行政院國家科學委員會專題研究計畫　期中進度報告

在含界面之異向彈性體內之集中力或差排的暫態分析(1/2)

計畫類別： 個別型計畫
計畫編號： NSC92-2212-E-002-067-
執行期間： 92 年 08 月 01 日至 93 年 07 月 31 日
執行單位： 國立臺灣大學應用力學研究所

計畫主持人： 吳光鐘

報告類型： 精簡報告
報告附件： 出席國際會議研究心得報告及發表論文
處理方式： 本計畫可公開查詢

中　華　民　國 93 年 5 月 12 日

行政院國家科學委員會專題研究計畫期中報告

在含界面之異向彈性體內之集中力或差排的暫態分析（1/2）

Transient analysis of a line force or dislocation in an anisotropic elastic solid with boundary (1/2)

計畫編號：NSC 92-2212-E-002-067

執行期限： 2003 年 8 月 1 日至 2004 年 7 月 31 日

主持人：吳光鐘　　台灣大學應用力學研究所

一、中文摘要

本計劃分析在均勻半空間或雙材料無限域內之集中力或差排的暫態反應。本計劃推廣一個 Wu (2000)所提求解異向動彈力的方法來處理該問題。此法之優點在於不需利用積分轉換即可求得顯式解。除此之外，顯式解中各個反射波及透射波亦可一目了然。本計劃第一年度考慮半空間的問題。

關鍵詞：彈性波傳，異向性材料，掩埋波源，半空間。。

Abstract

A transient analysis of a line force or dislocation in an anisotropic elastic homogeneous halfspace or a bimaterial infinite space is made in this project. A new method proposed by Wu (2000) for anisotropic elastodynamics is extended to treat the problem. A major advantage of the method is that explicit solution can be derived without the use of integral transforms. Furthermore, individual reflected and refracted waves can be clearly identified in the solution. In the first year a halfspace is considered.

Keywords: elastic wave propagation, anisotropic material, buried source, half-space.

2、INTRODUCTION

The Stroh formalism is widely recognized as an elegant and powerful method for two-dimensional anisotropic elastostatics. A distinctive feature of the Stroh formalism is that the general solution is provided in terms of the eigenvalues and eigenvectors of a six-dimensional eigenvalue problem. The general solution contains three arbitrary complex functions. The functions can often be found by taking advantage of the orthogonality relations among the eigenvectors in conjunction with theories of analytic functions. The readers are referred to [1] for more detailed discussions.

Elastodynamic problems are usually studied by integral transform methods. In these methods a formal solution in the transform space is first obtained. Considerable effort is then involved in the inversion from the transform space to the physical space. For isotropic media, the Smirnov-Soloblev method [2] is an alternative method to solve self-similar elastodynamic problems without the need of integral transforms. The Smirnov-Soloblev method has been extended to general anisotropic elastic materials by

Wu [3]. The formulation by Wu is similar to Stroh's formalism and preserves many of its advantages.

Self-similar problems are problems for which the displacements are homogeneous functions of time t and position $\mathbf{x}$. The corresponding physical systems thus involve neither a characteristic length nor a characteristic time. In the present problem of a buried source in a half-space the depth of the source appears as a characteristic length. It is therefore non-self-similar and the formulation developed by Wu [3] cannot be applied. In this paper we modify Wu's formulation to treat the problem of interest. The present formulation also casts the two-dimensional anisotropic elastodynamic problem into a six-dimensional eigenvalue problem and the general solution is directly expressed in terms of the eigenvalues and eigenvectors in the time domain.

3、FORMULATION

The equations of motion expressed in terms of displacements are

(1) $\mathbf{Q}\mathbf{u}_{,11} + (\mathbf{R} + \mathbf{R}^T)\mathbf{u}_{,12} + \mathbf{T}\mathbf{u}_{,22} = \rho\ddot{\mathbf{u}}$

where the matrices $\mathbf{Q}$, $\mathbf{R}$, and $\mathbf{T}$ are related to elastic constants C_{ijks} by $Q_{ik} = C_{i1k1}$, $R_{ik} = C_{i1k2}$, $T_{ik} = C_{i2k2}$. Let the displacement be assumed as $\mathbf{u}(x_1, x_2, t) = \mathbf{u}(w)$ with the variable $w(x_1, x_2, t)$ implicitly defined by

(2) $\phi(w, x_1, x_2, t) = wt - x_1 - p(w)x_2 - q(w) = 0$

where $p(w)$ is the function of w stipulated by equation (1) and $q(w)$ is an arbitrary given function of w. The special case $q(w) = 0$ has been discussed by Wu [3]. Let $\mathbf{u}'(w)$ be expressed as $\mathbf{u}'(w) = f'(w)\mathbf{a}(w)$ where $f(w)$ is an arbitrary scalar function of w and $f'(w)$ its derivative with respect to w. It follows that $\mathbf{u}(w)$ is a solution of equation (1) if

(3) $\mathbf{D}(p, w)\mathbf{a}(w) = \mathbf{0}$

where $\mathbf{D}(p, w)$ is given by $\mathbf{D}(p, w) = \mathbf{Q} + p(\mathbf{R} + \mathbf{R}^T) + p^2\mathbf{T} - \rho w^2\mathbf{I}$. The general solution of equation (1) may be represented as

(4) $\mathbf{u}_{,1} = 2\,\mathrm{Re}\{\sum_k f_k'(w_k)(\partial w_k / \partial x_1)\mathbf{a}_k(w_k)\}$

(5) $\mathbf{u}_{,2} = 2\,\mathrm{Re}\{\sum_k f_k'(w_k)(\partial w_k / \partial x_2)\mathbf{a}_k(w_k)\}$

(6) $\dot{\mathbf{u}} = -2\,\mathrm{Re}\{\sum_k w_k f_k'(w_k)(\partial w_k / \partial x_1)\mathbf{a}_k(w_k)\}$

where f_k is an arbitrary function of w, $k = 1,2,3$ or $4,5,6$. The choice of the range of k depends on whether up-going rays or down-going rays are considered. The general solutions of the stress vectors $\mathbf{t}_1$ and $\mathbf{t}_2$ can be expressed as

(7) $$\begin{aligned}\mathbf{t}_1 = 2\,\mathrm{Re}\{&\sum_k f_k'[\rho w_k^2(\partial w_k / \partial x_1)\mathbf{a}_k(w_k) \\ &- p_k(\partial w_k / \partial x_1)\mathbf{b}_k(w_k)]\}\end{aligned}$$

(8) $\mathbf{t}_2 = 2\,\mathrm{Re}\{\sum_k f_k'(\partial w_k / \partial x_1)\mathbf{b}_k(w_k)\}$

where

(9) $\mathbf{b}_k(w) = (\mathbf{R}^T + p_k(w)\mathbf{T})\mathbf{a}_k(w)$

Equation (9) can be cast into the following six-dimensional eigenvalue problem

(10) $\mathbf{N}(w)\boldsymbol{\xi}(w) = p(w)\boldsymbol{\xi}(w)$

where $\mathbf{N}(w) = \begin{pmatrix} \mathbf{N}_1 & \mathbf{N}_2 \\ \mathbf{N}_3(w) & \mathbf{N}_1^T \end{pmatrix}$, $\boldsymbol{\xi}(w) = \begin{pmatrix} \mathbf{a}(w) \\ \mathbf{b}(w) \end{pmatrix}$,

$\mathbf{N}_1 = -\mathbf{T}^{-1}\mathbf{R}^T$ $\quad$ $\mathbf{N}_2 = \mathbf{T}^{-1}$,
$\mathbf{N}_3(w) = \mathbf{R}\mathbf{T}^{-1}\mathbf{R}^T - \mathbf{Q} + \rho w^2\mathbf{I}$. Equation (10) is in the same form as that in Stroh's formalism for steady state motion [4]. The p and ξ are the eigenvalue and right eigenvector, respectively, of $\mathbf{N}$

4. BURIED DYNAMIC SOURCES IN AN INFINITE MEDIUM

Consider a line force $\mathbf{F}$ and a dislocation with Burgers vector $\boldsymbol{\beta}$ which appear at $x_1 = 0$ and $x_2 = h$ at time $t = 0$ and stay constant thereafter in an initially stress-free infinite medium. The configuration is shown in Figure 1. The associated jump conditions are given by

(11) $\mathbf{t}_2^+(x_1,h,t) - \mathbf{t}_2^-(x_1,h,t) = -\delta(x_1)H(t)\mathbf{F}$

(12) $\mathbf{u}_{,1}^+(x_1,h,t) - \mathbf{u}_{,1}^-(x_1,h,t) = -\delta(x_1)H(t)\boldsymbol{\beta}$

where δ is the Dirac delta function, H is the Heaviside step function, and superscripts + and – denote the limiting values as $x_2 \to h^+$ and $x_2 \to h^-$, respectively. The solution for the line force has been obtained by Wu [3] and that for the line dislocation may be derived similarly. The result is

(13) $\dot{\mathbf{u}}^{(0)} = -(1/\pi)\,\mathrm{Im}\{\sum_{k=1}^{3} c_k(\partial w_k/\partial x_1)\mathbf{a}_k\}$

(14) $\mathbf{t}_2^{(0)} = (1/\pi)\,\mathrm{Im}\{\sum_{k=1}^{3} (c_k/w_k)(\partial w_k/\partial x_1)\mathbf{b}_k\}$

where $c_k = \mathbf{a}_k^T\mathbf{F} + \mathbf{b}_k^T\boldsymbol{\beta}$, w_k in equation (2) is determined by taking $q(w) = -p(w)h$ so that $w_k = y_1 + p_k(w_k)y_2$ with $y_1 = x_1/t$, $y_2 = (x_2 - h)/t$.

5. BURIED DYNAMIC SOURCES IN A HALF-SPACE

Consider a line force $\mathbf{F}$ and a dislocation with Burgers vector $\boldsymbol{\beta}$ which appear at $x_1 = 0$ and $x_2 = h$ at time $t = 0$ and stay constant thereafter in the half-space $x_2 > 0$. The conditions on the boundary $x_2 = 0$ is given by

(15) $\mathbf{t}_2(x_1,t) = \mathbf{0}$

The resulting $\dot{\mathbf{u}}$ and $\mathbf{t}_2$ may be expressed as $\dot{\mathbf{u}} = \dot{\mathbf{u}}^{(0)} + \dot{\mathbf{u}}^{(1)}$ and $\mathbf{t}_2 = \mathbf{t}_2^{(0)} + \mathbf{t}_2^{(1)}$ where $\dot{\mathbf{u}}^{(0)}$ is the velocity due to the sources in an infinite medium and $\dot{\mathbf{u}}^{(1)}$ is the velocity due to the reflected waves from the free surface. Let

$$w_{jk}t = x_1 + p_j(w_{jk})x_2 - p_k(w_{jk})h$$

Note that at $x_2 = 0$, $w_{jk}(x_1,t) = w_k(x_1,t)$. $\dot{\mathbf{u}}^{(1)}(x_1,x_2,t)$ and $\mathbf{t}_2^{(1)}(x_1,x_2,t)$ can be expressed as

(17)

$$\dot{\mathbf{u}}^{(1)} = -(1/\pi)\sum_{k=4}^{6}\sum_{j=1}^{3} \begin{matrix}\mathrm{Im}\{(\partial w_{jk}/\partial x_1)\mathbf{A}(w_{jk})\mathbf{I}_j\mathbf{B}^{-1}(w_{jk}) \\ \hat{\mathbf{B}}(w_{jk})\mathbf{I}_{k-3}(\hat{\mathbf{A}}^T(w_{jk})\mathbf{F} + \hat{\mathbf{B}}^T(w_{jk})\boldsymbol{\beta})\}\end{matrix}$$

(18)

$$\mathbf{t}_2^{(1)} = (1/\pi)\sum_{k=4}^{6}\sum_{j=1}^{3} \begin{matrix}\mathrm{Im}\{(1/w_{jk})(\partial w_{jk}/\partial x_1)\mathbf{B}(w_{jk})\mathbf{I}_j\mathbf{B}^{-1}(w_{jk}) \\ \hat{\mathbf{B}}(w_{jk})\mathbf{I}_{k-3}(\hat{\mathbf{A}}^T(w_{jk})\mathbf{F} + \hat{\mathbf{B}}^T(w_{jk})\boldsymbol{\beta})\}\end{matrix}$$

where $\mathbf{I}_j = \mathbf{e}_j\mathbf{e}_j^T$ ($\mathbf{e}_j$ is the unit vector in the x_j – direction),

$\mathbf{A}(w) = [\mathbf{a}_1(w) \quad \mathbf{a}_2(w) \quad \mathbf{a}_3(w)]$,
$\mathbf{B}(w) = [\mathbf{b}_1(w) \quad \mathbf{b}_2(w) \quad \mathbf{b}_3(w)]$,
$\hat{\mathbf{A}}(w) = [\mathbf{a}_4(w) \quad \mathbf{a}_5(w) \quad \mathbf{a}_6(w)]$,
$\hat{\mathbf{B}}(w) = [\mathbf{b}_4(w) \quad \mathbf{b}_5(w) \quad \mathbf{b}_6(w)]$

6. NUMERICAL EXAMPLE

For fixed x_1, x_2 and t expand $\phi(w,t)$ in equation (2) about w_0 up to the second order terms by Taylor's series,

(19) $\phi(w) = \phi_0 + (\frac{\partial\phi}{\partial w})_0\Delta w + \frac{1}{2}(\frac{\partial^2\phi}{\partial w^2})_0(\Delta w)^2 = 0$

where $\Delta w = w - w_0$, and $(f)_0 = f(w_0)$. Equation (19) can be regarded as a quadratic equation of Δw:

(20) $a(\Delta w)^2 + 2b\Delta w + c = 0$

In equation (20), a, b, and c are given by

$a = -p''_j(w_0)x_2 + p''_k(w_0)h$,
$b = t - p'_j(w_0)x_2 + p'_k(w_0)h$, $c = 2\phi_0$. Equation (20) yields $\Delta w = (-b + \sqrt{b^2 - ac})/a$ for a given w_0. Let $w_1 = w_0 + \Delta w$. If $|\phi(w_1)| < \varepsilon$, where ε is a preset error, then w_1 is accepted as the solution of w for given x_1, x_2 and t. Otherwise Equation (20) is solved again with w_0 replaced by w_1. The process is repeated until the error criterion is met.

The vertical displacement due to a unit vertical line impulse has been calculated for silicon. The result is expressed in the following dimensionless form:

$(V_0)_y = [(t\pi C_{44})/\tau](\mathbf{G}_{0f})_{22}$,

$(V_1)_y = (V_0)_y + [(t\pi C_{44})/\tau](\mathbf{G}_{1f})_{22}$

where $\tau = tc_0/h$, $c_0 = \sqrt{C_{44}/\rho}$, and ρ is density. The elastic constants of silicon used for calculations were C_{11}=165Gpa, C_{12}=63Gpa and C_{44}=79Gpa. The (x_1,x_3)-plane is on the (111) surface and the x_1-axis is in the $[1\bar{1}0]$ direction. The result of $(V_1)_y$ as a function of τ for $x_1/h = 1.6$ and $x_2/h = 1.2$ is given in Figure 2. It is seen that all wave arrivals are accurately captured.

7. CONCLUSIONS

The formulation developed by Wu [3] is generalized to treat buried dynamic sources in an anisotropic elastic half-space. The displacement or traction fields in time domain have been obtained without using integral transforms. The numerical result shows that the dynamic responses can be efficiently calculated and the complicated wave phenomena accurately captured.

8. REFRENCES

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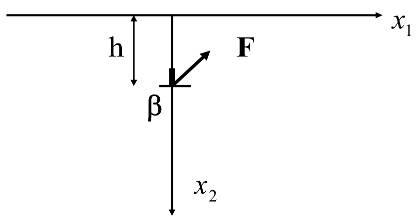

Figure 1. Configuration of the problem of interest.

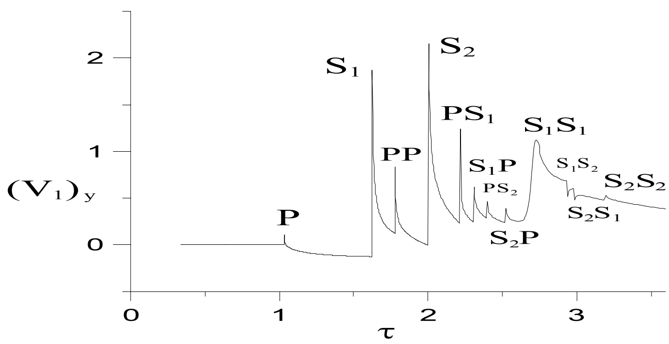

Figure 2 Dimensionless vertical displacement $(V_1)_y$ for $x_1/h = 1.6$ and $x_2/h = 1.2$