# 行政院國家科學委員會專題研究計畫 成果報告

## 公共財的穩定網路結構
## 研究成果報告(精簡版)

計 畫 類 別 ： 個別型
計 畫 編 號 ： NSC 95-2415-H-002-022-
執 行 期 間 ： 95 年 08 月 01 日至 96 年 07 月 31 日
執 行 單 位 ： 國立臺灣大學經濟學系暨研究所

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處 理 方 式 ： 本計畫可公開查詢

中 華 民 國 96 年 10 月 29 日

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# 行政院國家科學委員會補助專題研究計畫 □ 成果報告 □期中進度報告

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計畫類別：■ 個別型計畫 □ 整合型計畫
計畫編號：NSC 95－2415－H－002－022
期間： 2006 年 08 月 01 日至 2007 年 07 月 31 日

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中 華 民 國 96 年 10 月 29 日

# A Stable Network Structure of Public Goods

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October 29, 2007

**Abstract**

We aim to analyze the relationship between public goods provision and network structures, and the stability of the networks. We determine the stable networks by examining the properties of pairwise stability. A proper form of utility function also is introduced in our model to replace the network value function, which often makes the equilibrium outcomes depended on the network structures. We claim that a weakly stable network always exists but some architectures of networks may not be supported by strong stability.

**Keywords**: Stable networks, public goods.
**JEL Classification**: C72, D62, H41.

## 1 Introduction

Network structures play a very important role in determining the outcome of economic situations. Also the science of social networks is a significant field of sociological study and has been extensively documented. In recent years, theoretical models of network can be found in a vast literature and widely examined in various applications, such like the relationship between social network structures and labour market outcomes, the trade and exchange of goods in non-centralized markets, research and collusive alliances among companies, election results in political party networks.

However, most of those studies emphasis that the network structure is the only key determinant of individuals' utility in the society. An individual's value function only depends on a full or a reduced structure of the network. Externalities are generated by particular linkages and special structures, rather than the provision of goods offered by connected components. I would like to argue that a piece of puzzle is missing under such assumptions. For instance, a music-loving student may connect her laptop to another user's computer through a campus

*I am grateful for financial support from the National Science Council, Taiwan (NSC95-2415-H002-022).

network, and gets permit to share music files in other persons' computers. Thus we can say that her utility is not only depended on the number of computers connected to her laptop, but also the storage size of music files in those computers. No one will get benefit by connecting to an empty storage computer. Once a certain network is formed, those students may start to consider whether they should go to buy a music CD or wait until other students buy and store their music CDs in their computers. This example shows us that a variant amount of public goods provision will affect an individual's value even when the network structure is fixed. I would like to consider a network formation model which can fit in the above situation and find what the equilibrium network structure is.

There is a well established and vast literature in network formation. One question often asked in literature is how we predict a network is likely to form when individuals have the discretion to decide their connections. One also would like to check how efficient is a network and how does an equilibrium formation of network depend on the way that the value of a network is allocated among the individuals. Jackson (2003) surveys the literature on the formation of networks in recent years and shows us a clear picture of the relationships between different formation models. Bala and Goyal (2000) present a model to explain how a network is formed under a non-cooperative game. Their basic notion of network formation is based on the situation that an individual trades off the costs of forming and maintaining connections against the potential rewards from making so. They provide a characterization of the architecture of equilibrium networks, and study the dynamics of network formation. They find that individual efforts to access benefits offered by others lead to the emergence of an equilibrium network. And the possible limiting networks are few simple architectures, or generalizations of these simple architectures. Watts (2001) studies the process of network formation in a dynamic framework and determines which network structures the process converges to. She shows that if individuals are myopic and the benefit from maintaining an indirect link of length two is greater than the net benefit from maintaining a direct link, then the unique efficient network is difficult to form. By assuming that individuals' payoffs are depended on the network of connections among them, Jackson and Watts (2002) analyze a dynamic formation and stochastic evolution of networks. In their model, individuals can from and serve links over time based on the improvement between the current and new networks. They claim that predictions still can be made in spite of the likelihood that the stochastic process will lead to any given networks. Although most of studies are focusing on the process of network formation, there are still few researches to analyze how the payoff to one individual is affected by another connected individual's action. Sundararajan (2005) presents a model of local network effects in which the payoff to a individual connected in a network is influenced by the actions of her neighbors. He studies the adoption of a network good that displays local network effects, and shows that the symmetric Bayes-Nash equilibria of an adoption game can be strictly Pareto-ranked based on a scalar neighbor-adoption probability value. Bramoulle and Kranton (2007) introduce a social learning model under a network. They show their structure can foster specialization and specialization can have welfare benefits. They also claim that the extreme specialized equilibrium is the only outcome. Yuan (2005) demonstrates the possible multiplicity of Nash equilibria in a local public goods network. Unlike the assumption of homogenous players in BK's model, I offer a heterogeneous setting of players' preferences. Thus my model will have a more general result in public goods network study.

In this paper, I will present a network formation model which involves two main issues. First, the social networks are formed by individual decisions. I suppose a link with another individual allows access to the benefits available to the latter. However, an indirect link won't generate externalities. This assumption especially fits many intellectual property cases. It is illegal to download video game software from another individual's computer, but it is legal to play with the host through network connections. This means an individual only can receive benefit from another person who she has a direct link with. Secondly, individuals also make decisions to choose the provision of goods under the social network structures.

# 2 Model

The outline assumption of the game of local public goods provision in networks comes from Yuan (2005). Let us consider a local public goods game with $n$ players. We assume the circumstance in which the local public goods are provided by means of private provision by players. Let $I = \{1, 2, \cdots, n\}$, $n \geq 2$ be the set of players. For any pair of players $i, j \in I$, the pair-wise relationship between the two players is represented by a variable $r_{ij} \in [0, 1]$. $r_{ij}$ is a parameter which represents the degree of externalities of the local public goods provided by player $j$ to player $i$. It shows how much service player $i$ can receive from one unit of player $j$'s provision. In other words, one unit of the local public goods provided by player $j$ will offer player $i$ the same quantity of service as $r_{ij}$ unit does when the $r_{ij}$ unit is provided by player $i$ herself. When $r_{ij} = 1$, we say that player $i$ can receive a full service of player $j$'s local pubic good provision. Or we can say player $j$'s provision is just like a purely public good to player $i$. A player always receives the whole service of her own provision, so $r_{ii} = 1$. When $r_{ij} = 0$, it refers to the case that player $i$ receives no service from player $j$'s provision, or we can say player $j$'s provision is like a private good to player $i$.

In this local public goods model, we assume that there is a boundary to limit the users of a specific local public good. If we suppose player $j$ provides a local public good, then $r_{ij} = 0$ when player $i$ is located outside the boundary.

Now suppose that each player only provides one specific local public good. Since two different kinds of local public goods may have two different effective ranges of externalities, one player could benefit by another player's provision but the reverse situation is not true. It means $r_{ij} = 1$ but $r_{ji} = 0$ for some $i, j \in I$. Different local public goods may also offer different degrees of externalities, so $r_{ij} \neq r_{ji}$ for some $i, j \in I$. We even can assume the territory or the degree of externalities varies with the level of a provision. The variable $r_{ij}$ then should be replaced by a function $r_{ij} : [0, \bar{x}_j] \to [0, 1]$.

To avoid our analysis involving those complex situations of the asymmetric externalities between different pubic goods or units, we assume that the local public good are perfectly substitute. Although the provisions from different positions should be viewed as distinct local public goods, we think that the territory (or the degree) of the externality is exactly same for any unit of provision at any position. This assumption immediately implies $r_{ij} = r_{ji} = 1$ for any $i, j \in I$.

Let $L_i$ be a set of players and defined by $L_i \equiv \{j \mid j \neq i, j \in I \text{ and } r_{ij} > 0\}$. We can say that any player who belongs to the set $L_i$ is located in the certain area where the residents receive the externality of player $i$'s provision. Since we have assumed that $r_{ij} = r_{ji}$ for any $i, j \in I$, then we can imply that $j \in L_i$ if and only if $i \in L_j$.

Let $G(I, r)$ be a simple graph in which at most one edge (i.e., either one edge or no edges) may connect any two vertices. A network $r$ is the collection of the pair-wise relationships, $r = \{r_{ij}\}_{i,j \in I}$. The players, on the integers $1, 2, \cdots, n$, are located on the vertices of a simple graph. When $r_{ij} = 1$, we say that the two players $i$, $j$ are linked and one edge will connect the two corresponding vertices. And $r_{ij} = 0$ refers to the case of no link. To avoid confusion, we assume no vertex is self-connected. $L_i$, the set of players who are linked to player $i$, is written by $L_i = \{j \mid j \neq i, j \in I \text{ and } r_{ij} = 1\}$. Notice that we do not consider player $i$ belonging to the set $L_i$. In other words, a player is not linked to herself. For any given game $G$ with a set of players $I$ and a network $r$, there always is a corresponding graph $G(I, r)$, and vice versa.

Let the local public good provided by a player be also indexed by the same set of integers. The player $i$'s provision of the local public good is denoted by $x_i$, $x_i \geq 0$. The utility function of player $i$ is:

$$u_i = V_i(X_i) - c_i(x_i), \tag{1}$$

where

$$X_i = \sum_{j=1}^{n} r_{ij} x_j = x_i + \sum_{j \in L_i} x_j. \tag{2}$$

$X_i$ denotes the total service quantity which player $i$ can receive from all players. We also define $X_{-i} = X_i - x_i$. $X_{-i}$ is the total service quantity which player $i$ can receive from the other players' provisions. Let $V_i(\cdot)$ be an increasing, differentiable and concave function. $c_i(\cdot)$ is the cost function for player $i$. Suppose

$$c_i(x_i) = c_i x_i. \tag{3}$$

The marginal cost of providing any unit of the local public good is constant, but each player has her own marginal cost. We also assume that $V_i'(0) > c_i$ for all $i \in I$. Let us define $\bar{x}_i$ by assuming that $\bar{x}_i$ solves $V_i'(x_i) = c_i$. Here $\bar{x}_i$ can be seen as the optional amount of consumption for player $i$ if she pays for her own provision. It is clear that no player $i$ will provide an amount of $x_i$ such that $x_i > \bar{x}_i$. Since $V_i$ is an increasing, differentiable and concave function and $V_i'(0) > c_i$, we know $\bar{x}_i$ must exist and it is unique and positive.

## 2.1 Equilibrium Provision

In the local public goods model, each player will choose a provision $x_i$ as her strategy. Any strategy $x_i > \bar{x}_i$ is strictly dominated for player $i$. Hence we can let the strategy space of $x_i$ is $S_i = [0, \bar{x}_i]$. This fits the first condition in proposition. We assumed the utility function (or payoff function) $u_i = V_i(X_i) - c_i x_i$ and $V_i(\cdot) : \mathbb{R}^+ \cup \{0\} \to \mathbb{R}$ is an increasing, differentiable and concave function. $c_i x_i$ is a linear function of $x_i$. So $u_i$ is a differentiable and concave function which fits the second proposition condition. Therefore we can guarantee that there always exists a pure strategy Nash equilibrium in the local public goods model.

Although the above proposition tells us that a pure strategy Nash equilibrium does exist in the local public goods game. We still need to find a certain method which can allow us to practically examine an equilibrium. In the following lemma, we will give the necessary and sufficient conditions for a pure strategy Nash equilibrium.

**Theorem 1** $(x_1^*, x_2^*, \cdots, x_n^*)$ *is a pure strategy Nash equilibrium if and only if: (1)* $X_i^* = \bar{x}_i$ *if* $x_i^* > 0$ *and (2)* $X_i^* \geq \bar{x}_i$ *if* $x_i^* = 0$.

**Proof.** *Necessity.* Suppose $(x_1^*, x_2^*, \cdots, x_n^*)$ is a Nash equilibrium, then $x_i^*$ should maximize player $i$'s payoff: $u_i = V_i(x_i + X_{-i}^*) - c_i x_i$ for any $i \in I$. For any $x_i^*$, we know either $x_i^* > 0$ or $x_i^* = 0$. The first order condition $V_i'(x_i + X_{-i}^*) - c_i = 0$ must hold when player $i$ chooses $x_i = x_i^* > 0$. By the definition of $\bar{x}_i$, we know $x_i^* + X_{-i}^* = X_i^* = \bar{x}_i$. Thus we have proven the necessary condition (1). If $x_i^* = 0$ and we suppose $X_i^* < \bar{x}_i$, then $V_i'(X_i^*) = V_i'(X_{-i}^*) > V_i'(\bar{x}_i) = c_i$ (because $X_i^* = x_i^* + X_{-i}^* = X_{-i}^*$ and $V_i$ is concave). This result is in conflict with the maximum condition of Nash equilibrium. So we know $X_i^* \geq \bar{x}_i$ when $x_i^* = 0$. We have proven the necessary condition (2).

*Sufficiency.* Suppose players have chosen a provision vector $(x_1^*, x_2^*, \cdots, x_n^*)$ which satisfies the conditions (1) and (2). For those $x_i^* > 0$, $X_i^* = \bar{x}_i$ implies $x_i^* = \bar{x}_i - X_{-i}^*$. After giving the other players' choices $(x_1^*, \cdots, x_{i-1}^*, x_{i+1}^*, \cdots, x_n^*)$, $\frac{\partial u_i}{\partial x_i} |_{x_i = x_i^*} = V_i'(X_i^*) - c = V_i'(\bar{x}_i\ ) - c = 0$ shows us $x_i^*$ is the best response for player $i$. For any other $x_i^* = 0$, $\frac{\partial u_i}{\partial x_i} |_{x_i=0} = V_i'(X_i^*) - c \leq V_i'(\bar{x}_i\ ) - c = 0$ (because $X_i^* \geq \bar{x}_i$ and $V_i$ is concave) also tells us the choice $x_i^* = 0$ is the best response for player $i$ by given $(x_1^*, \cdots, x_{i-1}^*, x_{i+1}^*, \cdots, x_n^*)$. Therefore any $(x_1^*, x_2^*, \cdots, x_n^*)$ which satisfies conditions (1) and (2) must be a Nash equilibrium. ■

**Definition 2** *An equilibrium* $(x_1^*, x_2^*, \cdots, x_n^*)$ *is specialized if* $x_i^* > 0$ *then* $x_j^* = 0$ *for all* $j \in L_i$*; An equilibrium* $(x_1^*, x_2^*, \cdots, x_n^*)$ *is distributed if* $\exists i \in I$ *st.* $x_i^* > 0$ *and* $x_i^* > 0$ *for some* $j \in L_i$.

## 2.2 Stable Equilibrium

By examining the possible provision profiles, we can easily find the multiplicity of equilibria in certain networks by using the theorem 1. For example, there three possible equilibria (2,0,2), (0,3,2) and (1,1,1) in a three-player star network case, when the network $g = \{r_{12}, r_{23}\}$ and $(\bar{x}_1, \bar{x}_2, \bar{x}_3) = (2, 3, 2)$. This makes the task to judge the pairwise stability extremely difficult because players may be uncertain about their payoffs once a new link is formed or an existed link has been vanished.

To solve the uncertain problem, we wish to reduce the number of equilibria in our setting. We consider stable equilibria by using the notion of Nash tatonnement. Define $b_i(\mathbf{x}_{-i})$ as player $i$'s best response function to a profile $\mathbf{x}_{-i}$ and define $b$ as the collection of these players' best responses $b = (b_1, \cdots, b_n)$. An equilibrium $\mathbf{x}$ is stable if and only if a positive number $\rho > 0$ such that for any vector $\boldsymbol{\varepsilon}$ satisfying $\forall |\varepsilon_i| < \rho$ and $x_i + \varepsilon_i \geq 0$ the sequence $\mathbf{x}^{(t)}$ defined by $\mathbf{x}^{(0)} = \mathbf{x} + \boldsymbol{\varepsilon}$ and $\mathbf{x}^{(t+1)} = b(\mathbf{x}^{(t)})$ converges to $\mathbf{x}$.

this standard notion yields a result that all distributed equilibria are unstable. However, multiple stable specialized equilibria may still exist.

**Theorem 3** *An equilibrium $\mathbf{x}^* = (x_1^*, x_2^*, \cdots, x_n^*)$ is stable if and only if $\mathbf{x}^*$ is specialized and for any $x_i^* = 0$, $X_{-i}^* > \bar{x}_i$ .*

**Proof.** If $\mathbf{x}^*$ is distributed, we know it is not stable by applying Bramoulle and Kranton's proof. Suppose now $\mathbf{x}^*$ is specialized but $\exists i$, st. $x_i^* = 0$, $X_{-i}^* = \bar{x}_i$. We can prove $\mathbf{x}^*$ is unstable still by applying B&K's proof. Since an equilibrium must be either distributed or specialized, and a specialized equilibrium is either $\exists i$, st. $x_i^* = 0$, $X_{-i}^* = \bar{x}_i$ or for any $x_i^* = 0$, $X_{-i}^* > \bar{x}_i$. We have proven the first two types of possible equilibria are not stable. If we can prove all equilibria in the third type are stable, then we can get both necessary and sufficient conditions. Consider that $\mathbf{x}^*$ is specialized and for any $x_i^* = 0$, $X_{-i}^* > \bar{x}_i$. Let $\rho = \frac{1}{2}\left[\min_{k \in \{i | x_i^* = 0\}} \{X_{-k}^* - \bar{x}_k\}\right]$. For $\forall |\varepsilon_i| < \rho$ and $x_i^* + \varepsilon_i \geq 0$, We know $\mathbf{x}^{*(1)} = \mathbf{x}^*$. Thus $\mathbf{x}^*$ is stable. ■

Now we know once the network $g$ is given, we can figure out all possible stable equilibria. However, a stable equilibrium does not exist in some given network.

# 3 Heterogeneity and Stability

Herterogenrity of players' preferences may cause multiplicity of stable equilibria in public goods provision. We know that the concept of stable networks must carefully be chosen and always connected to the sense of stability of provision. Thus we may need to make more assumptions on players' preferences in order to define stable networks of public goods provision.

**Definition 4** *A network of public goods $g$ is strongly pairwise stable with respect to a provision profile $\mathbf{x}$ if (1) $\mathbf{x}$ is a stable equilibrium under $g$; (2) for any $i \in I$*

*and $j \in L_i$, $u_i(\mathbf{x}) \geq u_i(\mathbf{x}')$ for any stable equilibrium $\mathbf{x}'$ under the network $g - r_{ij}$; and (3) for any $i \in I$ and $j \notin L_i$, if $u_i(\mathbf{x}) \geq u_i(\mathbf{x}')$ then $u_j(\mathbf{x}) < u_i(\mathbf{x}')$ for any stable equilibrium $\mathbf{x}'$ under the network $g + r_{ij}$.*

**Definition 5** *A network of public goods $g$ is weakly pairwise stable with respect to a provision profile $\mathbf{x}$ if (1) $\mathbf{x}$ is a stable equilibrium under $g$; (2) for any $i \in I$ and $j \in L_i$, $u_i(\mathbf{x}) \geq u_i(\mathbf{x}')$ for at least one stable equilibrium $\mathbf{x}'$ under the network $g - r_{ij}$; and (3) for any $i \in I$ and $j \notin L_i$, if $u_i(\mathbf{x}) \geq u_i(\mathbf{x}')$ then $u_j(\mathbf{x}) < u_i(\mathbf{x}')$ for at least one stable equilibrium $\mathbf{x}'$ under the network $g + r_{ij}$.*

We have defined two different concepts of network stability: strongly pairwise stability and weakly pairwise stability. The strongly pairwise stability guarantees that no player can make herself possibly better off by severing any single link, and any two players never make them both possibly better off by forming a new link. We also can consider the strongly stability as a result of conservatively risk-aversion. Players take no risk when they play a network formation game under the concept of strong stability. On the other hand, the weakly pairwise stability presents players' risk-loving attitude. Even there is only a small chance that the player possibly makes herself better off, she will prefer to form or severe a link to make the chance possible.

It is not too difficult to understand that weakly pairwise stability is a necessary condition of strongly pairwise stability. We now need to prove that weakly pairwise stable networks always exist.

**Theorem 6** *A network of public goods $g$ is always weakly pairwise stable with respect to some provision profile $\mathbf{x}$.*

**Proof.** We need to know how to find a profile $\mathbf{x}$ which is able to make $g$ weakly pairwise stable. We develop a procedure to solve this problem. First we assign $x_i = \bar{x}_i$ if $\bar{x}_i = \max\{\bar{x}_1, \cdots, \bar{x}_n\}$ and $x_j = 0$ for all $j \in L_i$. Then consider a new network $g - \{r_{ij}\}$, which is the remaining network after exclude the player $i$ and $i$'s neighbour. Then we repeat the assigning process again until all $x_i$ has been assigned a number. Then we can prove that the network $g$ will be weak pairwise stable with respect to the profile $\mathbf{x}$. We will leave this proof to the Appendix. ■

We can give an example to show how a network $g$ can be weakly pairwise but failed to be strongly pairwise stable. Consider a network $g = \{r_{12}, r_{13}\}$ and $(\bar{x}_1, \bar{x}_2, \bar{x}_3) = (2, 3, 2)$. We can understand that network $g$ with respect to $(0, 3, 0)$ is weakly pairwise stable but not strongly pairwise stable. But the $g = \{r_{12}, r_{13}, r_{23}\}$ is a strongly pairwise stable network. To find the result, we can check all the possible cases no matter there is newly forming a link or sever an existed link. All possible change have been described by the figures among Fig. 1 and Fig. 4.

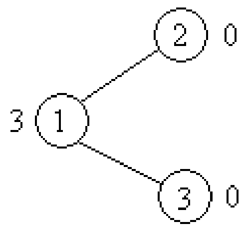


Fig. 1

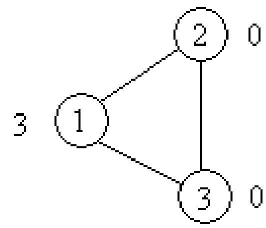


Fig. 2

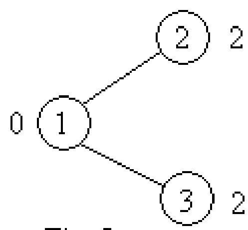


Fig. 3

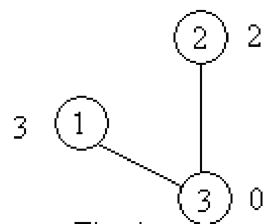


Fig. 4

## 4 Conclusion

This study built a model of public goods provision with endogenous network formation under the concept of pairwise stability. We first generalize the meaning of specialized and distributed equilibria under the setting of heterogeneous preferences. In the notion of Nash tatonnement, we find that we may still have multiple stable equilibria. We also find that the a maximal independent set of order 2 of networks is no longer the necessary and sufficient condition for stable equilibria. Our findings differ from those obtained by Bramoulle and Kranton (2007) for a homogeneous preferences model.

In our analysis of pairwise stability we identified the strong and weak stability between risk aversion dominance and risk loving dominance. When players' link decisions become risk averse, an endogenous network is much easier to be form through the formation game, and we proven that a weakly stable network always exist in the public goods provision model. However, if players link decisions become risk dominated, the concept of strong stability leads to a result of possibly empty set of stable networks.

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