MULTIRESOLUTION MOSAIC

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ABSTRACT

Mosaic techniques have been used to combine two or more signals into a new one with an invisible seam, and with as little distortion of each signal as possible. Multiresolution representation is an effective method for analyzing the information content of signals, and it also fits a wide spectrum of visual signal processing and visual communication applications. Wavelet transform is one kind of multiresolution representations, and has found a wide variety of application in many aspects, including signal analysis, image coding, image processing, computer vision and etc.

Due to its characteristic of multiresolution signal decomposition, wavelet transform is used in this paper to do the image mosaic by choosing the width of mosaic transition zone proportional to the frequency represented in the band. Both 1-D and 2-D signal mosaics will be described, and some factors which affect the mosaics will be discussed.

1. INTRODUCTION

The mosaic techniques have been used to combine two or more signals into a new one , which should be quite smooth and invisible around the seam, and should still preserve the characteristic of each one with as little distortion of each signal as possible. In advertisement, image mosaic has been numerously applied to create fancy synthetic images from possibly unrelated components.

It is clear that when two different signals are joined into one, the different magnitude around the seam will make the boundary quite noticeable. Intuitively, a weighted average technique applied within the transition zone should do the work. Within the transition zone, the value of points will be computed as a weighted average of the corresponding points in each signal. For example, two signals $l(x)$ and $r(x)$ are to be combined into $f(x)$ at the point x, where $l(x)$ is on the left side and $r(x)$ on the right side of the transition zone T. Let $w(x)$ be a monotonically decreasing weighting function, as shown in Fig. 1, then $f(x)$ is given by $f(x) = w(x)l(x) + (1 - w(x))r(x)$.

With appropriate choice of weighting function $w(x)$, the weighted average technique will result in smooth transition, but invisible seam is not ensured. On the other hand, width of T affects the seam lot. If T is small compared to signal features, then the seam is still visible as a step across two signals. If T is large compared to signal features, then features from both signals may appear superimposed within the zone [1].

Multiresolution spline technique [1] has been proposed to produce the image mosaic effect, in which Laplacian pyramid [2] is used, and a weighted average within the transition zone which is proportional in size to the wave length represented in the band is adopted. The weighted average concept using different transition zones according to different bands quite fits human visual sensibility, and will be adopted in our method.

Due to the characteristic of multiresolution signal decomposition and/or representation, Discrete Wavelet Transform (DWT) is used to accomplish the work of image mosaic in this paper.

Wavelet theory has emerged recently as a new mathematical tool for multiresolution signal representation, and have found a wide variety of application in many aspects, including signal analysis [10], image coding [9], image processing, computer vision [8], computer graphics [7], [11] and etc. It treats both the continuous and discrete-time cases and provides a very general technique that can be applied to many tasks in signal processing. Wavelet transform is defined by decomposing the signal into a family of functions which are the transition and dilation of an unique function. So, it is also one kind of multiresolution representations. Especially, wavelet represents the signal feature in both time and frequency simultaneously, and it is quite desirable for signal mosaic.

In the case of Discrete Wavelet Transform (DWT), both time and frequency-scale parameters are discrete. It is noted that, as far as the structure of computation is concerned, the DWT is in fact as an octave-band filter bank; which means that the DWT can be implemented by using the subband coding scheme. Moreover, in DWT, the undesired oversampling nature of the Laplacian pyramid

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disappears. And this is the reason the DWT is utilized in this work.

2. MULTIRESOLUTION REPRESENTATION

Multiresolution signal representation is an effective method for analyzing the information content of signals [3]. The basic concept is to decompose the signal spectrum into its subspectra, and each subspertrum component can then be treated individually based on its characteristic. For example, most nature signals will have predominantly low-frequency components, thus the low-band components contain most of significant information, while for a texture the most significant information often appears in its middle-band component [4].

The basic idea of multiresolution decomposition based on wavelet is described below.

A multiresolution analysis, with scaling function $\phi(x)$ and the mother wavelet $\psi(x)$, consists of a sequence of subspaces $\{V_m\}$ of $L^2(R)$ which have the following properties:

1) **containment**:

$$\ldots V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \ldots$$
$$\leftarrow \textit{coarser} \quad \textit{finer} \rightarrow$$

2) **completeness**:

$$\cap_m V_m = \{0\}$$
$$\cup_m V_m = L^2(R)$$

3) **scaling property**:

$$f(x) \in V_m \Leftrightarrow f(2x) \in V_{m-1}$$

4) **orthonormality**:

The dilated and translated versions of the scaling function $\phi_{m,n}(x) = 2^{-m/2}\phi(2^{-m}x - n)$ is an orthonormal basis for V_m. Let W_m be the orthogonal complement in V_{m-1} of V_m, i.e. $V_{m-1} = V_m \oplus W_m$, then the space W_m is spanned by the $\psi_{m,n}(x)$.

For a signal $f(x)$, then the coefficients $\langle \psi_{m,n}, f \rangle$ describe the approximation loss of f when resolution goes from m to m-1. The multiresolution decomposition is translated into computation of the $d_{m,n}(f) = \langle \psi_{m,n}, f \rangle$, and does not require the explicit forms $\phi(x)$ of and $\psi(x)$.

A M-level wavelet decomposition can be written as

$$\begin{aligned} f_0(x) &= \sum_n c_{0,n}\phi_{0,n}(x) \\ &= \sum_n c_{1,n}\phi_{1,n}(x) + \sum_n d_{1,n}\psi_{1,n}(x) \\ &= \cdots \\ &= \sum_n \left(c_{M,n}\phi_{M,n}(x) + \sum_{m=1}^{M} d_{m,n}\psi_{m,n}(x)\right) \end{aligned} \tag{1}$$

where $c_{0,n}$ are the given starting signal and $c_{m+1,n}$ and $d_{m+1,n}$ are related to $c_{m,n}$ by :

$$\begin{aligned} c_{m+1,n} &= \sum_k c_{m,k} h(k-2n) \\ d_{m+1,n} &= \sum_k c_{m,k} g(k-2n) \end{aligned} \tag{2}$$

As described above, the signal $d_{m+1,n}$ is just the discrete wavelet transform coefficient at resolution $m+1$, and represents the difference information between $c_{m,n}$ and its smoothed down-sampled approximation $c_{m+1,n}$.

Thus, this provide a recursive algorithm for wavelet decomposition through low-pass filter $h(k)$ and high-pass filter $g(k)$. Similarly, a recursive algorithm for signal synthesis can be derived based on the wavelet coefficients $d_{m,n}$, $1 \le m \le M$ and $c_{M,n}$ as:

$$c_{m,k} = \sum_n c_{m+1,n} h(k-2n) + \sum_n d_{m+1,n} g(k-2n) \tag{3}$$

These relations are shown in Fig. 2 .

3. MULTIRESOLUTION MOSAIC

In psychophysics and the physiology of human vision, evidence has been gathered showing that the retinal image is decomposed into several spatially oriented frequency channels. This explains why multiresolution decomposition methods are so popular in computer vision and image processing research and why Multiresolution Spline [1] approach works well for image mosaic. Our work was also motivated by the above fact originally.

Since the low-frequency content of a signal are often sufficient in many instances (such as the content of an image), and the detail information resembles the high-frequency components (such as edge of an image), thus, the width of the transition zone T is chosen according to the wave length represented in each band. That is, for lower frequency components, the width of transition zone T is chosen to be larger than that of higher frequency components. This implies that low-frequency components "bleed" across the boundary of mosaic region further than high-frequency components do.

Actually, as described in the above section, the signal

$c_{m+1,n}$ at resolution $m+1$ is a smoothed down-sampled approximation of $c_{m,n}$ at resolution m, and $d_{m+1,n}$ is just the detail (or difference) information between $c_{m,n}$ and $c_{m+1,n}$. Therefore, using the same width of transition zone between detail component of resolution m and its down-sampled components in resolution m+1 means the actual transition zone of the low- frequency components is larger than that of the high-frequency.

To simplify and generalize arbitrary shape mosaic both for 1-D and 2-D signals, the transition zone *T* and the weighting function are not explicitly expressed, in stead, another multiresolution structure of mask signal is introduced. The mask signal is a binary representation which describes how two signals will be combined. For example, two signals *A* and *B* will be combined to form a mosaic signal, and the mask signal *S* is a binary signal in which all points inside the mosaic region are set to 1 and those outside the mosaic region are set to 0. As the way to generate a sequence of lower resolutional signal (not the detail signal) described in the above section, the mask signal *S* is low-pass filtered and subsampled to construct its multiresolution structure $c'_{M,n}(S),\cdots,c'_{2,n}(S),c'_{1,n}(S)$, and then each smoothed version of the mask signal will be used as the weighting function in its corresponding resolution level. Note that the low-pass filter used to construct multiresolution structure of the mask signal need not to be the same as the one DWT used, that is why here we use $c'_{M,n}(S)$ instead of $c_{M,n}(S)$.

Now, both 1-D and 2-D DWT-based signal mosaics will be described.

A. 1-D signal mosaic

Suppose two 1-D signals *A(x)* and *B(x)* will be combined together to form a new signal *C(x)*, and complete overlapping of the two signals is assumed. A mask signal *S(x)* is designed to describe the mosaic region. Then the steps of 1-D signal mosaic are as follows:

step 1: Perform DWT on the two signals *A* and *B*, thus two sequences $c_{M,n}(A),d_{M,n}(A),\cdots,d_{2,n}(A),d_{1,n}(A)$ and $c_{M,n}(B),d_{M,n}(B),\cdots,d_{2,n}(B),d_{1,n}(B)$ will be obtained respectively.

step 2: Choose an low pass filter *h'* and use *h'* to generate a sequence of low-passed and subsampled signals $c'_{M,n}(S),\cdots,c'_{2,n}(S),c'_{1,n}(S)$ of the mask signal *S*.

step 3: Construct a sequence of signals $c_{M,n}(C),d_{M,n}(C),\cdots,d_{2,n}(C),d_{1,n}(C)$ by using the $c'_{M,n}(S),\cdots,c'_{2,n}(S),c'_{1,n}(S)$ as the weighting functions in the corresponding resolution level, that is

$$c_{M,n}(C)=c_{M,n}(A)*c'_{M,n}(S)+c_{M,n}(B)*(1-c'_{M,n}(S))$$
$$d_{k,n}(C)=d_{k,n}(A)*c'_{k,n}(S)+d_{k,n}(B)*(1-c'_{k,n}(S)),\quad 1\le k\le M. \tag{4}$$

step 4: Perform inverse DWT on $c_{M,n}(C),d_{M,n}(C),\cdots,d_{2,n}(C),d_{1,n}(C)$ to obtain the mosaic signal *C*.

An example is given in Figure 3.

B. Extension to 2-D image mosaic

There are various extensions of 1-D wavelet transform to higher dimensions and Mallat's method is adopted in this works. The 2-D wavelet basis function can then be expressed by the tensor product of two 1-D wavelet basis functions along the horizontal and vertical directions. Then, the scaling function is

$$\phi(x,y)=\phi(x)\phi(y) \tag{5}$$

and the three 2-D wavelets are defined as

$$\psi^{H}(x,y)=\phi(x)\psi(y)$$
$$\psi^{V}(x,y)=\psi(y)\phi(x)$$
$$\psi^{D}(x,y)=\psi(x)\psi(y) \tag{6}$$

And the corresponding 2-D image decomposition and reconstruction are shown in Figure 4.

Similar to the 1-D case, a 2-D mask signal S is designed to describe the 2-D mosaic region. Thus, the 2-D image mosaic steps are as follows:

step 1: Perform 2-D DWT of two images A and B, thus two sequences

$c_{M,n}(A),d^{V}_{M,n}(A),d^{H}_{M,n}(A),d^{D}_{M,n}(A),\cdots,d^{V}_{1,n}(A),d^{H}_{1,n}(A),d^{D}_{1,n}(A)$

and

$c_{M,n}(B),d^{V}_{M,n}(B),d^{H}_{M,n}(B),d^{D}_{M,n}(B),\cdots,d^{V}_{1,n}(B),d^{H}_{1,n}(B),d^{D}_{1,n}(B)$

will be respectively obtained.

step 2: Choose a low-pass filter *h'(x,y)*, which could be separable (i.e. *h'(x,y)=h'(x)h'(y)*, *h'(x)* is 1-D low-pass filter), and use *h'*(x,y) to generate a sequence of low-passed and subsampled signals $c'_{M,n}(S),\cdots,c'_{2,n}(S),c'_{1,n}(S)$ of the mask signal S. An example of mask signal and *h'(x)* is shown in Fig. 5.

step 3: Construct a sequence of signals

$c_{M,n}(C),d^{V}_{M,n}(C),d^{H}_{M,n}(C),d^{D}_{M,n}(C),\cdots,d^{V}_{1,n}(C),d^{H}_{1,n}(C),d^{D}_{1,n}(C)$

by using the $c'_{M,n}(S),\cdots,c'_{2,n}(S),c'_{1,n}(S)$ as the weighting functions in the corresponding resolution level, that is

$$
\begin{aligned}
c_{M,n}(C) &= c_{M,n}(A)*c'_{M,n}(S)+c_{M,n}(B)*(1-c'_{M,n}(S)),\\
d^{V}_{k,n}(C) &= d^{V}_{k,n}(A)*c'_{k,n}(S)+d^{V}_{k,n}(B)*(1-c'_{k,n}(S)),\\
d^{H}_{k,n}(C) &= d^{H}_{k,n}(A)*c'_{k,n}(S)+d^{H}_{k,n}(B)*(1-c'_{k,n}(S)),\\
d^{D}_{k,n}(C) &= d^{D}_{k,n}(A)*c'_{k,n}(S)+d^{D}_{k,n}(B)*(1-c'_{k,n}(S)),\\
& 1\le k\le M.
\end{aligned} \tag{7}
$$

step 4: Perform inverse DWT on $c_{M,n}(C), d^{V}_{M,n}(C), d^{H}_{M,n}(C), d^{D}_{M,n}(C), \cdots, d^{V}_{1,n}(C), d^{H}_{1,n}(C), d^{D}_{1,n}(C)$ to obtain the mosaic signal C.

An example is given in Figure 6.

Besides, this 2-D decomposition provides subimages corresponding to different resolution levels and orientations (i.e. horizontal, vertical and diagonal subimages), hence the weighted average techniques with specific orientation concern make image mosaic more flexible as users like.

4. DISCUSSION

The multiresolution mosaic using wavelet transform method is proposed in this work. The question about performance measurement, arbitrary shape mosaic, decomposition level M and texture mapping in this proposed method are worthy of giving further discussion and are as follows:

4.1 Performance measurement

The mosaic techniques used to combine two signals should result in a quite smooth and invisible seam, while preserve the characteristic of each signal with as little distortion as possible. But, it is too hard to measure if a mosaic technique works well, even a smooth transition does not ensure an invisible seam. Actually, how a mosaic technique works is quite subjective. Therefore, multiresolution signal processing, as is driven by visual signal processing, is used here to do the signal mosaic.

4.2 Arbitrary shape mosaic

As described in section 3, multiresolution subsignals of the mask signal S are used as the weighting functions in the corresponding resolution level, that is, the low-pass filter h' which contracts the multiresolution structure decides the final weighting functions. As shown in Figure 3, three different low-pass filters h' are used to perform the mosaic. The shape of h' in Fig. 3(d) is Gaussian-like but broader than that of Gaussian, while that in Fig. 3(f) is in triangular shape. Accordingly, they corresponds to the weighting functions with smoother, normal and sharper transition zone. However, as illustrated in Figs. 3(d), (e) and (f), the influences of different low-pass filters h' are limited in multiresolution case.

4.3 Decomposition level M

According to the multiresolution mosaic method, the actual transition zone of the low frequency components is boarder than that of the high-frequency components. Therefore, for larger decomposition level M, the actual transition zone of the lowest frequency component is boarder too. The smoother transition will be obtained across the seam at the price of losing some individual properties, as shown in Figure 3(d), (e) and (f).

4.4 Texture Mapping

If one of the original images is an texture image, then the texture mapping effect can be achieved by the proposed method as shown in Figs. 7(a) and (b). Another form of image mosaic - changing the grain of an image without affecting its character, can also be supported by the method. In Fig. 7(c), for example, the low-frequency components of Lena are preserved as the low-frequency components of the mosaic image, and only the high frequency components of Lena and texture image are used to combine with the mask signal, that is

$$
\begin{aligned}
c_{M,n}(C) &= c_{M,n}(A)\\
d_{k,n}(C) &= d_{k,n}(A)*c'_{k,n}(S)+d_{k,n}(B)*(1-c'_{k,n}(S)),\\
& 1\le k\le M.
\end{aligned} \tag{8}
$$

Moreover, in texture analysis and segmentation, multiresolution techniques [4], [5], [6] have been utilized to characterize multiple oriented spatial frequencies of texture efficiently. In the above image mosaic step, combine only parts of the transformed components with specific orientation would change the grain of an image as the specific orientation of the texture. For example, image mosaic with vertical (or diagonal) grain of the texture can be obtained by combining vertical (or diagonal) components, as shown in Figs. 7(d) and (e). More complex grain of texture can be synthesized with the aid of wavelet packets [4] and nonrectangular sampling geometry [12].

5. CONCLUSION

Wavelet transform is proposed as a tool for image mosaic in this paper. A binary mask image is used to simplify and generalize the arbitrary shape mosaic. It is shown that some special mosaic effects can be achieved by combining components with specific frequency or orientation by using the proposed method.

The wavelet transform mosaic techniques could be extended to 3-D mosaic follow the similar way. Also, for video sequences, two video sequences can be combined or temporally overlapped to achieve special effects with proper design of each corresponding mask image.

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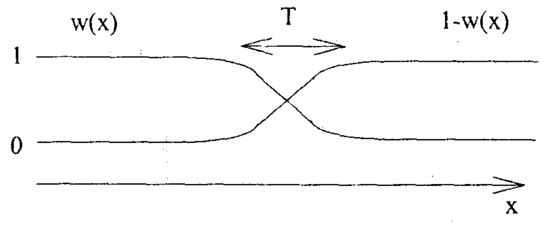


Figure 1: The weighted average function w(x).

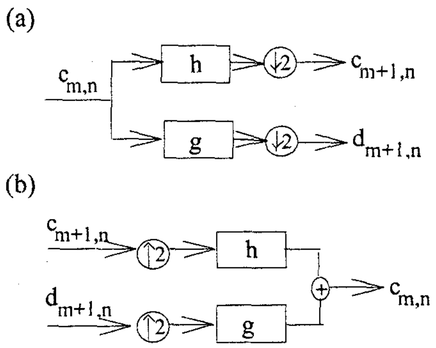


Figure 2: One stage of multiresolution signal decomposition (a) and reconstruction (b).

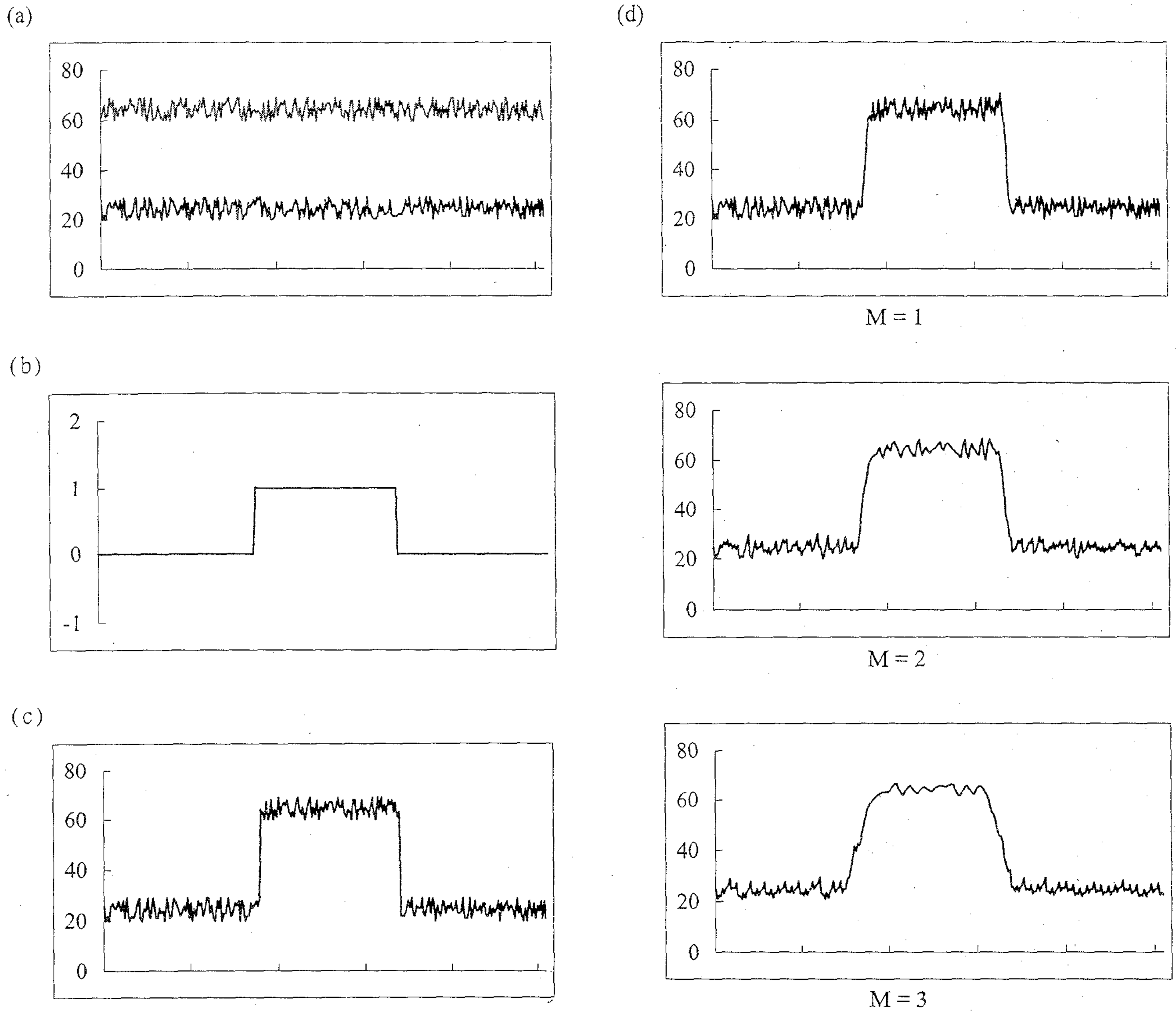


Figure 3: 1-D signal mosaic; (a) two 1-D signals A and B; (b) the mask signal S; (c) a directly combined signal without any mosaic technique; (d)(e)(f) the mosaic signals with scale number M=1, 2, and 3, and the low-pass filter h' of (d) corresponds to a smooth weighting function, and those of (f) corresponds to a sharp weighting function, while low-pass filter h' of (e) corresponds to a weighting function which is sharper than those of (d) and smoother than those of (f).

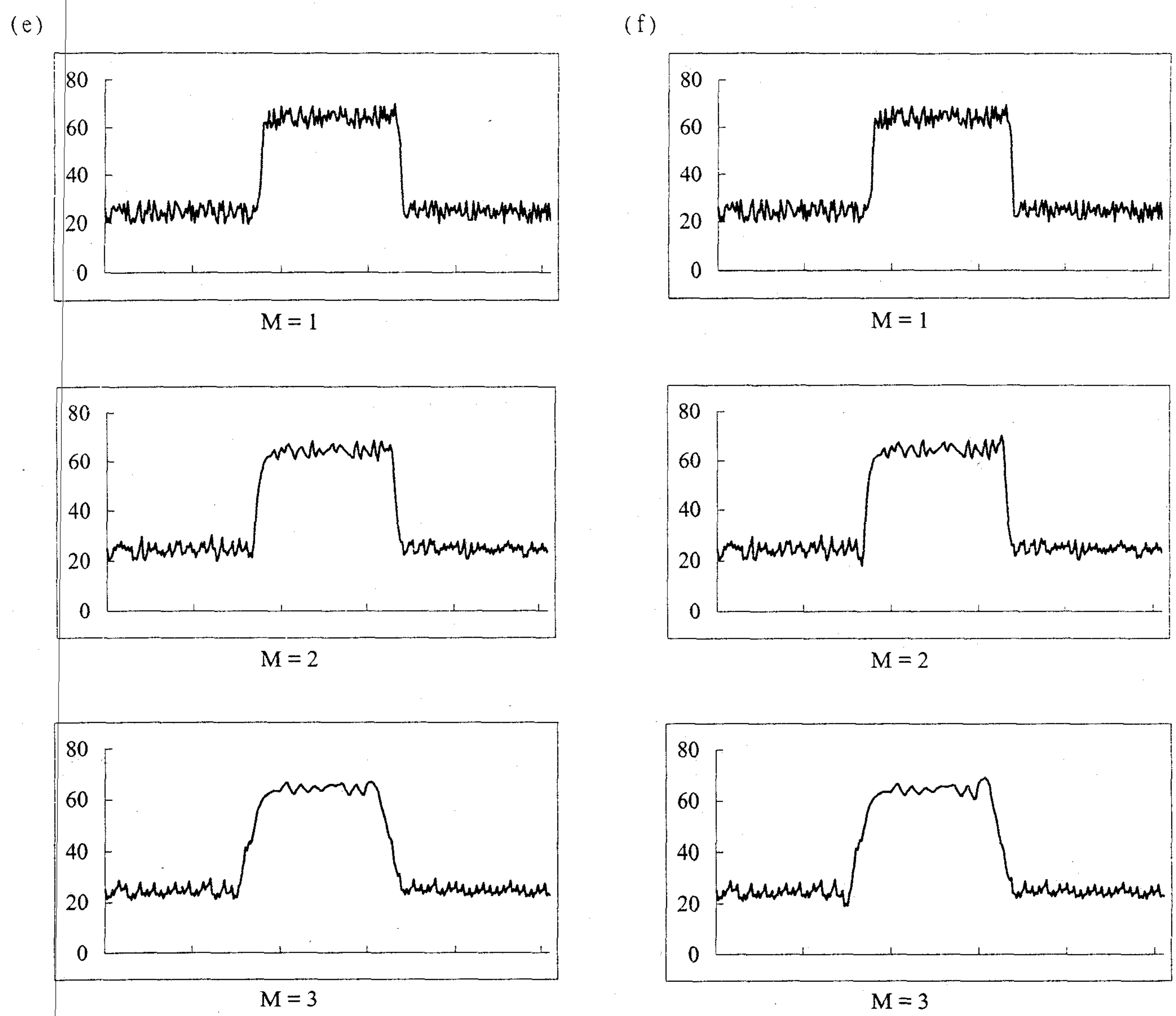
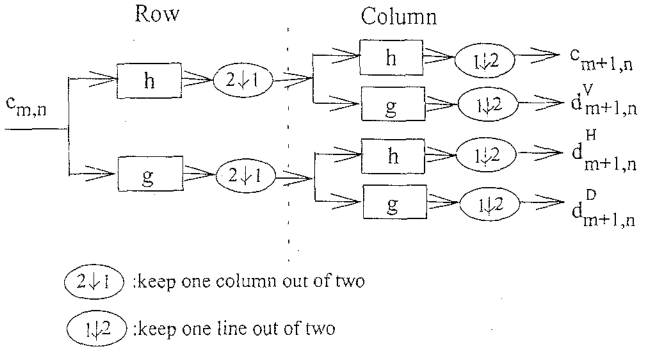


Figure 3 (continued)

(a)



(b)

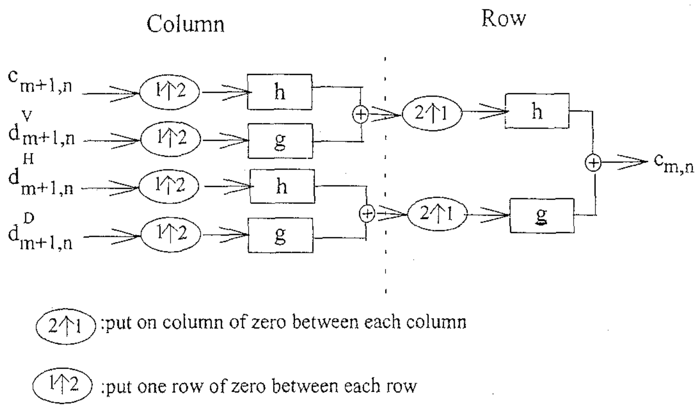


Figure 4 : One stage of multiresolution image decomposition (a) and reconstruction (b).

(a)

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(b)

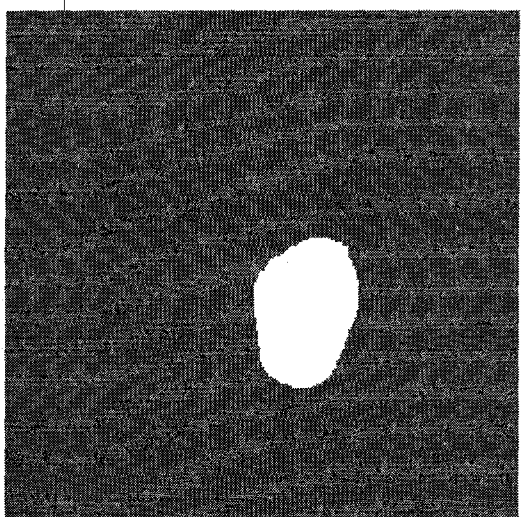
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0.0625	0.25	0.25	0.0625	0	0
0	0	0.25	1	1	0.25	0	0
0	0	0.25	1	1	0.25	0	0
0	0	0.0625	0.25	0.25	0.0625	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Figure 5: (a) Example of an 8×8 2-D mask signal S, (b) the low-passed and subsampled signal $c'_{1,n}(S)$ by applying an triangular low-pass filter h'={0.25,0.5,0.25}.

(a)

(b)

(c)



(d)

Figure 6: Image mosaic; (a) Lena; (b) fruit; (c) the mask image; (d) the mosaic image, where Lena's eyes, lips and nose are embedded into the red pepper.

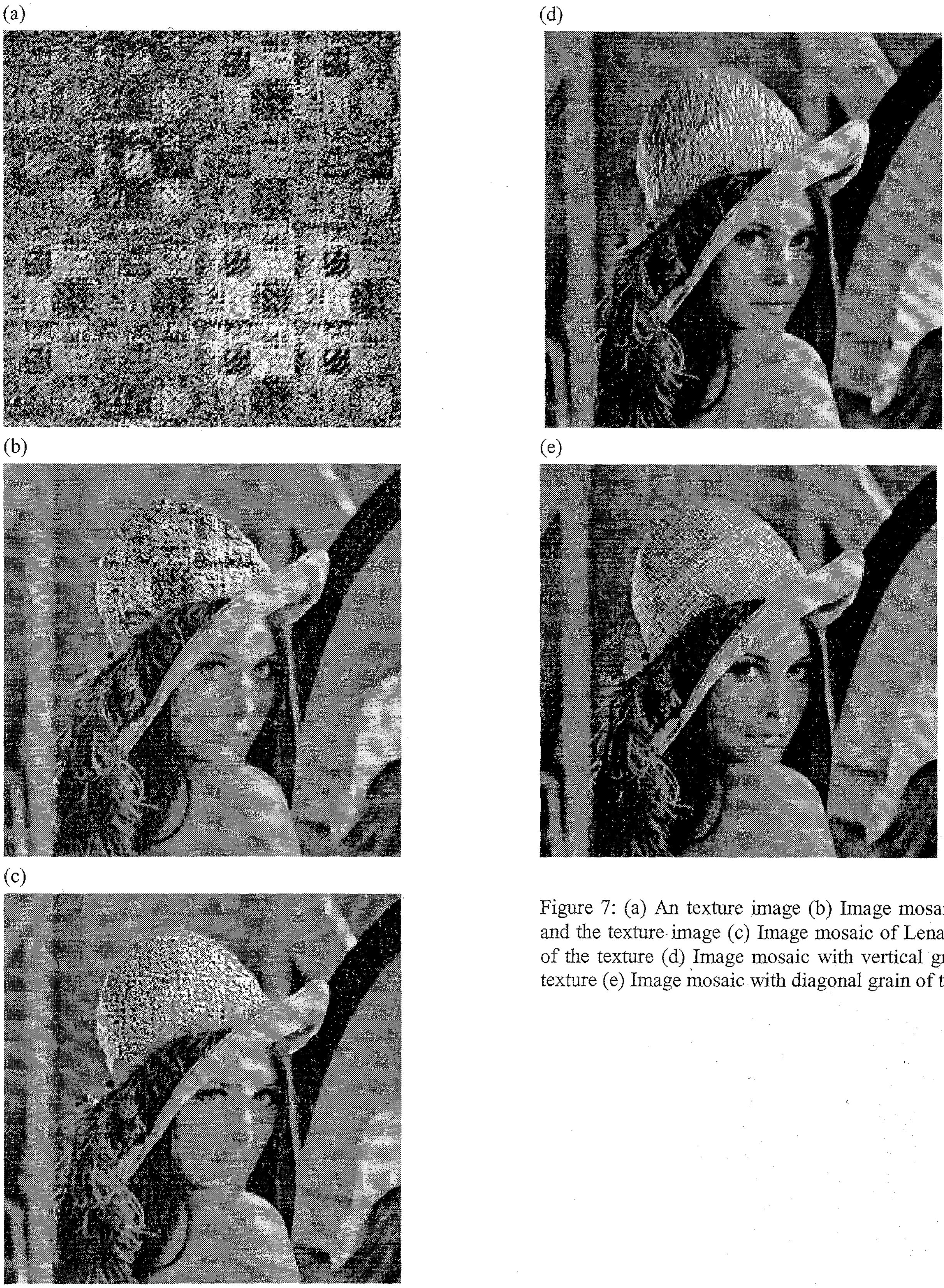


Figure 7: (a) An texture image (b) Image mosaic of Lena and the texture image (c) Image mosaic of Lena and grain of the texture (d) Image mosaic with vertical grain of the texture (e) Image mosaic with diagonal grain of the texture.