

ROBUST ADAPTIVE ARRAY BEAMFORMING IN TWO DIMENSIONS

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ABSTRACT

An effective technique for robust adaptive beamforming using one-dimensional (1-D) linear array has been proposed by one of the authors in [1]. It was shown that the resulting beamformer can cure the difficulties of using conventional robust techniques. In this paper, we investigate the extension of the above technique to two dimensions (2-D). We present two algorithms for the implementation of the 2-D robust technique. The comparison based on computational complexity is made. Computer simulations are given for illustration.

I. INTRODUCTION

Adaptive array beamformer designed for receiving the desired signal in specified beam direction while rejecting noise and other directional jammers is widely used in radar, sonar, and many communication systems. However, its performance is very sensitive to the accuracies of signal and array situations because an optimum adaptive beamformer is designed under the assumptions of ideal signal characteristics and exact array configurations. It has been shown that the effectiveness of an adaptive beamformer can be destroyed even if a small mismatch arises in practical circumstances.

Many techniques have been presented to tackle the degradation effect due to steering errors [2-9]. These conventional methods are based on either imposing directional derivative constraints to allow direct control of array sensitivity or reducing the sensitivity of array performance to the steering errors. However, the drawback is that the abilities of eliminating noise and interference are inevitably deteriorated. Recently, one of the authors has proposed an effective approach for robust adaptive beamforming in [1] to deal with the case of using 1-D linear array. In practice, it is possible that there exists more than one signal coming from different directions but having the same angle with respect to the broadside of a 1-D linear array. In this circumstances, the

1-D array cannot distinguish these different signals. Hence, 2-D arrays are required for the purpose of beamforming. In this paper, We extend the technique of [1] to two dimensions. It is shown that the resulting 2-D adaptive array beamformer is robust in the presence of steering errors.

II. THE 2-D ROBUST ADAPTIVE BEAMFORMING

The 2-D array is assumed to be rectangular with size $M \times N$ and the array sensors are equally spaced with interelement spacings along the X and Y axes are d_x and d_y , respectively.

Its configuration is shown in Figure 1. The directional angles of the j -th jammer and the signal are denoted by $(\theta_k, \phi_k), k=1, 2, \dots, K$ and (θ_s, ϕ_s) , respectively. The input of the mn -th array sensor, which is located at the intersection of the m -th row and the n -th column, is given as

$$u_{mn}(t) = \alpha_s(t) \exp[jk_c(md_x u_s + nd_y v_s)] + \sum_{j=1}^K \alpha_j(t) \exp[jk_c(md_x u_j + nd_y v_j)] + n_{mn}(t) \quad (1)$$

where $\alpha_s(t)$ and $\alpha_j(t)$ denotes the complex envelopes of the desired signal and the j -th jammer, respectively, k_c the wavenumber, and $u_s = \sin \theta_s \cos \phi_s$, $v_s = \sin \theta_s \sin \phi_s$, $u_j = \sin \theta_j \cos \phi_j$, $v_j = \sin \theta_j \sin \phi_j$, and $n_{mn}(t)$ is the complex noise of mn -th sensor. Using (1), the input signal vector of the m -th row of the 2-D array, which is of size $N \times 1$, is given as

$$\begin{aligned} \underline{U}_{mn}(t) &= [u_{m1}(t), \dots, u_{mN}(t)]^T \\ &= \alpha_s(t) \underline{s}_{sm} + \sum_{j=1}^K \alpha_j(t) \underline{s}_{jm} + \underline{n}_m(t) \end{aligned} \quad (2)$$

where \underline{s}_{sm} is the signal phase vector at the m -th row,

$$\underline{s}_{sm} = [s_{sm1}, \dots, s_{smN}]^T = [\exp[jk_c(md_x u_s + d_y v_s)], \dots, \exp[jk_c(md_x u_s + Nd_y v_s)]]^T \quad (3)$$

\underline{s}_{jm} is the phase vector of the j -th jammer, and $\underline{n}_m(t)$ is the noise vector. The input vector of the 2-D rectangular array can be defined as the $MN \times 1$ vector given by

$$\underline{U}(t) = [\underline{U}_1(t)^T, \dots, \underline{U}_M(t)^T]^T = \alpha_s(t) \underline{s}_s + \sum_{j=1}^K \alpha_j(t) \underline{s}_j + \underline{n}(t) \quad (4)$$

where \underline{s}_s is the phase vector of the desired signal

$$\underline{s}_s = [\underline{s}_{s1}^T, \dots, \underline{s}_{sM}^T]^T \quad (5)$$

and $\underline{n}(t)$ is the noise vector. Then the covariance matrix is given as

$$\mathbf{R} = E \{ \underline{U}(t) \underline{U}^\dagger(t) \} \quad (6)$$

where \dagger denotes the conjugate transposition and E the expectation.

Consider the m -th row of the 2-D $M \times N$ array. Assume that the tolerance error of the steering error, which is given as \underline{s}_{dm} with steering angle (θ, ϕ) , due to the mismatch is smaller than the distance between steering and jammer phase vectors. Based on the fact that the output signal-to-noise ratio (SNR) of an optimized adaptive beamformer becomes maximum when the steering vector coincides with the signal phase vector, we first find the optimum weight vector \underline{W}_{om}^1 for a given steering vector by minimizing the array output power with main beam constraint. After obtaining the \underline{W}_{om}^1 , we adjust the steering vector in a prescribed class S_c of possible arrival phase vectors of the signal. This procedure is repeated until the output SNR of the optimized adaptive beamformer reaches a maximum. Therefore, the resulting optimization problem can be written as [1]

$$\begin{aligned} & \text{Maximize} \{ \text{Minimize } E(|\underline{W}_m^\dagger \underline{U}_m(t)|^2) \} \\ & \underline{s}_{dm} \in S_c \quad \underline{W}_m^\dagger \underline{s}_{dm} = 1 \\ & \text{Subject to } |\underline{s}_{dmi}| = 1, \text{ for } i=1, 2, \dots, N \end{aligned} \quad (7)$$

where $S_c \equiv \{ \underline{s}_{dm} \mid \|\underline{s}_{dm} - \underline{s}_{sm}\|^2 \leq \delta \}$, δ is the tolerance error for robustness. Therefore, we correct the steering vector error by iteratively adjusting the steering vector during the optimization process. The adaptive algorithm presented in [1] for 1-D robust adaptive beamforming can be employed to search the correct steering vector \underline{s}_{sm} . Then the corresponding optimum weight vector is given as [10]

$$\underline{W}_{om} = \mathbf{R}_{um}^{-1} \underline{s}_{sm} / \underline{s}_{sm}^\dagger \mathbf{R}_{um}^{-1} \underline{s}_{sm} \quad (8)$$

for $m=1, 2, \dots, M$, where $\mathbf{R}_{um} \equiv E[\underline{U}_m(t) \underline{U}_m^\dagger(t)]$. Finally, all of the M resulting 1-D optimum weight vectors are used to form an $MN \times 1$ array weight vector \underline{W}_o which is given as

$$\underline{W}_o = [\underline{W}_{o1}^T, \dots, \underline{W}_{oM}^T]^T \quad (9)$$

and then obtain the associated 2-D array beam pattern.

An alternative approach is to only save the corrected steering vector \underline{s}_{sm} after the 1-D robust processing. Then all of the M corrected steering vectors are used to form an $MN \times 1$ steering vector as in (5). Then we compute the corresponding $MN \times 1$ array weight vector \underline{W}_o which is given as

$$\underline{W}_o = \mathbf{R}^{-1} \underline{s}_s / \underline{s}_s^\dagger \mathbf{R}^{-1} \underline{s}_s \quad (10)$$

Comparing the computational complexity, we note from (8) and (10) that the second algorithm faces the problem of computing the inverse of \mathbf{R} with size $MN \times MN$ in (10). This leads to heavy computational burden. By contrast, using the first algorithm requires the computation of inverse of \mathbf{R}_{um} with size $N \times N$ only in (8). In the next section, we thus only present the simulation examples based on the first algorithm.

III. SIMULATION EXAMPLES

In this section, several computer simulations are presented for illustration. We consider a 7×7 2-D rectangular array. Both the interelement spacings d_x and d_y are set to be half-wavelength ($\lambda/2$). The desired signal is at $(\theta_s, \phi_s) = (0^\circ, 0^\circ)$ while two incoherent jammers are located at $(\theta_1, \phi_1) = (65^\circ, 20^\circ)$ and $(\theta_2, \phi_2) = (45^\circ, 40^\circ)$, respectively. The noise received at the array sensors is assumed to be spatially white with $\text{SNR} = 20$ dB and JNR (jammer-to-noise ratio) = 10 dB.

A. Example of Steering Angle Mismatch.

In this case, the steering angle error is $(\Delta\theta, \Delta\phi) = (0.5^\circ, 0.5^\circ)$. Figure 2 shows the 2-D array beam patterns under the steering angle error and using the proposed robust technique after 500 iterations, respectively. The ideal result for this case is shown in Figure 3. Table 1 lists the array gains for this case. Comparing these results, we note that the proposed robust technique can effectively cure the 2-D array performance degradation due to steering angle error. Moreover, the beam pattern using the robust technique is as good as the ideal beam pattern.

B. Example of Sensor Position Error

In this case, we assume that the steering vector error is only due to 2-D random perturbation in sensor positions. The random position errors are Gaussian with mean = 0 and variance = 0.15 half-wavelength in each dimension. Figure 4 shows the resulting 2-D array beam pattern when the 2-D array suffers the 2-D random perturbation and the resulting 2-D array beam pattern using the proposed robust approach after 500 iterations, respectively. Table 2 lists the array gains for this case. Again, we note from these results that the proposed robust technique demonstrates its effectiveness in dealing with the degradation effect due to sensor position mismatch and provides a 2-D beam pattern as good as that of the ideal one shown in Figure 3.

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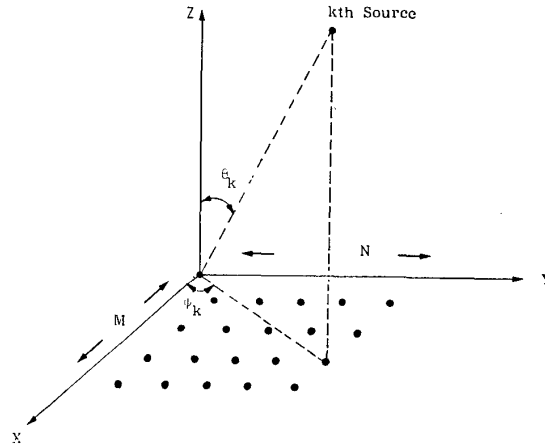


Figure 1 The 2-D MxN Rectangular Array

Table 1 Array Gains (dB) for the Case of Steering Angle Error

signal position array situation	(0°, 0°)	(65°, 20°)	(45°, 40°)
Ideal Array	0.0	-86.7	-90.8
Array with Angle Error	-23.9	-71.2	-53.2
Array after Robust Method	0.0	-84.1	-74.0

Table 2 Array Gains (dB) for the Case of Position Error

signal position array situation	(0°, 0°)	(65°, 20°)	(45°, 40°)
Ideal Array	0.0	-64.0	-74.5
Array with Position Error	0.0	-21.2	-22.6
Array after Robust Method	0.0	-63.0	-73.5

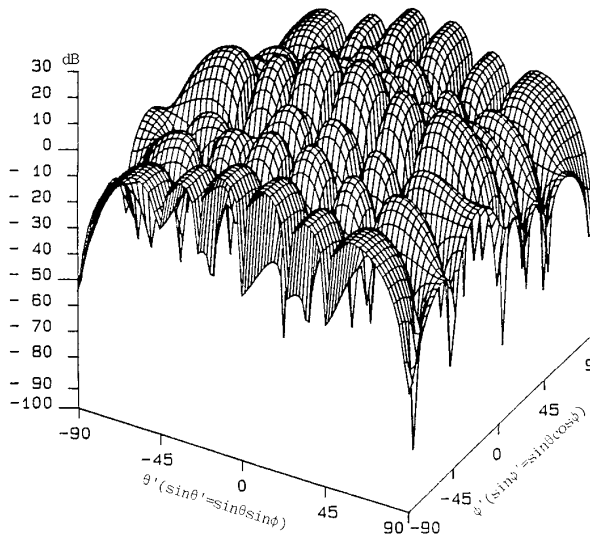


Figure 2 (a) The Array Beam Pattern in The Case of Steering Angle Error

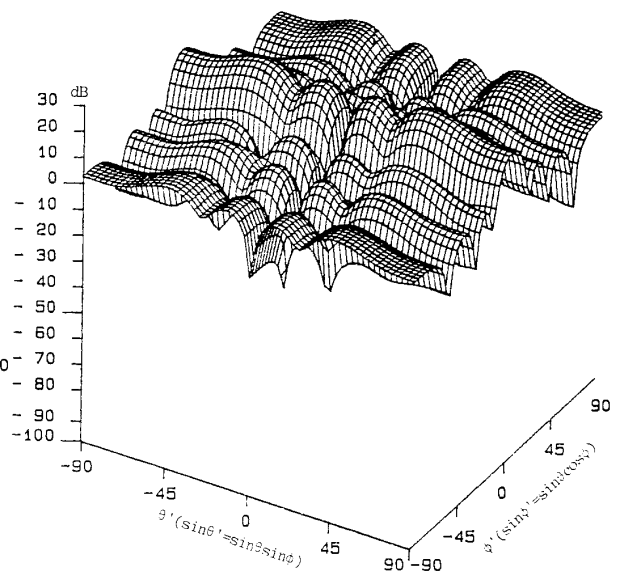


Figure 4 (a) The Array Beam Pattern in The Case of Position Error

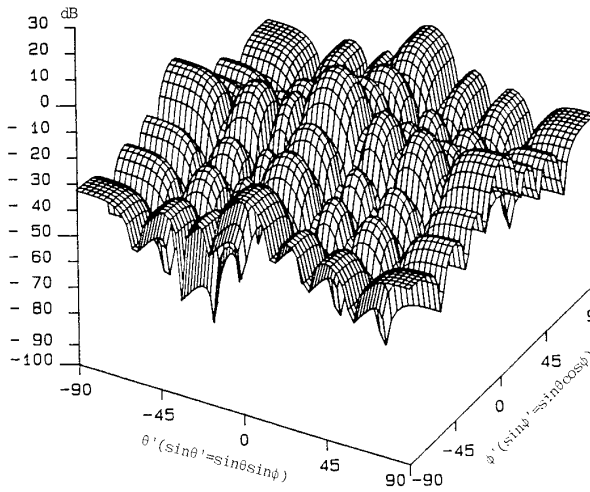


Figure 2 (b) The Array Beam Pattern After Robust Processing

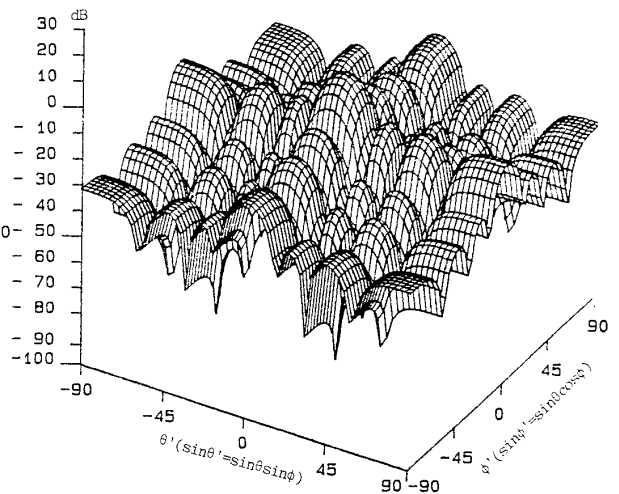


Figure 4 (b) The Array Beam Pattern After Robust Processing

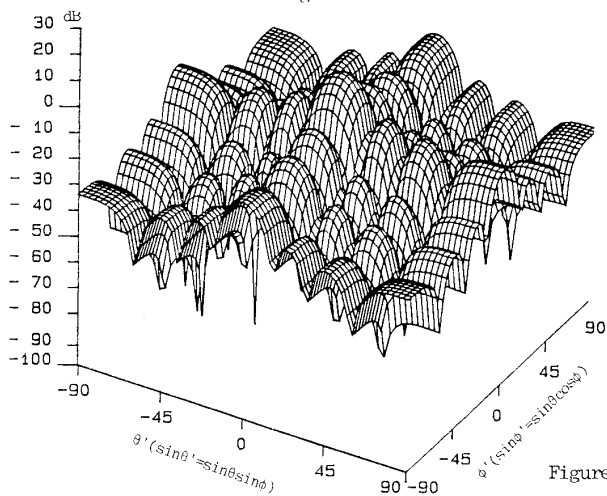


Figure 3 The 2-D Ideal Array Beam Pattern