



Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Design of Geneva mechanisms with curved slots for non-undercutting manufacturing

Jyh-Jone Lee *, Bin-Heng Jan

Department of Mechanical Engineering, National Taiwan University, 1, Roosevelt Road, Section 4, Taipei 106, Taiwan

ARTICLE INFO

Article history:

Received 11 January 2008

Received in revised form 13 August 2008

Accepted 1 September 2008

Available online xxxx

Keywords:

Geneva mechanism with curved slots

Theory of envelope

Undercutting

Double point

ABSTRACT

In this work, a systematic method using the theory of envelope to determine the geometry of Geneva mechanisms with curved slots is presented. In addition, the undercutting as well as double-point condition in the curve is investigated. The design charts free of undercutting and double point are also plotted based on the input driving angle and offset angle. A mock-up is built to demonstrate the feasibility and availability of the design theory.

© 2008 Elsevier Ltd. All rights reserved.

1. Introduction

Conventional Geneva mechanisms are commonly used as an indexing mechanism where an intermittent motion is required. A particular feature of this mechanism is that for each number of stops, there is a particular motion period and a corresponding motion law, whereas with a conventional cam mechanism there is virtually a free choice of motion period and motion law. The design and manufacturing of a conventional Geneva mechanism is generally simple and inexpensive because there is no specially curved profile on any of the components except straight lines and circular arcs. However, due to the discontinuity of the acceleration at the beginning and ending positions, the shortcoming of using conventional Geneva mechanisms is the large impact when the driving crank engages and disengages with the wheel slot. This feature, together with high acceleration peak, particularly in those smaller numbers of stops makes the dynamic performance of the mechanism inferior to that of a good standard cam mechanism. To improve the dynamic performance, several methods have been proposed. Dijkstra [1] used compound mechanisms to eliminate the non-zero accelerations at the initial and ending stage. Fenton [2] also used Geneva mechanisms in series such that the motion of the last wheel accelerates and decelerates smoothly regardless of the motion status at the intermediate mechanisms. This method inevitably needs extra components for the system. Another method proposed by Sadek et al. [3] and Cheng and Lin [4] was to employ certain damping element in the mechanism to reduce the shock. Likewise, specially designed extra components are required for the mechanism. A third method performed by Fenton et al. [5] and Lee [6] was to change the geometry for the wheel slot. Curved slots with designed motion law were applied as in the standard cam mechanisms. In this approach, the Geneva mechanism with curved slots is basically thought of as an inverse cam mechanism. However, basic equations for the surface geometry of the slots were not discussed in their work, resulting in unknown conditions for manufacturing, such as undercutting, double point in the curve, etc. Figliolini and Angeles [7] studied the force transmission of the Geneva mechanism with curved slots.

* Corresponding author. Tel.: +886 2 33662722; fax: +886 2 23631755.

E-mail address: jjlee@ntu.edu.tw (J.-J. Lee).

Nomenclature

a	distance between crank rotating centre and wheel rotating centre
b	distance between crank rotating centre and roller centre
ℓ	base radius of the wheel
N	number of stops
$\mathbf{n}_f^p$	common normal at the contact point
$\mathbf{r}_i$	position vector of slot with respect to coordinate system i
r_a	radius of roller
$\mathbf{v}_i^p$	velocity of point p relative to coordinate system i
α	surface parameter of roller
ψ	pressure angle
θ_d	crank angle
θ_{d0}	half of the range of crank angle
θ_{off}	offset angle
θ_w	wheel angle
θ_{w0}	half of the index angle of wheel
$\theta'_{w(w)}$	derivative of θ_w with respect to θ_d
$\omega_i^{(w)}$	angular velocity of wheel with respect to coordinate system i
ω_d	angular speed of the driving crank

Lee and Huang [8] derived the equation for the geometry of curved slots. Nonetheless, the undercutting condition for such mechanism was not much addressed. In light of such necessity, the objective of this paper is aimed to investigate the geometry characteristics when the slots are curved according to a motion law. Specifically, the mathematic equation for the curved profile will be developed using the theory of envelope. The criteria for non-undercutting as well as double-point analysis for the curved slots will be studied. A mock-up is also designed and experimented according to the results of analysis. This mock-up also demonstrates the feasibility and availability of the design theory.

2. Geometry design for the profile

In two-dimensional space, a family of regular planar curves can be represented in parametric form as

$$\mathbf{r}(\alpha, \theta) = x(\alpha, \theta)\mathbf{i} + y(\alpha, \theta)\mathbf{j} \quad (1)$$

where α and θ denote independent parameters of the curves, and $\mathbf{i}$ and $\mathbf{j}$ are respectively the unit vectors of coordinates axes x and y . The envelope of the family of planar curves is the simple and regular curve that is tangency with every curve of the family. Thus, the envelope can be determined by the following equations [9,10]:

$$\mathbf{r}(\alpha, \theta) \quad (2)$$

$$\left(\mathbf{k} \times \frac{\partial \mathbf{r}}{\partial \theta} \right) \cdot \frac{\partial \mathbf{r}}{\partial \alpha} = 0 \quad (3)$$

when $\mathbf{k}$ is the unit vector of the z -axis which is perpendicular to the planar curves.

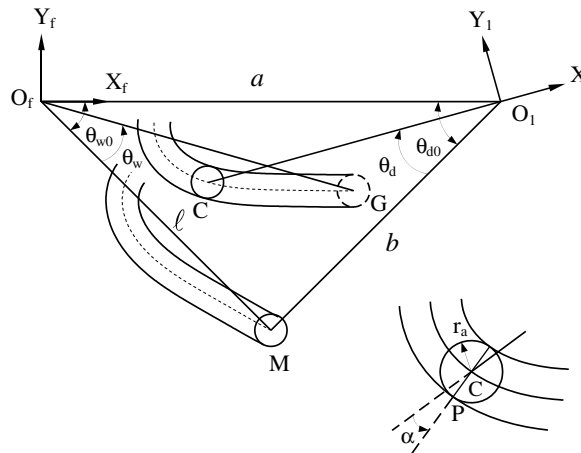


Fig. 1. Geometry definition of the Geneva mechanism with curved slots.

Fig. 1 shows the Geneva mechanism with curved slots. Assume the wheel has N identical and equally spaced curved slots and the roller is of cylindrical shape. Symbols O_f and O_1 are respectively the centres of rotation of the wheel and the driving crank. In the beginning, the roller engages with the slot at the point M . The distance between the wheel centre O_f and roller centre at the beginning entry point M , O_fM , is denoted as the base radius of the wheel, ℓ ; the distance between two rotating centres, O_fO_1 , is denoted as a ; and the distance between O_1 and M is denoted as b . After time elapses, the crank drives the wheel to a position where the roller centre comes to point C and M to G . Thus, the angular position of the crank, θ_d is the angle measured from O_1M to O_1C while the angular position of the wheel θ_w is the angle from O_fM to O_fG . It can also be noted that the geometric parameters of mechanism can be related as

$$\frac{\ell}{\sin(\theta_{d0})} = \frac{b}{\sin(\theta_{w0})} = \frac{a}{\sin(\pi - \theta_{d0} - \theta_{w0})} \quad (4)$$

where θ_{w0} is half of the indexing angle and is equal to π/N , and θ_{d0} is half of the crank driving angle. To obtain the envelope of the locus of roller curves the coordinate systems are defined as: a coordinate system $X_f-O_f-Y_f$ is attached to the fixed frame with the origin located at the centre of rotation of the wheel and coordinate system $X_1-O_1-Y_1$ is attached to the crank with the origin located at the crank centre and X_1 pointing away from the roller centre. Thus, the position vector of a point P on the roller surface with respect to the fixed frame can be represented as

$$\mathbf{r}_f^p = [a - b \cos(\theta_{d0} - \theta_d) - r_a \cos(\theta_{d0} - \theta_d + \alpha), -b \sin(\theta_{d0} - \theta_d) - r_a \sin(\theta_{d0} - \theta_d + \alpha), 0]^T \quad (5)$$

where the subscript 'f' denotes the coordinate system in which the vector is represented, r_a is the radius of roller and α is the surface parameter of the roller. Since the envelope can be obtained from the locus of the roller, the positions of the roller must be traced with respect to a fixed frame. This process can be done by kinematic inversion, i.e., first fixing the mechanism, and then rotating the wheel and crank about the centre of wheel so that O_fG is coincident with its initial position O_fM . This yields:

$$\mathbf{r}_f^{p/o} = \mathbf{R} \cdot \mathbf{r}_f^p \quad (6.1)$$

where $\mathbf{R} = \begin{bmatrix} \cos(\theta_w) & \sin(\theta_w) & 0 \\ -\sin(\theta_w) & \cos(\theta_w) & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is the rotation matrix. After deploying, Eq. (6.1) becomes

$$\mathbf{r}_f^{p/o} = [a \cos(\theta_w) - b \cos(\theta_w + \theta_d - \theta_{d0}) - r_a \cos(\theta_w + \theta_d - \theta_{d0} - \alpha), -a \sin(\theta_w) + b \sin(\theta_w + \theta_d - \theta_{d0}) + r_a \sin(\theta_w + \theta_d - \theta_{d0} - \alpha), 0]^T \quad (6.2)$$

Substituting the above equation into Eq. (3), and simplifying, yields:

$$\left(\mathbf{k} \times \frac{\partial \mathbf{r}_f^{p/o}}{\partial \theta_d} \right) \cdot \frac{\partial \mathbf{r}_f^{p/o}}{\partial \alpha} = \left(\mathbf{k} \times \begin{bmatrix} -a \sin(\theta_w) \theta'_w + b \sin(\theta_w + \theta_d - \theta_{d0})(1 + \theta'_w) + r_a(1 + \theta'_w) \sin(\theta_w + \theta_d - \theta_{d0} - \alpha) \\ -a \cos(\theta_w) \theta'_w + b \cos(\theta_w + \theta_d - \theta_{d0})(1 + \theta'_w) + r_a(1 + \theta'_w) \cos(\theta_w + \theta_d - \theta_{d0} - \alpha) \\ 0 \end{bmatrix} \right) \cdot \begin{bmatrix} -r_a \sin(\theta_w + \theta_d - \theta_{d0} - \alpha) \\ -r_a \cos(\theta_w + \theta_d - \theta_{d0} - \alpha) \\ 0 \end{bmatrix} = 0 \quad (7.1)$$

or

$$\alpha = \tan^{-1} [a \theta'_w \sin(\theta_{d0} - \theta_d) / (b(1 + \theta'_w) - a \theta'_w \cos(\theta_{d0} - \theta_d))] \quad (7.2)$$

where θ'_w is the derivative of θ_w with respect to θ_d . There are two solutions; that is if $\alpha = \alpha^*$ is a solution, $\alpha = \pi + \alpha^*$ is also a solution. These two values correspond to two different points when the roller is in contact with the slot. Eq. (7.2) is also called equation of meshing and is identical to the one derived by conjugate surface method [8]. From the above discussion, it can be seen that the design parameters include geometric parameters (N , ℓ , r_a , θ_{d0}) and the motion law of wheel θ_w . Once the design parameters are specified, the profile of the slots is determined by Eqs. (6.2) and (7.2).

2.1. Offset of the roller entry

If the design parameters, base radius of wheel, $O_fM(\ell)$, and initial angle of the driving crank, θ_{d0} , are remained invariant, the entry point that the roller engages with the wheel may be made an offset to its old point as shown in Fig. 2. As the roller engages with the slot at a new point M' , the angle between the line O_fM' and the centre line O_fO_1 becomes $(\theta_{w0} + \theta_{off})$, where θ_{off} is called the offset angle included by O_fM and O_fM' and O_1' is the new crank rotating centre. As the crank finishes one cycle motion, the roller exits from another exit point M'' . The relations of new centre distance $O_fO_1'(a')$, the new crank length $O_1'M'(b')$ and the design parameters become

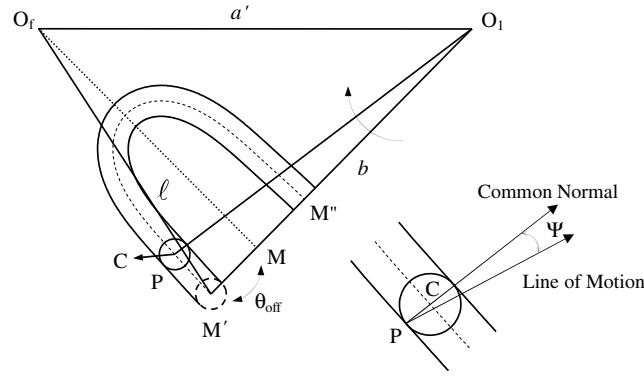


Fig. 2. Offset of the entry position.

$$a' = \ell \sin(\pi - \theta_{d0} - \theta_{w0} - \theta_{off}) / \sin(\theta_{d0}) \quad (8.1)$$

$$b' = \ell \sin(\theta_{w0} + \theta_{off}) / \sin(\theta_{d0}) \quad (8.2)$$

where $O_f M' = \ell$. Thus, by replacing a with a' and b with b' in previous equations, the geometry of the slot with an offset can be obtained. It is shown that this offset in profiles does not affect the motion characteristics, yet may yield singularities in curves as will be discussed in the following section.

2.2. Pressure angle

The pressure angle is defined as the angle between the line of motion and the common normal to the profile. The pressure angle is conventionally regarded as an important index for motion and force transmission. As shown in Fig. 2, the line of motion of a contact point P on wheel can be regarded as the direction of the velocity of that point on wheel and is

$$\begin{aligned} \mathbf{V}_f^p &= \omega_f^{(w)} \times \mathbf{O}_f \mathbf{P} \\ &= \theta_w' \omega_d [a \sin(\theta_w) - b \sin(\theta_w + \theta_d - \theta_{d0}) - r_a \sin(\theta_w + \theta_d - \theta_{d0} - \alpha), a \cos(\theta_w) - b \cos(\theta_w + \theta_d - \theta_{d0}) \\ &\quad - r_a \cos(\theta_w + \theta_d - \theta_{d0} - \alpha), 0]^T \end{aligned} \quad (9)$$

where $\omega_f^{(w)}$ is the angular velocity of wheel with respect to coordinate system $X_f-O_f-Y_f$. On the other hand, the common normal at the contact point is

$$\mathbf{n}_f^p = [\cos(\theta_w + \theta_d - \theta_{d0} - \alpha), -\sin(\theta_w + \theta_d - \theta_{d0} - \alpha), 0]^T \quad (10)$$

Thus, the pressure angle can be calculated as the angle between the direction of the velocity of the contact point on wheel and the direction of the common normal

$$\cos \psi = \mathbf{n}_f^p \cdot (\mathbf{V}_f^p / |\mathbf{V}_f^p|) \quad (11.1)$$

Substituting Eqs. (9) and (10) into the above equation yields:

$$\cos \psi = [a \sin(\theta_{d0} - \theta_d + \alpha) - b \sin(\alpha)] / [a^2 + b^2 + r_a^2 - 2ab \cos(\theta_{d0} - \theta_d) - 2ar_a \cos(\theta_{d0} - \theta_d + \alpha) + 2br_a \cos(\alpha)]^{1/2} \quad (11.2)$$

3. Criteria of undercutting

While machining the profile of slots, if a singular point exists on the envelope, then undercutting will occur. To develop the undercutting condition, the method presented in Litvin [9,10] is utilized in this work. A necessary condition of existence of singular points of the slot profile is

$$\mathbf{V}^{(1)} + \mathbf{V}^{(12)} = 0 \quad (12)$$

where $\mathbf{V}^{(1)}$ denotes the velocity of the contact point on roller and $\mathbf{V}^{(12)}$ is the relative velocity between the two contact points at contact location. Eq. (12) also implies the following two determinants be equal to zero:

$$\Delta_1 = \begin{vmatrix} \frac{\partial x_1^{(1)}}{\partial \alpha} & -V_{x1}^{(12)} \\ f_\alpha & -\frac{\partial f}{\partial \theta_d} \frac{d\theta_d}{dt} \end{vmatrix} = 0 \quad (13.1)$$

$$\Delta_2 = \begin{vmatrix} \frac{\partial y_1^{(1)}}{\partial \alpha} & -V_{y1}^{(12)} \\ f_\alpha & -\frac{\partial f}{\partial \theta_d} \frac{d\theta_d}{dt} \end{vmatrix} = 0 \quad (13.2)$$

where (x_1, y_1) are the components of the position vector of the envelope, $(V_{x1}^{(12)}, V_{y1}^{(12)})$ are the components of the relative velocity between the contact points, and f is the equation of meshing. Both velocity vectors are represented in the roller coordinate system $X_1-O_1-Y_1$. The relative velocity can be further derived from

$$\mathbf{V}_1^{(12)} = \mathbf{V}_1^{(1)} - \mathbf{V}_1^{(2)} = \omega_1^{(1)} \times \mathbf{r}_1 - \omega_1^{(w)} \times \mathbf{O}_f \mathbf{P} = (\omega_1^{(1)} - \omega_1^{(w)}) \times \mathbf{r}_1 + \omega_1^{(w)} \times \mathbf{O}_1 \mathbf{O}_f \quad (14)$$

where $\omega_1^{(1)}$ is the angular velocity of driving crank and

$$\omega_1^{(1)} = -\omega_d \mathbf{k}_1 \quad (15.1)$$

$$\omega_1^{(w)} = \omega_d \theta'_w \mathbf{k}_1 \quad (15.2)$$

$$\mathbf{O}_1 \mathbf{O}_f = -a \cos(\theta_{d0} - \theta_d) \mathbf{i}_1 + a \sin(\theta_{d0} - \theta_d) \mathbf{j}_1 \quad (15.3)$$

$$\mathbf{r}_1 = (-b - r_a \cos(\alpha)) \mathbf{i}_1 - r_a \sin(\alpha) \mathbf{j}_1 \quad (15.4)$$

Substituting Eqs. (15.1)–(15.4) into Eq. (14), yields:

$$V_{1x}^{(12)} = -r_a(1 + \theta'_w) \omega_d \sin(\alpha) + a \omega_d \theta'_w \sin(\theta_d - \theta_{d0}) \quad (16.1)$$

$$V_{1y}^{(12)} = (1 + \theta'_w) \omega_d (b + r_a \cos(\alpha)) - a \omega_d \theta'_w \cos(\theta_d - \theta_{d0}) \quad (16.2)$$

Again, substituting Eqs. (16.1), (16.2), (15.4), and (7.2) into Eq. (13.1) and (13.2), and rearranging, yields:

$$\begin{aligned} \Delta_1 = & r_a \sin(\alpha) [a \theta''_w \sin(\theta_{d0} - \theta_d + \alpha) - a \theta'_w \cos(\theta_{d0} - \theta_d + \alpha) - b \theta''_w \sin(\alpha)] - [r_a(1 + \theta'_w) \sin(\alpha) - a \theta'_w \sin(\theta_d - \theta_{d0})] \\ & \times [[b \cos(\alpha)(1 + \theta'_w) - a \theta'_w \cos(\theta_{d0} - \theta_d + \alpha)]] = 0 \end{aligned} \quad (17.1)$$

$$\begin{aligned} \Delta_2 = & -r_a \cos(\alpha) [a \theta''_w \sin(\theta_{d0} - \theta_d + \alpha) - a \theta'_w \cos(\theta_{d0} - \theta_d + \alpha) - b \theta''_w \sin(\alpha)] - [-(b + r_a \cos(\alpha))(1 + \theta'_w) \\ & + a \theta'_w \cos(\theta_d - \theta_{d0})] \times [[b \cos(\alpha)(1 + \theta'_w) - a \theta'_w \cos(\theta_{d0} - \theta_d + \alpha)]] = 0 \end{aligned} \quad (17.2)$$

where θ''_w is the twice derivative of θ_w with respect to θ_d . Thus, according to the criteria of undercutting, the singularity of the envelope exists when Eqs. (17.1) and (17.2) hold. The coordinates of the singular point can be obtained by solving the simultaneous equations, Eq. (7.2) and the following equation:

$$F_1 = \Delta_1^2 + \Delta_2^2 = 0 \quad (18)$$

3.1. Double points of the profile

The criteria of undercutting can be governed by Eqs. (18) and (7.2). However, a type of the profile that does not have undercut may also yield impracticality in engineering. As shown in Fig. 3, the convex (inner) curve intersects itself and the tangents have different slopes at intersection of which position is near by the roller entry and exit points during one cycle motion. This profile has double-point or crunode. At a double-point, the roller may lose its contact with the slot wall and the wheel will not be controlled by the crank.

Therefore, such designs should also be avoided. Although a sophisticated differential geometry can be applied to the analysis of double point, a simpler numerical approach to check if the double-point exists in the curve is adopted here. From Eq. (6.2), let the x and y component of the position vector of the profile respectively be

$$x_f(\theta_d) = a \cos(\theta_w) - b \cos(\theta_w + \theta_d - \theta_{d0}) - r_a \cos(\theta_w + \theta_d - \theta_{d0} - \alpha) \quad (19.1)$$

$$y_f(\theta_d) = -a \sin(\theta_w) + b \sin(\theta_w + \theta_d - \theta_{d0}) + r_a \sin(\theta_w + \theta_d - \theta_{d0} - \alpha) \quad (19.2)$$

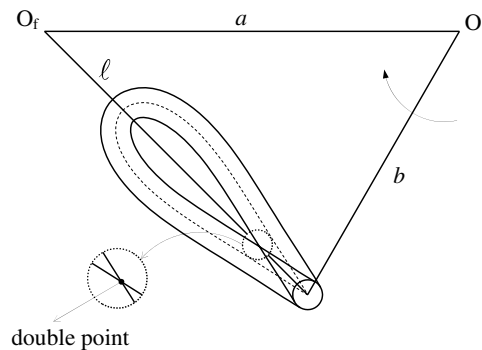


Fig. 3. Double point in the profile.

The coordinate of the double point can be obtained by solving simultaneous equations, Eq. (7.2) and the following equation via numerical method

$$F_2 = [x_f(\theta_{d1}) - x_f(\theta_{d2})]^2 + [y_f(\theta_{d1}) - y_f(\theta_{d2})]^2 = 0 \quad (20)$$

where $0 \leq \theta_{d1} \leq \theta_{d0}$, and $\theta_{d0} \leq \theta_{d2} \leq 2\theta_{d0}$.

4. Numerical example and mock-up design

In this section, the influence of design parameters on the undercutting condition is investigated. According to the undercutting criteria, Eq. (18), given the number of slots, base wheel length ℓ , the roller radius r_a , and half crank angle range

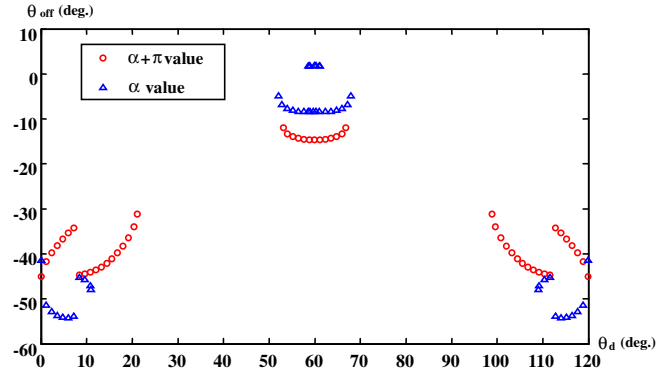


Fig. 4. Undercutting existing range when θ_{off} varies ($N = 4$, $r_a = 5$ mm, $\ell = 70$ mm, $\theta_{d0} = 60^\circ$).

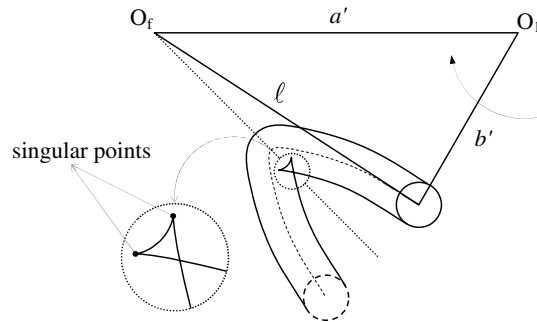


Fig. 5. Singular points in the convex curve ($N = 4$, $r_a = 5$ mm, $\ell = 70$ mm, $\theta_{off} = -12^\circ$, $\theta_{d0} = 60^\circ$).

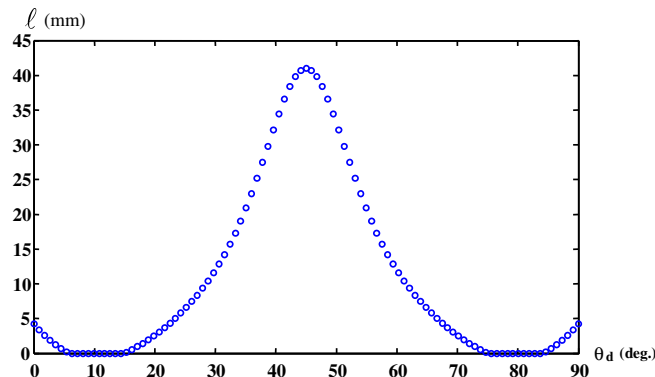


Fig. 6. Undercutting existing range when base wheel length ℓ varies ($N = 4$, $r_a = 5$ mm, $\theta_{off} = 12^\circ$, $\theta_{d0} = 45^\circ$).

θ_{d0} , the undercutting may be realized. To illustrate the effects of the design parameters, we have used some numerical data that can reveal unfavorable designs. For clarity, a four-slot wheel ($N = 4$) is to be designed using the cycloidal motion curve. In the first example, the roller radius r_a , base wheel length ℓ , and half crank angle range θ_{d0} are assumed as 5 mm, 70 mm, and 60° , respectively. The offset angle θ_{off} versus crank angle θ_d that results in undercutting can be obtained by solving Eqs. (18) and (7.2) and is plotted as shown in Fig. 4. This plot gives us an overlook for the undercutting area when offset angle θ_{off} varies once the design parameters are given. To summarize from the numerical data, undercutting exist when θ_{off} lies within the following areas: $\theta_{off} \leq -31.2^\circ$, $-14.65^\circ \leq \theta_{off} \leq -11.95^\circ$, $-8.43^\circ \leq \theta_{off} \leq -4.99^\circ$ and $1.65^\circ \leq \theta_{off} \leq 1.77^\circ$. An example of undercutting in the convex (inner) part of the slot is also demonstrated in Fig. 5 where $\theta_{off} = -12^\circ$ and is in accordance with what Fig. 4 shows.

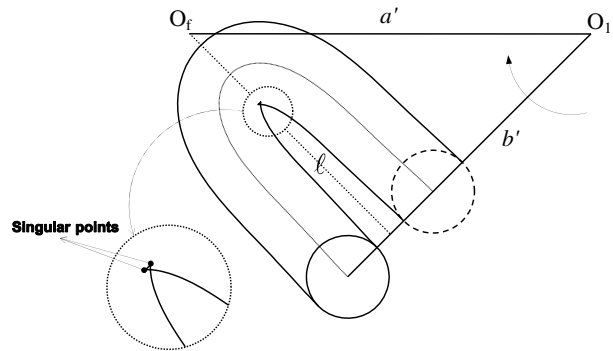


Fig. 7. Singular points in the convex curve ($N = 4$, $r_a = 5$ mm, $\ell = 35$ mm, $\theta_{off} = 12^\circ$, $\theta_{d0} = 45^\circ$).

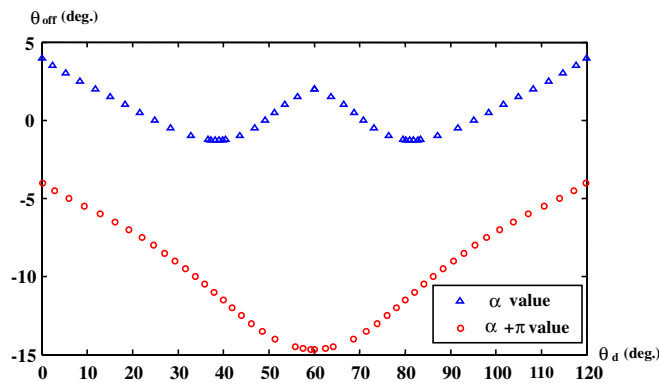


Fig. 8. Double point in the curves ($N = 4$, $r_a = 5$ mm, $\ell = 70$ mm, $\theta_{d0} = 60^\circ$).

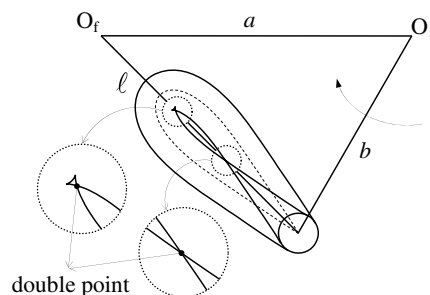


Fig. 9. Double point in the convex curve ($N = 4$, $r_a = 5$ mm, $\ell = 70$ mm, $\theta_{off} = 0^\circ$, $\theta_{d0} = 60^\circ$).

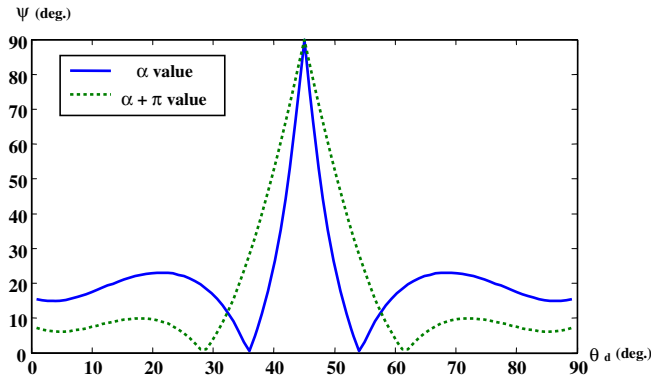


Fig. 10. Pressure angle ($N = 4$, $r_a = 5$ mm, $\ell = 70$ mm, $\theta_{\text{off}} = 12^\circ$, $\theta_{d0} = 60^\circ$).

The second example illustrates another undercutting situation when the base wheel length ℓ varies once the other design parameters are given as $\theta_{w0} = 45^\circ$, $\theta_{d0} = 45^\circ$, $\theta_{\text{off}} = 12^\circ$, and $r_a = 5$ mm. Fig. 6 shows the undercutting area and Fig. 7 demonstrates the singular points when the design parameters are chosen within the undercutting area in Fig. 6.

4.1. Double point

The double-point region can be analyzed using Eq. (20) via numerical method. Fig. 8 shows the search results of double-point region when θ_{off} varies providing other design parameters $\ell = 70$ mm, $r_a = 5$ mm, and $\theta_{d0} = 60^\circ$ are the same as in the first example. It can be observed that double-points exit in the following ranges: $-1.29^\circ \leq \theta_{\text{off}} \leq 3.95^\circ$ and $-4^\circ \leq \theta_{\text{off}} \leq -14.65^\circ$. Fig. 9 also shows the existing of double points in design when the parameters are chosen within the double-point area in Fig. 8. From the above discussion, it can be summarized from Figs. 4 and 8 that to design a Geneva mechanism with curved slots for non-undercutting and double-point free, the offset value must be chosen other than the undercutting areas aforementioned, that is, it must be chosen from the range $-31.2^\circ \leq \theta_{\text{off}} \leq -14.65^\circ$, $-11.95^\circ \leq \theta_{\text{off}} \leq -8.43^\circ$, $-4.99^\circ \leq \theta_{\text{off}} \leq -1.29^\circ$ and $3.95^\circ \leq \theta_{\text{off}}$.

4.2. Pressure angle

Fig. 10 shows the pressure angle ψ versus crank angle θ_d when the parameters are ($N = 4$, $r_a = 5$ mm, $\ell = 70$ mm, $\theta_{d0} = 60^\circ$, $\theta_{\text{off}} = 12^\circ$). It can be seen that the pressure angle may reach 90° about at mid position during one cycle motion of the crank. Yet, the wheel still can move at this position. Such behavior is contrary to the conventional cam mechanisms whose pressure angle usually cannot exceed 45° . This is because for the curved slot the component of the contact force acting at the right-angles to the direction of wheel motion is also very small at this transient position and hence induces very little friction in the wheel bearing. Nonetheless, the wheel requires its own inertia momentum to pass through this transient position. A mock-up has been built to demonstrate the feasibility and availability of the design theory. The design parameters are chosen as $N = 4$, $\ell = 70$ mm, $r_a = 5$ mm, $\theta_{\text{off}} = 12^\circ$, and $\theta_{d0} = 45^\circ$. A 3D solid modeling of the design (Fig. 11) was built to ensure the

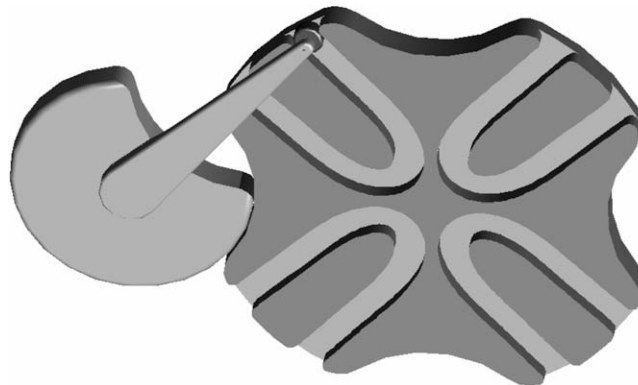


Fig. 11. 3D solid modeling of the design.

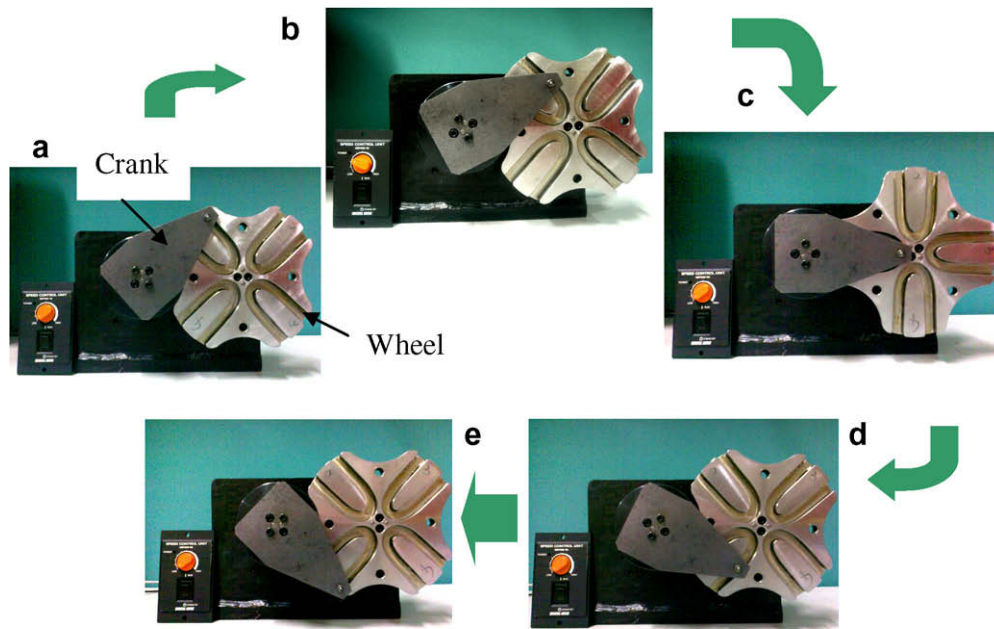


Fig. 12. Embodiment of the design and operation sequence.

design parameters are manufacturable. Fig. 12 shows the embodiment of the design and its operation sequence. Note that the posture in sequence c has pressure angle 90° .

5. Conclusion

In this paper, the profile of the curves for the Geneva mechanism with curved slots is designed by theory of envelope. The mathematical expression in parametric form for the curves and equations of non-undercutting are derived. The criteria for undercutting and double points in the design are also developed. Numerical examples have been performed to demonstrate these effects on the design of mechanism. It is clear that the results of this work indicate the importance of parameters selection for the geometry of the slots and its subsequent manufacturing. Also, the results of this work can provide ground rules for optimization work of the mechanism if necessary. It can be noted that this systematic procedure can be of practical value to the design and manufacturing of Geneva mechanisms with curved slots.

Acknowledgment

The financial support of the National Science Council (NSC 94-2212-E-002-026) of Taiwan is acknowledged.

References

- [1] E.A. Dijkman, Jerk-free Geneva wheel driving, *Journal of Mechanisms* 1 (1966) 235–283.
- [2] E.A. Fenton, Geneva mechanisms connected in series, *ASME Journal of Engineering for Industry* 97 (1975) 603–608.
- [3] E.A. Sadek, J.L. Lloyd, M.R. Smith, A new design of Geneva drive to reduce shock loading, *Mechanism and Machine Theory* 25 (1990) 589–595.
- [4] C.Y. Cheng, Y. Lin, Improving dynamic performance of the Geneva mechanism using non-linear spring elements, *Mechanism and Machine Theory* 30 (1995) 119–129.
- [5] R.G. Fenton, Y. Zhang, J. Xu, Development of a new Geneva mechanism with improved kinematic characteristics, *ASME Journal of Mechanical Design* 113 (1991) 40–45.
- [6] H.P. Lee, Design of a Geneva mechanism with curved slots using parametric polynomials, *Mechanism and Machine Theory* 33 (3) (1998) 321–329.
- [7] G. Figliolini, J. Angeles, Synthesis of conjugate Geneva mechanisms with curved slots, *Mechanism and Machine Theory* 37 (2002) 1043–1061.
- [8] J.J. Lee, K.F. Huang, Geometry analysis and optimal design of Geneva mechanisms with curved slots, *Journal of Mechanical Engineering Science, Proceedings of the Institution of Mechanical Engineers, Part C* 218 (4) (2004) 449–459.
- [9] F.L. Litvin, *Theory of Gearing*, NASA, Washington, DC, 1989.
- [10] F.L. Litvin, *Gear Geometry and Applied Theory*, Prentice-Hall, New Jersey, 1994.