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Globally Adaptive Decentralized Control of Time-Varying Robot Manipulators

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Abstract **– In this paper, we develop a globally adaptive decentralized control scheme of time-varying robot manipulators for trajectory tracking control. Since the proposed adaptive control law is in a decentralized manner, only low-cost hardware is required for implementation. Furthermore, even if the time-varying parameters of the robot manipulator change arbitrarily fast, both the position and velocity tracking errors of the manipulators will converge to zero after employment of the proposed adaptive control law. For practical implementation, the adaptive law can be combined with a leakage term so that there is no chattering in the motion of the manipulator but at the price a residual set of the tracking errors as mentioned above, of which the size can be however made smaller by use of some larger design parameters. Finally, in order to illustrate the performance of the proposed scheme, simulation results are also provided, which turn out to be quite satisfactory.**

I. INTRODUCTION

Research in the area of trajectory tracking control of robot manipulators has been widespread. Many industrial tasks of the robot manipulator such as material handling and part assembly involve such a problem. On the other hand, in many situations, some of the unknown parameters such as the mass of the payload or the mass of the links, may be time varying. For example, robotic pouring and filling operations belong to this case. A large number of control design exist in the literature that works well for both cases with the both known and unknown constant parameters [Slotine and Li, 7; Spong, 9]. However, it is worth noting that, because the time-varying parameters of robot manipulators enhance complexity of the overall systems, the control schemes developed for the time-invariant robot manipulators are not suitable for the time-varying ones [Reed and Ioannou, 6].

During recent years, quite a few research works have been devoted to the trajectory tracking control of time-varying robot manipulators. In Tao [12], a robust adaptive controller is proposed to guarantee asymptotic tracking in the presence of time-invariant parameters. However, the transient performance is related to the speed of parameter variations and cannot be arbitrarily improved. In Tomei [14], a robust adaptive control law is proposed which ensures the arbitrary transient performance. In Song and Middleton [8], a robust switching-type control law is proposed for the robot manipulators with time- varying parameters. In this research, the property of Hadarmad product is applied such that the dynamic model can be expressed in a parameterized form and the zero asymptotical tracking error can be obtained. In Su [11], an adaptive sliding mode control is proposed which guarantees asymptotic convergence of the tracking error. In the work by Pagilla *et al.* [5], with bounded time-varying parameters an adaptive control law is designed, of which the effectiveness is demonstrated by providing experimental results. In the literature, different control laws have been proposed for the tracking control of time-varying robot manipulators; however, most of them are in centralized manner. Since the decentralized controller structure is adopted by the majority of modern robots in favor of its computation simplicity and low-cost hardware setup [Fu, 2; Liu, 3; Tang *et. al.*, 13], how to best improve the tracking performance of time-varying robot manipulators through decentralized control becomes an interesting research topic.

In this study, a globally adaptive decentralized trajectory control law is proposed for time-varying robotic manipulators.

II. PROBLEM STATEMENT

In this section, the dynamic model of a time-varying robot manipulator is introduced and the difference between the time-invariant case and the time-varying one is also described. The present dynamic model is derived under on the assumption that the generalized constraints are independent of time but the payload mass can be time varying. It is also under on the assumption that the time-varying payload mass has no contribution to producing the generalized force. The dynamical model of robot manipulators is derived by Euler-Lagrange equations [Spong and Vidyasagar, 10]. Let $q = (q_1, \ldots, q_n)^T$ be the set of position variables of n joints of the robot manipulator. The scalar functions $K : R^n \times R^n \times [0, \infty) \to R$ and $P : R^n \times [0, \infty) \to R$ denote the kinetic energy and the potential energy, respectively, where apparently the kinetic energy $K : R^n \times R^n \times [0, \infty) \to R$ can be explicitly expressed as defined as follows

$$K = \tfrac{1}{2}\dot{q}^T M(q,t)\dot{q} \tag{1}$$

where $M : R^n \times [0, \infty) \to R^{n\times n}$ is the inertial matrix of the robot manipulator. It is worth noting that the inertial matrix M and the potential energy P are explicitly dependent on t such that the robot manipulator is called the time-varying one. The Lagrangian L is obtained by taking the difference between the kinetic energy K and potential energy P; i.e., $L = K - P$. After substituting the Lagrangian L into the Euler-Lagrange equations, the overall dynamics of the robot manipulator can be obtained as follows [Tao, 12]:

$$M(q,t)\ddot{q} + \tfrac{\partial M(q,t)}{\partial t}\dot{q} + C(q,\dot{q},t)\dot{q} + g(q,t) = \tau \tag{2}$$

where the (k, j) element, $1 \le k, j \le n$, of $C : R^n \times R^n \times [0, \infty) \to R^{n\times n}$ is:

$$c_{kj} = \sum_{i=1}^{n} \tfrac{1}{2}\left(\tfrac{\partial m_{kj}}{\partial q_i} + \tfrac{\partial m_{ki}}{\partial q_j} - \tfrac{\partial m_{ij}}{\partial q_k}\right)\dot{q}_i \tag{3}$$

with m_{ij} being the (i, j) element of M, and $g(q, t) = \partial V(q, t)/\partial q$. This dynamic model has the following properties that will be used in the controller design [Su, 11]:

Property 1: The inertial matrix M is a symmetric and positive definite matrix, which satisfies $\mu_m I \le M \le \mu_M I$, $\forall q \in R^n$, $\forall t \in [0, \infty)$, for some constants $\mu_m, \mu_M > 0$.

Property 2: The matrix C satisfies $\|C\| \le \mu_C \|\dot{q}\|$, $\forall q \in R^n$, $\forall \dot{q} \in R^n$, $\forall t \in [0, \infty)$, for some constant $\mu_C > 0$.

Property 3: $z^T(\dot{M} - \frac{\partial M}{\partial t} - 2C)z = 0$, $\forall z \in R^n$, $\forall t \in [0, \infty)$.

It is worth noting that, if the inertial matrix M is not explicitly independent of t, then Property 3 can be rewritten as: $z^T(\dot{M} - 2C)z = 0$, $\forall z \in R^n$, which is well-known in the literature of robot manipulators.

Property 4: The vector g satisfies $\|g\| \le \mu_G$, $\forall q \in R^n$, $\forall t \in [0, \infty)$, for some constant $\mu_G > 0$.

Compared with the time-invariant robot manipulators, besides that the parameters in the dynamic model of the time-variant robot manipulators are considered as time functions, additional terms related to the rate of change of time-varying parameters are also derived such that *Property 3* of time-varying robot manipulators is different from that of time-invariant robot manipulators.

Now let $q_d(t)$ denote the desired position trajectory for tracking, which is generally chosen twice differentiable to guarantee smoothness of the motion. Let $\tau^T = [\tau_1, \ldots, \tau_n]$. If the ith element, $i \in \{1, \ldots, n\}$, of the control input is only a function of joint configuration and velocity of the ith joint, the control input τ is called a decentralized control. In this study, a decentralized control scheme is to be developed such that $q(t) \to q_d(t)$ and $\dot{q}(t) \to \dot{q}_d(t)$ as $t \to \infty$.

III. CONTROLLER DESIGN

In this section, one adaptive decentralized control schemes will be developed for the control of a general n-link rigid time-varying manipulator. Let $q_d(t)$ denote the desired position trajectory. The following error signals are defined as $e = q - q_d$ and $s \equiv \dot{e} + \Lambda e$ where the feedback gain matrix Λ is a positive definite matrix. Now the dynamics defined by the signals e and s can be derived as:

$$\dot{e} = -\Lambda e + s \tag{4a}$$

$$M(q,t)\dot{s} = -C(q,\dot{q},t)s + \tau - v(t,q,\dot{q}) \tag{4b}$$

where

$$\begin{aligned} v(t,q,\dot{q}) = {} & M(q,t)(\ddot{q}_d + \Lambda\dot{e}) + C(q,\dot{q},t)(\dot{q}_d + \Lambda e) \\ & + g(q) - \tfrac{\partial M(q,t)}{\partial t}\dot{q} \end{aligned} \tag{5}$$

behaves as the perturbation term. In the following, without loss of generality, several technical assumptions are made to pose the problem in a tractable manner.

Assumption 1: The feedback gain matrix Λ is a diagonal positive definite matrix; that is, $\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_n) > 0$ for $\lambda_i > 0$, $i \in \{1, \ldots, n\}$.

Assumption 2: The desired position trajectory $q_d(t)$ and its time derivatives $\dot{q}_d(t), \ddot{q}_d(t)$ are all bounded time-varying signals.

Assumption 3: The time-varying parameters of robot manipulators are bounded and smooth functions in time. Furthermore, there is a positive constant $\eta > 0$ such that $\left\|\frac{\partial M}{\partial t}\right\| \le \eta$, $\forall q \in R^n$, $\forall t \in [0, \infty)$.

In this study, we start with investigating the following lemma. In this sequel, we adopts the truncated L_∞ norm defined as $\|x\|_{T,\infty} \equiv \max_{1\le i\le n} \sup_{t\in[0,T]}|x_i(t)|$ for all real vector-valued functions $x \in C^n[0, T]$ for some $T > 0$. To assess the tendency of growth, the following lemma will provide us useful facts to develop the control scheme in this study.

Lemma 1: *Under Assumption* 3, *if there exists a constant* $T > 0$ *such that* $\|s\|_{T,\infty}$ *exists, then there are positive constants* α_1, α_2, α_3, *and* α_4 *such that for* $t \in [0, T]$:

$$\|e(t)\| \le \alpha_1\|e_0\| + \alpha_2\|s\|_{T,\infty} \tag{6}$$

$$\|\dot{e}(t)\| \le \alpha_3\|e_0\| + \alpha_4\|s\|_{T,\infty} \tag{7}$$

where $e_0 = e(0)$ *is the initial condition of* e, *and positive constants* β_1, β_2, *and* β_3 *are such that for* $t \in [0, T]$:

$$\|\bar{v}(t,q,\dot{q})\| \le \beta_1 + \beta_2\|s\|_{T,\infty} + \beta_3\|s\|_{T,\infty}^2 \tag{8}$$

where $\bar{v}(t,q,\dot{q}) \in [0,\infty)\times \mathrm{R}^n \times \mathrm{R}^n \to \mathrm{R}^n$ *is defined as follows*:

$$\bar{v}(t,q,\dot{q}) = v(t,q,\dot{q}) - \tfrac{1}{2}\tfrac{\partial M(q,t)}{\partial t}s \tag{9}$$

Proof: Consider the linear system defined by $\dot{e} = -\Lambda e + s,\ e(0) = e_0$. Since the matrix $-\Lambda$ is Hurwitz, we obtain (6) for $t \in [0, T]$ where α_1 and α_2 are some positive parameters. Furthermore, according to the definition of the linear systems, we then obtain (7) for $t \in [0, T]$ where α_3 and α_4 are appropriate positive parameters. Next, according to the definition of the perturbation term v in (5) and Assumption 3, we have (8) for $t \in [0, T]$, where β_1, β_2, and β_3 are some chosen positive parameters. ■

Now, an adaptive decentralized control scheme is presented to solve this tracking control problem. Consider the control law $\tau^{\mathrm{T}} = [\tau_1, \ldots, \tau_n]$ given as follows:

$$\tau_i = \begin{cases} -\frac{1}{\varepsilon_i}\hat{\theta}_{i\_1}^2 s_i - \theta_{i\_2}s_i - \theta_{i\_3}s_i^3 & \text{if } \hat{\theta}_{i\_1}|s_i| \le \varepsilon_i \\ -\hat{\theta}_{i\_1}\operatorname{sgn}(s_i) - \theta_{i\_2}s_i - \theta_{i\_3}s_i^3 & \text{if } \hat{\theta}_{i\_1}|s_i| > \varepsilon_i \end{cases} \tag{10}$$

for $i = 1, \ldots, n$, where the parameter $\hat{\theta}_{i\_1}$, $i \in \{1, \ldots, n\}$ needs to be adjusted on line, and $\theta_{i\_j} > 0$, $i \in \{1, \ldots, n\}$ and $j \in \{2, 3\}$, and $\operatorname{sgn}(\cdot)$ denotes the signum function [Khalil, 4]. Furthermore, the auxiliary signal ε_i, $i \in \{1, \ldots, n\}$, satisfies:

$$\dot{\varepsilon}_i = -p_i\varepsilon_i, \quad \varepsilon_i(0) > 0 \tag{11}$$

where $p_i > 0$. It is worth noting that the time-varying signal ε_i, $i \in \{1, \ldots, n\}$, are always positive, which are used as the time-varying boundary layers. In order to account for the parametric uncertainty from the manipulator and the desired position trajectories, we choose the adaptive law as follows

$$\dot{\hat{\theta}}_{i\_1} = \gamma_{i\_1}|s_i| \tag{12}$$

where $\hat{\theta}_{i\_1}(0) \ge 0$, and the adaptive gain $\gamma_{i\_1} > 0$, $i \in \{1, \ldots, n\}$. With (10)-(12), the closed-loop system becomes differential equations with discontinuous right-hand sides [Filippov, 1]. A considerable amount of works has been carried out to deal with situations like this. Here, we base our results for these differential equations on Filippov's concept. Note that the adaptive control law (10)-(12) is apparently in a decentralized manner, and its performance can be summarized into the following theorem.

Theorem 2 (Adaptive Decentralized Control Scheme): *Under Assumption* 1~3, *consider the error dynamics of the robotic manipulator with the adaptive control law* (10)-(12) *defined above. Then, all signal are bounded, and, both, the position tracking error* $e(t)$ *and the velocity tracking error* $\dot{e}(t)$ *will converge to zero as* $t \to \infty$.

Proof: The proof proceeds in the following two steps.

Step 1: Prove the signal $s(t)$ *is ultimately bounded.* Consider the Lyapunov-like function as follows:

$$V(t) = \tfrac{1}{2}s^{\mathrm{T}}Ms \tag{13}$$

Consider $\|s\|_{T,\infty} < l_1$ for some finite $T > 0$. Then, after taking the time derivative of (13) along $s(t)$ for $t \in [0, T]$, we obtain the following result:

$$\dot{V} = s^{\mathrm{T}}\tau - s\bar{v} \le s^{\mathrm{T}}\tau + \|s\|(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2) \tag{14}$$

where the function $\bar{v}$ is defined as that in (9). Then, the following two different cases will be obtained:

Case 1: $\hat{\theta}_{i\_1}|s_i| \le \varepsilon_i$ for $t \in [0, T]$,

$$\begin{aligned}\dot{V} \le &-\sum_{i=1}^{n}\tfrac{1}{\varepsilon_i}\hat{\theta}_{i\_1}^2 s_i^2 - \theta_{2,\min}\|s\|_2^2 - \tfrac{1}{n}\theta_{3,\min}\|s\|_2^4 \\ &+ \|s\|_2(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2)\end{aligned} \tag{15}$$

Case 2: $\hat{\theta}_{i\_1}|s_i| > \varepsilon_i$ for $t \in [0, T]$,

$$\begin{aligned}\dot{V} \le &-\sum_{i=1}^{n}\hat{\theta}_{i\_1}|s_i| - \theta_{2,\min}\|s\|_2^2 - \tfrac{1}{n}\theta_{3,\min}\|s\|_2^4 \\ &+ \|s\|_2(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2)\end{aligned} \tag{16}$$

where $\theta_{j,\min} = \min\{\theta_{1\_j}, \ldots, \theta_{n\_j}\}$, $j \in \{2, 3\}$. From both Case 1 and 2, we can conclude that for $t \in [0, T]$:

$$\dot{V} \le -\theta_{2,\min}\|s\|_2^2 \tag{17}$$

when $\|s\|_2 \ge [\frac{n(\beta_1+\beta_2 l_1+\beta_3 l_1^2)}{\theta_{3,\min}}]^{1/3}$, which implies that there exists $l_2 > 0$ such that for $t \in [0, T]$, $\|s(t)\|_2 \le l_2 < l_1$ if l_1 is sufficiently large. Similar to the previous argument, l_2 can be defined as $l_2 = \sqrt{\frac{\mu_M}{\mu_m}}[\frac{n(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2)}{\theta_{3,\min}}]^{1/3}$ where l_1 is sufficiently large, and hence one can show that $s(t)$ is actually ultimately bounded so that there exists $l_3 > 0$ such that $l_3 = \sup_{t \in [0, \infty)} \| s(t) \|_2$.

Step 2: Prove all signals are bounded and the signal $s \to 0$ as $t \to \infty$. Now consider the Lyapunov-like function as follows:

$$V(t) = \tfrac{1}{2} s^{\mathrm{T}} M s + \sum_{i=1}^{n} [\tfrac{1}{2} \gamma_{i\_1}^{-1} (\hat{\theta}_{i\_1} - \theta_{i\_1}^{*})^2 + p_i^{-1} \varepsilon_i] \tag{18}$$

where $\theta_{i\_1}^{*}$, $i \in \{1, \ldots, n\}$ is the desirable but unknown parameter. After taking the time derivative of (18) along the solution trajectories of the closed-loop system, the following two different cases will be obtained:

Case 1: $\hat{\theta}_{i\_1} |s_i| \le \varepsilon_i$ for $t \in [0, \infty)$,

$$\begin{aligned} \dot{V} \le & -\sum_{i=1}^{n} \tfrac{1}{\varepsilon_i} \hat{\theta}_{i\_1}^2 s_i^2 - \theta_{2,\min} \|s\|_2^2 - \tfrac{1}{n} \theta_{3,\min} \|s\|_2^4 \\ & + \|s\|_2 (\beta_1 + \beta_2 l_3 + \beta_3 l_3^2) - \theta_{1,\min}^{*} \|s\|_2 \end{aligned} \tag{19}$$

Case 2: $\hat{\theta}_{i\_1} |s_i| > \varepsilon_i$ for $t \in [0, \infty)$,

$$\begin{aligned} \dot{V} \le & -\theta_{2,\min} \|s\|_2^2 - \tfrac{1}{n} \theta_{3,\min} \|s\|_2^4 \\ & + \|s\|_2 (\beta_1 + \beta_2 l_3 + \beta_3 l_3^2) - \theta_{1,\min}^{*} \|s\|_2 - \sum_{i=1}^{n} \varepsilon_i \end{aligned} \tag{20}$$

where $\theta_{1,\min}^{*} = \min\{\theta_{1\_1}^{*}, \ldots, \theta_{n\_1}^{*}\}$. From both Case 1 and 2, we conclude that for sufficiently large $\theta_{1,\min}^{*}$ we obtain:

$$\dot{V} \le -\theta_{2,\min} \|s\|_2^2 \tag{21}$$

for $t \in [0, \infty)$, which implies all signals are bounded and $\|s(t)\|_2$ is L_2. Finally, to show the zero convergence of the tracking error $e(t)$ and $\dot{e}(t)$, we need to show $\|s(t)\|_2$ is uniformly continuous and then apply Barbălat's Lemma [4]. Sufficiently, we investigate boundedness of the signal $\dot{s}(t)$ from (4), (5), and (10), and easily we can verify such a condition. Hence, the zero convergence is thus insured. ■

Remark: In Theorem 2, we would like to emphasize that here in our proposed scheme, the control gain θ_{i_j}, $i \in \{1, \ldots, n\}$ and $j \in \{1, 2\}$, only needs to be chosen positive, and the adaptive decentralized control law (10)-(12) will then successfully drive the tracking errors $e(t)$ and $\dot{e}(t)$ of the closed-loop system to converge to zero globally. Hence, this control law can be claimed to be a global one.

Due to the numerical noise caused by convergence of ε_i, $i \in \{1, \ldots, n\}$, to zero, practical implementation of the scheme should be considered, e.g., using addition of the leakage term [2]. Now, consider another modified adaptive law as follows:

$$\dot{\hat{\theta}}_{i\_1} = \gamma_{i\_1}(|s_i| - \sigma \hat{\theta}_{i\_1}) \tag{22}$$

where $\hat{\theta}_{i\_1}(0) \ge 0$ and the adaptive gains $\gamma_{i\_1} > 0$, $i \in \{1, \ldots, n\}$, and the leakage-term constant $\sigma > 0$. Note that the adaptive law (22) is apparently in a decentralized structure, too. The performance of adaptive control law (10), (11) and (22) can be summarized into the following corollary.

Corollary 3 (Adaptive Decentralized Control Scheme with the leakage term): *Under Assumption 1~3, consider the error dynamics of the robotic manipulator with the adaptive control law* (10), (11) and (22). *Then, all signal are bounded, and, the position tracking error $e(t)$ will converge to a residue set whose size can be reduced by use of larger $\theta_{2,\min}$, and $\gamma_{1,\min}$, where* $\theta_{2,\min} = \min\{\theta_{2\_1}, \ldots, \theta_{2\_n}\}$, *and* $\gamma_{1,\min} = \min\{\gamma_{1\_1}, \ldots, \gamma_{n\_1}\}$.

Proof: This proof is similar to that in the proof of Theorem 2, and, hence, is neglected here. ■

IV. SIMULATION RESULTS

In order to demonstrate the performance of the proposed adaptive decentralized controller, several numerical results are provided now. A two-link planar robot manipulate with two revolute joints is considered here [Spong and Vidyasagar, 10; Su, 11]. Let, for $i = 1, 2$, d_i denote the mass of link i, l_i denote the length of link i, l_{ci} denote the distance from the previous joint to the center of mass of link i, and I_i denotes the moment of inertia of Link i about an axis coming out of the page, passing center of mass of Link i. The inertial matrix M in (1) has four elements

$$\begin{aligned} m_{11} &= d_1 l_{c1}^2 + d_2 (l_1^2 + l_{c2}^2 + 2 l_1 l_{c2} \cos q_2) + I_1 + I_2 \\ m_{12} &= m_{21} = d_2 (l_{c2}^2 + l_1 \cos q_2) + I_2 \\ m_{22} &= d_2 l_{c2}^2 + I_2 \end{aligned} \tag{23}$$

and the potential energy P is as follows:

$$P(q,t) = (d_1 l_{c1} + m_2 l_1) g_0 \sin q_1 + m_2 l_{c2} g_0 \sin(q_1 + q_2) \tag{24}$$

where $g_0 = 9.8$ is the gravitational magnitude. The matrix C in (2) has four elements as follows:

$$\begin{aligned} c_{11} &= \dot{q}_2 h, \quad c_{12} = (\dot{q}_1 + \dot{q}_2) h, \\ c_{21} &= -\dot{q}_1 h, \quad c_{22} = 0 \end{aligned} \tag{25}$$

where $h = -d_2 l_2 l_{c2} \sin q_2$. If we consier on an unknown time-varying load carried by the robot as part of the second link, then the parameters m_2, l_{c2}, and I_2 will explicitly be time-dependent. In this case, the time rate of the change of the inertia matrix M becomes

$$\frac{\partial M}{\partial t}=\begin{bmatrix} 2\dot{\theta}_4\cos q_2+\sum_{i=1}^{3}\dot{\theta}_i & \dot{\theta}_1+\dot{\theta}_3+\dot{\theta}_3\cos q_2 \\ \dot{\theta}_1+\dot{\theta}_3+\dot{\theta}_4\cos q_2 & \dot{\theta}_1+\dot{\theta}_3 \end{bmatrix} \quad (26)$$

where $\theta_1 = d_2 l_{c2}^2$, $\theta_2 = d_2 l_1^2$, $\theta_3 = I_2$, $\theta_4 = d_2 l_2 l_{c2}$, which satisfies Assumption 3. We assume that the two physical links are thin cylinders with uniform density and identical length. The parameters used in this simulation study are listed as follows: $d_1 = 10$, $l_1 = 1$, $l_{c1} = 0.5$, $I_1 = 5/6$, $d_2 = 5 + \Delta d_2(t)$, $l_2 = 1$, $l_{c2} = 0.5 + \Delta l_{c2}(t)$, $I_2 = 5 d_2 l_2^2/12$ where $\Delta d_2(t) = 0.5\cos(0.2t)$, $\Delta l_{c2}(t) = 0.2\cos(0.2t)$. For practical consideration, the adaptive control scheme with the leakage term is employed. The desired trajectory $q_d(t) = [q_{d1}(t), q_{d2}(t)]^T$ is defined by $q_{d1}(t) = \sin t$, $q_{d2}(t) = \sin t$. The initial conditions used in the numerical study are $q_1(0) = -\pi$, $q_2(0) = 0$, $\dot{q}_1(0) = \dot{q}_2(0) = 0$, $\dot{q}_1(0) = \dot{q}_2(0) = 0$, $\hat{\theta}_{1\_1}(0) = \hat{\theta}_{2\_1}(0) = 0$, $\varepsilon_1(0) = \varepsilon_2(0) = 1$. The control gains, adaptive gain constants, and auxiliary signals are, respectively, defined as $\lambda_1 = \lambda_2 = 6$, $\theta_{1\_2} = \theta_{2\_2} = 50$, $\theta_{1\_3} = \theta_{2\_3} = 1$, $\gamma_1 = \gamma_2 = 50$, $\sigma = 0.5$, and $p_1 = p_2 = 0.5$. After compute simulations, Figures 1 and 2 illustrate the numerical results of tracking for Links 1 and 2, respectively. From Figures (1a), (2a), (1b) and (2b), it can be concluded is found that the position and velocity tracking errors approach to a residual set as the time approaches infinity. Figures (1c) and (2c) demonstrate that the updated parameters due to the adaptive law are kept bounded with time. Finally, Figures (1d) and (2d) illustrate that the two control inputs pertaining to this adaptive control scheme also remain bounded.

V. CONCLUSION

In this paper, we develop a globally adaptive decentralized control scheme for the time-varying robot manipulator performing tasks of trajectory tracking. Because the proposed adaptive control law is in a decentralized manner, only low-cost hardware is required for implement. Even if the time-varying parameters of robot manipulators change fast, both the position and velocity tracking error of robot manipulators will also converge to zero by use of the proposed adaptive control law. For practical implementation mentioned, the adaptive law can be combined with a leakage term such that there is no chatting in the control to manipulators and the tracking errors of joint positions and velocities of robot manipulator will converge to residual set whose size can be made smaller by use of larger design parameters. Finally, in order to illustrate the performance of the proposed scheme, satisfactory simulation results are also provided. All the simulation results are satisfying.

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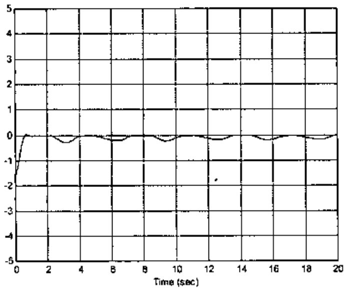
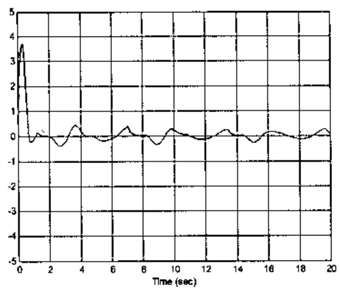
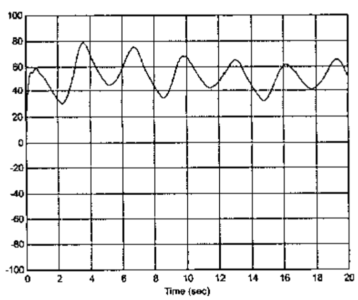


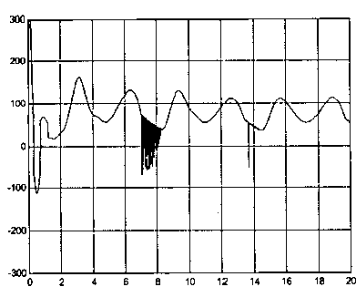
(1a) Position tracking error e_1



(1b) Velocity tracking error $\dot{e}_1$

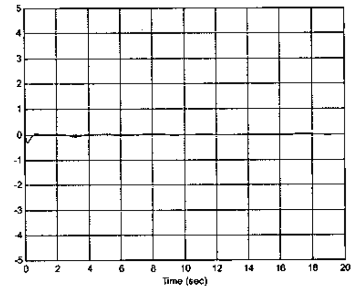


(1c) Estimated parameter $\hat{\theta}_{1\_1}$

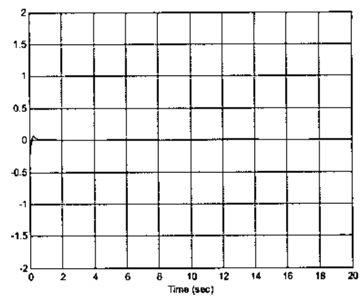


(1d) Control input τ_1

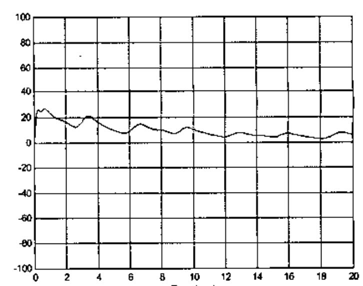
Fig. 1 Numerical results of Link 1



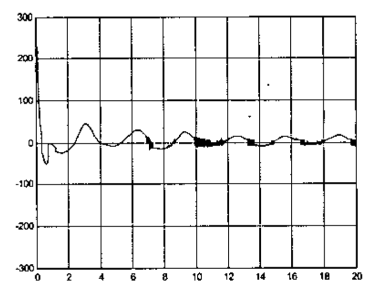
(2a) Position tracking error e_2



(2b) Velocity tracking error $\dot{e}_2$



(2c) Estimated parameters $\hat{\theta}_{2\_1}$



(2d) Control input τ_2

Fig. 2 Numerical results of Link 2