CHINESE JOURNAL OF PHYSICS VOL. 34, NO. 2-11 APRIL 1996

Shapiro Steps Modulated by Magnetic Fields in YBCO Step-Edge Junctions†

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(Received September 1, 1995)

The effect of the magnetic field on Shapiro steps is studied in $YBa_2Cu_3O_y$ step-edge junctions. Theoretically, we show the predictions of the voltage source model and the current source model for Shapiro steps modulated by magnetic flux. Experimentally, the magnetic field dependence of the height of Shapiro steps was measured at a fixed microwave power. Experimental results are qualitatively consistent with the predictions of the current source model.

PACS. 74.50.+r – Proximity effects, weak links, tunneling phenomena, and Josephson effects

Microwave-induced constant-voltage steps, the so called Shapiro steps, have been studied widely in conventional and high-T_c superconducting junctions [1,2]. But there are few works to study the current-voltage (I-V) charactcristics of a Josephson junction simultaneously coupled with applied magnetic field and microwave [2,3]. In this report, we theoretically and experimentally investigate the effect of the applied magnetic field on Shapiro steps.

Two superconductors separated by a thin insulator plotted in Fig. 1 form a Josephson junction, and permit tunneling supercurrent to flow across the junction. We consider a magnetic field H applied to the junction along the y direction. The fundamental Josephson equations are:

$$J = J_c \sin\varphi\,, \tag{1}$$

$$d\varphi/dt = 2eV/\hbar\,, \tag{2}$$

$$d\varphi/dx = (2e/\hbar c)(\lambda_L + \lambda_R + l)H_y\,, \tag{3}$$

† Refereed version of the contributed paper presented at the 1995 Taiwan International Conference on Superconductivity, August 8-11, 1995, Hualien, Taiwan, R.O.C.

where φ is the phase difference of the superconducting order parameters across the junction, J_c is the amplitude of the supercurrent density J, t is the time, e is the electron charge, V is the voltage across the junction, $\hbar$ is the Boltzman constant h divided by 2π, c is the velocity of light in the air, λ_L and λ_R are the London penetration depths of the two superconductors, l is the thickness of the insulating layer, and H_y is the amplitude of the applied magnetic field.

There are two different equivalent-circuit models to describe the behavior of a Josephson junction driven by a microwave field. One is the voltage source model. The other is the current source model, in which Josephson junction is represented by the resistively-shunted junction (RSJ) model.

When the dimension of the Josephson junction is much smaller than the microwave wavelength, the phase difference across the junction becomes

$$\varphi(x, t) = 2eV_0t/\hbar + (\omega_f v/\omega_r V_0) \sin \omega_r t + (2e\Lambda/\hbar c) H_y x + \varphi_0 \tag{4}$$

where V_0 is the d.c. voltage, v is the effective voltage amplitude of the applied microwave, ω_r is the microwave angular frequency! ω_r is $2eV_0/\hbar$, $\Lambda = \lambda_L + \lambda_R + l$, and φ_0 is a constant.

We substitute Eq. (4) into Eq. (1) and assume the junction width is smaller than Josephson penetration depth λ_J (the so called small junction). Finally, the maximum amplitude of the nth Shapiro step is given by

$$I_n = I_c \sin \varphi_0 |J_n(a)| |\sin(\pi\Phi/\Phi_0)/(\pi\Phi/\Phi_0)| \tag{5}$$

where I_c is the critical current. $J_n(a)$ is the integral Bessel function. a is $\omega_r v/\omega_r V_0$, Φ is the total flux in the junction. and Φ_0 is the fluxon.

In Eq. (5), In is zero when Φ/Φ_0 is integer. The zeroth-order (n = 0) and the first-order ($n=$ 1) Shapiro steps modulated by the magnetic flux are plotted in Fig. 2a.

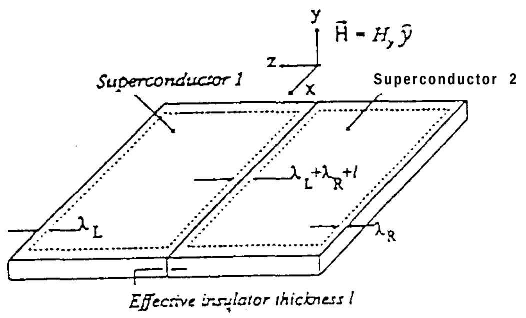
The equivalent circuit of the RSJ current source model for a Josephson junction driven by microwave can be described by the following equation

$$I_{dc} + I_{rf} \sin \omega_r t = (\hbar/2eR_n) d\varphi/dt + I_c \sin \varphi \tag{6}$$

where I_{dc} is the input dc current: I_{rf} is the microwave-induced current, ω_r is the microwave angular frequency, I_c is the supercurrent, and R_n is the normal state resistance of the junction. We let $I_{dc}/I_c = i_0$, $I_{rf}/I_c = i_{rf}$, $\omega_r\omega_c = \Omega$, $\omega_c t = \tau$, and $\omega_c = 2eI_cR_n/\hbar$, *then* Eq. *(6)* becomes a dimensionless equation:

$$i_0 + i_{rf} \sin \Omega\tau = d\varphi/d\tau + \sin \varphi. \tag{7}$$

In Eq. (7), both $i_{rf}(H) = I_{rf}/I_c(H)$ and R(H) = $\omega_r/\omega_c(H)$ are the functions of the applied magnetic field, and $\omega_c(H) = 2eI_c(H)R_n/\hbar$. Simply consider the case of a small junction and constrain the value of $\Omega(H)$ in a region of 0.2 $\leq$ R(H) $\leq$ 1. The simulation results for the zeroth-order and the first-order steps are plotted in Fig. 2b. Fig. 2b shows that the height of the zeroth order Shapiro steps is local minimum, whereas the height of the first order Shpario steps is about a local maximum, and vice versa.



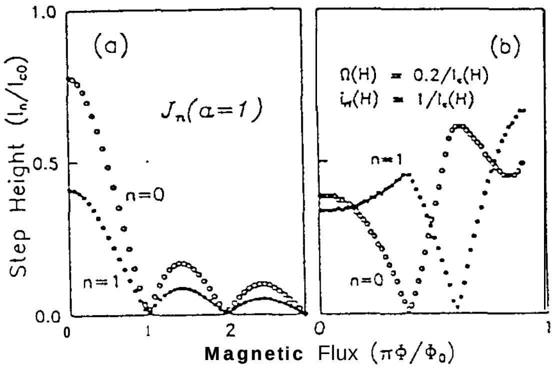


FIG. 1. A Josephson *junction* consists of two superconductors separated by an effectively thin insulator. The magnetic field H is applied to the junction along the y direction. λ_L and λ_R are the London depths respectively. in the two superconductors, and l is the effective insulator thickness.

FIG. 2. The magnetic field dependences of the height of the zeroth-order and the first-order Shapiro steps expressed in (a) the voltage source model and (b) the current source model.

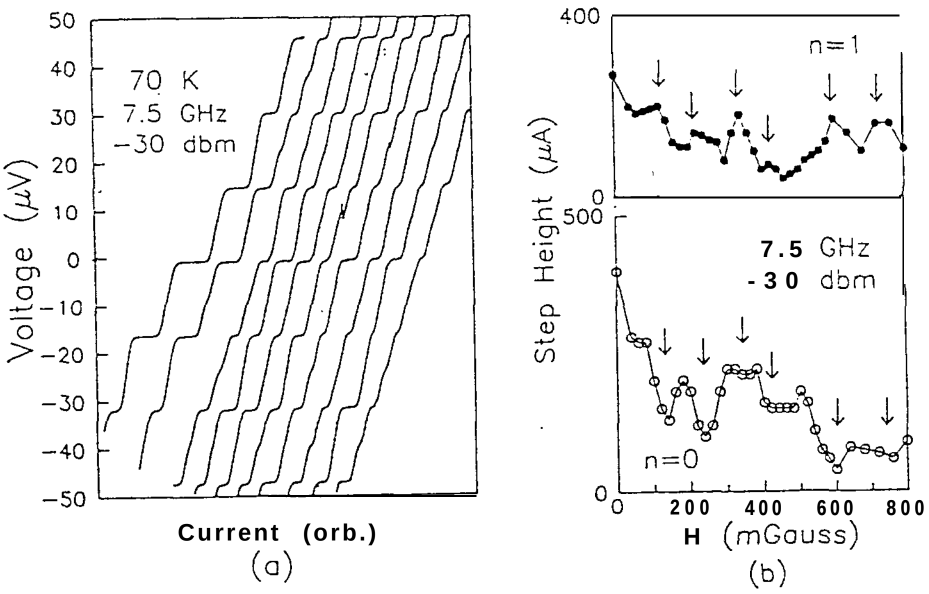


FIG. 3. **(a) I-V** curves were measured at a fixed microwave power -30 dbm and at varied applied magnetic fields. The microwave-induced step height was modulated oscillatorily by the applied fields. From left to right, the applied magnetic fields is increasing. (b) The open and the solid circles are experimental data for the magnetic field dependence of the height of the zeroth and the first order Shapiro steps respectively.

YBCO step-edge Josephson junctions were prepared by the photolithographic technique and the ion milling. The detail processes were published in previous papers [2]. In this report, we only show the experimental results.

Fig. 3a shows the I-V characteristic curves at a fixed microwave power -30 dbm and varied magnetic fields. The height of Shapiro steps was modulated by the applied magnetic fields. The magnetic field dependence of the height of the zeroth-order and the first-order Shapiro steps at a fixed microwave power is shown in Fig. 3b. It is observed that the local minimum in the zeroth order Shapiro steps corresponds to a local maximum in the first order steps as indicated by arrows and vice versa. For this sample, the width of the bridge is 40 μm, a value much smaller than the wavelength of about 500 μm at f = 10 GHz. so the microwave field can be considered to be uniform across the junction. In such situation, we can rule out the complexity of the standing wave effect. Although the critical current vs. the magnetic field is not a simple Fraunhofer relation, the experimental results are qualitatively consistent with the predictions of the current source model.

The I-V characteristics of a YBCO step-edge Josephson junction simultaneously coupled with the applied magnetic field and the microwave were studied in a situation that the field of the microwave is uniform across the junction. The experimental data favor the predictions of the current source model.

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