0.04983	0.01495	0.00245	0.03443	0.08608
$\tau = 0.10$	0.05427	0.03015	0.00904	0.00351	0.02462	0.05628
$\tau = 0.50$	0.01683	0.01010	0.00336	0.00000	0.00733	0.01467
$\tau = 0.90$	0.00996	0.00498	0.00249	0.00000	0.00491	0.00983
$N = 10$	0.03344	0.02020	0.00505	0.01050	0.03151	0.03151
$N = 20$	0.01683	0.01010	0.00336	0.00000	0.00733	0.01467
$N = 30$	0.01212	0.00606	0.00301	0.00000	0.00703	0.01055

Note: R, r, π, τ, and N are respectively the long-term real interest rate, the short-term real interest rate, the ratio of impatient depositors, the liquidation cost, and the number of depositors. In each cell of the probability, if the parameters are not specified, the default values of π, τ, and N are set to be 0.50, 0.50, and 20, respectively. Moreover, the utility function specification implies that $c_1^* = r$, $c_2^* = R^2$, and $I^* = 1 - \pi$ in the simulation.

on early withdrawing to ensure that others are indifference between withdrawing early and late. Moreover, Figure 2 also shows that the probability increases as the number of depositors decreases. The intuition behind this is that, when N decreases, the external effect of one early withdrawing action is increasing for other depositors.

We now show that the simulation are robust under different values of π, τ, and N. In each simulation of the probability, if the parameters are not specified, the default values of π, τ, and N are set to be 0.50, 0.50, and 20, respectively. The simulation results are summarized in Table 2. It shows that the probability is an increasing function of π but a decreasing function of τ, and N. That is, varying π has an adverse effect comparing to that of varying τ and N on the probability. A higher π means small number of patient depositors and hence the external effect of one early withdrawing action is increasing for other patient depositors, as the effect of decreasing N mentioned above. A higher τ will lead to larger liquation costs when unexpected early withdrawing occurs and thus the probability will decrease due to such cost for depositors. The simulation also shows that the probability of bank runs is a decreasing function of short-term real interest rates

Table 3: The Probability of Bank Runs under CARA Utility Function

	$R = 1.06$			$r = 1.00$		
	$r = 1.01$	$r = 1.03$	$r = 1.05$	$R = 1.01$	$R = 1.03$	$R = 1.06$
$\tau = 0.10$	0.22748	0.20213	0.17678	0.22181	0.23115	0.24049
$\tau = 0.50$	0.04214	0.03612	0.03010	0.00602	0.01580	0.02558
$\tau = 0.90$	0.02107	0.01881	0.01655	0.00452	0.00904	0.01356

Note: R, r, and τ are respectively the long-term real interest rate, the short-term real interest rate, and the liquidation cost. The utility function specification implies that $c_1^* = 2(1 - I^*)r$, $c_2^* = 2I^*R^2$, and $I^* = (2\ln R - \ln r + 2r)/2(r + R^2)$ in the simulation.

but an increasing function of long-term real interest rates. Consequently, the relation between the probability and the term structure still remains under different values of long-term real interest rate, short-term real interest rate, proportion of impatient depositor, liquidation cost, and the number of depositors.

The specification of equation (5) is a simple case of the constant relative risk aversion (CRRA) utility function. It is nature to ask whether the main results still hold in other specifications. Therefore, we simulate the probability of bank runs under a different case of the constant absolute risk aversion (CARA) utility function as follows:

$$u(c) = 1 - \exp(-c). \tag{7}$$

Substituting this utility function into the problem of (2) yields the optimal solution as $I^* = (2\ln R - \ln r + 2r)/2(r + R^2)$, $c_1^* = 2(1 - I^*)r$, and $c_2^* = 2I^*R^2$. According to this solution and given $N = 20$ and $\pi = 0.50$, we simulate the probability from equation (4) and then report the results in Table 3. As we can see, the relation between the probability and the term structure remains under this utility function. That is, in Table 3, the probability is still a decreasing function of short-term real interest rates but an increasing function of long-term real interest rates.

4 Discussion

In this section, we discuss the empirical work relating to the model and theoretical extensions for future research. The simulation described above shows that the probability of bank runs is a decreasing function of short-term real interest rates but an increasing function of long-term real interest rates. The empirical work of Hagen and Ho (2007) also finds that low short-term real interest rates would raise the probability of banking crises in the data of 47 countries over the period from 1980 to 2001. Hagen and Ho (2007) explain the result by words that, after periods of low interest rates, the monetary contraction for combating inflation induces a higher probability of banking crises. In contrast, Arena (2008) using the bank-level data from East Asia and Latin America finds that, during crises, failing banks in East Asia have higher deposit interest rates than those of surviving banks, and failing banks in Latin America have both higher deposit and loan rates than those of surviving ones. Similarly, Schumacher (2000) suggests that banks with higher deposit interest rates tend to fail in the currency crises of Argentina after the Mexican devaluation of December 20, 1994.

For policymakers, it is desired to know which rate and how its variation could raise the likelihood of bank runs, but the evidences described above seem to conflict with each other. Nevertheless, they could be consistently explained in two aspects. First, the interest rates used in Hagen and Ho (2007) are from the 60b line in IFS database, i.e., the money market interest rates, which are exactly the short-term rate in our model. A low short-term rate raising the probability of bank runs is exactly the model prediction. Second, the deposit and loan rates used in Arena (2008) have longer maturities than the money market interest rates. Besides, the returns of loans will contain liquidation costs if they are interrupted early. Therefore, it is better to view the loan rates in Arena (2008) as the long-term interest rate in our model. A high long-term rate raising the probability of bank runs is also the model prediction.

The relation between the term structure and the bank runs can be extended in an alternative model with asymmetric information. In our representative bank model, a lower short-term or a higher long-term real interest rate creates an environment that the early withdrawing behaviors have a greater likelihood to occur. Nevertheless, if banks have heterogeneous assets, there remains a possibility that some banks with worse assets may provide higher interest rates to attract depositors and thus the probability of bank runs may be different among those banks. In practice, interest rates offered by banks could be significantly different in the pre-crisis periods, but such phenomenon could not be characterized in a systemic banking model.

In the literature, the non-systemic model of bank runs is often developed with settings of asymmetric information, such as Jacklin and Bhattacharya (1988), Chari and Jagannathan (1988), Jacklin (1993), Alonso (1996), and Chen and Hasan (2008). In addition, as pointed by Wallace (1988), the first-come, first-served rule is an important sequential-service constraint to banks. Chen (1999) models a contagious bank runs by highlighting the information externalities and the first-come, first-served rule. Moreover, according to the asymmetric and dynamic nature of the banking system, Schotter and Yorulmazer (2009) provide an interesting experimental investigation showing that there exists a welfare improvement by introducing some informed insiders who know the quality of the bank. For future research, those models with asymmetric information might be extended with highlighting the role of the term structure of interest rates. We believe such extension could properly explain the empirical findings from bank-level data.

5 Concluding Remarks

The term structure of interest rates and the phenomenon of bank runs are widely but separately studied in the literature. In this study, we introduce a simple term structure of real interest rates in the Diamond and Dybvig (1983) model. Assuming depositors to use the symmetric mixed strategy, the probability of bank runs can be constructed in the equilibrium. When a bank consists of dominant depositors, the interaction of them is properly modeled by the probability of bank runs in the symmetric mixed-strategy Nash equilibrium. Our simulation shows that a low short-term real interest rate or a high long-term real interest rate would raise the probability.

Finally, as Estrella (2005) points out, theoretical explanations for the predictive power of the term structure on the real economic activities are not complete in the literature. In our model, the investment technologies and their returns are exogenously determined and hence the simulation only shows how these exogenous variables affect the probability of bank runs. Thus the model provides no explanation for the predicting power. If the returns of investments could be endogenously determined, the model may provide some explanations for the relation between the term structure and the real economic activities. This extension is left for future research.

Appendix

A. The Deposit Contract Is Generally Accepted

In this appendix, we show that the expected utility of $\{c_1^*, c_2^*\}$ is strictly greater than that of $\{c_1', c_2'\}$, i.e., $E[u(c_1^*, c_2^*)] > E[u(c_1', c_2')]$ and hence the optimal deposit contract is generally accepted. We first introduce an alternative deposit contract of $\{\tilde{c}_1, \tilde{c}_2\}$ that satisfies the constraint of (2). Since $\{c_1^*, c_2^*\}$ is the optimal solution in (2), we thus have $E[u(c_1^*, c_2^*)] > E[u(\tilde{c}_1, \tilde{c}_2)]$. If there exists an alternative deposit contract of $\{\tilde{c}_1, \tilde{c}_2\}$ such that $E[u(\tilde{c}_1, \tilde{c}_2)] > E[u(c_1', c_2')]$, we then obtain $E[u(c_1^*, c_2^*)] > E[u(c_1', c_2')]$.

The constraint of (2) implies that $\{\tilde{c}_1, \tilde{c}_2\} = \{(1-\tilde{I})r/\pi, \tilde{I}R^2/(1-\pi)\}$ and the problem (3) implies that $\{c_1', c_2'\} = \{(1-I')r + I'(1-\tau), (1-I')r^2 + I'R^2\}$. Let $\tilde{c}_2 = c_2'$, we must have $\tilde{I} = (1-\pi)[(1-I')r^2 + I'R^2]/R^2$. Note that $\partial\tilde{c}_1/\partial\tilde{I} < 0$, $\partial c_1'/\partial I' < 0$, and $\tilde{I}, I' \in [0, 1]$. That is, $\tilde{c}_1$ and c_1' are monotonically decreasing in $\tilde{I}$ and I', respectively. Therefore, if $\tilde{c}_1 > c_1'$ for $I' = 0$ and for $I' = 1$ as well, then we will have $\tilde{c}_1 > c_1'$ for any $I' \in [0, 1]$. It follows that $E[u(\tilde{c}_1, \tilde{c}_2)] > E[u(c_1', c_2')]$ because $\tilde{c}_1 > c_1'$ and $\tilde{c}_2 = c_2'$.

When $\tilde{c}_2 = c_2'$ and $I' = 0$, we have $\tilde{I} = (1-\pi)r^2/R^2$ and then

$$\tilde{c}_1 - c_1' = r\left[\frac{1}{\pi}\left(1 - \frac{(1-\pi)r^2}{R^2}\right) - 1\right] > 0,$$

because $1 \leq r < R$ and $0 < \pi < 1$. Note that we need to show the value in the square brackets is positive in this case. Let $f(\pi) = (1-(1-\pi)r^2/R^2)/\pi$. Clearly, $f(\pi)$ is monotonically decreasing in $\pi \in (0, 1)$ because of $\partial f(\pi)/\partial\pi < 0$. In addition, we have $f(\pi \to 0) = \infty$ and $f(\pi \to 1) = 1$, and this implies that $f(\pi) - 1 > 0$ for $\pi \in (0, 1)$. Thus, we surely have $\tilde{c}_1 - c_1' > 0$ in this case. Similarly, when $\tilde{c}_2 = c_2'$ and $I' = 1$, we have $\tilde{I} = 1 - \pi$ and also obtain

$$\tilde{c}_1 - c_1' = r + \tau - 1 > 0,$$

because $1 \leq r < R$ and $0 < \tau < 1$.

We find an alternative deposit contract that $E[u(\tilde{c}_1, \tilde{c}_2)] > E[u(c_1', c_2')]$, and hence $E[u(c_1^*, c_2^*)] > E[u(c_1', c_2')]$ is proved. In general, the expected utility of $\{c_1^*, c_2^*\}$ is strictly greater than that of $\{c_1', c_2'\}$.

B. The Existence and Uniqueness of the Equilibrium

In this appendix, we show the existence and uniqueness of the equilibrium p^* in (6). Recall that p^* is determined by the following equation (6):

$$
\begin{aligned}
1+\ln r^2 = {} & \left(1+\ln R^2\right)(1-p)^{N(1-\pi)-1} \\
& + \sum_{m=1}^{N(1-\pi)-1} u\left(\max\left\{\frac{N(1-\pi)(1-\tau)-mr}{N(1-\pi)-m}, 0\right\}\right) \times \\
& n(m)p^m(1-p)^{N(1-\pi)-1-m}.
\end{aligned}
$$

Given the values of parameters, the left-hand side of (6) is a constant term, and the right-hand side of (6), denoted by $g(p)$, is a function of p. If we can show that $g(p)$ is greater than $1+\ln r^2$ when $p=0$; $g(p)$ is smaller than $1+\ln r^2$ when $p=1$; and $g(p)$ is monotonically decreasing in $p \in (0, 1)$, then there exists an unique equilibrium of p^* in (6).

First, when $p=0$, $g(p) = 1+\ln R^2$ is greater than $1+\ln r^2$ because $1 \le r < R$. Second, when $p=1$, $g(p)=0$ is smaller than $1+\ln r^2$. Third, there are two main terms in $g(p)$, denoted by $g_1(p)$ and $g_2(p)$, and we will show both of them are monotonically decreasing in $p \in (0, 1)$ as follows. For the first term of $g(p)$, we have

$$
\frac{\partial g_1(p)}{\partial p} = -(N(1-\pi)-1)(1+\ln R^2)(1-p)^{N(1-\pi)-2} < 0.
$$

For the second term of $g(p)$, we have

$$
\begin{aligned}
\frac{\partial g_2(p)}{\partial p} = {} & \sum_{m=1}^{N(1-\pi)-1} (m)u\left(\max\left\{\frac{N(1-\pi)(1-\tau)-mr}{N(1-\pi)-m}, 0\right\}\right) \times \\
& n(m)p^{m-1}(1-p)^{N(1-\pi)-1-m} \\
& - \sum_{m=1}^{N(1-\pi)-1} (N(1-\pi)-1-m) \times \\
& u\left(\max\left\{\frac{N(1-\pi)(1-\tau)-mr}{N(1-\pi)-m}, 0\right\}\right) \times \\
& n(m)p^m(1-p)^{N(1-\pi)-m-2},
\end{aligned}
$$

which consists of two plausibly symmetric terms, simply denoted by $h_1(p)$ and $h_2(p)$. If those two terms are exactly the same, we shall have $h_1(p) -$

$h_2(p) = 0$. However, those two terms are instead asymmetric and we really have $h_1(p) < h_2(p)$ for the following reasons. Note that there exists a critical value $\bar{m}$ such that

$$u\left(\max\left\{\frac{N(1-\pi)(1-\tau)-mr}{N(1-\pi)-m}, 0\right\}\right) = 0$$

when $m \geq \bar{m}$. We thus have $h_1(p) = h_2(p)$ when only consider the summation of $m \geq \bar{m}$. In contrast, since $\sum_{m=1}^{\bar{m}} m < \sum_{m=1}^{\bar{m}} (N(1-\pi) - 1 - m)$, we have $h_1(p) < h_2(p)$ when consider the other summation of $m < \bar{m}$. Hence, $\partial g_2(p)/\partial p = h_1(p) - h_2(p) < 0$ is obtained.

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銀行擠兌與利率結構

陳思寬
台灣大學國際企業學系
高一誠
中原大學企業管理學系

文獻上關於銀行擠兌與利率期限結構分別有諸多探討。本文將簡單的利率期限結構導入銀行擠兌模型, 並允許存款者使用對稱之混合策略, 以求出均衡時的銀行擠兌機率。模型的模擬結果顯示出, 較低的短期實質利率或較高的長期實質利率, 皆會提高銀行擠兌機率。此理論結果與近期的實證研究結論一致。

關鍵詞: 銀行擠兌機率, 利率期限結構, 對稱混合策略
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